The Deaths of Very Massive Stars

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Abstract The theory underlying the evolution and death of stars heavier than $10\,M_\odot$ on the main sequence is reviewed with an emphasis upon stars much heavier than $30\,M_\odot$. These are stars that, in the absence of substantial mass loss, are expected to either produce black holes when they die, or, for helium cores heavier than about $35\,M_\odot$, encounter the pair instability. A wide variety of outcomes is possible depending upon the initial composition of the star, its rotation rate, and the physics used to model its evolution. These heavier stars can produce some of the brightest supernovae in the universe, but also some of the faintest. They can make gammaray bursts or collapse without a whimper. Their nucleosynthesis can range from just CNO to a broad range of elements up to the iron group. Though rare nowadays, they probably played a disproportionate role in shaping the evolution of the universe following the formation of its first stars.

1 Introduction

Despite their scarcity, massive stars illuminate the universe disproportionately. They light up regions of star formation and stir the media from which they are born. They are the fountains of element creation that make life possible. The neutron stars and black holes that they make are characterized by extreme physical conditions that can never be attained on the earth. They are thus unique laboratories for nuclear physics, magnetohydrodynamics, particle physics, and general relativity. And they are never quite so fascinating as when they die.

Here we briefly review some of aspects of massive star death. The outcomes can be crudely associated with three parameters - the star's mass, metallicity, and rotation rate. In the simplest case of no rotation and no mass loss, one can delineate five outcomes and assign approximate mass ranges (in some cases *very* approximate mass ranges) for each. These masses then become the section heads for the first part of this chapter. 1) From 8 to 30 M_{\odot} on the main sequence (presupernova helium core masses up to 12 M_{\odot}), stars mostly produce iron cores that collapse to neutron stars leading to explosions that make most of today's observable supernovae and

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heavy elements. Within this range there are probably islands of stars that either do not explode or explode incompletely and make black holes, especially for helium cores from 7 to 10 M_{\odot} . 2) From 30 to 80 M_{\odot} (helium core mass 10 to 35 M_{\odot}), black hole formation is quite likely. Except for their winds, stars in this mass range may be nucleosynthetically barren. Again though there will be exceptions, especially when the effects of rotation during core collapse are included. 3) 80 to (very approximately) 150 M_☉ (helium cores 35 to 63 M_☉), pulsational-pair instability supernovae. Violent nuclear-powered pulsations eject the star's envelope and, in some cases, part of the helium core, but no heavy elements are ejected and a massive black hole of about $40 \,\mathrm{M}_{\odot}$ is left behind. 4) $150 - 260 \,\mathrm{M}_{\odot}$ (again very approximate for the main sequence mass range, but helium core 63 to 133 M_☉), pair instability supernovae of increasing violence and heavy element synthesis. No gravitationally bound remnant is left behind. 5) Over 260 M_☉ (133 M_☉ of helium), with few exceptions, a black hole consumes the whole star. Rotation generally shifts the main sequence mass ranges (but not the helium core masses) downwards for each outcome. Mass loss complicates the relation between initial main sequence mass and final helium core mass.

The latter part of the paper deals with some possible effects of rapid rotation on the outcome. In the most extreme cases, gamma-ray bursts are produced, but even milder rotation can have a major affect on the light curve and hydrodynamics if a magnetar is formed.

2 The Deaths of Stars 8 M_{\odot} to 80 M_{\odot}

2.1 Compactness as a Guide to Outcome

The physical basis for distinguishing stars that become supernovae rather than planetary nebulae, and that are therefore, in some sense, "massive", is the degeneracy of the carbon-oxygen (CO) core following helium core burning. Stars with dense, degenerate CO cores develop thin helium shells and eject their envelopes leaving behind stable white dwarfs, while heavier stars go on to burn carbon and heavier fuels. A mass around 8 M_{\odot} is usually adopted for the transition point. The effects of degeneracy linger, however, on up to at least 30 M_{\odot} at oxygen ignition, and to still heavier masses for silicon burning. Even at 80 M_{\odot} , the center of a massive star has become degenerate by silicon depletion.

Were the core fully degenerate and composed of nuclei with equal numbers of neutrons and protons, its maximum mass would be the cold Chandrasekhar mass, 1.38 M_{\odot} . This cold Chandrasekhar mass is altered however, both by electron capture reactions, which tend to reduce it, and the high temperatures necessary to burn oxygen and silicon, which increase it (Chandrasekhar, 1939; Hoyle & Fowler, 1960; Timmes, Woosley, & Weaver, 1996). For main sequence stars from 8 to 80 M_{\odot} , the iron core mass at the time it collapses varies from about 1.3 to 2.3 M_{\odot} (baryonic

mass), with the larger values appropriate for more massive stars. Surrounding this degenerate core is a nested structure of shells that cause adjustments to the density structure. For very degenerate cores with energetic shells at their edges, the presupernova structure resembles that of an asymptotic giant branch star - a compact core surrounded by thin burning shells and a low density envelope with little gravitational binding energy. The matter outside of the iron core is easily ejected in such stars, and it is easy to make a supernova out of them, even with an inefficient energy source like neutrinos. Heavier stars with less degenerate cores and shells farther out, on the other hand, have a density that declines more slowly. These mantles of heavy elements, where ultimately most of the nucleosynthesis occurs, are more tightly bound and the star is more difficult to blow up.

O'Connor & Ott (2011) have defined a "compactness parameter", $\xi_{2.5}$ = 2.5/R_{2.5}, that is a quantitative measure of this density fall off. Here R_{2.5} is the radius, in units of 1000 km, of the mass shell in the presupernova star that encloses 2.5 M_{\odot}. The fiducial mass is taken to be well outside the iron core but deep enough in to sample the density structure around that core. It makes little difference whether this compactness is evaluated at the onset of hydrodynamical instability or at core bounce (Sukhbold & Woosley, 2014). Figure 1 shows $\xi_{2.5}$ as a function of main sequence mass for stars of solar metallicity. O'Connor and Ott and Ugliano et al (2012) have both shown that it becomes difficult to explode the star by neutrino transport alone if $\xi_{2.5}$ becomes very large. The critical value is not certain and may vary with other properties of the star, but in Ugliano's study is usually 0.20 to 0.30. By this criterion, it may be difficult to explode stars in the 22 to 24 M $_{\odot}$ range (at least) as well as all stars above about 30 M $_{\odot}$ that do not lose substantial mass along the way to their deaths. The latter especially includes stars with very subsolar metallicity.

There are a number of caveats that go along with this speculation. The structure of a presupernova star is not fully represented by a single number and its compactness is sensitive to a lot of stellar physics, including the treatment of semiconvection and convective overshoot mixing and mass loss and the nuclear reaction rates employed (Sukhbold & Woosley, 2014). Rotation and magnetic fields will change both the presupernova structure and its prospects for explosion by non-neutrino processes. Finally, the surveys of how neutrino-powered explosions depend on compactness have, so far, been overly simple and mostly in 1D, though see recent work by Janka and colleagues (Janka et al, 2012; Janka, 2012; Müller, Jamka, & Heger, 2012). Still the simplification introduced by this parametrization is impressive and reasonably consistent with what we know about the systematics of supernova progenitors.

