A study on the turbulent transport of an advective nature in the fluid plasma

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Advective nature of the electrostatic turbulent flux of plasma energy is studied numerically in a nearly adiabatic state. Such a state is represented by the Hasegawa-Mima equation that is driven by a noise that may model the destabilization due to the phase mismatch of the plasma density and the electric potential. The noise is assumed to be Gaussian and not to be invariant under reflection along a direction \hat{s} . It is found that the flux density induced by such noise is anisotropic: While it is random along \hat{s} , it is not along the perpendicular direction \hat{s}_{\perp} and the flux is not diffusive. The renormalized response may be approximated as advective with the velocity being proportional to $(k\rho_s)^2$ in the Fourier space \vec{k} .

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I. INTRODUCTION

Hasegawa-Mima (HM) model of the electrostatic fluctuations in the turbulent plasmas is a minimal foundation for the dynamics of the nonlinear interactions of drift waves in the adiabatic state [1]. The emergence and the subsequent role of the zonal flow is just one of many areas of the complicated plasma transport where the HM dynamics successfully provide pedagogical explanations [2, 3]. For large scale fluctuations, however, the plasma is not believed to be adiabatic. The so-called Terry-Horton (TH) equation supplements the HM model by adding ad hoc non-adiabatic electron response as a way of making the perturbations linearly unstable due to inverse Landau damping [4]. With such linear growth and artificial damping one may study the saturated states resulting from the dual cascades of the energy and the enstrophy. The HM model may be extended to include the collisional aspect of the drift waves by not neglecting the electron motion along the magnetic field line so that linearly unstable modes are present self-consistently and that the energy is transferred to locally adjacent scale and is eventually dissipated at small scale [5, 6]. Such collisional drift wave (CDW) description of the plasma may be shown to reduce to the HM equation in the adiabatic limit. For a quantitative analysis of the CDW model numerical simulation is necessary. However, the fluctuations of the CDW model that are close to the adiabatic state are very slow to reach a saturation numerically.

Instead of dealing with the self-consistent CDW model in assessing the non-adiabatic effect on the transport one may choose to approach conceptually similarly to the TH equation but more realistically by introducing a simulated noise that represents energy pumping of the linearly unstable CDW's. As CDW's are not isotropic, such noise needs to reflect the dependency on the directions. Specifically we assume that the noise is Gaussian with the spectrum of power-law type that is widely used in the renormalization-group (RG) approach to turbulence [7, 8]. The noise is assumed to be short-correlated in time compared with typical time scale of the plasma fluctuations as well as reflection non-invariant or parity non-conserving (PNC) along a direction \hat{s} as a symmetry breaking element. It has been shown through RG calculations that such PNC noise leads to the renormalized plasma response of advective nature. This work is, thus, an extension of Ref. [8] by including numerical aspect to the analysis. The present work finds out that the induced energy flux can be, indeed, advective. The direction, however, is not along \hat{s} , but along the perpendicular direction to \hat{s} . In Sec. II the model and the results of the numerical simulation are described and we conclude in Sec. III.

II. SIMULATION MODEL AND RESULTS

The forced HME for the electrostatic field φ in the Fourier space is

$$\partial_t \left(1+k^2\right)\varphi_k + ik_y\varphi_k - \frac{1}{2}\sum_{\vec{k}=\vec{p}+\vec{q}} M_{\vec{k},\vec{p},\vec{q}}\varphi_p\varphi_q + \nu k^4\varphi_k = f_k,\tag{1}$$

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FIG. 1: Real parts of typical turbulent energy flux R(k) with $\alpha = 0.5$ is shown for two cases of uv pumping, $\delta = 0.5$ and $\delta = 1.5$ with the PNC direction along \hat{y} .

where the space scale is normalized to ρ_s and the time unit is ρ_s/v_d with the electron diamagnetic drift speed v_d , $M_{\vec{k},\vec{p},\vec{q}} = (\hat{z} \cdot \vec{p} \times \vec{q}) \left(p^2 - q^2\right)$ is the coupling between three modes, and ν is the viscosity. The Gaussian random forcing $f_k(t)$ is delta-correlated in time and the power spectrum P of the noise in the d-dimensional Fourier space

$$P(\vec{k}) = Dk^{6-d-2\delta} \left(1 + i\alpha \hat{s} \cdot \hat{k} \right).$$
⁽²⁾

As the sum of P over all \vec{k} is proportional to the input rate of the energy, $W = (1/2) \sum_{\vec{k}} (1+k^2) |\varphi_k|^2$, the power-law form of P, with a proper exponent δ , may be thought of as an approximation to a part of localized realistic power spectrum. The coefficient α in Eq. (2) measure the strength of the PNC noise.

