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Reflection and Refraction of SH-Waves at a Corrugated Interface between Two-Dimensional Transversely Isotropic Half-Spaces

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A two-dimensional reflection/transmission problem for SH-waves at a corrugated interface between homogeneous transversely isotropic half-spaces is considered. Rayleigh's method is adopted and expressions for reflection and transmission coefficients are obtained in closed form for the first-order approximation of the corrugation. Numerical computations for a particular model have been performed.

1. Introduction

Theoretical study of the dynamic characteristics of seismic waves reflected/refracted by surfaces is of great practical importance. Keeping in view the fact that earthquake-generated seismic waves encounter mountain basins, mountain roots and salt and ore bodies in their paths, it is doubtless that such irregularities affect the reflection and transmission of elastic waves through the earth. Seismic prospecting is widely carried out in regions of complex geological structures, and the interfacial curvature must be taken into account for a more accurate interpretation of the results. In reality, the boundaries of the earth medium can never be perfectly flat and are always stochastically irregular to a certain extent. Our research studies the motivation of the situation if the Mohorovicic discontinuity is not a horizontal plane, but has an irregular shape. The model is based on this type of geophysically interesting situation. Since the body waves carry information about the internal structure of the earth, the study of reflection and refraction of elastic waves is therefore of great importance in seismology.

There are several methods to deal with the problems of wave scattering from a corrugated surface. Sato (1955) treated the problem of elastic wave scattering from a corrugated, traction-free surface of a semi-infinite elastic medium using Rayleigh's method. The same method was adopted by Asano (1960, 1961, 1966), who investigated the problems of reflection and transmission of body waves across a periodic interface between two dissimilar half-spaces. In Rayleigh's method, expressions in the boundary conditions containing the function defining the corrugated boundary are expanded in

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a Fourier series, and the unknown coefficients in the solutions are determined to any given order of approximation in terms of a small parameter characteristic of the boundary. The reflection of body waves from an arbitrary, not necessarily periodic, rough surface of a semi-infinite solid was discussed by Abubakar (1962a) and Dunkin and Eringen (1962) among others, each of who applied a perturbation method to arrive at the results. The perturbation method of approximation is based on the assumption that the curvature of the boundary surface is small everywhere. In this method, the amplitudes of superposed scattered plane waves is expressed into sums of terms whose orders of magnitude are proportional to the powers of the amplitude of the rough surface, which is assumed small. It is noted that the first few terms of the power series will give an approximate solution of the problem. Abubakar (1962b, c) devoted two papers to work out the problems of reflection and refraction of SH-waves at an irregular interface using the perturbation method. Gilbert and Knopoff (1960) used a perturbation scheme for investigating the problem of scattering of seismic waves by an irregular surface. Kuo and Nafe (1962) obtained the period equation for Rayleigh waves in a solid layer overlying a solid half-space separated by a sinusoidal interface. DeNoyer (1961) has treated the problem of Love-wave dispersion in a layered medium, in which the layer thickness varies as a sine function of distance. Aki and Larner (1970) discussed the surface motion of a layered medium having an irregular interface due to incident plane SH-waves. The Aki-Larner technique actually stems from Rayleigh's technique, in which the scattered wavefield is represented as a linear combination of plane waves with discrete horizontal wave numbers including inhomogeneous waves. The book by Ogilvy (1961) is worth noting regarding the subject of wave scattering from rough surfaces.

Musgrave (1960) treated the problem of reflection and refraction of plane elastic waves at a plane boundary between aelotropic media. Henneke (1972) studied the effect of anisotropy on the reflection and refraction of stress waves at a plane boundary in anisotropic media. Thapliyal (1974) studied the effect of anisotropy on the reflection of SH-waves from an anisotropic transition layer that lies between two isotropic half-spaces. Keith and Crampin (1977) derived the formulae for calculating the energy partition among waves generated by plane waves incident on a plane boundary between two generally anisotropic media. Saini and Singh (1977) investigated the problem of reflection and refraction of SH-waves at a plane interface between two homogeneous elastic media, one isotropic and the other transversely isotropic. Delay and Hron (1977) discussed the problem of reflection and refraction of elastic waves (P and SV) in transversely isotropic media. Hanyga (1980) discussed the outgoing waves and boundary value problem of anisotropic elasticity. Mandal and Mitchell (1986) obtained eigen solutions, using the recursive scheme of scattered operators and numerical wave number integration technique, in transversely isotropic medium. Rokhlin *et al.* (1986) and Mandal (1991) studied the reflection and transmission of elastic waves at a plane interface between two generally anisotropic media. Mandal and Toksoz (1990) developed an algorithm for generating complete waveforms in general anisotropic media and computed complete wavefields therein due to a point source. A report related to wave motion in anisotropic media has also been reviewed by Crampin (1981) and Pao (1983).

