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ABSTRACT

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1. INTRODUCTION AND SUMMARY

Economists have long asked whether investors who misperceive asset returns can survive in a competitive asset market such as a stock or a currency market. The classic answer, given by Friedman (1953), is that they cannot. Friedman argued that mistaken investors buy high and sell low, as a result lose money to rational investors, and eventually lose all their wealth. In response, Figlewski (1979) pointed out that it might take irrational investors a very long time to lose their entire wealth, but he agreed that in the long run those who choose their portfolios irrationally are doomed. Similar conclusions have been reached even by advocates of the importance of traders with incorrect expectations—or "noise traders"—for the determination of asset prices (Shiller, 1984; Kyle, 1985; Black, 1986; and Campbell and Kyle, 1986). Without continuous injections of new "noise money," noise traders must on average lose wealth and eventually disappear.

In an earlier paper (De Long, Shleifer, Summers, and Waldmann, 1987; hereafter DSSW) we questioned the presumption that traders who misperceive returns do not survive. Since noise traders who are on average bullish bear more risk than do investors holding rational expectations, as long as the market rewards risk-taking such noise traders can earn a higher expected return even though they buy high and sell low on average. The relevant risk need not even be fundamental: it could simply be the risk that noise traders' asset demands will become even more extreme tomorrow than they are today and bring losses to any investor betting against them. Because Friedman's argument does not take account of the possibility that noise traders' misperceptions lead them to take on more risk, it cannot be correct as stated.

But this objection to Friedman does not settle the matter, since expected returns are not an appropriate measure of long run survival. For even if they have a higher expected wealth, noise traders who take on more risk might end up bankrupt with high probability and extremely wealthy with low probability (Samuelson, 1971). To adequately analyze whether noise traders are likely to persist in an asset market, one must describe the long run distribution of their and rational investors' wealth, and not just expected returns.

In this paper, we take a first step in considering the long run distribution of wealth and examine a model in which noise traders do not affect prices. If they did affect prices, the returns on assets would depend on the distribution of wealth between noise traders and rational investors. This added complication would make obtaining analytical solutions for our model very difficult. The assumption that noise traders do not affect prices enables us to deal with the implications of their misperceptions for the long-run distribution of their wealth rather than just for expected returns, but not with Friedman's concern that noise traders buy high and sell low. Strictly speaking, we provide comparative statics results for long-run wealth distributions taking prices as given.

To describe the long run evolution of rational investor and noise trader wealth, we adopt the following definitions of "survival" and "dominance":

• Survival: A given group of investors x "survives in the long run" if its share of the economy's total wealth does not approach zero almost surely as time passes, i.e. if:

(1) There are $\varepsilon_1, \varepsilon_2 > 0$ such that for all times t: $\text{Prob}\left\{\omega_t^x > \varepsilon_1\right\} > \varepsilon_2$,

where ω_t^x is the share of the economy's total wealth at time t that belongs to investor group x.

• Dominance: A given group of investors x "dominates" another group y if after sufficient time the probability that group x has a higher share of wealth than group y is greater than 1/2. That is, no matter what the initial relative wealth levels ω_0^x and ω_0^y of the two groups,

(2) There is a t_0 such that for every time $t>t_0$: $\text{Prob}\left\{\omega_t^x > \omega_t^y\right\} > \frac{1}{2}$.

As long as the distribution of gross returns is the same across periods and entails no risk of losing all one's wealth, (2) implies (2'):

(2') For every positive integer n, there is a t_n such that for every time $t > t_n$:

$$\text{Prob}\left\{ \omega_t^x > \omega_t^y \right\} > \frac{n-1}{n}$$

In the context of our model, (1) and (2) hold if and only if the geometric mean growth rate of wealth is higher for group x than for group y. Subject to the assumptions that return distributions are unchanging and do not allow for a negative 100% return, if group x survives, then it dominates. But the distinction between the two concepts is worth preserving for situations in which return distributions do change over time.

We analyze the evolution of the wealth of noise traders and rational investors using these definitions of "survival" and "dominance" in a model with infinitely lived investors. We allow for the possibility that excessive risk taking brings noise traders virtually certain ruin. We also take account of the fact that noise traders falsely believe that they can earn excess returns, as a result overestimate their wealth, and possibly consume too much thereby reducing their survival prospects.

In our model, noise traders falsely perceive that a particular asset is mispriced and take positions in it to exploit this perceived mispricing without properly hedging market risk. For many plausible forms of misperceptions, especially about return variances, they bear additional market risk and so earn an extra expected return. Moreover, small mistakes concerning the valuation of a particular security are not very costly to noise traders. On the margin the extra market risk they take is almost completely offset by the higher expected return, and the extra diversifiable risk they bear reduces their utility only slightly. An immediate implication is that an investor who realizes that his theory about the mispricing of an individual stock is likely to be inaccurate would still use this theory to allocate at least part of his wealth.