Figure 1 suggests that stars below 22 M_{\odot} should be, for the most part, easy to explode using neutrinos alone and no rotation. This is consistent with the observational limits that Smartt (2009) and Smartt et al (2009) placed upon about a dozen presupernova progenitor masses as well as the estimated mass of SN 1987A. It also is a minimal set of masses if the solar abundances are to be produced (Brown & Woosley, 2013). The compactness of stars between 22 and about 35 M_{\odot} is highly variable though due to the migration outwards of the carbon and oxy-

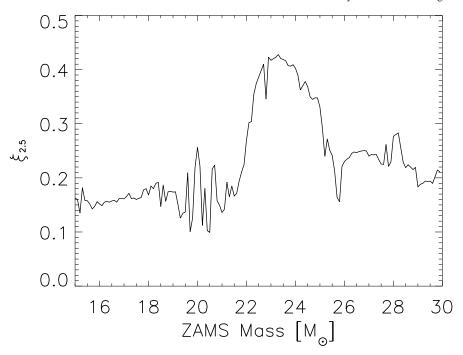


Fig. 1 Compactness parameter for presupernova stars of solar metallicity as a function of main sequence mass (Sukhbold & Woosley, 2014). Stars with smaller $\zeta_{2,3}$ explode more easily.

gen burning shells (Sukhbold & Woosley, 2014). For a standard choice of stellar physics, there exists an island of compact cores between 26 and 30 M_{\odot} that might allow for islands of "explodability". This would help with nucleosynthesis and also possibly have implications for the properties of the Cas A supernova remnant. Cas A, like SN 1993J and 2001gd (Chevalier & Soderberg, 2010), is thought to be the remnant of a relatively massive single star that lost most of its hydrogenic envelope either to a wind or a binary companion, yet its remnant contains a neutron star. If the mass loss was to a companion, as is currently thought, then the progenitor mass was probably less than 20 M_{\odot} , but if a star of 30 M_{\odot} could explode after losing most of its envelope, this might provide an alternate, solitary star explanation.

On the other hand, binary x-ray sources exist and the black holes in them are thought to be quite massive (Özel et al, 2010; Wiktorowicz, Belczynski, & Maccarone, 2014). Stars above 35 M_{\odot} either make black holes if their mass loss during the Wolf-Rayet stage is small, or some variant of Type Ibc supernovae if it is large and shrinks the carbon oxygen core below about 6 M_{\odot} .

Probably the greatest omission here is the effect of rotation and the need to produce gamma-ray bursts in a subset of stars. We also have said nothing about the fate of stars over $80~M_{\odot}$. Both topics will be covered in later sections.

2.2 8 M_{\top} to 30 M_{\top}; Today's Supernovae and Element Factories

For reasonable choices of initial mass function, stars in this mass range are responsible for most of the supernovae we see today and for the synthesis of most of the heavy elements. This does not preclude many of these stars from making black holes, but the supernovae we see are in this range. Baring binary interaction, including mergers, or low metallicity, such stars are, at death, red supergiants, and so the most common supernovae are Type IIp. Explosion energies range from 0.5 to 4×10^{51} erg with a typical value of 9×10^{50} erg (Kasen & Woosley, 2009). These values are the kinetic energy of all ejecta at infinity and the actual energy requirement for the central engine may be larger, especially for more massive stars with large binding energies in their mantles. The light curves and spectra of the models are consistent with observations, to the extent that models for SN IIp can even be used as "standard candles" based upon the expanding photosphere method.

Including binary interactions, one can account for the remainder of common (non-thermonuclear) supernovae, including Type Ib, Ic, IIb, etc (Dessart et al, 2011, 2012). These events typically come from massive stars in the 12 - 18 M_{\odot} range that lose their binary envelopes and die as stripped down helium cores of 3 to 4 M_{\odot} . On the low end, the explosion ejects too little ^{56}Ni to be a bright optical event. Heavier stars are rarer and may not explode. If they do their light curves are broader and fainter than typical Ib and Ic supernovae.

The nucleosynthesis produced by solar metallicity stars in this mass range has been explored many times (Woosley & Weaver, 1995; Woosley, Heger, & Weaver, 2002; Woosley & Heger, 2007; Thielemann, Nomoto, & Hashimoto, 1996; Nomoto et al, 2006; Limongi, Straniero, & Chieffi, 2000; Chieffi & Limongi, 2004, 2013; Hirschi, Meynet, & Maeder, 2005; Nomoto, Kobayshi, & Tominaga, 2013). While the results from the different groups studying the problem vary depending upon the treatment of critical reaction rates, mass loss, semiconvection, convective overshoot, and rotationally induced mixing, some general conclusions may be noted.

- The majority of the elements and their isotopes from carbon (Z = 6) through strontium (Z = 38) are made in solar proportions in supernovae with an average production factor of around 15 (IMF averaged yield expressed as a mass fraction and divided by the corresponding solar mass fraction). The iron group, Ti through Ni, is underproduced in massive stars by a factor of several, which is consistent with the premise that most of the solar abundances of these species were made recently in thermonuclear (Type Ia) supernovae. In the distant past, the oxygen to iron ratio was larger, and massive stars probably produced the iron group in very low metallicity stars.
- For a reasonable choice for the critical 22 Ne(α ,n) 25 Mg reaction rate, the light sprocess up to A = 90 is made well in massive stars, but only if the upper bound for the masses of stars that explode is not too low (Brown & Woosley, 2013). The heavy component of the p-process above A = 130 is also produced in massive stars, but the production of the lighter p-process isotopes (A = 90 130) remains

- a mystery, especially the origin of the abundant closed shell nucleus 92 Mo (Z = 42, N = 50).
- While oxygen is definitely a massive star product, the elemental yield of carbon (12 C) is sensitive to how mass loss is treated and requires for its production the inclusion of the winds of stars heavier than 30 M $_{\odot}$. Red giant winds, AGB mass loss, and planetary nebulae also produce 12 C, perhaps most of it, as well as all of 13 C and 14 N. 15 N and 17 O are not sufficiently produced in massive stars and may be made in classical novae.
- ¹¹B and about one-third of ¹⁹F are made by neutrino spallation in massive star supernovae. ⁶Li, ⁹Be, and ¹⁰B do not appear to be substantially made, and probably owe their origin to cosmic ray spallation in the interstellar medium. Some but not all of 7Li is made by neutrino spallation.
- Certain select nuclei like ⁴⁴Ca, ⁴⁸Ca, and ⁶⁴Zn are underproduced and may require alternate synthesis

In addition to the previously mentioned uncertainties affecting presupernova evolution, assumptions about the explosion mechanism also play a major role. Fundamentally important is just which masses of stars eject their mantles of heavy elements and which collapse to black holes while ejecting little new elements. For a given presupernova structure, a shock that imparts ~10⁵¹ erg of kinetic energy to the base of the ejecta, none of which fall back, will give a robust pattern of nucleosynthesis whether that energy is imparted by a piston or as a thermal "bomb". The approximation used by many, however, that the explosion across all masses can be parametrized by a constant kinetic energy at infinity is too crude and needs revisiting. Stars of different masses have different binding energies, compactness parameters, and iron core masses. Rotation probably has a major effect on the explosion, especially of the more massive stars. The next stage of modeling will need to take into account these dependencies.

2.3 Stars 30 M_☉ to 80 M_☉; Black Hole Progenitors

While the jury is still out regarding the mass-dependent efficiency of an explosion mechanism that includes realistic neutrino transport, rotation, magnetic fields, and relativity in three dimensions, the existence of stellar mass black holes and the absence of observable supernova progenitors with high mass implies that at least some stars do not explode and eject all of their heavy element inventory. Until such time as credible models exist, a reasonable assumption is that the success of the explosion is correlated with the compactness (O'Connor & Ott, 2011; Ugliano et al, 2012). By this criterion, one expects the central regions of stars with *helium cores* much larger than about 10 M_{\odot} and lighter than 35 M_{\odot} to collapse (Sukhbold & Woosley, 2014) to black holes. Above 10 M_{\odot} of helium, or about 30 M_{\odot} on the main sequence, the iron core is large, typically over 2.0 M_{\odot} and the compactness parameter is large. Above 35 M_{\odot} , or about 80 M_{\odot} on the main sequence, one encounters the pulsational pair instability (Section 3).

For solar metallicity stars, mass loss may reduce the presupernova mass of the star to a level where it can frequently explode. If it does and the entire envelope has been lost, the explosion will be some sort of Type Ib or IC supernova. Because of the large mass, the light curve would be broad, and not as bright as most observed SN Ibc. The remnant would probably be a neutron star. It is unclear if such events have been observed, though Cas A might be a candidate.

