

## THE $k$ -RAINBOW BONDAGE NUMBER OF A DIGRAPH

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### Abstract

Let  $D = (V, A)$  be a finite and simple digraph. A  $k$ -rainbow dominating function ( $k$ RDF) of a digraph  $D$  is a function  $f$  from the vertex set  $V$  to the set of all subsets of the set  $\{1, 2, \dots, k\}$  such that for any vertex  $v \in V$  with  $f(v) = \emptyset$  the condition  $\bigcup_{u \in N^-(v)} f(u) = \{1, 2, \dots, k\}$  is fulfilled, where  $N^-(v)$  is the set of in-neighbors of  $v$ . The weight of a  $k$ RDF  $f$  is the value  $\omega(f) = \sum_{v \in V} |f(v)|$ . The  $k$ -rainbow domination number of a digraph  $D$ , denoted by  $\gamma_{rk}(D)$ , is the minimum weight of a  $k$ RDF of  $D$ . The  $k$ -rainbow bondage number  $b_{rk}(D)$  of a digraph  $D$  with maximum in-degree at least two, is the minimum cardinality of all sets  $A' \subseteq A$  for which  $\gamma_{rk}(D - A') > \gamma_{rk}(D)$ . In this paper, we establish some bounds for the  $k$ -rainbow bondage number and determine the  $k$ -rainbow bondage number of several classes of digraphs.

**Keywords:**  $k$ -rainbow dominating function,  $k$ -rainbow domination number,  $k$ -rainbow bondage number, digraph.

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