Having established that many plausible types of noise traders earn higher expected returns than do rational investors, we ask what happens to their share of wealth in the long run. Even though noise traders bear extra risk and are likely to consume more than they would with rational expectations, for many plausible misperceptions of returns noise traders survive and come to dominate rational investors, in the sense described above. Specifically, we show that noise traders as a group might survive and dominate rational investors even when on average a rational investor dominates any noise trader of a fixed type in wealth. Excess consumption, excess bearing of market risk because of a failure to properly hedge, and excess bearing of idiosyncratic risk together impart a downward drift to each individual noise trader's wealth relative to that of an average rational investor. But the aggregate wealth of noise traders relative to that of rational investors need not tend toward zero, for the downward drift imparted by idiosyncratic risk does not affect noise traders' collective wealth. If idiosyncratic risk is large, each individual noise trader with high probability fails to survive in the market, but noise traders as a whole can nevertheless survive. Evolution may leave an ever-shrinking army of ever-richer fools who collectively dominate the market.

Our model considers the long-run evolution of relative wealth in an environment in which noise traders do not affect prices. But it can also be interpreted in a context where noise traders do exert pressure on prices and thus, as Friedman indicates, buy high and sell low. As the noise trader share of wealth drops, the price pressure they exert and the degree to which they buy high and sell low drop also. For these reasons, the conditions necessary for the dominance of noise traders when they do not affect prices translate into conditions necessary for their survival (but not dominance) when they do.

Section 2 motivates our assumptions about noise traders' misperceptions of returns by discussing the misperceptions of subjects of psychological experiments. Section 3 lays out a one period model and calculates the expected returns earned by noise traders and rational investors. It also shows that the utility cost of being a noise trader is small. Section 4 considers a dynamic model of wealth accumulation with infinitely lived rational investors and noise traders, and explores noise traders' chances for long run survival in the market. Section 5 reinterprets our conditions for the dominance of noise traders in the case where their trades do not affect prices as conditions for the long run survival in the marketplace of noise traders when their trades do affect prices. Section 6 concludes.

2. THE PLAUSIBILITY OF MISPERCEPTIONS

In this paper we assume that noise traders are poor assessors of probability distributions, especially of variances. Moreover we assume that the misperceptions of different noise traders about a particular asset are correlated, for if all traders confused about the returns on a stock have different misperceptions, their trades will cancel out.[1] We justify this assumption by summarizing some psychological evidence on systematic judgment errors made by experimental subjects.

Experiments reveal that individuals are consistently poor assessors of probabilities. They use a variety of heuristics to estimate probabilities that can lead to biases (Tversky and Kahneman, 1974) that are not random but instead correlated across subjects. People agree which particular player has a "hot hand" (Gilovich, Valone, and Tversky, 1985), and they see the same nonexistent trends and patterns in artificially generated as in real stock price series (Andreassen and Kraus, 1987).

We focus on one of the best documented baises: the tendency to underestimate variances and be overconfident (Alpert and Raiffa, 1959; Einhorn and Hogarth, 1978; Lichtenstein, Fischoff, and Phillips, 1982). Experts and novices alike are too certain about their predictions given the true odds of being wrong. Alpert and Raiffa's (1959) original finding that business school students are overconfident has been confirmed for many different populations using a variety of questions on which respondents had varying degrees of expertise (Tversky and Kahneman, 1971; Slovic, 1981). CIA analysts, experienced psychologists, and physicians are all overconfident. Overconfidence does not arise from lack of concern by experimental subjects for the accuracy of their distributions: students were more overconfident when their performance was linked to grades than when it was not. Moreover, overconfidence gets worse, not better, when the difficulty of the task increases (Langer, 1975).

In addition, overconfidence is likely to become more extreme over time as those who succeed attribute their success to their own skill and judgment. In Langer's words, "heads I win, tails it's chance." In asset markets, the richest individuals may well be those who placed large bets on

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[1] Pagano (1987) studies thin markets where, even though noise traders' misperceptions are uncorrelated, their trades need not cancel out. We assume that markets are thick enough that the law of large numbers applies.

very risky gambles and won. Their success would naturally tend to reinforce their confidence in their own hunches whether or not such confidence is justified.

We see the psychological literature as providing a suggestive sketch of how noise traders tend to behave. First, they might misperceive expected returns, although it is hard to predict in what way. Perceptions of risks and opportunities might depend on past patterns of prices and volume in not very rational ways and be strongly correlated across agents. Second and most important, no matter what return investors expect many of them are likely to be overconfident. Investors are likely both to have hunches and to underestimate the risk that they are assuming when they bet their portfolios on their hunches. In subsequent sections, we analyze the market performance of noise traders who behave in this fashion.

3. A ONE-PERIOD MODEL

This section develops a one-period model that serves as the basis for the multiperiod, infinite-horizon model considered in section IV. We first present our assumptions about noise traders' beliefs. We then compute the distributions both of rational investors' and noise traders' wealth as a function of each type's perceptions of asset returns. We show that noise traders earn higher expected returns than rational investors for a large set of possible misperceptions. Last, we show that the utility cost of being a noise trader is low.

3.1 Assumptions of the Model

Investment opportunities consist of one safe asset paying a known gross return $(1+r)$, and a continuum of risky assets indexed by i in the interval $[0,1]$. The return on the risky asset i is:

$$R_i = \rho + \eta + \varepsilon_i \,, \qquad (3)$$

where ρ is the average dividend paid on all risky assets, and η and the ε_i are uncorrelated mean zero random variables satisfying $E(\eta) = E(\varepsilon_i) = 0$, $E(\eta^2) = \sigma_\eta^2$, and $E(\varepsilon_i^2) = \sigma_i^2$. Under these assumptions all assets have a market β of one. This simplifies the algebra without loss of generality. Returns are assumed to be exogenous, with no investor having an effect on the price of any risky asset. The supply of assets is thus assumed to be infinitely elastic.