Even if the core of the star collapses to a black hole, its death is not necessarily nucleosynthetically barren or unobservable. The black hole could result from fall back and the envelope may still be ejected. Even if the presupernova star does not explode at all, its evolution will still have contributed to nucleosynthesis by its wind, which may be appreciable (Hirschi, Meynet, & Maeder, 2005). If only the hydrogenic layers are ejected, these winds can be a rich source of ¹²C, ¹⁶O and, at low metallicity, ¹⁴N (Meynet, 2002). If the wind eats deeply into the helium core, ¹⁸O and ²²Ne can also be ejected, but the winds of such stars are devoid of heavier elements like silicon and iron.

If the star rotates sufficiently rapidly, a gamma-ray burst may result (Section 6.2) or a magnetar-powered supernova. Even for non-rotating stars, it is debatable whether the star can simply disappear without a trace. The sudden loss of mass energy from the protoneutron star can trigger mass ejection and a very subluminous supernova (Lovegrove & Woosley, 2013). Pulsations or gravity waves generated in the final stages of evolution may partly eject the envelope. Even a weak explosion might produce a potentially observable bright spike as its shock wave erupts through the surface of the star (Piro, 2013). In a tidally locked binary or a low metallicity blue supergiant with diminished mass loss, sufficient angular momentum may exist in the outermost layers of the star to pile up in an accretion disk around the new black hole producing some sort of x-ray and gamma-ray transient (Woosley & Heger, 2012; Quataert & Kasen, 2012).

2.4 Yesterday's Metal Poor Stars

Stars with lower metallicity, as may have predominated in the early universe, can have different presupernova structures for a variety of reasons (Sukhbold & Woosley, 2014). Most importantly, metallicity affects mass loss, especially for the more massive stars. If the amount of mass lost is low or zero, the presupernova star including its helium core, is larger, and that has a dramatic effect on its compactness and explodability. A vastly different outcome is expected for e.g., a 60 M_{\odot} star that retains most of its hydrogen envelope and dies with a helium core of 24 M_{\odot} , and one that loses all of its envelope as well as most of its helium core to die with a total mass of 7.3 M_{\odot} . This small mass is obtained with current estimates of mass loss for solar metallicity stars (Woosley, Heger, & Weaver, 2002). Indirect effects can also come into play. Because a low metallicity star loses less mass, it loses less angular momentum and thus dies rotating more rapidly. Indeed, there is some suggestion from theory that massive stars are all born rotating near break up and only slow as a conse-

quence of evolution (expansion) and mass loss (Rosen, Krumholz, & Ramirez-Ruiz, 2012).

Very low metallicity may also enhance the probability of forming more massive stars (Abel et al, 2002). Whether this results in much more massive stars than are being born today is being debated. While this is an important issue for the frequency of first generation stars with masses over 80 M_{\odot} (Section 3), an equally important question is whether the IMF for the first generation stars might have been "bottom-light", that is producing a deficiency of stars below some characteristic mass, say ${\sim}30~M_{\odot}$ (Tan & McKee, 2004). Since this would remove the range of masses responsible for most supernovae and nucleosynthesis today, the early universe would have been quite a different place.

Even assuming the exact same masses of stars and explosions as today, nucleosynthesis would be distinctly different in low metallicity stars. The amount of neutrons available to produce all isotopes except those with Z = N depends on the "neutron excess", $\eta = \Sigma(N_i - Z_i)(X_i/A_i)$, where Z_i , N_i , and A_i are the proton number, neutron number and atomic weight of the species "i" and X_I is its mass fraction. At the end of hydrogen burning all CNO (essentially the metallicity of the star) has become ¹⁴N. Early in helium burning this becomes ¹⁸O by the reaction sequence ¹⁴N(α , γ) ¹⁸F(e^+v) ¹⁸O. The weak interaction here is critical as it creates a net neutron excess that persists throughout the rest of the star's life and limits the production of neutron rich isotopes (like ²²Ne, ²⁶Mg, ³⁰Si etc) and odd-Z elements (like Na, Al, P). Other weak interactions in later stages of evolution also increase η , so that by the time one reaches calcium, the dependence on initial metallicity is not so great, but one does expect an affect on the isotopes from oxygen through phosphorus.

Assuming that the IMF was unchanged and using the same explosion model as for solar metallicity stars (but suppressing mass loss) gives an abundance set that agrees quite well with observations of metal-deficient stars in the range -4 < [Z/Z $_{\odot}$] < -2 (Lai et al, 2008). All elements from C through Zn are well fit without the need for a non-standard IMF or unusually high explosion energy.

Below $[Z/Z_{\odot}]$ = -4, one becomes increasing sensitive to individual stellar events and to the properties of the first generation stars. If the stars below 30 M_{\odot} are removed from the sample, the nucleosynthesis is set by a) the pre-collapse winds of stars in the 30 - 80 M_{\odot} range; b) the results of rotationally powered explosions with uncertain characteristics; and c) the contribution of pulsational pair- and pair-instability supernovae (see below). If only a) and c) contribute appreciably, the resulting nucleosynthesis could be CNO rich and very iron poor.

The light curves of metal deficient supernovae below $80~M_\odot$ are likely to be different - some of the time. If the stars die as red supergiants, then very similar Type IIp supernovae will result, but more of the stars are expected to die as blue supergiants with light curves like SN 1987A (Heger & Woosley, 2010). Rotation can alter this conclusion, however, as it tends to increase the number of red supergiants compared with blue (Maeder & Meynet, 2012). To the extent that the massive stars retain their hydrogenic envelope, Type Ib and Ic supernovae will be suppressed, though of course a binary channel remains a possibility.

3 Pulsational Pair Instability Supernovae (80 to 150 M_{\odot})

The pair instability occurs during the advanced stages of massive stellar evolution when sufficiently high temperature and low density lead to a thermal concentration of electron-positron pairs sufficient to have a significant effect on the equation of state. Only the most massive stars have sufficiently high entropy to encounter this instability. Making the rest mass of the pairs in a post-carbon burning star takes energy that might have otherwise contributed to the pressure. As a result, for a time, the pressure does not rise rapidly enough in a contracting stellar core to keep pace with gravity. The structural adiabatic index of the core dips below 4/3 and, depending on the strength of the instability, the core contracts more or less rapidly to higher temperature, developing considerable momentum as it does so. As temperature rises, carbon, oxygen and, in some cases, silicon burn rapidly. The extra energy from this burning, plus the eventual partial recovery from the instability when the pairs become highly relativistic, causes the pressure to rebound fast enough to slow the collapse. If enough burning occurs before the infall momentum becomes too great, the collapse is reversed and an explosion is possible. For stars that are too big though, specifically for helium non-rotating cores above 133 M_{\odot} , the collapse continues to a black hole.

When an explosion happens, it can be of two varieties. If enough burning occurs to unbind the star in a single pulse, a "pair-instability supernova" results (Section 4). If not, the core of the star expands violently for a time and may kick off its outer layers, including any residual hydrogen envelope. It then slowly contracts until the instability is encountered again and the core pulses once more. The process continues until enough mass has been ejected and entropy lost as neutrinos that the pair instability is finally avoided and the remaining star evolves smoothly to iron core collapse. Typically this requires a reduction of the helium and heavy element core mass to below $40~{\rm M}_{\odot}$. These repeated thermonuclear outbursts can have energies ranging from "mild", barely able to eject even the loosely bound hydrogen envelope of a red supergiant, to extremely large, with over 10^{51} erg in a single pulse. On the high energy end, collisions of ejected shells can produce very bright transients. The observational counterpart is "pulsational pair-instability supernovae" (PPSN).

