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MONEY, REAL INTEREST RATES, AND OUTPUT: A REINTERPRETATION OF POSTWAR U.S. DATA

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ABSTRACT

This paper reexamines both monthly and quarterly U.S. postwar data to investigate if the observed comovements between money, real interest rates, prices and output are compatible with the money--real interest--output link suggested by existing monetary theories of output, which include both Keynesian and equilibrium models.

The major empirical findings are these;

1) In both monthly and quarterly data, we cannot reject the hypothesis that the ex ante real rate is exogenous, or Granger-causally prior in the context of a four-variable system which contains money, prices, nominal interest rates and industrial production.

2) In quarterly data, there is significantly more information contained in either the levels of expected inflation or the innovation of this variable for predicting future output, given current and lagged output, than in any other variable examined (money, actual inflation, nominal interest rates, or ex ante real rates). The effect of an inflation innovation on future output is unambiguously negative.

The first result casts strong doubt on the empirical importance of existing monetary theories of output, which imply that money should have a causal role on the ex ante real rates. The second result would appear incompatible with most demand driven models of output.

In light of these results, we propose an alternative structural model which can account for the major dynamic interactions among the variables. This model has two central features: i) output is unaffected by money supply; and ii) the money supply process is motivated by short-run price stability.

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Money, Real Interest Rates, and Output: A Reinterpretation of Postwar U.S. Data

Does money matter? This paper reexamines the time series evidence that changes in money supply have been an important factor in generating postwar U.S. business cycles. Specifically, we investigate whether the observed comovements between money, real interest rates, prices, and output are compatible with existing monetary theories of income determination, which include both traditional Keynesian analysis as well as the newer informationally constrained equilibrium theories. The main empirical findings cast strong doubts on the empirical importance of these theories for understanding recent U.S. experience. Rather, we find that most of the dynamic interactions among the key variables can be best explained as arising from an economic structure in which changes in money supply on the order of those which have been historically observed do not affect output. Thus, we conclude that monetary instability has not played an important role in generating fluctuations.

The paper focuses on the money-real interest rate-output link suggested by monetary theories of output. In both Keynesian theory and the equilibrium models money affects current activity by altering perceptions of intertemporal terms of trade. The Keynesian "liquidity preference" theory posits that the link between money and interest rates is direct and causal; the nominal interest rate changes are changes in the real rate relevant for a firm's investment decision and hence output. In the newer equilibrium theories, the connection between money, perceived real interest rates and output is more subtle. In the theories of Lucas (1972) and Barro (1978, 1980), fluctuations are portrayed as stemming from the response of labor supply to perceived temporary abnormal rates of return. A key assumption of the newer theories is that there are barriers to information flows and agents use observed nominal price signals as imperfect summaries from the rest of the world. These theories imply that monetary phenomena may effect perceptions of the real rate only to the extent that such disturbances are not directly perceived as such, but are confused with real changes.

The empirical relationship between money, nominal interest rates, and output has recently been studied by Sims (1979, 1980). Using a four-variable autoregressive system (money, prices, output, and a short-term interest rate), Sims found that upward innovations in interest rates were followed by a decline in production after a lag of about six months, reaching a minimum about 18 months later for postwar U.S. data. Equally striking is the fact that the inclusion of interest rates leads to the rejection of Sims' earlier finding that the money stock is strongly Granger-causally prior for income. When interest rates are omitted, monetary innovations explain 37 percent of the 48-month forecast error variance for industrial production; when interest rates are added, the proportion falls to 4 percent. Sims concludes (p. 253) "some of the observed comovements of industrial production and money stock are attributed to common responses to surprise changes in the interest rate." Although the magnitudes and timing of the response differ among the several samples studied, the relationship appears in both prewar and postwar U.S. data and postwar French, U.K., and German data.

From the standpoint of most monetary theories of output, these empirical results are anomalous since the nominal interest rate is a poor proxy for the theoretically meaningful ex ante real rate of interest. As Fama (1975) has shown, a substantial part of the movement in short-term interest rates, at least over the postwar U.S. experience, can be attributed to changes in expected inflation.

In this paper, we attempt to reexamine the empirical relationship between money, interest rates, and output in postwar U.S. quarterly and monthly data, emphasizing the distinction between movements in expected (ex ante) real interest rate movements and movements in expected inflation rates. We look at both unrestricted and restricted vector autoregressions in an effort to find evidence of the channels through which money, inflation, and nominal interest rates affect output.

