TRIGONOMETRIC APPROXIMATION OF PERIODIC SIGNALS BELONGING TO GENERALIZED WEIGHTED LIPSCHITZ $W'(L_r, \xi(t)), (r \ge 1)$ — CLASS BY NÖRLUND-EULER $(N, p_n)(E, q)$ OPERATOR OF CONJUGATE SERIES OF ITS FOURIER SERIES

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Abstract. Approximation theory has been an established field of mathematics in the past century. Analysis of signals or time functions is of great importance, because it conveys information or attributes of some phenomenon. The engineers and scientists use properties of Fourier approximation for designing digital filters. In the present paper, an attempt is made to determine a theorem on the degree of approximation of a function \tilde{f} , conjugate to a 2π -periodic signal belonging to the generalized weighted Lipschitz $W'(L_r, \xi(t))$, $(r \ge 1)$ -class by product $(N, p_n)(E, q)$ summability, which in turn generalizes the results of Mishra et al. [17]. In support of our theorem, we illustrated examples and deduced some corollaries from our main result of this paper.

Mathematics subject classification (2010): 41A10, 42B05, 42B08, 40G05.

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