Depending upon rotation, the electron-positron pair instability begins to have a marked effect on the post-carbon burning evolution of massive stars with negligible mass loss when their main sequence mass exceeds about 70 - 80 M_{\odot} . (Extremely efficient rotationally-induced mixing leading to chemically homogeneous chemical evolution can reduce the threshold main sequence mass still further to approximately the threshold helium core mass (Chatzopoulos & Wheeler, 2012)). For solar metallicity, stars this massive are usually assumed to lose all their hydrogen envelope and part of their cores along the way and thus avoid the instability. Suffice it to say that if the combined effects of mass loss and rotation allow the existence of a helium core mass in excess of 34 M_{\odot} at carbon depletion, the pair instability will have an effect. To get a full-up pair instability supernova, one needs a helium core mass of about 63 M_{\odot} which might correspond, depending upon the treatment of convection physics, to a main sequence star around 150 M_{\odot} . In between, lies the PPSN. As we

shall see, the final evolution of such stars can be quite complicated because of the many pulses, but they have the merit that the explosion hydrodynamics is simple.

3.1 Pulsationally Unstable Helium Stars

While the observable display is quite sensitive to whether the presupernova star retains its hydrogen envelope or not, the number, energies, and duration of the pulses driven by the pair instability is determined entirely by the helium core mass. One can thus sample the broad properties of PPSN using only a grid of bare helium cores. This has the appealing simplicity of removing the uncertain effects of convective dredge up and rotational mixing during hydrogen burning and reducing the problem to a one parameter family of outcomes. Table 1 and Figure 2 summarize some recent results for helium cores of various masses.

Initially, the instability is quite mild and only happens very close to the end of the star's life, after it has already completed core oxygen burning and is burning oxygen in a shell. For larger helium core masses, a few pulses contribute sufficient energy (about 10^{48} erg), that starting at around 34 M_{\odot} , the hydrogen envelope is ejected, but little else. The low energy ejection of the envelope produces very faint, long lasting Type IIp supernovae. The continued evolution of such stars yields an iron core of about 2.5 M_{\odot} that almost certainly collapses to a black hole with a mass nearly equal to the helium core mass. Thus the ejection of the envelope and its nucleosynthesis are the only observables for a distant event.

Table 1 Pulses from Helium Core Explosions of Different Masses (M_☉)

Mass	N Pulse	Duration	Energy	Rem. Mass
32	weak	4.0(3)	1.6(45)	32
34	12	6.5(3)	1.5(48)	33.93
36	many	1.4(4)	9.2(48)	35.81
38	many	8.7(4)	1.1(50)	37.29
40	many	2.8(5)	2.7(50)	38.24
42	18	3.3(5)	2.4(50)	39.72
44	10	9.0(5)	5.8(50)	39.94
46	10	2.2(6)	6.6(50)	41.27
48	7	6.4(6)	9.2(50)	41.52
50	4	7.1(7)	8.1(50)	42.80
52	4	4.3(8)	8.1(50)	45.87
54	2	5.4(10)	1.6(51)	43.35
56	2	1.3(11)	1.6(51)	40.61
58	2	3.0(11)	3.7(51)	17.06
60	2	1.3(11)	2.7(51)	36.60
62	2	5.3(11)	7.1(51)	5.33
64	1	-	4.7(51)	0
66	1	-	6.8(51)	0

Moving on up in mass, the pulses have more energy, start earlier, and increase in number until, above 42 M_☉, their number starts to decline again. Figure 2 shows that in the mass range 36 M_{\odot} to about 44 M_{\odot} a major pulse is typically preceded by a string of smaller ones that grow in amplitude until a single violent event causes a major change in the stellar structure. Recovery from this violent event requires a Kelvin-Helmholtz time scale ($\tau_{\rm KH} \sim GM^2/RL$) for the core to contract back to the unstable temperature, around 2×10^9 K. If the pulse is a weak one, the luminosity in the Kelvin-Helmholtz time scale is the neutrino luminosity and is large, making the time scale short. If the pulse decreases the central temperature below a half-billion degrees however, radiation transport enters in and the time scale becomes long. On the heavier end of this mass range, the total energy of pulses is a few times 10^{50} erg, but their overall duration is less than a week. Since this is less than the time required for the ejected matter to become optically thin, the collisions are usually finished before any supernova becomes visible. Depending upon the presence of an envelope, one expects, for these cases, a rather typical Type Ib or IIp light curve, with some structure possible in the case of the bare helium core because of its short shock transversal time (Section-3.2). When the pulses are over, a large iron core is again produced, and, some time later, the remaining core of helium and heavy elements probably becomes a black hole.

For still heavier helium core masses, 44 to 52 M_{\odot} , the total energy of the pulses becomes that of a typical supernova, but spread over several pulses that require from weeks to years to complete. An important alignment of time scales occurs in this mass range. For the masses and energies ejected, average shell speeds for the first pulse are a few thousand km s⁻¹ (much less if a hydrogen envelope is in the way). At this speed, a radius of $\sim \! 10^{16}$ cm is reached in about a year, which is comparable to the interval between pulses. Repeated supernovae and supernovae with complex light curves are thus possible. The photospheric radii of typical supernovae in nature are a few times 10^{15} cm, this being the distance where the expanding debris most efficiently radiate away their trapped energy on an explosive time scale. Since the ejecta of a given pulse will consist of material moving both slower than and faster than the average, and because each pulse is typically more energetic than its predecessor, shells collide at radii $10^{15}-10^{16}$ cm (Figure 3).

These collisions convert streaming kinetic energy to optical light with high efficiency. In principle, a substantial fraction of the total kinetic energy of the pulses can be radiated, especially if the shells all run into a slowly moving hydrogen shell ejected in the first pulse. Stars in this mass range, in the most extreme cases, can thus give repeated supernovae with up to 10^{51} erg of light.

Still more energetic and less frequent pulsations happen at higher mass, but now the presence of the envelope becomes critical. Without a hydrogen envelope, the time between pulses is so long that the collisions happen at very large radii, 10^{17} - 10^{18} cm. For these very large radii, the result would not be so different from an ordinary 10^{51} erg supernova running into an unusually dense interstellar medium. Both the very large radii and long time scales preclude any resemblance to ordinary optical supernovae, but the events might instead present as bright radio and x-ray transients.

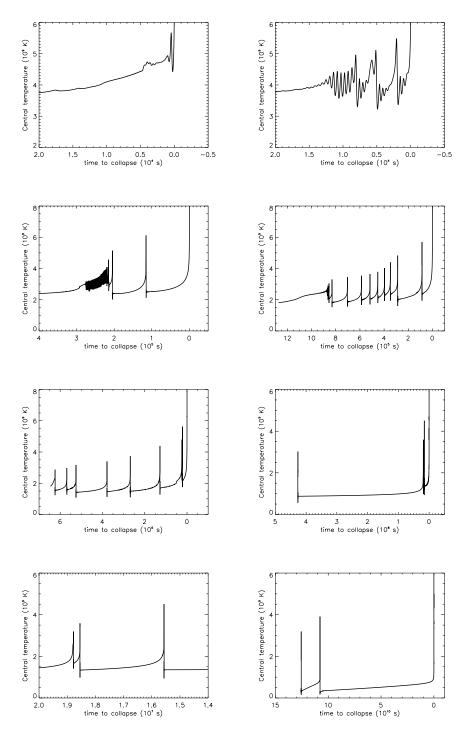


Fig. 2 Pair-driven pulsations cause rapid variations in the central temperature (10^9 K) near the time of death for helium cores of 32, 36, 40, 44, 48, 52 (on two different time scales) and 56 M_{\odot} (left to right; top to bottom). The log base 10 of the time scales (s) in each panel are respectively 4, 4, 5, 5, 6, 8, 7, and 10. The last rise to high temperature marks the collapse of the iron core to a compact object. More massive cores have fewer, less frequent, but more energetic pulses. All plots begin at central carbon depletion.