In this section, we focus on a single type of noise trader who misperceives the return distribution of a single risky asset i. In section 4 we consider a continuum of types of noise traders, with each type misperceiving the return distribution on only one of the continuum of risky assets. We index noise trader types by the same i that indexes risky assets. Noise traders of type i correctly perceive the distribution of returns of every asset except i, but they falsely believe that the distribution of asset i's net returns is given not by (3) but by:

$$(4) \qquad (\hat{R})_i = r + \mu(\rho - r) + \tau(\eta + \varepsilon_i) \ ,$$

for some parameters μ and τ. A caret ($\hat{}$) above a variable denotes the noise traders' perception of the variable.

The parameters μ and τ allow noise traders to have different misperceptions of the mean and variance of the returns on asset i. A noise trader's μ describes his opinion about the mean return on asset i. If $\mu \neq 1$ then noise traders of type i misperceive the expected return on asset i:

$$(5) \qquad E(\hat{R})_i = \mu(\rho - r) + r \neq E(R_i) = \rho \ .$$

If μ is greater (less) than one, then noise traders overestimate (underestimate) asset i's expected return. The parameter τ describes the opinion about the standard deviation of the return on asset i. If $\tau \neq 1$, then noise traders misperceive both asset i's idiosyncratic variance and its market β:

$$(6) \qquad (\hat{\sigma})_i^2 = \tau^2 \sigma_i^2 \neq \sigma_i^2 \ ,$$

$$(7) \qquad (\hat{\beta})_i = \tau \neq 1 \ .$$

Note that noise traders have the same misperception of each component of the variance of the return on asset i.

Given his own perception of the distribution of returns, each investor maximizes:

$$(8) \qquad E(U) = E(W_1) - \frac{\gamma}{2W_0(1 + r)} \sigma_w^2 \ ,$$

where W_1 is the wealth of the investor at the end of the period, σ^2_w is its variance, W_0 is the investor's initial wealth, and expectations are taken using each investor's own beliefs. By assumption, the local degree of absolute risk aversion is inversely proportional to the investor's end of period wealth.that appears in the denominator of (8). In continuous time, maximizing (8) is equivalent to maximizing a constant relative risk aversion utility function. As long as both mean excess returns (ρ-r) and variances are small, and excess returns are not large relative to variances, (8) is a good approximation to constant relative risk aversion utility.

Because noise traders affect asset quantities but not prices, we can calculate the equilibrium portfolio allocations of noise traders and rational investors separately. Rational investors maximizing (8) hold equal infinitesimal amounts of each risky asset to avoid idiosyncratic risk. They therefore invest a share of their wealth $\alpha(1+r)$ in the equally-weighted market portfolio of risky assets, where:

$$\alpha = \frac{\rho - r}{\gamma\sigma^2_\eta} \tag{9}$$

Rational investors invest the rest of their wealth in the riskless asset.

Noise traders do not confine their investments to positions in the riskless asset and the diversified equal-weighted risky market portfolio. They also perceive an additional investment opportunity in asset i. Because noise traders believe that asset i is mispriced, they choose to hold it in a proportion different from its infinitesimal share of the risky market portfolio. This perceived mispricing of asset i does not, however, make noise traders wish to hold a different amount of the common risk factor η. Noise traders hedge their holdings of i using the market portfolio so that they (falsely) believe that their additional investment in i has no effect on their exposure to aggregate market risk.

The net result is that noise traders' portfolios are made up of three pieces. The first is their investment of $\alpha(1+r)W_0$ in the equally weighted risky market portfolio, which is identical to the risky market holdings of rational investors. The second is their holding of the riskless asset. The third is their investment in the perceived zero-β portfolio (henceforth PZBP) for asset i. A unit of this PZBP consists of a position long one unit of asset i and short τ units of the market. This unit has a net cost of $(1-\tau)$, carries what noise traders believe to be no exposure to market risk, carries in noise traders' opinion and in fact unit exposure to the idiosyncratic risk ε_i, and has in noise traders' estimation a non-zero expected return.

Since its true β is not zero, the PZBP actually earns an excess expected return relative to the riskless rate:

$$(10) \qquad R_i^z - r = (1-\tau)(\rho - r + \eta) + \varepsilon_i \ ,$$

But noise traders (falsely) believe that this PZBP has a different excess return that arises not from its covariance with the market but from their false perception that asset i is mispriced. Noise traders expect this excess return on the PZBP to be:

$$(11) \qquad (\bar{R})_i - r \ = \ (\mu - \tau)(\rho - r) + \tau\varepsilon_i \ .$$

If $\mu - \tau > 0$, then noise traders believe that asset i is underpriced, and so they go long its PZBP. If $\mu - \tau < 0$ then noise traders think that asset i is overpriced, and they sell short its PZBP. Since noise traders (falsely) believe that PZBP risk is orthogonal to the market risk that they already bear, they hold a quantity $\lambda_i(1+r)W_0$ of the PZBP, where:

$$(12) \qquad \lambda_i \ = \ \frac{(\mu - \tau)(\rho - r)}{\gamma\tau^2\sigma_i^2} \ .$$

The difference between noise traders' and rational investors' share of wealth held in the riskless asset is $(1+r)\lambda_i$.

