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Noncommutative Dynamics and E-Semigroups

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Preface

These days, the term Noncommutative Dynamics has several interpretations. It is used in this book to refer to a set of phenomena associated with the dynamical evolution of quantum systems of the simplest kind that involve rigorous mathematical structures associated with infinitely many degrees of freedom. The dynamics of such a system is represented by a one-parameter group of automorphisms of a noncommutative algebra of observables, and we focus primarily on the most concrete case in which that algebra consists of all bounded operators on a Hilbert space.

If one introduces a natural causal structure into such a dynamical system, then a pair of one-parameter semigroups of endomorphisms emerges, and it is useful to think of this pair as representing the past and future with respect to the given causality. These are both E_0-semigroups, and to a great extent the problem of understanding such causal dynamical systems reduces to the problem of understanding E_0-semigroups. The nature of these connections is discussed at length in Chapter 1. The rest of the book elaborates on what the author sees as the important aspects of what has been learned about E_0-semigroups during the past fifteen years. Parts of the subject have evolved into a satisfactory theory with effective tools; other parts remain quite mysterious.

Like von Neumann algebras, E_0-semigroups divide naturally into three types: I, II, III. The type I examples are now known to be classified to cocycle conjugacy by their numerical index. It is also known that examples of type II and III exist in abundance (there are uncountably many cocycle conjugacy classes of each type), but we are a long way from a satisfactory understanding: we have surely not seen all the examples of type II or III, and we still lack effective cocycle conjugacy invariants for distinguishing between the ones we have seen.

This subject makes significant contact with several areas of current interest, including quantum field theory, the dynamics of open quantum systems, and probability theory, both commtative and noncommutative. Indeed, Powers' first examples of type III E_0-semigroups were based on a construction involving quasi-free states of the C^*-algebra associated with the infinite-dimensional canonical anticommutation relations. More recently, the product systems constructed by Tsirelson are based on subtle properties of "noises" of various types, both Gaussian and non-Gaussian, that bear some relation to Brownian motion and white noise. When combined with appropriate results from the theory of E_0-semigroups, the examples of product systems based on Bessel processes give rise to a continuum of examples of E_0-semigroups of type II, and an E_0-semigroup that cannot be paired with itself. The Tsirelson–Vershik product systems discussed in Chapter 14 lead to a continuum of type III examples that are mutually non-cocycle-conjugate.

It appears to me that the current state of knowledge about these matters can be likened to the state of knowledge of von Neumann algebras in the late sixties, in the period of time after Powers' proof that there are uncountably many nonisomorphic type III factors but before the revolutionary developments of the seventies, which began with the discovery, based on the Tomita–Takesaki theory, that a type III factor is an object that carries with it an intrinsic dynamical group, and culminated with Connes's classification of amenable factors. I believe that there are exciting developments in the future of E_0-semigroups as well.

The book contains new material as well as reformulations of results scattered throughout the literature. For example, we have based our discussion of dilation theory on certain aspects of noncommutative dynamics that are common to all dynamical systems, allowing us to deduce the existence of dilations of quantum dynamical semigroups from very general considerations involving continuous free products of C^*-algebras. We have freed the discussion of the interaction inequality of Chapter 12 from the context of semigroups of endomorphisms in order to place it in an appropriate general context, in which the central result becomes an assertion about the convergence of eigenvalue lists along a tower of type I factors in $\mathcal{B}(H)$. Chapter 13 contains a technically complete discussion of Powers' examples of type III E_0-semigroups that brings out the role of Toeplitz and Hankel operators and quasi–continuous functions, and provides a new concrete criterion for the absence of units. Finally, the theory of spectral C^*-algebras presented in Chapter 4 has been simplified and rewritten from scratch.

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