The paper is organized as follows: In Section I we review the basic results of Sims' four variable vector autoregressions on postwar monthly U.S. data. In Section II we construct proxies for the ex ante real rate which allow us to separate the effects of changes in the expected real rate from changes in expected inflation. In this section, we use a two-stage procedure which first estimates expected prices, and then uses these estimates as data in a vector autoregression to distinguish the effects of expected inflation and real rates on output. We cannot unambiguously attribute the effects of nominal interest rate innovations documented in the first section to either expected inflation innovations alone or to real rate innovations alone. However, there is strong evidence to suggest that upward innovations in expected inflation have a depressing effect on future output which is not readily explained by demand driven models of output. These systems also suggest that the real rate appears to be largely exogenous.

In light of these descriptive statistical results, we go on in Section III to test a number of specific hypotheses concerning money, real rates, and output which are suggested by particular structural models. We first examine the influence of output, money, and prices on the ex ante real short rate. Specifically, we test whether past money, prices, and income have any additional predictive content for current expected real rates given past real rates. We cannot reject the hypothesis that the real rate is exogenous. Although money is useful for predicting nominal rates, it does not appear to influence future real rates. This finding casts strong doubt on the money, real interest link implied by either Keynesian IS-LM analysis and the informationally based equilibrium models. Because the real rate is unobserved, the hypothesis that the real rate is exogenous to money and prices takes the form of nonlinear cross-equation restriction on a vector autoregressive system which is estimated by a maximum likelihood method. We then test a number of hypotheses concerning the relationship between interest rates and output. These tests are designed to pass on the validity of alternative theories of the transmission mechanism between financial variables and real variables. In particular, we are interested in reexamining the evidence that changes in money supply have real effects on output.

Our most robust finding of Section III is that expected inflation has more explanation for output than does the level of the expected real interest rates. This finding is difficult to interpret in the context of both IS-LM analysis and most intertemporal versions of the equilibrium models. In light of this, in Section IV we construct an alternative structural model which is consistent with the data. The model has the central features that the money supply process is governed by the desire for short run price stability; and changes in money supply do not affect output. A test of this model is implemented. Although the results are not definitive, we find the model to be surprisingly consistent with the data. The fifth section is the conclusion.

I. Review of Earlier Work

Using a multivariate, linear time series model, Sims (1979, 1980) showed that nominal interest innovations explain a substantial fraction of variance in industrial production. Furthermore, the inclusion of interest rates decreases significantly the fraction of variance in industrial production attributed to innovations in the money supply.

Table I shows the decomposition of variance of industrial production in both a three variable (industrial production, IP; money stock, M1; and consumer prices less shelter, CPI) and a four variable (plus end-of-month nominal interest rate on Treasury bills with one month to maturity,[1/] BILLS1) vector autoregression at various time horizons. In both systems the data is in logs and is monthly for the period 1949:1 to 1981:12. Twelve lags of each variable were estimated.

In the three variable system, the test of the hypothesis that all twelve lags of money have zero coefficients in the output equation may be rejected at the 1 percent level (the F-test has a marginal significance level of .004), but this is not true in the four variable system (marginal significance of .018). As can be seen in Table I, the dominance of interest rate innovations over money innovations becomes stronger as the time horizon for predicting output lengthens. This accords with Sims' finding that the response of output to interest rate innovations is essentially flat for about six months, followed by a smooth decline reaching a minimum of about 18 months later.

Table I

Decomposition of Variance of Industrial Production
In Three and Four Variable Systems[a]

Forecast Horizon (months)	3 Variable System			4 Variable System			
	IP	CPI	M1	IP	CPI	M1	BILLS1
1	100.0	0.0	0.0	100.0	0.0	0.0	0.0
12	71.4	5.4	23.2	77.9	3.6	5.7	12.8
24	51.5	16.7	31.9	55.2	10.8	3.9	30.1
36	48.9	19.8	31.2	51.7	11.8	3.1	33.3
48	50.5	19.3	30.1	50.4	11.4	2.8	35.4

a. Entries give the percentage of forecast error variance accounted for by orthogonalized innovations in the listed variables. The orthogonalization order is as they are listed.