3.2 The Difference in Expected Returns

Given these holdings, the expected end-of-period wealth of a noise trader of type i is:

$$(13) \qquad E(W_1^n) = \ W_0^n(1 + r) \left\{ 1 + \frac{(\rho - r)^2}{\gamma\sigma_\eta^2} + \frac{(1-\tau)(\mu - \tau)(\rho - r)^2}{\gamma\tau^2\sigma_i^2} \right\} \ .$$

The expected end-of-period wealth of a rational investor is:

$$(14) \qquad E(W_1^s) = \ W_0^s(1 + r) \left\{ 1 + \frac{(\rho - r)^2}{\gamma\sigma_\eta^2} \right\} \ .$$

The first term inside the brackets in (13) and (14) captures the return all market participants would earn if only the safe asset existed. The second term captures the return that everybody earns because they can invest in the risky market as well as in the riskless asset. The third term captures the difference in expected return that the noise traders earn because they misperceive the distribution of returns on asset i, take a non-zero position in asset i's PZBP, and so bear a different amount of market risk than they intend.

If $\tau=1$—if noise traders perceive β_i correctly whether or not they misperceive the mean return on asset i—then noise traders earn the same expected return as do rational investors because the true expected excess return on the PZBP of asset i is zero. Noise traders do, however, bear a positive amount of asset i's idiosyncratic risk and as a result hold inefficient portfolios.

If $\mu=\tau$ then expected returns are again equal. Noise traders hold the same portfolio as rational investors because noise traders believe that asset i is correctly priced. Because their belief that it has a larger β offsets their perception of its higher excess return, they do not hold any of asset i's PZBP.

If $\mu=1$ and $\tau\neq1$—if noise traders correctly perceive the mean return on asset i but misperceive the variance—then they always earn a higher expected return. In this case noise traders necessarily hold portfolios that carry a larger degree of systematic risk than do rational investors. If noise traders underestimate β_i they think that asset i is underpriced, go long its PZBP, and so hold more of the risky market than do rational investors because their underestimation of β_i gives the PZBP a positive covariance with the market. If noise traders overestimate β_i they think asset i is overpriced, sell short its PZBP, and as a result hold more of the risky market than do rational investors because their overestimation of β_i gives their PZBP a negative covariance with the market.

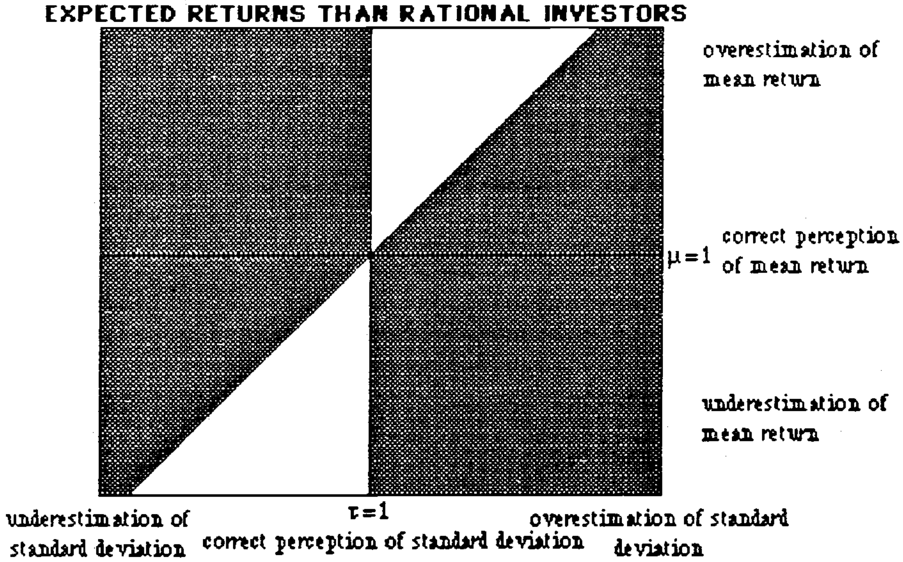
Proposition 1: A noise trader who misperceives only the variance of returns on a single risky asset earns higher expected returns than does a rational investor.

Proof: By inspection of equations (13) and (14).

Note that both overconfident and underconfident noise traders can earn higher expected returns. As we suggested in section II, empirically overconfidence, meaning $\tau < 1$, is the more important case. Interestingly, if there are restrictions on short sales, then underconfident noise traders find themselves unable to hold their optimal portfolio since it involves selling asset i short. By contrast overconfident investors need only to buy asset i and reduce their holdings of the market, without actually selling the market short for τ close to one. As a result in the presence of restrictions on short sales, overconfident investors end up holding more market risk but underconfident investors do not.

Equations (13) and (14) reveal that noise traders earn lower expected returns than do rational investors only if $(1-\tau)(\mu-\tau)$ is negative. Noise traders earn higher expected returns on three-fourths of the plane in (μ,τ) space. If noise traders' μ's and τ's are symmetrically randomly distributed around the point (1, 1), then the probability that a randomly selected noise trader earns a higher expected return is three fourths. If misperceptions of standard deviations are larger in magnitude than misperceptions of means, then the likelihood that a given noise trader earns higher expected returns is even greater.