Eq. (1) is numerically integrated in time by advancing a time step Δt with Runge-Kutta 2nd-order scheme starting from the zero initial conditions. If Δt is taken to be shorter than the auto-correlation time τ of the noise, the $\delta(t-t')$ part of the auto-correlation function of the noise is approximated as $1/\Delta t$. The noise is numerically generated by using two variables θ and ξ [9, 10]. They are uniformly distributed over $(0, 2\pi)$ such that

$$\left\langle \theta_{\vec{k}}(t)\theta_{\vec{k}'}(t+\Delta t)\right\rangle = \left\langle \xi_{\vec{k}}(t)\xi_{\vec{k}'}(t+\Delta t)\right\rangle = 0 \tag{3}$$

and between different \vec{k} 's they are independent of each other. However, $\theta_{\vec{k}}(t)$ and $\xi_{\vec{k}}(t + \Delta t)$ are assumed to be correlated according to Eq. (2). The noise is, then, expressed as

$$f_{\vec{k}}(t) = a_{\vec{k}}e^{i\theta_{\vec{k}}(t)} + ib(\vec{k})e^{i\xi_{\vec{k}}(t)}$$
(4)

and between two successive time steps the auto-correlation becomes

$$\left\langle f_{\vec{k}}(t)f_{\vec{k}'}^{*}(t+\Delta t) \right\rangle = \left\{ |a_{\vec{k}}|^{2} + |b_{\vec{k}}|^{2} - ia_{\vec{k}}b_{\vec{k}}^{*} \left\langle e^{i[\theta_{\vec{k}}(t) - \xi_{\vec{k}}(t+\Delta t)]} \right\rangle + ia_{\vec{k}}^{*}b_{\vec{k}} \left\langle e^{i[\xi_{\vec{k}}(t) - \theta_{\vec{k}}(t+\Delta t)]} \right\rangle \right\} \delta_{\vec{k}\vec{k}'}$$

$$(5)$$



FIG. 2: Imaginary parts of the enstrophy turbulent flux $\langle Q \rangle_{\rm rh}$ in the right half plane $k_x > 0$ are plotted versus k for the cases of $\alpha = 0, \pm 0.3, \pm 0.5$ with $\delta = 0.5$ and $\hat{s} = \hat{y}$.

It is convenient to choose $\theta_{\vec{k}}(t) = \xi_{\vec{k}}(t + \Delta t)$. Then, $\langle e^{i[\theta_{\vec{k}}(t) - \xi_{\vec{k}}(t + \Delta t)]} \rangle = 1$ and $\langle e^{i[\xi_{\vec{k}}(t) - \theta_{\vec{k}}(t + \Delta t)]} \rangle = 0$. Thus, from Eqs. (2) and (5), $a_{\vec{k}}$ and $b_{\vec{k}}$ are determined by

$$a_{\vec{k}}^2 + b_{\vec{k}}^2 - ia_{\vec{k}}b_{\vec{k}} = Dk^{6-d-\delta}(\Delta t)^{-1}(1 + i\alpha\hat{k}\cdot\hat{s}).$$
(6)

A case of uv forcing is simulated where the noise is applied in the scale that is smaller than ρ_s^{-1} , that is $k_f < 1$ in our unit of k being equal to ρ_s^{-1} . We set $L = 2\pi \times 1024$, N = 4096 and consider a uv forcing of $\delta = 0.5$ with amplitude $f_0 = \sqrt{D} = 0.5$ in the range $1/1024 \le k \le 960/1024$ with $\hat{s} = \hat{y}$. Additional parameters are the time step $\Delta t = 5 \times 10^{-4}$ and the viscosity $\nu = 10^{-4}$. Hyper-viscose damping term of the form $4.4 \times 10^2 k^{16} \varphi_k$ is introduced in Eq. (1) that localizes uv forcing. The Eckman-like friction term $0.005\varphi_k$ is added in Eq. (1) that, by providing a necessary damping, secures the saturation of the energy transfer toward smaller k [11].