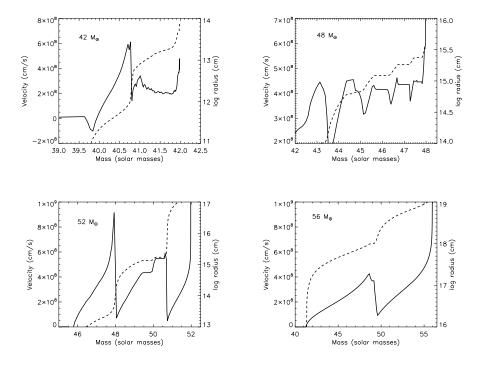


Fig. 3 Velocities (solid lines) and radii (dashed lines) of ejected shells for four helium cores producing mass ejection by the pulsational pair mechanism. The velocities are evaluated at various times when the collision between shells is underway. For the 42 and 48 M_{\odot} models, this was near iron core collapse. For 52 M_{\odot} , it was at central silicon depletion, and for 56 M_{\odot} , after a strong silicon flash, but before the re-ignition of silicon. Some merging of pulses has already occurred. Regions of flat velocity imply spatially thin, high density shells that may be unstable in two or three dimensions.

In the presence of an envelope, the first pulse does not eject matter with such high speed and, given the large variation in speed from the inner part of the moving shell to its outer extremity, substantial energy could still be emitted by explosions in this mass range by shells colliding inside of 10^{16} cm making a bright Type II supernova.

Pulses continue until the helium core has lost enough mass to be stable again. This gives a range of remnant masses typically around 34 to 46 M_{\odot} (Table 1). The iron core masses and compactness parameters for these stars are both very large, so it seems very likely that black holes will result for the entire range of stars making PPSN, all having typical masses around 40 M_{\odot} .

3.2 Light Curves for Helium Stars

Light curves for a sample of helium core explosions are shown in Figure 4 and illustrate the characteristics discussed in the previous section. For the lighter helium cores, the pulses only eject a small amount of matter with low energy. Shell collisions are over before light escapes from the collision region. The light curve for the 26 M_{\odot} helium core is typical for this mass range - a subluminous "supernova" of less than 10^{42} erg s $^{-1}$ lasting only a few days. These might be looked for in the case of stars that have lost their envelopes prior to exploding. In a star with an envelope, as we shall see later, the situation would be very different. Even the small $(10^{49}~\text{erg})$ kinetic energy would unbind the envelope producing a long, faint Type IIp supernova.

For the $42M_{\odot}$ helium core, a brighter, longer lasting transient is produced, but still only a single event, albeit a structured one. The total duration of pulses is about 2 days, followed by a 2 day wait until the core collapse. The last pulse is a particularly violent one. The light curve (Figure 4) shows a faint outburst occurring as many smaller pulses merge and the first big of mass is ejected, followed by a longer more luminous peak as that main pulse runs into the prior ejecta. Both of these transients are quite blue since the collisions are occurring at small radius, a few times 10^{14} cm.

By $48~M_{\odot}$, the shell collisions are becoming sufficiently energetic and infrequent that the light curve fractures into multiple events. The collisions are now happening at around 10^{15} cm and should be quite bright optically. At $52~M_{\odot}$, one sees repeated individual supernovae. Figure 4 merely shows the brightest one from this object. Activity at the 10^{41} erg level started two years before.

It should be noted, though, that all these 1D light-curve calculations are quite approximate and need to be repeated in a multi-dimensional code with the appropriate physics, especially for cases where the shells collide in an optically thin regime. KE-PLER, a one dimensional implicit hydrodynamics code with flux-limited radiative diffusion does an admirable job in a difficult situation. In 1D however, the snow-plowing of a fast-moving shell into a slower one generates a large spike in density, with variations of many orders of magnitude in density between one zone and an adjacent one. For a time this thin shell corresponds to the photosphere. The "linearized" equations of hydrodynamics do not behave well in such clearly non-linear circumstances and the outcome of a multi-dimensional calculation may be qualitatively different. This is an area of active research.

3.3 Type II Pulsational Pair Instability Supernovae

The retention of even a small part of the original hydrogen envelope significantly alters the dynamics and appearance of PPSN. For example, what wold have been a brief, faint transient for a $36~M_{\odot}$ helium core (Figure 4), provides more than enough energy to eject the entire envelope of a red supergiant. A great diversity of outcomes

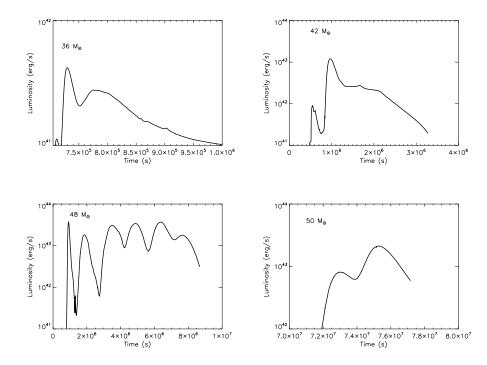
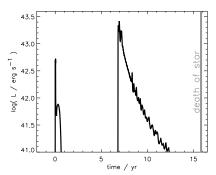


Fig. 4 Bolometric light curves from pulsational pair instability supernovae derived from bare helium cores of 36, 42, 48 and 50 M_{\odot} . A wide variety of outcomes is possible. For the 36 and 42 M_{\odot} models the photospheric radius is inside 10^{15} cm and the transients will be blue. For the higher two masses, the photosphere is near 10^{15} cm and the transients might have colors more like an ordinary supernova.

is possible depending upon the mass of the envelope and helium core and the radius of the envelope

Most striking are the "ultra-luminous supernovae" of Type IIn that happen when very energetic pulses from the edge of the helium core strike a slowly moving, previously ejected hydrogen envelope. A similar (Type I) phenomenon could happen for bare helium cores, but probably with a shorter-lived, less luminous light curve owing to the smaller masses involved. An example is shown in Figure 6 based upon the evolution of a 110 M_{\odot} star (Woosley, Blinnikov, & Heger, 2007). By the end of its life this star had shrunk to 74.6 M_{\odot} (using a wholly artificial mass loss rate), of which 49.9 M_{\odot} was the helium core. This core experienced three violent pulsations. The first ejected almost all of the hydrogen envelope, leaving 50.7 M_{\odot} behind. This envelope ejection produced a rather typical Type IIp supernova although with a slower than typical speed and luminosity (Figure 5). By 6.8 years later, the stellar remnant had contracted to the point that it experienced the pair instability again. Two more pulses, occurring in rapid succession, ejected an additional 5.1 M_{\odot} with



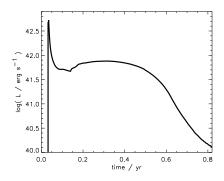


Fig. 5 Light curves of the two supernovae produced by the 110 M_{\odot} PPSN (Woosley, Blinnikov, & Heger, 2007). The first pulse ejects the envelope and produces the faint supernova shown in greater detail on the right. 6.8 years later the collision of pulses 2 and 3 with that envelope produces another brighter outburst (see Figure 6)

a total kinetic energy of 6×10^{50} erg. Pulses 2 and 3 quickly merged and then run into the ejected envelope (Figures 5 and 6).

These light curves were calculated using 1D codes in which the collision of the shells again produced a very large density spike. When the calculation was run again in 2D, but without radiation transport (Figure 6), a Rayleigh-Taylor instability developed that led to mixing and a greatly reduced density contrast. The combined calculation of multi-D hydro coupled to radiation transport has yet to be carried out, so the light curves shown here are to be used with caution, but a multi-dimensional study would probably give a smoother light curve.

3.4 Nucleosynthesis

The nucleosynthesis from PPSN is novel in that it is heavily weighted towards the light species that are ejected in the shells. For present purposes, given the large iron cores, we assume that all matter not ejected by the pulsations becomes a black hole. This assumption could be violated if rapid rotation energized some sort of jet-like outflows (e.g., a gamma-ray burst), but otherwise it seems reasonable.

