

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

911

Ole G. Jørsboe
Leif Mejlbro

The Carleson-Hunt Theorem
on Fourier Series



Springer-Verlag
Berlin Heidelberg New York 1982

Authors

Ole Groth Jørsboe

Leif Mejlbro

Department of Mathematics, Technical University of Denmark

DK-2800 Lyngby, Denmark

AMS Subject Classifications (1980): 43A50

ISBN 3-540-11198-0 Springer-Verlag Berlin Heidelberg New York

ISBN 0-387-11198-0 Springer-Verlag New York Heidelberg Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to "Verwertungsgesellschaft Wort", Munich.

© by Springer-Verlag Berlin Heidelberg 1982

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.

2141/3140-543210

CONTENTS.

PREFACE	1
CHAPTER I	3
1. Interpolation theorems.	4
2. The Hardy-Littlewood maximal operator.	10
3. The Stein-Weiss theorem.	15
4. Carleson-Hunt's theorem.	17
CHAPTER II	23
5. The operators P_y and Q_y .	24
6. Existence of the Hilbert transform and estimates for the Hilbert transform and the maximal Hilbert transform.	33
7. Exponential estimates for the Hilbert transform and the maximal Hilbert transform.	40
CHAPTER III	45
8. The dyadic intervals and the modified Hilbert transforms.	46
9. Generalized Fourier coefficients.	51
10. The functions $S_n^*(x; f; \omega^*)$ and the operator M^* .	60
CHAPTER IV	69
11. Construction of the sets S^* and V_L^* .	70
12. Construction of the $P_k(x; \omega)$ -functions and the sets G_k and Y^* and X_k^* and X^* .	74
13. Estimates of $P_k(x; \omega)$ and introduction of the index set G_k^* .	80
14. Construction of the splitting $\Omega(p^*, r)$ of ω^* .	87
15. Construction of the sets T^* and V^* and E_N .	91
16. Estimation for elements $p^* \notin G_{rL}^*$.	101

IV

17. Final estimate of $S_n^*(x; \chi_F^0; \omega_{-1}^*)$.	110
18. Proof of theorem 4.2.	118
REFERENCES	122
INDEX	123