As a further check of the robustness of this nominal interest rate--output link,[2/] the four variable system was reestimated separately for the two periods 1955:1 to 1971:12 and 1972:1

to 1981:8. For this comparison only six lags of each variable were included. This further restriction is motivated by the desire to perform hypothesis tests described in the next section in which long lag lengths are computationally unwieldy. In a likelihood ratio test the hypothesis of six lags versus 12 lags is not rejected (marginal significance .06). The six lag system indicates that the interest rate output relationship is stable over time. In Figure I, the moving average response of each of the four variables to an innovation in nominal interest rates is presented for each period. In Figure II, the response of industrial production to an innovation in each of the four variables is shown. In both periods, output declines in response to interest rate innovations. This response is much quicker in the more recent period; there is no discernable lag and the response is strongest at the 12-month horizon. In the earlier period, a six-month lag is evident and the maximum impact is at the 24-month horizon. In both periods, interest rate innovations are followed by a decrease in nominal balances.

Although a standard test of structural stability of coefficients is overwhelmingly rejected (Chi-Square test with 96 degrees of freedom = 180.92, marginal significance < 10^{-6}),[3/] the qualitative properties of the impulse responses look remarkably similar. This similarity should give pause to those who argue that the preponderance of "supply shocks" in the more recent period has radically altered the dynamic interactions between money, prices, interest rates, and output.