FIGURE 1: AREA OF (μ,τ) SPACE WHERE NOISE TRADERS EARN HIGHER EXPECTED RETURNS THAN RATIONAL INVESTORS



3.3 The Private Cost of Noise Trading

Rational investors and noise traders with equal initial wealth W_0 have a difference in true expected end of period utility levels that could be offset by a certain addition of:

$$(15) \qquad W_0(1+r)\left\{ \frac{(\mu-\tau)^2(\rho-r)^2}{2\gamma\tau^2\sigma_i^2}\left(1+\frac{\sigma_n^2(1-\tau)^2}{\sigma_i^2}\right)\right\},$$

to noise traders' end of period wealth. The difference in the true utility levels of the two groups is therefore second order in the misperception parameters μ and τ.

Proposition 2: The utility cost to a noise trader of his misperception of the distribution of returns is second-order in magnitude of his misperception.
Proof: By inspection of equation (15).

Since the cost of being a noise trader is second-order in misperceptions, there is a sense in which the efficiency of the stock market protects and encourages noise traders. An investor with a Bayesian prior that puts positive weights on both the rational and the noise trader views of the world would not want to hold the same portfolio as a rational investor. For if the noise trader view of the world is correct, then holding the market portfolio is not optimal and a small purchase of the PZBP yields a first-order increase in utility. If the rational view of the world is correct, then assets are priced correctly and so a purchase of a small amount of the PZBP leads to only a second-order utility loss. An investor who attaches any positive probability to the correctness of the noise trader belief that asset i is mispriced should therefore take at least a small position in the PZBP that exploits this mispricing.

At the margin, it is not expensive to be a noise trader. An investor who attaches a positive probability to his truly having some special insight about expected returns or variances should act on this information and not just hold the market. If an investor enjoys the stock-picking process itself,[1] he should take positions according to his opinions about mispricings even if he understands that his opinions do not reflect any informational edge over the market. And any investor genuinely uncertain about the efficiency of the market should commit at least some of his assets to his own

[1]Lease, Lewellen, and Schlarbaum (1974) find that most individual investors pick their own stocks because they like to, even though most do not expect to beat the market.

favorite stocks.

4. A MULTI-PERIOD MODEL

We assume that both noise traders and rational investors have infinite horizon constant relative risk aversion utility functions, and optimally choose their consumption and investment plans given their beliefs. We assume that noise traders of type i continue to misperceive the returns on asset i by the same amount in every period: they do not learn from their mistakes. We consider the evolution of the wealth of a continuum of noise traders, where each noise trader misperceives the return distribution on a different asset i. Finally, we assume for simplicity that noise traders correctly perceive the means of all return distributions ($\mu = 1$). This assumption greatly simplifies the algebra and reflects the lack of evidence on the sign of μ.

Even if noise traders are likely to earn higher expected returns in any one period, they might not survive and come to dominate the market. Three factors keep higher expected returns from translating immediately into a higher share of long run wealth.

First, noise traders who (falsely) believe they have a profit-making trading opportunity overestimate their permanent income and as a result consume too much. This slows down their wealth accumulation.

Second, having a higher period-by-period expected return is not identical to long-run dominance in wealth. As the time horizon increases, the distribution of the average per period gross return earned by an investor who places constant wealth shares in different assets approaches a log normal and becomes highly skewed. It might then be the case that with high probability noise traders become poorer than rational investors, but with low probability noise traders become vastly richer. Noise traders' wealth share might asymptotically approach zero with probability one—they might fail to "survive" in the market on our definition—even if they have a higher expected wealth (Samuelson, 1971).

Last, each individual type of noise trader holds an inefficient portfolio. Noise traders of type i bear a finite amount of idiosyncratic risk of asset i, and so their portfolios have more variance than necessary to attain their actual level of expected returns. This risk further increases the variance of noise traders' returns and so leaves them with an even smaller probability of having a high relative wealth share.

To consider the evolution of noise traders' wealth taking into account these three factors, we first show that idiosyncratic risk reduces the survival probabilities of individual noise traders but not of noise traders as a whole. We then embed the one-period model of the previous section in an infinite period context and consider how the skewness of the distribution of expected returns affects noise traders' survival prospects. Third, we analyze how excess consumption impedes noise traders' wealth accumulation. We then arrive at conditions for the long-run survival and dominance of noise traders.

4.1 Noise Traders' Individual and Aggregate Wealth

The extra risk imparted by the inefficiency of noise traders' portfolios is eliminated if we examine the aggregate wealth of all noise traders with misperceptions distributed over different stocks. If noise traders of each type i misperceives the variance of stock i by the same τ, then noise traders as a whole bear no idiosyncratic risk and hold an efficient portfolio.

The variance of the returns earned by each noise trader is affected by his exposure to the idiosyncratic risk of asset i:

$$\text{(16)} \qquad \text{Variance}_i^n = \frac{(\rho - r)^2}{\gamma^2 \sigma_\eta^2} + \frac{2(1-\tau)^2(\rho - r)^2}{\gamma^2 \tau^2 \sigma_i^2} + \frac{(1-\tau)^2(\rho - r)^2 \sigma_\eta^2}{\gamma \tau^4 \sigma_i^4} + \frac{(1-\tau)^4(\rho - r)^2}{\gamma^2 \tau^4 \sigma_i^2} .$$

The last term, however, disappears from the expression for the variance of returns earned by noise traders as a whole. As far as noise traders in the aggregate are concerned, the "consumption" and "systematic variance" effects are the only ones that drive a wedge between having a higher expected return and coming to dominate the market. Noise traders in the aggregate might then survive in the marketplace. In this case, the wealth share of a randomly-selected noise trader type eventually falls with probability one, but the wealth of a small fraction of the noise trader population is increasing fast enough to give them a rising aggregate share of the economy's wealth.[1]