In the above investigations of corrugated interfaces, the elastic media considered were isotropic and homogeneous. Anisotropy is a well-established phenomenon within the earth and has been widely detected in field data, as well as in laboratory experiments (e.g., Babuska, 1981; Crampin *et al.*, 1984; Gaiser *et al.*, 1984). Transverse isotropy is the most important type of anisotropy encountered, in which elastic properties involved are the same in any direction perpendicular to an axis but are different parallel to this axis. We therefore consider the problem of reflection and refraction of SH-waves at a corrugated interface between two transversely isotropic, homogeneous elastic solids. Rayleigh's method is adopted and the expressions for the reflection and transmission coefficients are obtained for the first-order approximation of the corrugation.

2. Formulation of the Problem and Its Solution

We consider two homogeneous transversely isotropic elastic half-spaces, H_1 and H_2. The elastic constants, densities and velocities in H_i are given by M_i, N_i, ρ_i, and β_{V_i}, β_{h_i} $(i=1, 2)$, respectively. The anisotropy factor is denoted by N_i/M_i. The x-axis and y-axis are horizontal and z-axis is pointing vertically downward. The equation of the corrugated interface between two considered solid half-spaces is given by

$$Z=\zeta\,, \tag{1}$$

where ζ is assumed to be a periodic function of x and independent of y, the mean value of which is zero (Fig. 1). The representation of ζ in the form of Fourier series can be expressed as (Sato, 1955)

$$\zeta=\sum_{n=1}^{\infty}\left[\zeta_n e^{\imath npx}+\zeta_{-n}e^{-\imath npx}\right]. \tag{2}$$

On expanding the right side of Eq. (2) and introducing the notations

$$\zeta_1=\zeta_{-1}=\frac{c_1}{2}\,, \qquad \zeta_{\pm n}=\frac{(c_n\mp\imath S_n)}{2}\,,$$

we obtain,

$$\begin{aligned}\zeta=c_1\cos(px)+c_2\cos(2px)+s_2\sin(2px)+\cdots\\ +c_n\cos(npx)+s_n\sin(npx)+\cdots.\end{aligned} \tag{3}$$

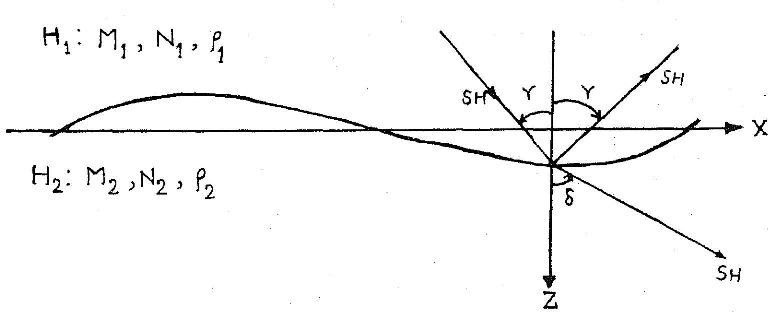

Fig. 1. Geometry of the problem.

In the special case when the interface is represented by one cosine term, that is

$$\zeta = c\cos(px)\,, \tag{4}$$

the wavelength of corrugation is $2\pi/p$.

The equation of motion for SH-waves in transversely isotropic homogeneous elastic medium can be written as (Sheriff and Geldart, 1995, p. 56)

$$\frac{\rho}{M}\frac{\partial^2 V}{\partial t^2} = \frac{N}{M}\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2}\,, \tag{5}$$

where M, N, and ρ are elastic properties of the medium and V denotes the y-component of the displacement. In order to solve Eq. (5), we use the method of separation of variables. Consider the time harmonic waves and let

$$V = X(x)Z(z)\,e^{\iota\omega t}\,, \tag{6}$$

where ω is the circular frequency. Insertion of Eq. (6) into Eq. (5) and separating the variables, we obtain

$$\frac{d^2X}{dx^2} + a^2X = 0\,, \tag{7}$$

$$\frac{d^2Z}{dz^2} - S^2Z = 0\,, \tag{8}$$

where

$$S^2 = \frac{N}{M}a^2 - \frac{\omega^2}{\beta_v^2}\,; \qquad \beta_v^2 = \frac{M}{\rho}\,, \tag{9}$$

and a is the x-component of the wave number given by (Gupta, 1965),

$$a = \frac{\omega\sin\theta}{\beta_h}\,; \qquad \beta_h^2 = \frac{N}{\rho}\,, \tag{10}$$

where θ is the angle between the wave normal and the z-axis.

Consider a plane SH-wave of unit amplitude and period $2\pi/\omega$ with an incident from the upper half-space. Let γ be the angle between the z-axis and the incident wave normal, and the direction of propagation wave being along the positive x-axis. If the boundary surface is on the plane, then it suffice to consider the only solutions of Eqs. (7) and (8), and hence solution (6) of Eq. (5) which gives

(i) for the incident of regularly reflected wave as

$$\{V^{\mathrm{inc}}, V^{\mathrm{refl}}\} = \{e^{-qz}, B_0\,e^{qz}\}e^{\iota\omega[t-(x\sin\gamma/\beta_{h_1})]}\,, \tag{11}$$

(ii) where the regularly refracted wave equals

$$V^{\mathrm{refr}} = D_0\,e^{-rz}\,e^{\iota\omega[t-(x\sin\delta/\beta_{h_2})]}\,, \tag{12}$$

$$q = \frac{\iota\omega}{\beta_{h_1}}\left(\frac{N_1}{M_1}\right)^{1/2}\cos\gamma\,, \tag{13}$$

and

$$r=\frac{\iota\omega}{\beta_{h_1}}\left[\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2}-\sin^2\gamma\right)\right]^{1/2}. \tag{14}$$

