Symmetry in Science

Joe Rosen

Symmetry in Science

An Introduction to the General Theory

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For Mira

Preface

This book is a revision and expansion of my book *A Symmetry Primer for Scientists*, published by Wiley in 1983. I expanded and revised that book mainly because, since it was published, my further quest for symmetry comprehension has led to a deeper understanding of the concepts and ideas underlying and supporting the formal theory of symmetry and its manifestation and application in science that was the main import of *A Symmetry Primer for Scientists.* Furthermore, during that period some of my ideas about symmetry in science have changed somewhat. So the expansion of the previous book was primarily by the addition of three new chapters of conceptual considerations at the end of the book. That allows the reader to choose an "application" track, whereby he or she leaves the conceptual chapters for the end, or skips them altogether, or to choose a "concept" track, in which the conceptual chapters are read before any formalism or mathematics is approached. The revision involved appending a summary of the six symmetry principles derived in the book, adding a summary section for each chapter, updating the Bibliography, and making various and numerous modifications and additions throughout.

As was its predecessor, this book is intended to fill a huge gap in the symmetry literature: Until the appearance of *A Symmetry Primer for Scientists* there was, to the best of my knowledge, not a single textbook devoted to the fundamentals of symmetry and its application in science. And since the appearance of *A Symmetry Primer for Scientists* no other such textbook has been published, again to the best of my knowledge. Indeed, that is in glaring contradiction to the undisputed importance of symmetry considerations in modern science. It is true that wherever its importance is greatly appreciated there exist textbooks on the application of symmetry in those fields, especially in quantum theory, crystallography, and chemistry. But the fact that those textbooks are almost always couched in terms of group theory and its application, rather than symmetry and its application, is symptomatic of the underlying problem this book and its predecessor are intended to alleviate: Just as technical mastery of the Schrödinger equation does not imply commensurate understanding of the physics represented by that equation, neither does proficiency in the application of group theory in some field of science imply insight into the symmetry considerations of which that group theory is the formal expression.

In fact, all too many science students, science teachers, and even scientists are uncomfortable, to say the least, with symmetry considerations. Students tend to be suspicious of symmetry arguments. They all too rarely take advantage of them and then all too often do it wrong. And when the lecturer, textbook, or thesis advisor uses a symmetry argument in a proof or to cut swiftly through to the solution of a problem, students feel cheated; something was pulled out of a hat. Science teachers often do not have a clear understanding of symmetry considerations, so they tend to avoid them, except when a symmetry argument is very obvious and compelling. And even those teachers who do understand and appreciate symmetry considerations rarely convey to their students a feeling for, if not some understanding of, the principles of symmetry and their generality and importance. Then we have the scientists, too many of whom tend to be doubtful of symmetry considerations, especially their own, because they are uncertain about how to use them correctly. And those doubtful scientists might at the same time be very proficient at the group-theoretical techniques involved in the application of symmetry in their own fields.

So what is sorely needed is a basic textbook on symmetry and its application in science, starting from scratch and presenting the material in an orderly manner, with problems for the reader to solve and a bibliography. Such a book should be able to serve as the primary text for a course on symmetry, as a supplementary text for science courses in which symmetry considerations play an important role, and as a self-study text for scientists, science teachers, and advanced science students who want to fill in what they may have missed and increase their symmetry sophistication. Detailed specific applications need not be included, since they can be found in abundance elsewhere. As you might suspect, this book attempts to fill just that role.

As for starting from scratch, the conceptual presentation of symmetry, the formal presentation of symmetry, and the presentation of group theory (the mathematical language of symmetry) and of the principles of symmetry do indeed do so. However, for some of the additional material and for examples and problems throughout the book the reader is assumed to possess a certain background familiarity with various mathematical subjects and with various physical phenomena. For example, some familiarity with elementary algebra, complex numbers, geometric concepts, linear algebra, or ordinary differential equations is assumed at various points. And, especially for the material on quantum systems, a more than introductory understanding of quantum theory is required.

Thus, in keeping with its many roles, not all of the book is accessible to all readers. The undergraduate science student should be able to handle all the core material (see below) except the quantum material and some of the examples and problems. The advanced graduate students should find almost all the material, examples, and problems accessible, as should the science teacher and scientist. And any reader, upon returning to this book after gaining experience and insight in science, should find that more of it has become accessible and some of it has become clearer.

The book is structured as follows. Chapter 1 serves as a brief gateway to symmetry by introducing the notion of symmetry in its generality. Chapters 2 and 3 are an introduction to group theory, the mathematical language of symmetry. They present a reasonable dose of group theory and certain other mathematical ideas and supply the reader with the mathematical ideas and language necessary for the succeeding chapters, where a symmetry formalism is developed in group-theoretical language, the language most suitable for it. For Chapters 2 and 3 to serve as a useful introduction to group theory they go somewhat further than is strictly necessary for the purpose of this book.