[1]This observation can be illustrated by considering the Forbes 400 list of the richest people in America. Most of the

4.2 Distinguishing Between High Expected Returns and Dominance

For the moment, we neglect consumption and consider only the returns on noise traders' and rational investors' portfolios. Assume that investors live forever, face an unchanging distribution of period-by-period returns, and exhibit constant relative risk aversion. Such investors devote the same portfolio share to a given asset each period. Their wealth is multiplied by an i.i.d. random variable $(1 + R_t)$ each period. Taking logs, the random variable $\ln(1 + R_t)$ is added to the log of wealth each period. The law of large numbers tells us that the average growth rate g of wealth is:

$$(17) \qquad g = E(\ln(1 + R_t)) \approx E(R_t) - \frac{V(R_t)}{2} \, ,$$

where $V(R_t)$ is the variance of returns.

To evaluate the relative survival chances of noise traders and rational investors, we therefore consider the difference in their geometric mean returns, approximately equal to:

$$(18) \qquad E(R^s - R^n) - \left(\frac{V^s - V^n}{2} \right) ,$$

where V is the period-by-period variance of the return on each type's portfolio. The second "drift" term reflects the likelihood that agents whose returns have a higher variance end up with lower wealth. Occasional large negative realization of returns decrease such investors' capital bases and reduce the future absolute change in their wealth so much that they might eventually have lower total wealth even if they earn higher period-by-period expected returns. As a result, investors for whom a larger drift outweighs their advantage of a higher expected return neither survive nor dominate the market in terms of wealth. If examined after a sufficiently long time interval, in an overwhelming proportion of cases investors with a higher geometric mean of returns are richer.

Forbes 400 hold extremely undiversified portfolios. They concentrate their wealth in their own particular industries in spite of the clear presence of large amounts of industry-specific risk which their skill and judgment cannot help them avoid. Moreover, the Forbes 400 became members of this particular club by, having already become very rich, continuing to hold portfolios subject to large amounts of idiosyncratic risk for which the idiosyncratic component of returns turned out to be vastly positive.

The difference between the geometric mean returns of the aggregates of noise traders and rational investors is:

$$(19) \qquad \frac{(1-\tau)^2(\rho - r)^2}{\gamma\tau^2\sigma_i^2}\left\{1 - \frac{1}{\gamma} - \frac{(1-\tau)^2\sigma_\eta^2}{2\gamma\tau^2\sigma_i^2}\right\}.$$

Because the leading common factor is always positive, the sign of the difference in geometric mean returns depends on the terms inside the brackets. The leading 1 inside the brackets reflects the greater expected returns that noise traders earn because of their unwittingly greater exposure to systematic risk. The second and third terms capture the increase in aggregate return variance that this exposure entails.

The third term is small if τ is close to one. In this case as long as investors are more risk averse than investors with logarithmic utility (have $\gamma>1$), the aggregate of noise traders who misperceive variances survive and come to dominate the market. For each $\gamma > 1$ there is some $\delta > 0$ such that if $|\tau-1| < \delta$ then noise traders in the aggregate have a higher geometric mean return. Such noise traders are confused about variances, but their confusion is sufficiently small that the higher expected return more than outweighs the larger drift induced by the greater variance.

Note, however, that if $\gamma\leq 1$ there is no misperception of the variance of the return distribution that delivers a higher geometric mean return for noise traders. This point is equivalent to the observation that an investor wishing to maximize the long-run average rate of return earned on his portfolio should choose portfolio shares as if he had logarithmic utility, i.e. $\gamma=1$ (Samuelson, 1971). An investor with $\gamma>1$ does not choose such a portfolio because he is sufficiently averse to low wealth realizations to forego at least some long-run expected return in order to reduce risk. This implies that all investors who take a position that bears marginally more systematic risk—even by mistake, as in the case of noise traders—have a higher geometric mean return, and therefore come to dominate rational investors in wealth.[1]

Conversely, an investor with $\gamma<1$ bears too much risk to maximize the long-run average rate of return on his portfolio. He values the occasional high realization of wealth enough to accept a

[1]Another implication of this is that no type of noise trader can dominate a rational investor with logarithmic utility.

substantial chance of having low wealth generated by a risky portfolio. Such an investor could increase his long-run average rate of return and improve his survival potential by reducing his holdings of the risky asset. We find the case $\gamma<1$ unattractive because investors less risk averse than log utility fall victim to the St. Petersburg paradox (Samuelson, 1976). Such investors are willing to pay an infinite amount for a gamble that pays zero with a probability arbitrarily close to one and pays finite amounts in every state of the world.

4.3 Consumption

We now turn to the effects of noise traders' misperceptions on their consumption. If investors live forever, maximize the same approximation to a constant relative risk aversion utility function as in section III, and face an unchanging distribution of returns, then their consumption is given by:

$$(20) \qquad c_t = W_t \left\{ \frac{\gamma - 1}{\gamma} \left(E(R_{t+1}) - \frac{\gamma(V(R_{t+1}))}{2} + \frac{\delta}{\gamma} \right) \right\},$$

where expectations are taken with respect to the perceived distribution of returns (Merton, 1969). Since all noise traders consume the same fraction of wealth, aggregation causes no problems.