Moreover, δ is the angle made by the refracted wave normal with the z-axis in the medium H_2, with B_0 and D_0 as the reflection and transmission coefficients for a plane interface. Snell's law is given by

$$\frac{\omega\sin\gamma}{\beta_{h_1}}=\frac{\omega\sin\delta}{\beta_{h_2}}. \tag{15}$$

Since the interface is corrugated, in addition to the regular reflected and refracted waves, it is necessary to take the effect of corrugation on reflection and refraction of waves into consideration and introduce the following waves:

(i) irregularly reflected waves whose spectrum of the nth order is given by

$$V^{\text{ir-refl}}=B_n e^{q_n z} e^{\iota\omega[t-(x\sin\gamma_n/\beta_{h_1})]}+B_n' e^{q_n' z} e^{\iota\omega[t-(x\sin\gamma_n'/\beta_{h_1})]}, \tag{16}$$

(ii) irregularly refracted waves whose spectrum of the nth order is given by

$$V^{\text{ir-refr}}=D_n e^{-r_n z} e^{\iota\omega[t-(x\sin\delta_n/\beta_{h_2})]}+D_n' e^{-r_n' z} e^{\iota\omega[t-(x\sin\delta_n'/\beta_{h_2})]}, \tag{17}$$

where γ_n, γ_n', δ_n, and δ_h' are given by the following spectrum theories (Asano, 1960),

$$\sin\gamma_n-\sin\gamma=\frac{np\,\beta_{h_1}}{\omega} \quad\text{and}\quad \sin\gamma_n'-\sin\gamma=-\frac{np\,\beta_{h_1}}{\omega}, \tag{18}$$

and two more relations obtained by replacing γ, γ_n, γ_n', and β_{h_1} by δ, δ_n, δ_n' and β_{h_2} respectively, and

$$q_n=\frac{\iota\omega}{\beta_{h_1}}\sqrt{\frac{N_1}{M_1}}\cos\gamma_n, \quad\text{and}\quad r_n=\frac{\iota\omega}{\beta_{h_1}}\sqrt{\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2}-\sin^2\gamma_n\right)}, \tag{19}$$

where the values of q_n' and r_n' can be obtained by replacing γ_n by γ_n' in the above values of q_n and r_n, respectively. B_n, B_n', D_n, and D_n' are unknown constants.

The total displacement (V_1) in the upper medium is the sum of those incidents, of regularly reflected and irregularly reflected waves. Thus, the combination of Eqs. (11) and (16) give

$$V_1=(e^{-qz}+B_0 e^{qz})e^{\iota\omega[t-(x\sin\gamma/\beta_{h_1})]}+\sum_n B_n e^{q_n z} e^{\iota\omega[t-(x\sin\gamma_n/\beta_{h_1})]}$$
$$+\sum_n B_n' e^{q_n' z} e^{\iota\omega[t-(x\sin\gamma_n'/\beta_{h_1})]}. \tag{20}$$

Similarly, combining Eqs. (12) and (17) for the total displacement (V_2) in the lower medium, we have

$$V_2 = D_0 e^{-rz} e^{\iota\omega[t-(x\sin\delta/\beta_{h_2})]} + \sum_n D_n e^{-r_n z} e^{\iota\omega[t-(x\sin\delta_n/\beta_{h_2})]}$$
$$+ \sum_n D'_n e^{-r'_n z} e^{\iota\omega[t-(x\sin\delta'_n/\beta_{h_2})]} \,. \tag{21}$$

Using relation (18), Eqs. (20) and (21) can be written as

$$V_1 = \left[e^{-qz} + B_0 e^{qz} + \sum_n B_n e^{q_n z} e^{-\iota npx} + \sum_n B'_n e^{q'_n z} e^{\iota npx} \right] e^{\iota\omega[t-(x\sin\gamma/\beta_{h_1})]} \tag{22}$$

and

$$V_2 = \left[D_0 e^{-rz} + \sum_n D_n e^{-r_n z} e^{-\iota npx} + \sum_n D'_n e^{-r'_n z} e^{\iota npx} \right] e^{\iota\omega[t-(x\sin\gamma/\beta_{h_1})]} \,, \tag{23}$$

where in writing Eqs. (22) and (23), relation (15) is used.

3. Boundary Conditions

The displacement-stress continuity conditions to be satisfied at the boundary surface $z=\zeta$ are

$$V_1 = V_2 \tag{24}$$

and

$$M_1\left(\frac{\partial V_1}{\partial z} - \frac{\partial V_1}{\partial x}\zeta'\right) = M_2\left(\frac{\partial V_2}{\partial z} - \frac{\partial V_2}{\partial x}\zeta'\right), \tag{25}$$

where ζ' is the derivative of ζ with respect to x.