Table 2 gives the approximate bulk nucleosynthesis, in solar masses, calculated for our standard set of helium cores models. For the lightest cores, the pulses lack sufficient energy to eject more than a small amount of surface material, which by assumption here is pure helium. It should be noted, however, that even these weak explosions would eject at least part of the hydrogen envelope of any red supergiant (typical binding energy less than 10^{48} erg). Since these envelopes often produce primary nitrogen by mixing between the helium core and hydrogen burning shell, an uncertain but possibly large yield of carbon, nitrogen, and oxygen (and of course

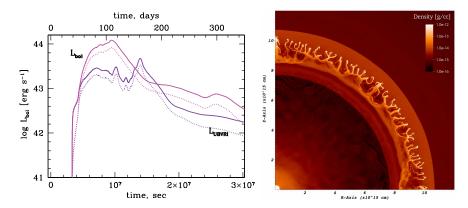


Fig. 6 Left: Light curve for the second very luminous outburst of the $110~M_{\odot}$ model (see Figure 5) of the two supernovae produced by the $110~M_{\odot}$ PPSN (Woosley, Blinnikov, & Heger, 2007). The brighter set of curves results hen the collision speed is artificially increased by a factor of 2 and resembles SN 2006gy. *Right*: 2D calculation of the explosion of a $110~M_{\odot}$ star as a PPSN. The dense shell produced in 1D by the collision of the ejecta from two pulse is Rayleigh-Taylor unstable. The resulting density contrast is much smaller.

hydrogen and helium) would accompany these explosions in a star that had not lost its envelope.

Moving up in mass, the violence of the pulses increases rapidly and more material is ejected, eventually reaching the deeper shells rich in heavier elements. In Table 2, total yields of less than 0.01 M_{\odot} have not been included with the single exception of the 66 M_{\odot} model which made 0.037 M_{\odot} of 56 Ni. The 64 and 66 M_{\odot} models are actually full up pair instability supernovae and leave no remnants, so perhaps including their yields here with the PPSN is a bit misleading.

If one folds these yields with an IMF to get an overall picture of the nucleosynthesis from a generation of PPSN, it is clear that the production (and the typical spectra of PPSN) will be dominated by H, (He), C, N, O, (Ne) and Mg and little else. In particular, PPSN make no iron-group elements. Given the dearth of strong He and Ne lines, one might expect that the generation of stars following a putative "first generation" of PPSN would show enhancements of C, N, O, and Mg and be "ultrairon poor". Of course *some* heavier elements could be made by stars sufficiently light (main sequence mass less than $20~M_{\odot}$?) to explode by the neutrino-transport process, or sufficiently heavy to make iron in a pair-instability supernova (helium core mass over $65~M_{\odot}$).

4 150 to 260 M_☉; Pair Instability Supernovae

The physics of pair instability supernovae (PISN) is sufficiently well understood that they can be accurately modeled in 1D on a desktop computer. A major ques-

Mass	Total	Не	С	О	Ne	Mg	Si	S	Ar	Ca
34	0.071	0.071	-	-	-	-	-	-	-	-
36	0.19	0.19	-	-	-	-	-	-	-	-
38	0.71	0.32	0095	0.17	0.096	0.032	-	-	-	-
40	1.76	0.50	0.29	0.53	0.32	0.11	-	-	-	-
42	2.28	0.60	0.43	0.70	0.41	0.14	-	-	-	-
44	4.06	0.85	0.79	1.36	0.80	0.26	-	-	-	-
46	4.73	1.02	0.94	1.61	0.90	0.27	-	-	-	-
48	6.48	1.34	1.40	2.30	1.15	0.30	-	-	-	-
50	7.20	1.58	1.60	2.61	1.16	0.26	-	-	-	-
52	6.13	1.55	1.33	2.29	0.81	0.16	0.001	-	-	-
54	10.64	1.65	1.83	5.32	1.35	0.41	0.074	-	-	-
56	15.38	1.74	2.06	9.41	1.52	0.50	0.15	-	-	-
58	40.93	1.85	2.87	30.5	2.64	1.42	1.49	0.17	0.020	0.015
60	23.39	1.89	3.10	15.0	2.49	0.60	0.28	0.058	0.008	0.005
62	56.67	1.95	2.87	37.5	2.60	1.43	6.39	2.99	0.51	0.44
64	64	1.92	3.62	44.1	3.60	2.12	5.35	2.41	0.43	0.38
66	66	1.79	3.60	42.8	3.99	2.07	7.11	3.49	0.60	0.53

Table 2 Nucleosynthesis in Ejected Shells (M_☉) from Helium Core Pulsational Explosions

tion though is their frequency in the universe. PISN come from a range of masses somewhat heavier than we expect for presupernova stars today. This is not to say that stars of over 150 M_{\odot} are not being born. See e.g., the review by Crowther reported in Vink et al (2013) which gives 320 M_{\odot} as the current observational limit. The issue is whether such large masses can be retained in a star whose luminosity hovers near the Eddington limit (Vink et al, 2011). Still observers claim to have discovered at least one PISN event (Galyam et al, 2009). Because the critical quantity governing whether a star becomes PISN is the helium core mass of the presupernova star (greater than 65 M_{\odot}), they are favored by diminished mass loss, i.e., at low metallicity, and may have been more abundant in the early universe.

A common misconception is that all PISN make a lot of 56 Ni and therefore are always very bright. As Figure 7 shows, large 56 Ni production and very high kinetic energies are limited to a fairly narrow range of exceptionally heavy and rare PISN. Most events will either present as a particularly energetic Type IIp supernova or a *subluminous* SN I. For an appreciable range of masses, less 56 Ni is produced than in, e.g., a SN Ia (about $0.7~{\rm M}_{\odot}$).

The nucleosynthesis of very low metallicity PISN is quite distinctive because they lack the excess neutrons needed to make odd-Z elements during the explosion. This is because the initial metallicity of the star, mostly CNO, is turned into ¹⁴N during hydrogen burning. During helium burning, ¹⁴N captures an alpha particle experiencing a weak decay to make ¹⁸O which has two extra neutrons. Subsequent burning stages rearrange these neutrons using them to make isotopes and elements that require an excess of neutrons over protons, like almost all odd Z elements do. During the collapse phase, the time is too short for additional weak interactions so the ejected matter ends up deficient in things like Na, Al, P, Cl, K, Sc, V, and Mn.

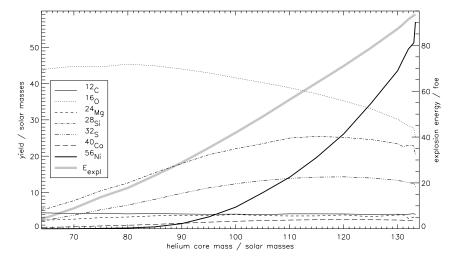


Fig. 7 Nucleosynthesis in pair-instability supernovae as a function of helium core mass. Also given is the explosion energy in units of 10^{51} erg (broad grey line) which rises steadily with mass. The dark solid line is 56 Ni synthesis which is not particularly large below 90 M_{\odot} (Heger & Woosley, 2002).

Very metal poor stars show no such anomalies and this suggests that the contribution of PISN to very early nucleosynthesis was small.

5 Above 260 M_☉

Stars heavier than 260 M_{\odot} , or more specifically non-rotating helium cores greater than $133\,M_{\odot}$, are expected to produce black holes, at least up to about $10^5\,M_{\odot}$. Starting around $10^5\,M_{\odot}$, hydrogenic stars encounter a post-Newtonian instability on the main sequence and collapse (Fowler & Hoyle, 1964). If these stars have near solar metallicity (above Z=0.005) then titanic explosions of 10^{56} - 10^{57} erg, powered by explosive hydrogen burning, can result for masses in the range 10^5 - $10^6\,M_{\odot}$ (Fuller, Woosley, & Weaver, 1986). Lacking a large initial concentration of CNO, stars in this mass range, collapse to black holes.

For lighter stars, $\sim \! 10^3$ - 10^5 M_{\odot} , hydrogen burns stably, but helium burning encounters the pair instability, and on the upper end, the post-Newtonian instability. Again black hole formation seems the most likely outcome, though this mass range has not been fully explored.

