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N. S. Narasimha Sastry · Manoj Kumar Yadav  
Editors

# Group Theory and Computation

*Editors*

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# Preface

This volume is a collection of invited articles by some of the leading and very active researchers in the theory of finite groups and their representations, and the Monster group, with an emphasis on computational aspects. Among the authors are participants who led workshops and delivered invited talks in the international program “Group Theory and Computational Methods” held at the International Center for Theoretical Sciences (ICTS), Bangalore, from November 05–14, 2016. The program comprised of two parts: workshops (November 05–09, 2016) and discussion meetings (November 11–14, 2016).

The workshops comprised of the following five minicourses of 6-h duration each:

- (1) Computational homological algebra
- (2) Computational representation theory
- (3) Computational aspects of finite  $p$ -groups
- (4) Computer algebra system GAP
- (5) Introduction to Monster simple group and Moonshine

The discussion meetings comprised of 20 invited talks by eminent mathematicians. They presented recent developments and problems of current mathematical interest on a variety of topics in group theory and related areas. Poster presentation session consisted of 11 posters by young researchers.

The topics of the articles include finite loops, non-abelian tensor product, periodic groups, character table of finite groups, computing subgroups using computer algebra system GAP, Majorana theory related to the Monster group, groups with abelian automorphism groups, unit groups of integral group rings, and Camina groups and generalizations.

We thank the authors for writing the articles and our colleagues for carefully refereeing them. We also thank the speakers and the participants of the workshops and discussion meetings mentioned above whose participation helped closer mathematical scrutiny of the themes discussed.

We thank ICTS, Bangalore, for the excellent facilities and support for the smooth and successful conduct of the program. We also thank Springer for publishing this volume, and Shamim Ahmad, for his friendly and efficient handling of the publication process.

Dharwad, India  
Allahabad, India

N. S. Narasimha Sastry  
Manoj Kumar Yadav

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