# Simultaneous Bloch and Josephson Oscillations, and Resistance Quantizations in Small Superconducting Double Junctions

Proc. 18th Int. Conf. on Low Temperature Physics, Kyoto, 1987
Japanese Journal of Applied Physics, Vol. 26 (1987) Supplement 26-3



# Simultaneous Bloch and Josephson Oscillations, and Resistance Quantizations in Small Superconducting Double Junctions

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A structure comprising three superconducting electrodes separated with two small Josephson junctions has been analyzed. It has been assumed that a dc voltage $\bar{V}$ is applied to the external electrodes of the structure, and a dc current $\bar{I}$ is fed to its middle electrode. The analysis has proved that the supercurrent flowing through the structure should oscillate simultaneously with the Josephson frequency $f_J = 2e\bar{V}/h$ and the Bloch frequency $f_B = \bar{I}/2e$ . Mutual phase locking of these oscillations leads to quantization the ratio $R = \bar{V}/\bar{I}$ in units $R_Q = h/(2e)^2$.

## 1. INTRODUCTION

Recently, it was predicted theoretically [1] and confirmed experimentally [2] that extremely small Josephson junctions exhibit a behavior quantum-mechanically dual to the usual Josephson effect. In particular, the junction fed by a dc current $\bar{I}$ generates coherent "Bloch" oscillations with the frequency $f_B = \bar{I}/2e$ rather than the usual Josephson oscillations with frequency $f_J = (2e/h)\bar{V}$ . A natural question arises, whether the two types of oscillations can coexist . The answer is presumably no if a single junction is considered . We will show, however, that even a slight modification of the system leads to a coexistence of the Bloch and Josephson oscillations. Moreover, interaction of the two processes results in a qualitatively new dynamics, in particular, in quantization of the voltage/current ratio.

## 2. SYSTEM AND MODEL

Let us consider a structure [3] of three supercoducting electrodes connected by two Josephson junctions with very small capacitances and conductances (for experimental estimates see ref.1)

$$C_{1,2} << e^2/2k_BT , \quad G_{1,2} << R_Q^{-1} , \tag{1}$$

where $R_Q = h/(2e)^2$ is the quantum unit of resistance. Let a voltage source fix a dc voltage $V_o$ across the external electrodes while the dc current source $I_o$ with a nonvanishing internal conductance G,

$$G_{1,2} << G << R_Q^{-1}, \tag{2}$$

feeds a current I into the middle electrode (Fig.1).

Neglecting the intrinsic junction damping one can present the Hamiltonian of the system in the form similar to that of ref.3

$$H = H_o - V_o(Q_1\frac{C_2}{C_+} + Q_2\frac{C_1}{C_+}) - (\hbar/2e)(I_o-I_x)\,\phi_2 + H_x , \tag{3}$$

$$H_o = \frac{Q^2}{2C_+} - E_1\cos\phi_1 - E_2\cos\phi_2 , \tag{4}$$

$$Q = Q_2 - Q_1 , \quad C_+ = C_1 + C_2 \tag{5}$$

where $Q_1$ and $Q_2$ are charges passed through the junctions 1 and 2 respectively; $E_1$ and $E_2$

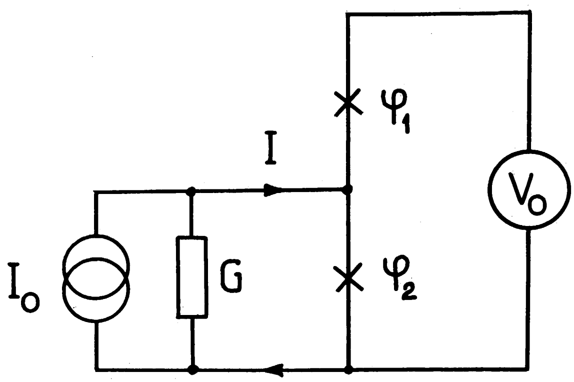


Fig. 1. The equivalent circuit of the system comprising the double junction, the dc voltage source, and the dc current source with internal conductance G.

(we will assume $E_1 \le E_2$), the Josephson coupling energy amplitudes; $\phi_1$ and $\phi_2$, the Josephson phase drops across the junctions, $H_x$, the Hamiltonian of the shunting conductance G which plays the role of environment, and $I_x$, the operator of current through the conductance G providing the environment-subsystem coupling via the variable $\phi_2$.

Earlier, it was shown [3] that in the particular case $V_o = 0$, $I_o = 0$ the supercurrent $I_s$ flowing through the double junction is a $2\pi$ -periodic function of the total Josephson phase difference $\phi_o = \phi_1 + \phi_2$ and a 2e-periodic function of the electric charge been injected into the middle electrode. It was also supposed that at $V_o \neq 0$, $I_o \neq 0$ the Josephson and Bloch oscillations will coexist and interact in the system. Here we will analyze this interaction quantitatively.