Figs. 1 and 2 are collections of plots regarding the turbulent flux. In Fig. 1, the typical energy flux expressed in terms of $k = |\vec{k}|$, $R(k) = \frac{1}{2} \sum_{k'=0}^{k} \sum_{\vec{k'}=\vec{p}+\vec{q}} M_{\vec{k'},\vec{p},\vec{q}} \varphi_{-k'} \varphi_{p} \varphi_{q}$ with $\delta = 0.5$ and $\alpha = 0.5$ is shown. The negative sign of R

means that the energy flows toward smaller k. Roughly in the region $0.04 \le k \le 0.4$ inertial range of constant energy flux is seen to form. For a comparison, the flux of another uv forcing, $\delta = 1.5$, is also plotted where the inertial range is not as vivid as the former case. It is checked that the shape of R(k) is almost independent of the measure of the PNC noise α as expected. The effects of the PNC noise α in the flux are observed in Fig. 2 where the imaginary part of the turbulent enstrophy flux is plotted. The enstrophy flux density Q is obtained upon multiplying both sides of Eq. (1) by ω_{-k} . If it is summed over all \vec{k} 's, its imaginary part cancels out. Therefore, In order to elucidate the role of the imaginary part of the flux, partial sums are taken over a semi-annular disks of small width δk and shown in Fig. 2: $Q_{rh}(k)$ in the right-half plane, $k_x \ge 0$, and $Q_{uh}(k)$ in the upper-half plane, $k_y \ge 0$. The flux $Q_{rh}(k)$ is defined as

$$Q_{rh}(k) = \Im_{\frac{1}{2}} \sum_{k'=(k-\delta k)}^{k} \sum_{k'_{x}\geq 0} M_{\vec{k}',\vec{p},\vec{q}} {k'}^{2} \varphi_{-k'} \varphi_{p} \varphi_{q}$$

$$\tag{7}$$



FIG. 3: Imaginary parts of the enstrophy turbulent flux $\langle Q \rangle_{\rm uh}$ in the upper half plane $k_y > 0$ are plotted versus k for the cases of $\alpha = 0$ and ± 0.5 with $\delta = 0.5$ and $\hat{s} = \hat{y}$.

and $Q_{uh}(k)$ correspondingly by the substitution of $\sum_{k'_x \ge 0}$ with $\sum_{k'_y \ge 0}$. It is implied in Eq. (7) that $\vec{k'} = \vec{p} + \vec{q}$ in the sum.

Both $Q_{rh}(k)$ and $Q_{uh}(k)$ are the enstrophy fluxes over the corresponding half annulus of width δk , which is 1/1024 in this case, in the (k_x, k_y) plane. Shown in Fig. 2 are the mean values of $\langle Q \rangle_{\rm rh}$ in terms of k for the cases of $\alpha = 0, \pm 0.3$ and ± 0.5 being averaged over 200 sampled data that are taken at every $100\Delta t$. Since for the isotropic forcing ($\alpha = 0$) $\langle Q \rangle_{\rm rh}$ is negligibly zero, $\langle Q \rangle_{\rm rh}$ with $\alpha \neq 0$ is purely induced by the PNC noise. The size of $\langle Q \rangle_{\rm rh}$ is proportional to α and it changes sign correspondingly to the sign of α .

The fluxes in the upper-half plane, $k_y \ge 0$, $\langle Q \rangle_{uh}$ are plotted in Fig. 3 for $\alpha = 0$ and ± 0.5 . Upon comparison with Fig. 2, it is observed that, for both the isotropic ($\alpha = 0$) and the PNC forcing irrespective of the sign of α , $\langle Q \rangle_{uh}$ seems to be little different from one another. It is clear that such anisotropic nature of $\langle Q \rangle_{uh}$ is the result of the nonlinear interactions between the drift waves and the PNC noise is not relevant to $\langle Q \rangle_{uh}$ [12]. Since the Hasegawa-Mima dynamics has an intrinsic direction of \hat{y} due to the gradient of the plasma density along \hat{x} , it is understandable for isotropic forcing that Q has odd parity under the reflection along \hat{y} leading $\langle Q \rangle_{rh}$ to vanish while leaving $\langle Q \rangle_{uh}$ finite. It may be concluded that the turbulent flux induced by the PNC noise is predominantly along the perpendicular direction to \hat{s} , that is \hat{x} in the present setup.