Insertion of Eqs. (22) and (23) in Eqs. (24) and (25) make use of the following substitutions,

$$\begin{aligned} q &= \iota Q\,, & r &= \iota R\,, \\ q_n &= \iota Q_n\,, & r_n &= \iota R_n\,, \\ q'_n &= \iota Q'_n\,, & r'_n &= \iota R'_n\,; \end{aligned} \tag{26}$$

where

$$Q = \frac{\omega}{\beta_{h_1}} \sqrt{\frac{N_1}{M_1}} \cos\gamma\,, \qquad R = \frac{\omega}{\beta_{h_1}} \sqrt{\left[\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)\right]}\,,$$
$$Q_n = \frac{\omega}{\beta_{h_1}} \sqrt{\frac{N_1}{M_1}} \cos\gamma_n\,, \qquad Q'_n = \frac{\omega}{\beta_{h_1}} \sqrt{\frac{N_1}{M_1}} \cos\gamma'_n\,, \tag{27}$$
$$R_n = \frac{\omega}{\beta_{h_1}} \sqrt{\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma_n\right)} \quad \text{and} \quad R'_n = \frac{\omega}{\beta_{h_1}} \sqrt{\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma'_n\right)}\,.$$

Therefore, we obtain

$$\begin{aligned}&e^{-\imath Q\zeta}+B_0 e^{\imath Q\zeta}+\sum_n B_n e^{\imath Q_n\zeta}e^{-\imath npx}+\sum_n B'_n e^{\imath Q'_n\zeta}e^{\imath npx}\\&=D_0 e^{-\imath R\zeta}+\sum_n D_n e^{-\imath R_n\zeta}e^{-\imath npx}+\sum_n D'_n e^{-\imath R'_n\zeta}e^{\imath npx}\end{aligned}\tag{28}$$

and

$$\begin{aligned}&M_1\left[\left(\frac{\omega\zeta'\sin\gamma}{\beta_{h_1}}-Q\right)e^{-\imath Q\zeta}+\left(\frac{\omega\zeta'\sin\gamma}{\beta_{h_1}}+Q\right)B_0 e^{\imath Q\zeta}+\sum_n B_n\left\{Q_n+\left(\frac{\omega\sin\gamma}{\beta_{h_1}}+np\right)\zeta'\right\}\right.\\&\left.\quad\times e^{\imath Q_n\zeta}e^{-\imath npx}+\sum_n B'_n\left\{Q'_n+\left(\frac{\omega\sin\gamma}{\beta_{h_1}}-np\right)\zeta'\right\}e^{\imath Q'_n\zeta}e^{\imath npx}\right]\\&=M_2\left[\left(\frac{\omega\zeta'\sin\gamma}{\beta_{h_1}}-R\right)D_0 e^{-\imath R\zeta}+\sum_n D_n\left\{\left(\frac{\omega\sin\gamma}{\beta_{h_1}}+np\right)\zeta'-R_n\right\}e^{-\imath R_n\zeta}e^{-\imath npx}\right.\\&\left.\quad+\sum_n D'_n\left\{\left(\frac{\omega\sin\gamma}{\beta_{h_1}}-np\right)\zeta'-R'_n\right\}e^{-\imath R'_n\zeta}e^{\imath npx}\right].\end{aligned}\tag{29}$$

4. Solution of the First Approximation

The corrugation of the surface $Z=\zeta$ is assumed to be very small, therefore, we have

$$e^{-\imath Q\zeta}=1-\imath Q\zeta\,,$$

and the terms of second and higher order in ζ have been neglected for the first approximation. The first approximation for B_0 and D_0 can be obtained by collecting the terms independent of x and ζ in Eqs. (28) and (29); therefore, we have

$$1+B_0=D_0\tag{30}$$

and

$$M_1 Q[-1+B_0]=-M_2 R D_0\,.\tag{31}$$

These formulae give the amplitudes of reflected and refracted waves at a plane boundary surface.

In order to obtain the solution for the first approximation for B_n and D_n, we collect the coefficients of $e^{-\imath npx}$ on both sides of Eqs. (28) and (29), thus obtaining

$$B_n-D_n=\imath(1-B_0)Q\zeta_{-n}-\imath D_0 R\zeta_{-n}\tag{32}$$

and

$$\begin{aligned}M_1 B_n Q_n+M_2 D_n R_n=\imath M_1\zeta_{-n}&\left[\frac{np\omega\sin\gamma}{\beta_{h_1}}-Q^2\right](1+B_0)\\&-\imath M_2\zeta_{-n}\left(\frac{np\omega\sin\gamma}{\beta_{h_1}}-R^2\right)D_0\,.\end{aligned}\tag{33}$$

Similarly, the solutions of the first approximation for B'_n and D'_n can be obtained by collecting the coefficients $e^{\iota npx}$ in Eqs. (28) and (29), giving

$$B_n' - D_n' = \iota(1-B_0)Q\zeta_n - \iota D_0 R\zeta_n \tag{34}$$

and

$$M_1 B_n' Q_n' + M_2 D_n' R_n' = -\iota M_1 \zeta_n \left[\frac{np\omega \sin\gamma}{\beta_{h_1}} + Q^2\right](1+B_0) + \iota M_2 D_0 \zeta_n \left(\frac{np\omega \sin\gamma}{\beta_{h_1}} + R^2\right); \tag{35}$$

where, in writing Eqs. (33) and (35), we have made use of the spectrum theorem given by relation (18). From Eqs. (32) to (35), the solution for the first approximation for B_n, D_n, B_n' and D_n' can be obtained, where B_0 and D_0 involved in these equations are to be found from Eqs. (30) and (31).