## 3. BASIC EQUATION

As the summary phase $\phi_o$ is directly controlled by $V_o$,

$$\phi_o(t) = (2e/\hbar)\int^t V_o dt' \equiv 2\pi f_J t + \phi_o(0) \tag{6}$$

this variable can be considered as a classical one. On the contrary, the combination $\phi=(\phi_2-\phi_1)/2$ should be considered as operator, so that $H_o$ can be presented as

$$H_o = \frac{Q^2}{2C_+} - E_J(t)\cos[\phi + \chi(t)] \tag{7}$$

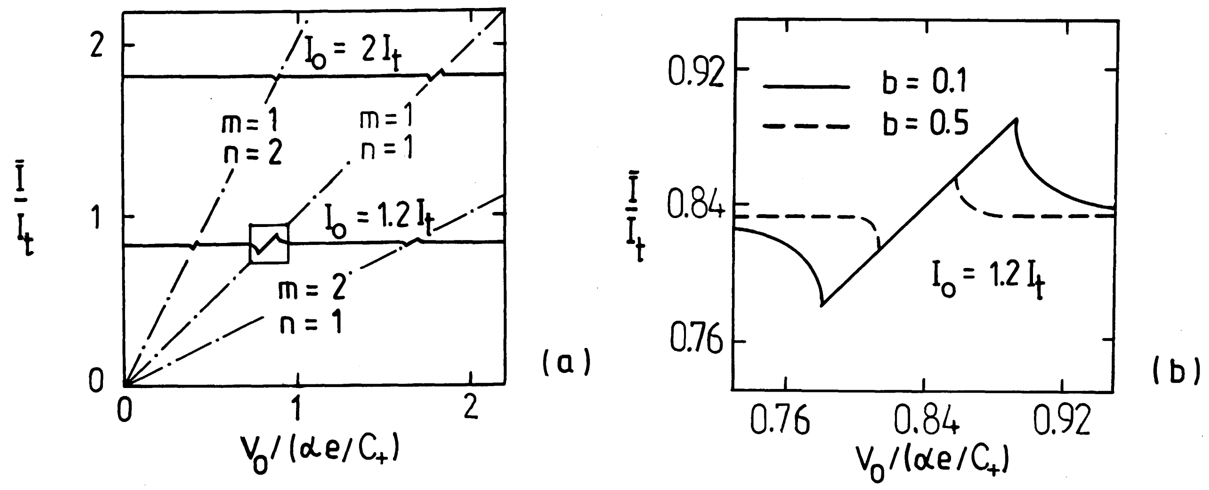


Fig. 2. The average current $\bar{I}$ flowing into the middle electrode as a function of $V_0$ for $\alpha = 0.1$ and two values of $I_0$; $I_t = eG/C_+$ is the threshold current 1) ; (a) general view for b = 0.1; (b) blowup of the main resistance step (m=n=1) vicinity for two values of b.

with

$$[\phi\ ,\ Q] = 2ei\ ,$$

where

$$E_J(t) = \left[E_1^2 + E_2^2 + 2E_1E_2\cos\phi_0(t)\right]^{1/2}, \qquad (8)$$

$$\chi(t) = \mathrm{Arctan}\,(b\tan\frac{\phi_0}{2}),\ b=(E_2-E_1)/(E_1+E_2). \qquad (9)$$

The Hamiltonian (7) is similar to that of a single Josephson junction. Its time-dependent eigenfunctions $\Psi_q^s(\phi,t)$ can be derived from the usual Bloch wave functions $\psi_q^s(\phi)$ [1] as

$$\Psi_q^s(\phi,t) = \psi_q^s[\phi-\chi(t)]\ \exp\left[-\frac{i}{\hbar}\int_0^t E^s(q,t')dt'\right], \qquad (10)$$

where q is the quasicharge; s, the energy band number; and $E^s(q,t)$, the s-band dispersion curve whose time dependence is provoked by time variation of $E_J$ (8). At low temperature and weak damping (1) the system is confined in the lower allowed energy band: s = 0. Proceeding just as in ref. 1 one can obtain in the limit $\alpha \equiv GR_Q << 1$ the basic equation for quasicharge q

$$\dot{q} = I\ , \quad \text{where}$$

$$I = I_0 - \tilde{I} - G\frac{\partial E^0(q,t)}{\partial q} + G\frac{\hbar}{2e}(\dot{\chi} - \pi f_J), \qquad (11)$$

where $\tilde{I}$ is the noise-current term obeying the Callen-Welton theorem. We see that Eq.(11) describes in the system both the Bloch oscillations with frequency $f_B = \bar{I}/2e$ and the Josephson oscillations with frequency $f_J = 2eV_0/h$.

## 4. RESULTS AND DISCUSSION

We have carried out numerical calculation of $\bar{I}$ using Eq.(11) in the limit $E_J << E_Q \equiv e^2/(2C)$. The result is presented in Fig.2 where $\bar{I}$ is ploted as a function of $V_0$. One can see a set of sloped steps pointing out a phase locking of the two types of oscillations which occur at $mf_B = nf_J$ , i.e.,

$$V_0/\bar{I} = (m/n)R_Q\ . \qquad (12)$$

Such quantization of a voltage/current ratio is similar to that taking place at the quantum Hall effect (see, e.g., ref.4 ) and can also be used in quantum metrology for development of the fundamental standard of resistance.

In our calculations we have neglected the noise term $\tilde{I}$ . In the limit (1) this noise would provide only weak rounding of the edges of the resistance steps, with an exponentially small influence upon the fundamental relation (12) at the middle of the steps.

To summarize, we have shown that the coexistence of the Bloch and Josephson oscillations and the resistance quantization suggested in ref. 3 should really take place in the system under analysis. Note, however, that according to Eq.(11) the resistance step magnitudes are relatively small ( $\Delta I/I_0 \sim \alpha$ ) within the limit $\alpha \to 0$ when this equation is valid. One needs to use more elaborate techniques (see, e.g., refs.5 and 6) to analyze the general case $\alpha \sim 1$.

## ACKNOWLEDGMENT

The authors are grateful to Dr. V. K. Semenov for helpful discussions.

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