Since $\langle Q \rangle$ in the left-hand plane $\langle Q \rangle_{\rm lh}$ is equal to $-\langle Q \rangle_{\rm rh}$, $\langle Q \rangle$ suggests advective nature along the direction of \hat{x} , perpendicular to the PNC direction. Upon anticipating $\langle Q \rangle_{\rm rh}$ to be advective with the velocity $\hat{x}U$, an advective flux $U \langle k_x \omega^2 \rangle_{\rm rh}$, where ω_k is the vorticity $-k^2 \varphi_k$, in the right-half plane is laid over $\langle Q \rangle_{\rm rh}$ in Fig. 4. Scaling the advecting velocity U with k^2 seems to fit $\langle Q \rangle_{\rm rh}$ to the advective flux. It is determined that, for $\alpha = -0.5$, $U = -40k^2$ and, for $\alpha = 0.3$, $U = 24k^2$. One notes that U is proportional to α and that the dependence of U on k^2 agrees with the prediction of the RG and the strongly magnetized MHD plasmas.

In order to check the validity of the k^2 dependence of the advecting velocity another case of uv forcing is considered where $k_f > 1$. The box size is $L = 2\pi \times 32$, N = 1024, the hyper-viscosity is set to be 10^{-12} with the uv forcing of $\delta = 0.5$ in the region $1/32 \le k \le 240/32$ and $f_0 = 3.6 \times 10^{-2}$. Similar to the former case of $k_f < 1$ an inertial range is found to exist and the dependence of $\langle Q \rangle$'s on α is verified. In Fig. 5 $\langle Q \rangle_{\rm rh}$ for the cases of $\alpha = -0.5$ and 0.3 are overlaid with $Uk_x \langle |\omega_k|^2 \rangle$ where the advecting velocity is $U = -0.025k^2$ and $U = 0.015k^2$, respectively. One may, thus, conclude that an effect of the PNC noise (2) on the convective nonlinear term in Eq. (1) may result in the



FIG. 4: Flux density $\langle Q \rangle_{\rm rh}$ in the right-half plane is compared with the advective flux density $U \langle k_x \omega^2 \rangle_{\rm rh}$ where U is advecting velocity along \hat{x} . U is found to be proportional to k^2 .

advective response. With the PNC direction along y, regarding the transport of the second-order moment like the energy and the enstrophy, one may approximate Eq. (1) as

$$\partial_t \left(1+k^2\right) \varphi_k + i \left(k_y + U k_x k^2\right) \varphi_k + \nu k^4 \varphi_k \approx f_{\rm PC},\tag{8}$$

where $U \propto k^2$ and $f_{\rm PC}$ is the parity-conserving part of the noise.

III. CONCLUSION

Motivated by observations that the fluctuations in the magnetic fusion devices are predominantly anisotropic as attested by the presence of various types of the drift waves, we set out to pedagogically study the advective characteristics of the turbulent transport. In this paper we simulate the dynamics of the electrostatic fluctuations in a magnetized plasma modeled by Hasegawa-Mima equation that is driven by an anisotropic noise. The noise is not reflection-invariant along the direction \hat{s} . Such noise is simulated numerically by considering, for each mode, two random variables that have time-delayed correlation to each other for a short time in a time-ordered fashion.

To be specific the electrostatic fluctuations are considered to be driven by the noise in the uv scale so that the plasma energy cascades toward smaller scale. In the region of the Fourier space \vec{k} where the energy flux is nearly constant owing to the uv noise power it is found that the imaginary part of the turbulent flux $\langle Q \rangle$ is advective. With the case of $\hat{s} = \hat{y}$, the advective flux $\langle Q \rangle_{uh}$ along the direction \hat{y} is arguably due to the strong interactions of the drift waves based on the fact that $\langle Q \rangle_{uh}$ is mostly independent of the PNC noise. However, the advective flux $\langle Q \rangle_{rh}$ along the direction \hat{x} arises because of the PNC noise. The advecting velocity U along \hat{x} is found to be proportional both to $(k\rho_s)^2$ and to the strength of the parity-non-conserving noise. Such dependence is shown to hold in a wide range of k, including $k\rho_s < 1$, as long as the plasma is forced from a uv region. The k^2 dependence of U is in accordance with the strongly magnetized MHD plasma forced by a PNC noise. As it is clear that the PNC noise may lead to the non-diffusive transport in the plasma, it is desirable to look into the presence of PNC properties in the turbulent plasmas. Work in such a direction is under way and results will be presented in another publication.



FIG. 5: PNC-induced turbulent enstrophy fluxes $\langle Q \rangle_{\rm rh}$ for the cases of $\alpha = -0.5$ and 0.3 are plotted at $\delta = 0.5$ and the box size $L = 2\pi \times 32$. For comparison, advective fluxes $Uk_x \langle |\omega_k|^2 \rangle$ with velocity $U = -0.025k^2$ and $U = 0.015k^2$ are overlaid correspondingly

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