5. Special Case

We consider a special case when the boundary surface is given by Eq. (4); that is,

$$Z = c\cos px\,. \tag{36}$$

We have already obtained the results for the corrugated interface of the form $Z=\zeta$. In the considered case, we have

$$\zeta_n = \zeta_{-n} = 0 \quad (n \neq 1) \quad \text{and} \quad \zeta_1 = \zeta_{-1} = c/2\,. \tag{37}$$

From Eqs. (30) and (31), one can obtain the solution of the first approximation for B_0 and D_0 as

$$B_0 = \frac{\Delta_2}{\Delta_1}, \qquad D_0 = \frac{\Delta_3}{\Delta_1}, \tag{38}$$

where

$$\Delta_{1,2} = \frac{M_1}{M_2}\left\{\frac{N_1}{M_1}\right\}^{1/2} \cos\gamma \pm \left[\frac{N_2}{M_2}\left\{\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right\}\right]^{1/2}$$

and

$$\Delta_3 = 2\frac{M_1}{M_2}\left\{\frac{N_1}{M_1}\right\}^{1/2} \cos\gamma\,.$$

The formulae in Eq. (38) are the expressions for reflection and transmission coefficients when the SH-wave is incident at a plane interface between the transversely isotropic elastic solid half-spaces.

To obtain the solution of the first approximation for B_1, D_1, B_1', and D_1', which are of much interest, we solve Eqs. (32)–(35) and obtain the following values:

$$B_1=\frac{d_1}{d}\,,\qquad D_1=\frac{d_2}{d}\,,\qquad B_1'=\frac{d_1'}{d'}\,,\qquad D_1'=\frac{d_2'}{d'}\,, \tag{39}$$

where

$$d_1=\frac{\imath c}{2}\left[\frac{R_1\omega}{\beta_{h_1}}\left[(1-B_0)\left(\frac{N_1}{M_1}\right)^{1/2}\cos\gamma-D_0\left\{\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2}-\sin^2\gamma\right)\right\}^{1/2}\right]\right.$$
$$\left.+\frac{M_1}{M_2}(1+B_0)\left(\frac{p\omega\sin\gamma}{\beta_{h_1}}-Q^2\right)-D_0\left(\frac{p\omega\sin\gamma}{\beta_{h_1}}-R^2\right)\right],$$

$$d_2=\frac{\imath c}{2}\left[\frac{M_1}{M_2}(1+B_0)\left(\frac{p\omega\sin\gamma}{\beta_{h_1}}-Q^2\right)-D_0\left(\frac{p\omega\sin\gamma}{\beta_{h_1}}-R^2\right)\right.$$
$$\left.+\frac{M_1}{M_2}\frac{Q_1\omega}{\beta_{h_1}}\left[D_0\left\{\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2}-\sin^2\gamma\right)\right\}^{1/2}-(1-B_0)\left(\frac{N_1}{M_1}\right)^{1/2}\cos\gamma\right]\right],$$

$$d_1'=\frac{\imath c}{2}\left[\frac{R_1'\omega}{\beta_{h_1}}\left[(1-B_0)\left(\frac{N_1}{M_1}\right)^{1/2}\cos\gamma-D_0\left\{\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2}-\sin^2\gamma\right)\right\}^{1/2}\right]\right.$$
$$\left.-\frac{M_1}{M_2}(1+B_0)\left(\frac{p\omega\sin\gamma}{\beta_{h_1}}+Q^2\right)+D_0\left(\frac{p\omega\sin\gamma}{\beta_{h_1}}+R^2\right)\right],$$

$$d_2'=\frac{\imath c}{2}\left[-\frac{M_1}{M_2}(1+B_0)\left(\frac{p\omega\sin\gamma}{\beta_{h_1}}+Q^2\right)+D_0\left(\frac{p\omega\sin\gamma}{\beta_{h_1}}+R^2\right)\right.$$
$$\left.-\frac{M_1}{M_2}\frac{Q_1'\omega}{\beta_{h_1}}\left[(1-B_0)\left(\frac{N_1}{M_1}\right)^{1/2}\cos\gamma-D_0\left\{\frac{N_2}{M_2}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2}-\sin^2\gamma\right)\right\}^{1/2}\right]\right],$$

$$d=\frac{M_1}{M_2}Q_1+R_1$$

and

$$d'=\frac{M_1}{M_2}Q_1'+R_1'\,.$$

If the special case of normal incidence is considered then

$$\gamma=\delta=0\,,\qquad \cos\gamma_1=\cos\gamma_1'\quad\text{and}\quad\cos\delta_1=\cos\delta_1'\,.$$

It is easy to varify, from Eqs. (32)–(35) that $B_1=B_1'$ and $D_1=D_1'$.