Chapter 4 starts the development of a general symmetry formalism, which is continued in the following chapters. It presents a formalism applicable to all, not necessarily even physical, systems. A special section on symmetry in quantum systems is included. Chapter 5 teaches the theory of application of symmetry in science. It contains some discussion of a somewhat philosophical nature, but most of it is utilitarian, and numerous examples are included. Those chapters comprise the core of the "practical" part of the book. The remainder of the "practical" part consists of Chapters 6 and 7, which require a more sophisticated reader than does most of the core material, Chapter 7 more than Chapter 6. Those chapters can be considered somewhat "extracurricular." They are not necessarily less important, however, and the first four sections of Chapter 7, discussing symmetry of the laws of nature and of initial and final states, symmetry in processes, and conservation, definitely should be read by the sufficiently sophisticated reader.

The "conceptual" part of the book consists of Chapters 8–10, which are a conceptual discussion of symmetry and its manifestation and application in science. Chapter 8 discusses symmetry at its most general, while symmetry in science is considered in Chapters 9 and 10.

Numerous problems are included in this book, following almost every section of all chapters except 1 and 6. The reader should, of course, attempt to solve them in order to enhance the learning process and for self-testing. The problems of Chapters 2 and 3, a few of which extend the presentation, and the problems of Chapters 8–10 should all be accessible to whomever the chapters are accessible to. The problems of Chapters 4, 5, and 7, on the other hand, are not all accessible to every reader. Some of the problems purposely leave leeway for the reader's interpretation and thus do not have unique solutions.

A rather extensive bibliography is included, whose principal purpose is to offer sources for parallel, supplementary, complementary, and subsequent reading. Being very self-contained, this book rarely makes direct reference to the Bibliography. When it does, a reference is designated by square brackets, [...].

For the person desiring preparatory reading to this book the cupboard is almost bare. As a non-textbook introduction to symmetry I dare suggest my own *Symmetry Discovered: Concepts and Applications in Nature and Science* [S22]. It is the only one I know of at its level and, like the present book, was also intended to fill a gap in the symmetry literature. For a more advanced non-textbook introduction to symmetry I highly recommend Hermann Weyl's modern classic, *Symmetry* [S31].

A symmetry course based on this book might be structured as follows. If you, the instructor, prefer the "application" track, with the conceptual material relegated to the end or left out altogether, then start with Chapter 1 (What Is Symmetry?). Follow with Chapters 2 (The Mathematics of Symmetry: Group Theory) and 3 (Group Theory Continued), if the students need an introduction to group theory. For a thorough introduction use all of both chapters. For a brief introduction that is sufficient for the rest of the course the essential sections, are in Chapter 2: 2.1. The Group Concept; 2.2. Mapping; 2.3. Isomorphism; 2.4. Equivalence Relation; 2.6. Subgroup; and in Chapter 3: 3.5. Generators; 3.7. Permutations, Symmetric Groups; and the material on equivalence class in Section 3.1. If the students have already been sufficiently introduced to group theory, mapping, and equivalence relation, Chapters 2 and 3 may be skipped, unless a rapid review is desired. In no case should one attempt to teach the subsequent chapters to students who are not sufficiently familiar with group theory, mapping, and equivalence relation.

Then present Chapters 4 (Symmetry: The Formalism) and 5 (Application of Symmetry). The last section of each chapter is only for students with a good formal background in quantum theory. After that Chapters 6 (Approximate Symmetry and Spontaneous Symmetry Breaking) and 7 (Symmetry in Processes, Conservation, and Cosmic Considerations) are for your picking and choosing. For these chapters a rather advanced science background is expected of the students.

At this point in the "application" track, if time and interest allow, the conceptual foundations of symmetry may be considered by presenting Chapters 8 (Symmetry: The Concept), 9 (Symmetry in Science), and 10 (More Symmetry in Science). This material can and should be pared down according to your taste and interest.

If you, the instructor, choose the "concept" track rather than the "application" track, preferring to start with the conceptual foundations of symmetry and its application in science and only then develop the formalism for application, start with Chapter 1 (What is Symmetry?) and continue with Chapters 8 (Symmetry: The Concept), 9 (Symmetry in Science), and 10 (More Symmetry in Science), adapting this material to your taste, interest, and needs as you find appropriate. Then present Chapters 2–7 as described above for the "application" track.

The most important feature of this book, it seems to me, is the statement and rigorous derivation of six principles of symmetry: the equivalence principle (Section 5.2); the symmetry principle (Section 5.3); the equivalence principle for processes (Section 7.2); the symmetry principle for processes (Section 7.2); the general symmetry evolution principle (Section 7.2); and the special symmetry evolution principle (Section 7.3). I know that the equivalence principle, the symmetry principle, and the special symmetry evolution principle have been stated previously in one version or another [A17, A5, M48]. About the other three principles I do not know. In any case, I am sure this (and in *A Symmetry Primer for Scientists*, the present book's predecessor) is the first time all principles have been presented together, in their full generality, and within a coherent framework. And I am sure also that this is the first time the principles have been rigorously derived from a premise as fundamental as the very existence of science. The six principles are summarized in the Summary of Principles, following Chapter 10.

I would like to express my thanks to the Department of Physics of The Catholic University of America, and especially to Larry Fagg, Jim Brennan, and Jack Leibowitz, for a most fruitful visit during 1990–1993, when this book was written.

Bethesda Joe Rosen

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