Noise traders in this case consume more than do rational investors. They (falsely) believe that their portfolios have a risk-adjusted net rate of return higher than those of rational investors by:

$$(21) \qquad \lambda_i^2 \tau^2 \sigma_i^2 = \frac{(1-\tau)^2(\rho - r)^2}{2\gamma\tau^2\sigma_i^2}.$$

Noise traders' misperceptions lead them to consume a fraction of their wealth higher than that consumed by rational investors by:

$$(22) \qquad \frac{c^n}{W^n} - \frac{c^s}{W^s} = \frac{(\gamma-1)(1-\tau)^2(\rho - r)^2}{2\gamma\tau^2\sigma_i^2}.$$

4.4 Conditions for the Long-Run Survival of Noise Traders

Combining (22) with (19) gives an expression for the difference of the geometric mean growth rates of noise traders' and rational investors' aggregate wealth. The set of investors with the higher geometric mean growth rate both survives in the market and comes to dominate in wealth. Noise traders come to dominate the market in the long run if:

$$(23) \qquad \frac{(1-\tau)^2(\rho - r)^2}{\gamma\tau^2\sigma_i^2}\left\{ 1 - \left[\frac{1}{\gamma} - \frac{(1-\tau)^2}{2\gamma\tau^2}\left(\frac{\sigma_n^2}{\sigma_i^2}\right)\right] - \frac{\gamma - 1}{2\gamma}\right\} > 0 .$$

Leaving aside the positive common factor, this expression consists of three pieces. The leading "1" reflects the higher expected returns earned by noise traders. The final piece arises from noise traders' excess consumption. Note that the additional consumption effect is always outweighed by the extra return effect. Even though noise traders consume a higher fraction of their wealth than do rational investors, the average rate of wealth accumulation of noise traders who misperceive variances alone is always higher than that of rational investors.

The middle two terms of (23) reflect the downward relative drift imposed on the geometric mean of noise traders' relative returns by the extra variance of their portfolios. Examination of (23) reveals that when $\gamma > 1$ noise traders with small misperceptions survive and come to dominate the market. We can further simplify (23) to:

$$(24) \qquad (\gamma - 1) - \left(\frac{1-\tau}{\tau}\right)^2\left\{\frac{\sigma_n^2}{\sigma_i^2}\right\} > 0 .$$

By inspection, if $\gamma > 1$ (24) holds for τ sufficiently close to one. Moreover, if idiosyncratic risk is large relative to market risk then (24) holds as long as τ is not too close to zero. Only noise traders whose misperceptions of returns are truly extraordinary would then fail to dominate the market. Equations (23) and (24) clearly demonstrate that for all parameters of the return distribution, there are plausible misperceptions by noise traders that allow them to survive and to dominate the market.

<u>Proposition 3</u>: For any parameters of the return distribution there exists a δ such that if $|\tau - 1| < \delta$,

then noise traders who misperceive variances with parameter t survive and come to dominate the market.

Proof: By inspection of equation (24).

Not only are noise traders who misperceive variances by only a small amount and are more risk averse than log guaranteed to survive in the market, but a wide range of noise traders who misperceive variances by a large amount also survive. The range of parameter values for which noise traders survive in the long run is substantial. Taking $\gamma=2$ and $\sigma_i/\sigma_\eta = 3$ as representative parameter values, we can compute that for any $\tau > 0.366$ noise traders' relative wealth will be almost certainly larger in the long run. As long as noise traders' underestimate the variance of asset returns by a fraction less than 87%, noise traders survive and dominate. For $\gamma=2$ and $\sigma_i/\sigma_\eta = 1$, noise traders' relative wealth grows without bound for any $\tau > 1/2$. Noise traders who have reasonable degrees of overconfidence make money, and all noise traders who are excessively uncertain make money if short sales are allowed.

5. INVASION

We have interpreted our model as a model of the long run survival and dominance of noise traders in an environment where they do not affect prices. Our results also apply, with a more restricted interpretation, in models in which noise traders exert price pressure and distort prices against themselves. Our conditions sufficient for the dominance of noise traders in a model in which they do not affect prices have another interpretation in a model where noise traders do affect prices as conditions sufficient for noise traders to be able to successfully invade the economy, in the sense that a small group of noise traders introduced into the economy will find that their wealth share tends to grow, not shrink, over time. Our sufficient conditions can further be interpreted as conditions sufficient for noise traders to survive in the long run, in the sense of having a share of the economy's wealth that is with finite probability bounded away from zero for all time.

When noise traders have an infinitesimal share of wealth, they distort prices and returns only an infinitesimal amount away from fundamental values. Hence if noise traders have a higher average rate of wealth accumulation, then they can "invade" even if they distort prices. If their wealth share is infinitesimal, noise traders will exert negligible price pressure and so their wealth share will tend to grow. Noise traders therefore survive, in the sense that their wealth share does not drop toward zero in the long run with probability one. Our analysis of the long run tendency of the noise trader share in models where they do affect prices is limited to these statements about "invasion" and "survival." We can make no statements about the conditions for noise trader dominance in this context, because as soon as they acquire a nontrivial share of wealth they begin to affect prices in a nontrivial way, and our model and analysis no longer apply.