In the case that anisotropy vanishes, we have

$$\begin{aligned}N_1&=M_1=\mu_1\,, & N_2&=M_2=\mu_2\,,\\ \beta_{h_1}&=\beta_{v_1}=\beta_1\,, & \beta_{h_2}&=\beta_{v_2}=\beta_2\,,\end{aligned} \tag{40}$$

and Snell's law, given by Eq. (15), reduces to

$$\sqrt{\sigma_1}\,h_1\sin\gamma=\sqrt{\sigma_2}\,h_2\sin\delta\,; \tag{41}$$

where the notations used are

$$\sigma_i=(\lambda_i+2\mu_i)/\mu_i\,,\qquad h_i^2=\rho_i\omega^2/(\lambda_i+2\mu_i)\,,\tag{42}$$

so that

$$\omega\beta_i^{-1}=\sqrt{\sigma_i}\,h_i\,.$$

With the help of relations (40) to (42), Eqs. (30) and (31) reduce to

$$B_0=\frac{\mu_1\sqrt{\sigma_1}\,h_1\cos\gamma-\mu_2\sqrt{\sigma_2}\,h_2\cos\delta}{\mu_1\sqrt{\sigma_1}\,h_1\cos\gamma+\mu_2\sqrt{\sigma_2}\,h_2\cos\delta}\tag{43}$$

and

$$D_0=\frac{2\mu_1\sqrt{\sigma_1}\,h_1\cos\gamma}{\mu_1\sqrt{\sigma_1}\,h_1\cos\gamma+\mu_2\sqrt{\sigma_2}\,h_2\cos\delta}\,,\tag{44}$$

and Eqs. (32) to (35) reduce to

$$B_n-D_n=\imath(1-B_0)\sqrt{\sigma_1}\,h_1\zeta_{-n}\cos\gamma-\imath D_0\sqrt{\sigma_2}\,h_2\zeta_{-n}\cos\delta\,,\tag{45}$$

$$\begin{aligned}\mu_1B_n\sqrt{\sigma_1}\,h_1\cos\gamma_n+\mu_2D_n\sqrt{\sigma_2}\,h_2\cos\delta_n&\\=\imath\mu_1\sqrt{\sigma_1}\,h_1\zeta_{-n}(np\sin\gamma-\sqrt{\sigma_1}\,h_1\cos^2\gamma)(1+B_0)&\\+\imath\mu_2\zeta_{-n}D_0(\sigma_2h_2^2\cos^2\delta-\sqrt{\sigma_1}\,h_1np\sin\gamma)\,,&\end{aligned}\tag{46}$$

$$B_n'-D_n'=\imath(1-B_0)\sqrt{\sigma_1}\,h_1\zeta_n\cos\gamma-\imath D_0\sqrt{\sigma_2}\,h_2\zeta_n\cos\delta\tag{47}$$

and

$$\begin{aligned}\mu_1\sqrt{\sigma_1}\,h_1B_n'\cos\gamma_n'+\mu_2D_n'\sqrt{\sigma_2}\,h_2\cos\delta_n'&\\=-\imath\mu_1\sqrt{\sigma_1}\,h_1\zeta_n(\sqrt{\sigma_1}\,h_1\cos^2\gamma+np\sin\gamma)(1+B_0)&\\+\imath\mu_2\zeta_nD_0(\sqrt{\sigma_1}\,h_1\,np\sin\gamma+\sigma_2h_2^2\cos^2\delta)\,.&\end{aligned}\tag{48}$$

Keeping in view the geometry of the problem given by Asano (1960) and making changes accordingly, we see that Eqs. (43) to (48) are the same as obtained by Asano (1960) in his problem of reflection and refraction of SH-waves incident at a corrugated interface between two elastic half-spaces.

To replace the corrugated interface by a plane interface, we put $\zeta=0$ in Eqs. (30) to (35), and removing the anisotropy with the help of Eq. (43), the problem reduces to the problem of SH-wave incident at plane interface $z=0$ between two homogeneous, isotropic elastic half-spaces. In this case B_n and D_n and their dashes become zero since they are proportional to ζ, and we obtain from Eq. (38)

$$B_0=\frac{m\cos\gamma-\sqrt{n^2-\sin^2\gamma}}{m\cos\gamma+\sqrt{n^2-\sin^2\gamma}}\qquad(n>\sin\gamma)$$

and

$$D_0 = \frac{2m\cos\gamma}{m\cos\gamma + \sqrt{n^2 - \sin^2\gamma}} \qquad (n > \sin\gamma)\,,$$

where

$$m = \frac{\mu_1}{\mu_2} \qquad \text{and} \qquad n = \frac{\beta_1}{\beta_2}\,.$$

These expressions are the same as given in Savarensky (1975, p. 284) for the relevant problem.

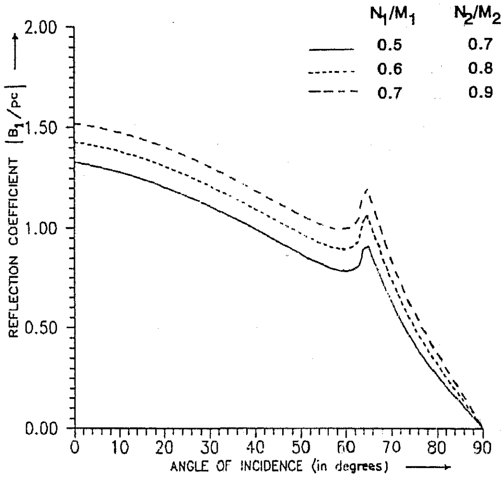

Fig. 2. Variation of modulus of B_1/pc with angle of incidence.