6 The Effects of Rotation

Rotation alters stellar evolution in two major ways. During presupernova evolution it leads to additional mixing processes that can stir up either regions of the star or the whole star. Generally the helium cores of rotating stars are larger and, since the nucleosynthesis and explosion physics of massive stars depends sensitively upon the helium core mass, the outcome of a smaller mass main sequence star with rotation can resemble that of a larger one without rotation. The mixing can also increase the lifetime of the star and its luminosity and bring abundances to the surface that might have otherwise remained hidden. In extreme cases, rotation can even lead to the complete mixing of the star on the main sequence, thus avoiding the formation of a supergiant and producing a very rapidly rotating presupernova star that might serve as a gamma-ray burst progenitor (Section 6.2).

The other way rotation changes the evolution is by affecting how the star explodes and the properties of the compact remnant it leaves behind. Calculations that use reasonable amounts of rotation and approximate the effects of magnetic torques in transporting angular momentum show that rotation may play an increasingly dominant role in the explosion as the mass of the star increases (Heger, Woosley, & Spruit, 2005). This is in marked contrast to the neutrino transport model which shows the opposite behavior (Section 2.1); heavier stars are more difficult to explode with neutrinos.

Table 3 Pulsar Rotation Rate Predicted by Models (Heger, Woosley, & Spruit, 2005)

Mass (M_{\odot})	Baryon (M_{\odot})	Gravitat (${ m M}_{\odot}$)	ti d nal (10 ⁴⁷ erg s)	BE (10 ⁵³ erg)	Pulsar P (ms)
12	1.38	1.26	5.2	2.3	15
15	1.47	1.33	7.5	2.5	11
20	1.71	1.52	14	3.4	7.0
25	1.88	1.66	17	4.1	6.3
35	2.30	1.97	41	6.0	3.0

Table 3 shows the expected rotation rates of pulsars derived from the collapse of rotating stars of various main sequence masses. The rotational energy of these neutron stars is given approximately by $10^{51}(5 \text{ms}/P)^{-2}$ erg, where it is assumed that the neutron star moment of inertia is $80 \text{ km}^2 \text{ M}_\odot(\text{Lattimer \& Prakash, } 2007)$. This implies that supernova over about 20 M_\odot or so have enough rotational energy to potentially power a standard supernova. Rapidly rotating stellar cores are also expected to give birth to neutron stars with large magnetic fields (Duncan and Thompson, 1992), thus providing a potential means of coupling the large rotation rate to the material just outside the neutron star. Calculations so far are encouraging (e.g. Akiyama et al, 2003; Burrows et al, 2007; Janka, 2012). No calculation has yet modeled the full history, of a rotational, or rotational plus neutrino powered supernova all the way

through from the collapse to explosion phase including all the relevant neutrino and MHD physics, but probably this will happen in the next decade.

In principle, the outcomes of rotationally powered supernovae and those powered by neutrinos should be very similar, though only rotation offers the prospect of making the explosion hyper-energetic (much greater than 10^{51} erg). To the extent that nucleosynthesis, light curves and spectra only depend upon the prompt deposition of $\sim 10^{15}$ erg at the center of a highly evolved red or blue supergiant, they will be indistinguishable. Rotation breaks spherical symmetry and may produce jets, but except in the case of gamma-ray bursts, it may be hard to disentangle effects essential to the explosion from those that simply modify an already successful explosion. There are interesting constraints on time scales, however, and hence on field strengths. Rotation or neutrinos must overcome a ram pressure from accretion that, in the case of high compactness parameter, may approach a solar mass per second. At a radius of 50 km, roughly typical of a young hot protoneutron star, it would take a field strength of over 10¹⁵ gauss to impede the flow. A similar estimate comes from nucleosynthesis. In order to synthesize ⁵⁶Ni, material must be heated to at least 4 and preferably 5×10^9 K. In a hydrodynamical model in which radiation dominates and 10⁵¹ erg is deposited instantly, this will only occur in a region smaller than 3000 km. It takes the shock, moving at typically 20,000 km s⁻¹, about 0.1 s to cross that region, after which it begins to cool off. To deposit 10⁵¹ erg in that time with a standard dipole luminosity (Lang, 1980) the field strength would need to exceed about 10¹⁶ gauss. This probably exceeds the *surface* fields generated by collapse alone. Whether the magneto-rotational instability can generate such fields is unclear, but it may take an exceptionally high rotation rate for this to all work out.

Perhaps the most common case is a neutrino-powered initial explosion amplified by rotation at later times. If that is the case though, a successful outgoing shock must precede any significant pulsar input. That starting point be difficult to achieve in stars with high compactness (Figure 1). In any case we do know that *some* massive stars do make black holes.

6.1 Magnetar Powered Supernova Light Curves

If magnetic fields and rotation can provide the $\sim 10^{51}$ erg necessary for the kinetic energy of a supernova, they might, with greater ease, deliver the 10^{48} or even 10^{50} erg needed to make a bright - or a really bright - light curve (Woosley, 2010; Kasen & Bildsten, 2010). At the outset, one must acknowledge the huge uncertainty in applying the very simple pulsar power formula (Lang, 1980),

$$\frac{dE}{dt} \approx 10^{49} B_{15}^2 P_{\rm ms}^{-4} \,{\rm erg \, s^{-1}},\tag{1}$$

to a situation where the neutron star is embedded in a dense medium and that is still be rapidly evolving. Doing this blindly, however, yields some interesting results (Figure 8). Since the energy is deposited late, it is less subject to adiabatic losses and

is emitted as optical light with high efficiency. For reasonable choices of magnetic field and initial rotation rate, the supernova can be "ultra-luminous", brighter than a typical SN Ia for a much longer time.

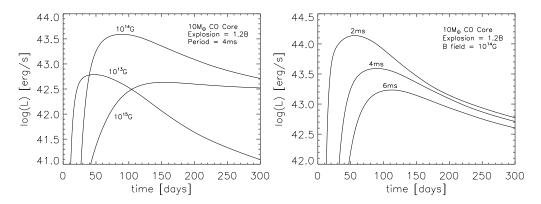


Fig. 8 Magnetar powered light curves for (left) different values of field strength (10^{14} , 10^{15} , and 10^{16} G at 4 ms) and (right) initial rotation periods (2, 4, 6 ms at 10^{14} G). The base event is the 1.2 $\times 10^{51}$ erg explosion of a 10 $\rm M_{\odot}$ carbon-oxygen core. (Sukhbold and Woosley, 2014, in preparation)

The magnetic fields required are not all that large and are similar to what has been observed for modern day magnetars (Mereghetti, 2008). In fact, too large a field results in the rotational energy being deposited too early. That energy then contributes to the explosion kinetic energy, but little to the light curve because, by the time the light is leaking out, the magnetar has already deposited most of its rotational energy. The rotation rates, though large, are also not extreme, not very different, in fact, from the predictions for quite massive stars (Heger, Woosley, & Spruit, 2005). If gamma-ray bursts are to be powered by millisecond magnetars with fields $\sim 10^{15}$ - 10^{16} G, and if ordinary pulsars have fields and rotational energies 100 to 1000 times less, one expects somewhere, sometime to make neutron stars with fields and rotational energies that are just ten times less. The long tails on the light curves are interesting and, lacking spectroscopic evidence or very long duration observations, might easily be confused with 56 Co decay (Woosley, 2010).

Depending upon the mass and radius of the star, the presence or absence of a hydrogenic envelope, and the supernova explosion energy, the resulting magnetar-illuminated transients can be quite diverse. The brighter events will tend to be of Type I because the supernova becomes transparent at an earlier time when greater rotational energy is being dissipated. The upper bound to the luminosity is a few times 10^{51} erg emitted over several months, or $\sim 10^{44.5}$ erg s⁻¹, but much fainter events are clearly possible. For Type II supernovae in red supergiants, the magnetar contribution may present as a rapid rise in brightness after an extended plateau (Maeda et al, 2007). The rise could be even more dramatic and earlier in a blue supergiant.

An interesting characteristic of 1D models for magnetar powered supernovae is a large density spike caused by the pile up of matter accelerated from beneath by radiation. In more than one dimension, this spike will be unstable and its disruption will lead to additional mixing that might have consequences for both the spectrum and the appearance of the supernova remnant.

