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Notes on Infinite Permutation Groups

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Preface

The oldest part of group theory is that which deals with finite groups of permutations. One of the newest is the theory of infinite permutation groups. Much progress has been made during the last two decades, but much remains to be discovered. It is therefore an excellent research area: on the one hand there is plenty to be done; on the other hand techniques are becoming available. Some research takes as its goal the generalisation, extension or adaptation of classical results about finite permutation groups to the infinite case. But mostly the problems of interest in the finite and infinite contexts are quite different. In spite of its newness, the latter has already developed a momentum and an ethos of its own. It has also developed strong links with logic, especially with model theory.

To bring this lively and exciting subject to the attention of mathematicians in Assam—both students and established scholars—a course of sixteen lectures was presented in August and September 1996 at the Indian Institute of Technology, Guwahati. In the available time it was not possible to explore more than a fraction of the area. The lectures were therefore conceived with restricted aims: firstly, to expound a useful amount of general theory; secondly, to introduce and survey just one of the rich areas of recent research in which considerable progress has been made, namely, the theory of Jordan groups. That limited aim is reflected in this book. Although it is a considerable expansion of the lecture notes (and we have included many exercises, which range in difficulty from routine juggling of definitions to substantial recent research results), it is not intended to introduce the reader to more than a small—though important and interesting—part of the subject.

A course at this level can only succeed if it is a collaboration between lecturers and audience. That was so in this case and although the four speakers are those listed as authors of this volume, credit would be spread much more widely if title-page conventions permit-

ted. In particular, we record our very warm thanks to the audience for being so enthusiastic and alert, and especially to the three note-takers Dr. B. K. Sharma, Ms. Shreemayee Bora and Ms. Shabeena Ahmed.

We are very grateful to Dr. S. Ponnusamy and Dr. K. S. Venkatesh for help with typesetting and with the figures. We wish to thank Prof. Moloy Dutta for help with typing and editing, and, Shabeena and Shreemayee, a second time, for help with proof-reading. We also thank the Editors of the TRIM series, specially Professors R. Bhatia and C. Musili, for accepting the book for publication in the series.

MB (Guwahati), HDM (Leeds), RGM (Reykjavík), ΠMN (Oxford): April 1997

Some more acknowledgements:

The three of us who came from overseas have warm thanks to record to various institutions that provided support: to the Royal Society of London (Macpherson and Neumann); to the Indian National Science Academy (Neumann); to the Indian Institute of Technology Guwahati (Macpherson, Möller and Neumann) for a very kind welcome, for academic facilities and for help with travel within India. Our warmest thanks and congratulations, however, are reserved for our colleague and co-author Dr Meenaxi Bhattacharjee. She conceived and organised the whole project, she provided delightful company and hospitality in Assam, and, with characteristic energy she has brought this book project to a satisfactory conclusion.

HDM (Leeds), RGM (Reykjavík), ΠMN (Oxford): April 1997

About the lecture course:

This book is based on lectures delivered by the four authors at the **'Lecture Course on Infinite Permutation Groups'** that was held at the Indian Institute of Technology (IIT) Guwahati, India from 6th August to 19th September, 1996. The course was originally conceived as an informal series of lectures coinciding with the visits of Dr. Neumann, Dr. Macpherson and Dr. Möller to the Institute during that period.

The lectures were aimed at graduate students, research scholars, teachers and research workers of mathematics drawn from the colleges, universities, research institutes and other academic institutions in this region. The lectures (each lasting 90 minutes) were held twice a week. The course generated a tremendous amount of interest and as many as 60 participants attended the lectures.

The first speaker was Dr. Möller who introduced the participants to the basics of permutation group theory. I spoke next on wreath products of groups. In his lectures, Dr. Neumann explained the general theory of Jordan groups. Dr. Macpherson delivered the concluding lectures in which he developed the theory further, leading to the classification of infinite primitive Jordan groups. He also introduced the audience to the basic concepts of model theory, and illustrated the connections between model theory and Jordan groups using the Hrushovski construction. The sixteen chapters of this book correspond roughly to the sixteen lectures delivered at the course.

We wish to record our thanks to Professors M. S. Raghunathan and S. G. Dani of the National Board of Higher Mathematics for extending financial support, and to the faculty and staff of IIT Guwahati for their help in organising the course. We are particularly grateful to the Director of IIT Guwahati, Prof. D. N. Buragohain, for providing all infra-structural facilities and for his kind encouragement and support. We are also grateful to the Head of the Department of Mathematics at IIT Guwahati, Prof. P. Bhattacharyya, for giving

us his whole-hearted help and guidance. On a personal note, my son Amlan deserves a special word of thanks for patiently tolerating my total preoccupation with the course last summer and then with the preparation of this book over the past few months.

I extend my warm and sincere thanks to each of my three co-authors—my teacher and *guru* Dr. Peter M. Neumann, my friend and collaborator Dr. Dugald Macpherson, and, my friend and *gurubhai* Dr. Rögnvaldur G. Möller—on my own behalf as well as on behalf of everyone else involved in the project, for accepting our invitation to visit Guwahati (and smilingly putting up with many odds during their Indian sojourn), for agreeing to speak at the course, for delivering such superb lectures and for being readily available to the participants for consultation at other times. By doing so they have given a rare opportunity to the participants to benefit from their expertise in the subject. Co-ordinating the course was both thrilling as well as challenging—and I have gained a lot from the experience.

MB (Guwahati): April 1997

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