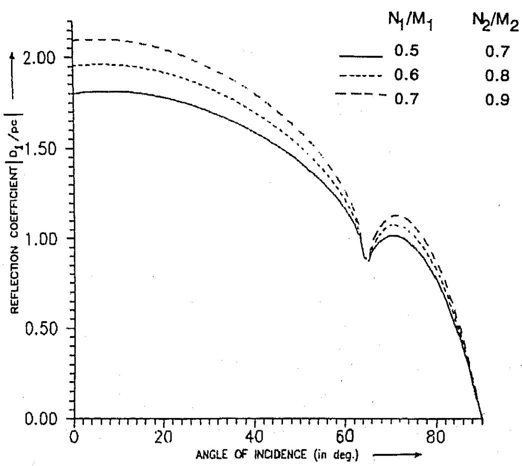

Fig. 3. Variation of modulus of D_1/pc with angle of incidence.

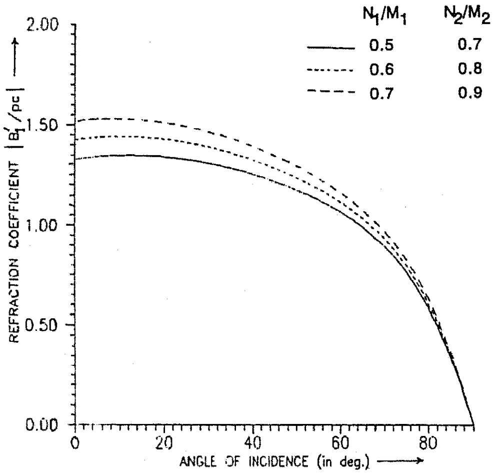

Fig. 4. Variation of modulus of B_1'/pc with angle of incidence.

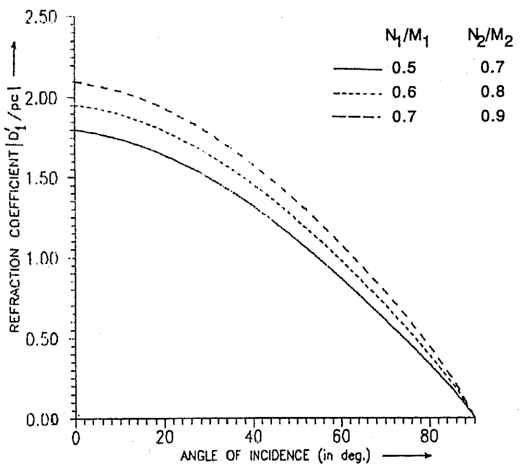

Fig. 5. Variation of modulus of D_1'/pc with angle of incidence.

If we put $\zeta=0$ (i.e., the effect of corrugation vanishes) in Eqs. (30) to (35), then we are left with reflection and transmission coefficients B_0 and D_0 given by Eq. (38), at a plane interface between two transversely isotropic half-spaces in welded contact. If we remove the anisotropy of half-space H_1, then putting $N_1=M_1=\mu_1$ so that $\beta_{h_1}=\beta_{v_1}=\beta_1$ in the formulae given in Eq. (38), we can obtain the same expression for reflection and transmission coefficients as obtained by Saini and Singh (1977) for the problem of reflection and transmission of SH-waves striking at the interface between two homogeneous media; one isotropic and the other transversly isotropic.

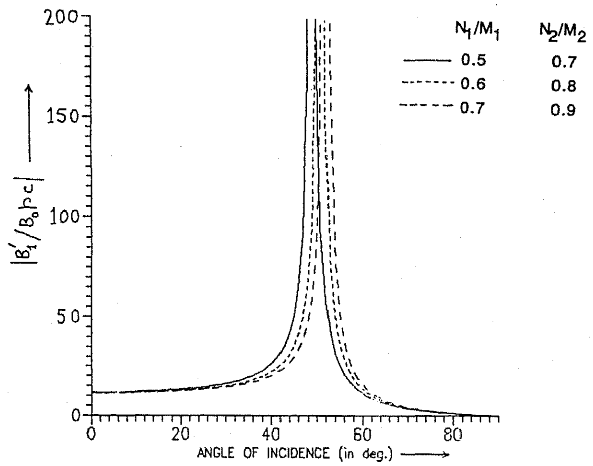

Fig. 6. Variation of modulus of B_1'/B_0pc with angle of incidence.