6.2 Gamma-Ray Bursts (GRBs)

In the extreme case of very rapid rotation and the complete loss of its hydrogenic envelope, the death of massive star can produce a common (long-soft) GRB. For a recent review see Woosley (2013). There are two possibilities for the "central engine" - a "millisecond magnetar" and a "collapsar". The former requires that the product of a successful supernova explosion be, at least for awhile, a neutron star, and that the power source is its rotational energy. The latter assumes the formation of a black hole with a centrifugally supported accretion disk. The energy source can be either the rotational energy of that black hole or of the disk, which is, indirectly, energized by the black hole's strong gravity.

Both models require that the progenitor star have extremely high angular momentum in and around the iron core. Loss of the hydrogen envelope could occur though a wind, binary mass exchange, or because extensive rotationally-induced mixing on the main sequence kept a red giant from ever forming. Loss of the envelope by a wind is disfavored because the existence of a lengthy red giant phase would probably break the rotation of the core to the extent that the necessary angular momentum was lost. One is this left with the possibility of a massive star that lost its envelope quite early in to a companion or a single star that experienced chemically homogeneous evolution (Maeder, 1987; Woosley & Heger, 2006; Yoon & Langer, 2005, 2006). The resulting Wolf-Rayet star must also not lose much mass or its rotation too will be prohibitively damped. This seems to exclude most stars of solar metallicity, so GRBs are relegated to a low metallicity population. The relevant mass loss rate depends upon metallicity (specifically the iron abundance) as Z^{0.86} (Vink & de Koter, 2005), and even mild reduction is sufficient to provide the necessary conditions for a millisecond magnetar.

The collapsar model is capable, in principle, of providing much more energy (up to $\sim 10^{54}$ erg) than the magnetar model (up to 3×10^{52} erg). The former is limited only by the efficiency of converting accreted mass into energy, which can be quite high for a rotating black hole, while the latter is capped by a critical rotation rate where the protoneutron star deforms and efficiently emits gravitational radiation. So far, there is no clear evidence for total (beaming corrected) energies above $10^{52.5}$ in any GRB, so both models remain viable. It is interesting that there may be some pile up of the most energetic GRBs and their associated supernovae around a few times 10^{52} . That might be taken as (mild) evidence in favor of the magnetar model. On the other hand, black hole production is likely in the more massive stars and it may be

difficult to arrange things such that all the matter always accretes without forming a disk (Woosley & Heger, 2012)

Since angular momentum is in short supply, it is definitely easier to produce a millisecond magnetar which requires a mass averaged of angular momentum of only 2×10^{15} erg s (for a moment of inertia $I=10^{45}$ g cm²), or a value at its equator of 6×10^{15} erg s (for a neutron star radius of 10 km). For comparison, the angular momentum for the last stable orbit of a Kerr black hole is $1.5\times 10^{16}\,\frac{M_{\rm BH}}{3\,{\rm M}_\odot}$ erg s and about three times larger for a Schwarzschild hole. The same sorts of systems that make collapsars thus also seem likely to make, at least briefly, neutron stars with millisecond rotation periods. How these rapid rotators make their fields and how the fields interact with the rapidly accreting matter in which they are embedded is a very difficult problem in 3D, general relativistic magnetohydrodynamics. Analytic arguments suggest however that large fields will be created (Duncan and Thompson, 1992) and that the rotation and magnetic fields will play a major role in launching an asymmetric explosion (Akiyama et al, 2003; Burrows et al, 2007).

Just which mass and metallicity stars make GRBs is an interesting issue. Even when the effects of beaming are included, the GRB event rate is a very small fraction of the supernova rate and thus the need for special circumstances is a characteristic of all successful models. These special circumstances include, as mentioned, the lack of any hydrogenic envelope and very rapid rotation. Without magnetic torques, the cores of most massive stars would rotate so rapidly at death that millisecond magnetars, collapsar, and presumably GRBs would abound. Any realistic model thus includes the effects of magnetic braking, even though the theory (Spruit, 2002; Heger, Woosley, & Spruit, 2005) is highly uncertain. In fact, most massive stars may be born with extremely rapid rotation, corresponding to 50% critical in the equatorial plane, because of their magnetic coupling to an accretion disk (Rosen, Krumholz, & Ramirez-Ruiz, 2012). The fact that most massive stars are observed to be rotating more slowly on the main sequence is a consequence of mass loss which would be reduced in regions with low metallicity. Since these large rotation rates are sufficient, again with uncertain parameters representing the inhibiting effect of composition gradients, to provoke efficient Eddington-Sweet mixing on the main sequence, GRBs should be abundant (too abundant?) at low metallicity. It is noteworthy that models for GRBs that invoke such efficient mixing on the main sequence do not require that the star be especially massive since, for low metallicity, the zero age main sequence mass is not much greater than the presupernova helium core mass (Woosley & Heger, 2006). A low metallicity star of only 15 M_☉ could become a GRB and a star of 45 M_☉ could become a pulsational pair instability supernova.

Using a standard set of assumptions, the set of massive stars that might make GRBs by the collapsar mechanism has been surveyed for a grid of masses and metallicities by Yoon & Langer (2006). Averaged over all redshifts they find a GRB to supernova event ratio of 1/200 which declines at low redshift to 1/1250. Half of all GRBs are expected to be beyond redshift 4. Given that magnetars might also make GRBs, or even most of them, these estimates need to be reexamined. In particular, the mean redshift for bursts may be smaller and the theoretical event rate higher.

7 Final Comments

As is frequently noted, we live in interesting times. Most of the basic ideas invoked for explaining and interpreting massive star death are now over 40 years old. This includes supernovae powered by neutrinos, pulsars, the pair-instability, and the pulsational pair instability. Yet lately, the theoretical models and observational data have both experienced exponential growth, fueled on the one hand by the rapid expansion of computer power and the shear number of people running calculations, and on the other, by large transient surveys. Ideas that once seemed "academic", like pair-instability supernovae and magnetar-powered supernovae are starting to find counterparts in ultra-luminous supernovae.

"Predictions" in such a rapidly evolving landscape quickly become obsolete or irrelevant. Still, it is worth stating a few areas of great uncertainty where rapid progress might occur. These issues have been with us a long time, but problems do eventually get solved.

- What range(s) of stellar masses and metallicities explode by neutrino transport alone. The community has hovered on the brink of answering this for a long time. Today some masses explode robustly and others show promise (Janka et al, 2012; Janka, 2012), but a comprehensive, parameter-free understanding is still lacking. The computers, scientists, and physics may be up to the task in the next five years. The compactness of the progenitor very likely plays a major role. It would be really nice to know.
- What is the relation between the initial and final (presupernova) masses of stars of all masses and metallicities. Suppose we knew the *initial* mass function at all metallicities (a big given). What is the *final* mass function for presupernova stars? We can't really answer questions about the explosion mechanism of stars of given main sequence masses without answering this one too. Our theories and observations of mass loss are developing, but still have a long way to go.
- What is the angular momentum distribution in presupernova stars? To answer
 this the effects of magnetic torques and mass loss must be included throughout
 all stages of the evolution a tough problem. Approximations exist, but they are
 controversial and more 3D modeling might help.
- Are the ultra-luminous supernovae that are currently being discovered predominantly pair instability, pulsational pair instability, or magnetar powered (or all three)? Better modeling might help, especially with spectroscopic diagnostics.
- Is the most common form of GRB powered by a rotating neutron star or by an accreting black hole? What are the observational diagnostics of each?
- Does "missing physics", e.g., neutrino flavor mixing or a radically different nuclear equation of state play a role in answering any of the above questions?

This small list of "big theory issues" of course connects to a greater set of "smaller issues" - the treatment of semiconvection, convective overshoot, and rotational mixing in the models; critical uncertain nuclear reaction rates; opacities; the complex interplay of neutrinos, magnetohydrodynamics, convection and general relativity in 3D in a real core collapse - well maybe that is not so small.

Obviously there is plenty for the next generation of stellar astrophysicists to do.

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