These results imply that population composed entirely of rational investors is not "evolutionarily stable" (Maynard Smith, 1982). If a small number of noise traders are introduced into the population, their relative wealth tends to grow. Noise traders can successfully "invade" the population. In a world in which investors occasionally "mutated" and changed from noise trader to rational investor or vice versa, it would be surprising to find a population composed almost entirely of rational investors.

6. CONCLUSION

In this paper we have presented a model of portfolio allocation by noise traders who form incorrect expectations chiefly about the variance of the return distribution of a particular asset. We showed that for many types of misperceptions, such noise traders not only earn higher returns than do rational investors but also survive and dominate the market in terms of wealth in the long run. Such long run success of noise traders occurs despite their excessive risk taking and excessive consumption. The case against their long run viability is by no means as clearcut as is commonly supposed.

The main limitation of our model is that it does not allow noise traders to affect prices. This paper therefore cannot address Friedman's main point: that noise traders buy high and sell low. In our earlier paper (DSSW, 1987) we have shown that noise traders can earn higher <u>expected</u> returns than rational investors even when they buy high and sell low. But the model of our earlier paper could not deal with survival and dominance. The next step in this literature, then, is to arrive at a tractable model in which noise traders affect prices, and in which survival and dominance can be analyzed. The answers afforded by such a model would go a long way toward settling the theoretical question raised by Friedman (1953).

REFERENCES

Alpert, Mark and Howard Raiffa (1959), "A Progress Report on the Training of Probability Assessors," in Kahneman, Daniel; Paul Slovic; and Amos Tversky, eds. (1982), Judgment Under Uncertainty: Heuristics and Biases (Cambridge, UK: Cambridge University Press).

Andreassen, Paul and Stephen Kraus (1987), "Intuitive Prediction by Extrapolation: The Stock Market, Exponential Growth, Divorce, and the Salience of Change," Harvard University mimeo.

Black, Fischer (1986), "Noise," Journal of Finance, **41**, 529-43.

Campbell, John and Albert Kyle (1986), "Smart Money, Noise Trading, and Stock Price Behavior," Princeton University mimeo.

De Long, J. Bradford; Andrei Shleifer; Lawrence Summers; and Robert Waldmann (1987), "Noise Trader Risk in Financial Markets," NBER Working Paper #2395.

Einhorn , Hillel and Robin Hogarth (1978), "Confidence in Judgment: Persistence of the Illusion of Validity," Psychological Review, **85**, 395-476.

Figlewski, Stephen (1979), "Market 'Efficiency' in a Market with Heterogeneous Information," Journal of Political Economy, **86**, 581-97.

Friedman, Milton (1953), "The Case for Flexible Exchange Rates," in Essays in Positive Economics (Chicago: University of Chicago Press).

Gilovich, T., R. Valone, and A. Tversky (1985), "The Hot Hand in Basketball: On the Misperception of Random Sequences," Cognitive Psychology, **17**, 295-314.

Kahneman, Daniel and Amos Tversky (1973), "On the Psychology of Prediction," Psychological Review, **80**, 237-51.

Kahneman, Daniel; Paul Slovic; and Amos Tversky, eds. (1982), Judgment Under Uncertainty: Heuristics and Biases (Cambridge, UK: Cambridge University Press).

Kyle, Albert (1985), "Continuous Auctions and Insider Trading," Econometrica, **53**, 1315-36.

Langer, E.J. (1975), "The Illusion of Control," Journal of Personality and Social Psychology, **32**, 311-28.

Lease, Ronald C; Wilbur G. Lewellen; and Gary G. Schlarbaum (1974), "The Individual Investor: Attributes and Attitudes," Journal of Finance, **29**, 413-33.

Lichtenstein, Sarah; Baruch Fischhoff; and Lawrence D. Phillips (1982), "Calibration of Probabilities: The State of the Art as of 1980," in Kahneman, Daniel; Paul Slovic; and Amos Tversky, eds. (1982), Judgment Under Uncertainty: Heuristics and Biases (Cambridge, UK: Cambridge University Press).

Maynard Smith, John (1982), Evolution and the Theory of Games (Cambridge, UK: Cambridge University Press).

Merton, Robert (1969), "Optimal Portfolio Selection by Dynamic Stochastic Programming," Review of Economics and Statistics, 51, 247-57.

Pagano, Marco (1987), "Endogenous Thinness and Stock Price Volatility," CEPR Working Paper.

Samuelson, Paul A. (1971), "The 'Fallacy' of Maximizing the Geometric Mean in Long Sequences of Investing or Gambling," Proceedings of the National Academy of Sciences, **68**, 207-10.

Samuelson, Paul A. (1976), "St. Petersburg Paradoxes," Journal of Economic Literature.

Shiller, Robert (1984), "Stock Prices and Social Dynamics," Brookings Papers on Economic Activity, **1984**, 457-98.

Slovic, Paul (1981), "Toward Understanding and Improving Decisions," in W. Howell, ed., Human Permformance and Productivity (Hillsdale, NJ: Erlbaum Press).

Stein, Jeremy (1987), "Informational Externalities and Welfare-Reducing Speculation," Journal of Political Economy 95 (December): 1123-45.

Tversky, Amos and Daniel Kahneman (1971), "Belief in the Law of Small Numbers," Psychological Bulletin, **2**, 105-10.

Tversky, Amos and Daniel Kahneman (1974), "Judgment Under Uncertainty: Heuristics and Biases," Science, **185**, 1124-31.