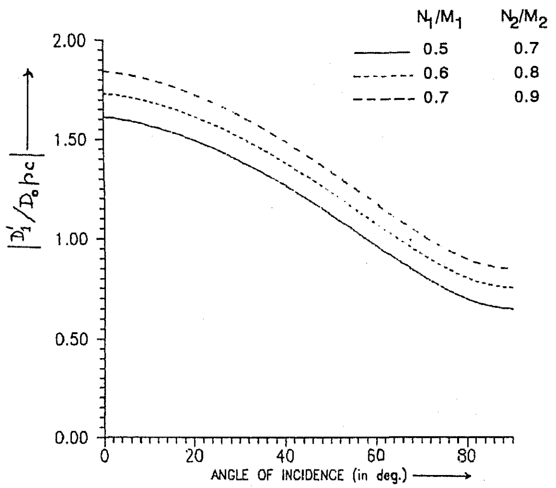

Fig. 7. Variation of modulus of D_1'/D_0pc with angle of incidence.

$$\frac{N_1}{M_1}=0.5,\ 0.6,\ 0.7\,, \qquad \frac{N_2}{M_2}=0.7,\ 0.8,\ 0.9\,,$$

$$\frac{M_1}{M_2}=2.0\,, \qquad \text{and} \qquad \frac{\beta_{h_1}}{\beta_{h_2}}=1.34\,.$$

6. Numerical Results and Discussion

To study the problem in greater detail, numerical calculations were performed for a specific model. The following values of relevant parameters were used:

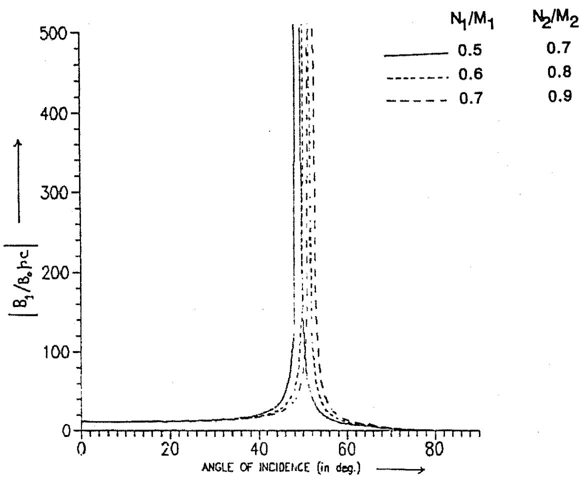

Fig. 8. Variation of modulus of B_1/B_0pc with angle of incidence.

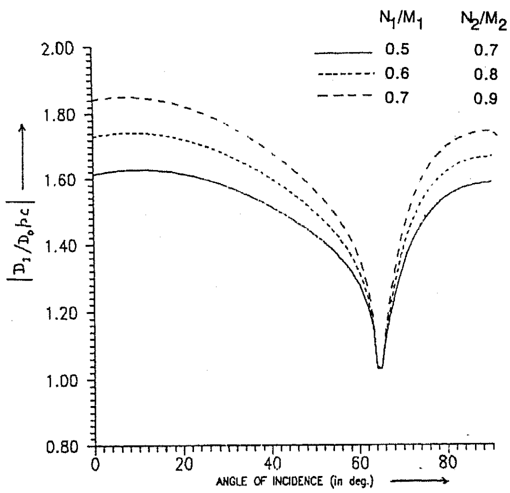

Fig. 9. Variation of modulus of D_1/D_0pc with angle of incidence.

Figures 2 to 5 show the variation of various reflection and refraction coefficients with angle of incidence γ, where $\omega/p\beta_{h_1}=10$. It can be seen from these figures that anisotropy has a significant effect on reflection/transmission coefficients, and the values of these coefficients increase with the increase in anisotropy factor N_i/M_i of the media. It is observed from Fig. 3 that the anisotropy has a negligible effect on refraction coefficient D_1/pc at a 62-deg angle of incidence. It can be seen from these figures that all the coefficients for the first-order approximation of the corrugation are zero at $\gamma=90$-deg angle of incidence. This shows that at grazing incidence (i.e., at $\gamma=90$ deg), there is no effect of corrugation of the interface and the reflection/refraction phenomenon of SH-waves take place similar to that at the plane interface. In this case, we obtain from Eq. (38) that $B_0=-1$ and $D_0=0$. Figures 6–9 show the variation of ratios of reflection and transmission coefficients with angle of incidence γ, when $\omega/p\beta_{h_1}=10$. It is observed that at the angle $\gamma=0$, $B_1/pc=B_1'/pc$ and $D_1/pc=D_1'/pc$, implying that $B_1=B_1'$ and $D_1=D_1'$ as already proved theoretically.

In conclusion, a mathematical study of the problem of reflection and refraction of SH-waves at a corrugated interface between two transversely isotropic media has been considered. We note that the solutions of the first-order approximation for B_n, D_n, B_n' and D_n', given by formulae (32) to (35), are proportional to ζ_n or ζ_{-n}. Hence, they depend upon the amplitude of corrugated interface. It was found that the values of reflection and transmission coefficients for the first-order approximation of corrugation increase, in general, with an increase in anisotropy of the media. However, for a corrugated interface, the major effect of anisotropy factor occurs at the $\gamma=0$-deg angle of incidence. If anisotropy in the media disappears, the results of Asano (1960) are obtained. The results of the problem of SH-wave incident at a plane interface between two homogeneous, isotropic elastic half-spaces have been obtained as a special case of our problem.

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