RESPONSES TO NOMINAL INTEREST RATE INNOVATIONS

FIGURE Ia. ESTIMATION PERIOD 1949:1-1971:12

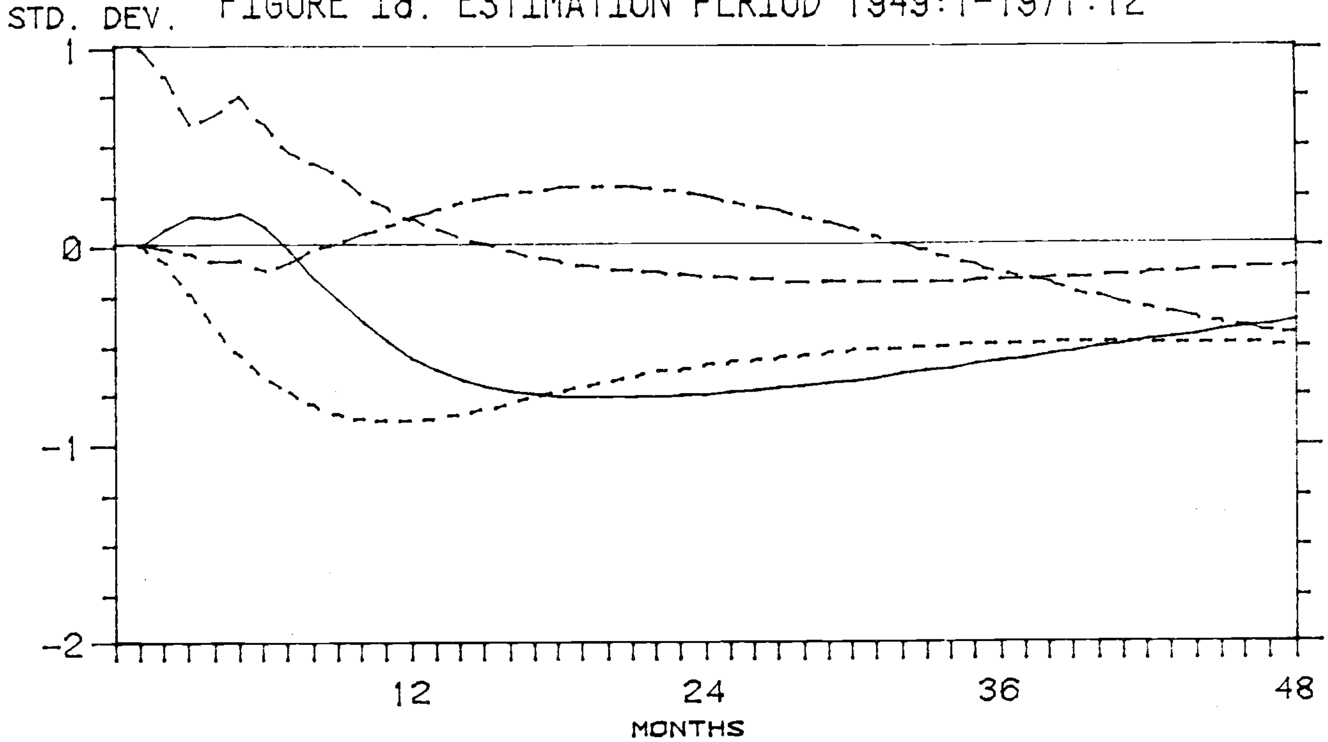
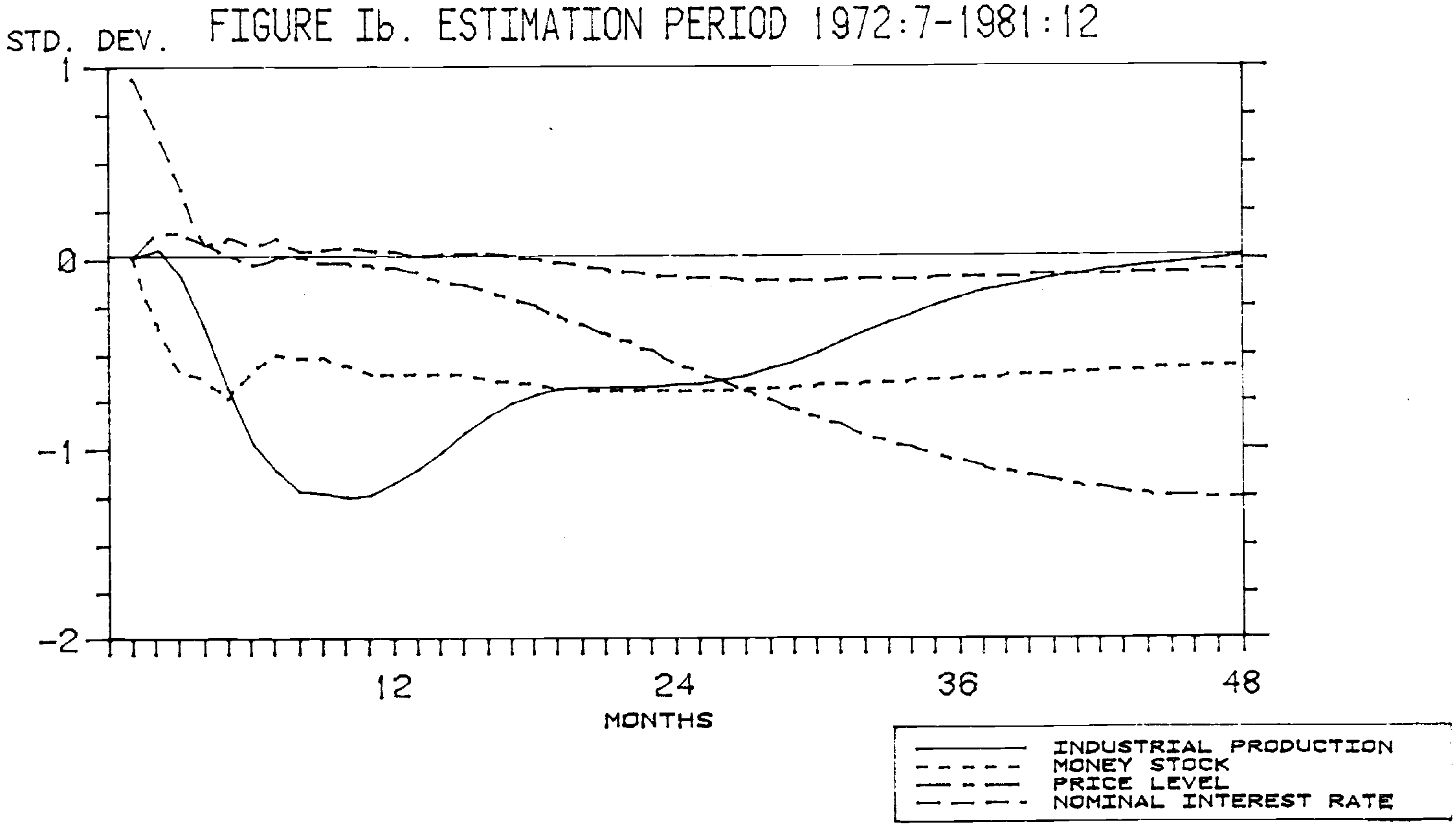


FIGURE Ib. ESTIMATION PERIOD 1972:7-1981:12



RESPONSES OF INDUSTRIAL PRODUCTION

FIGURE IIa. ESTIMATION PERIOD 1949:1-1971:12

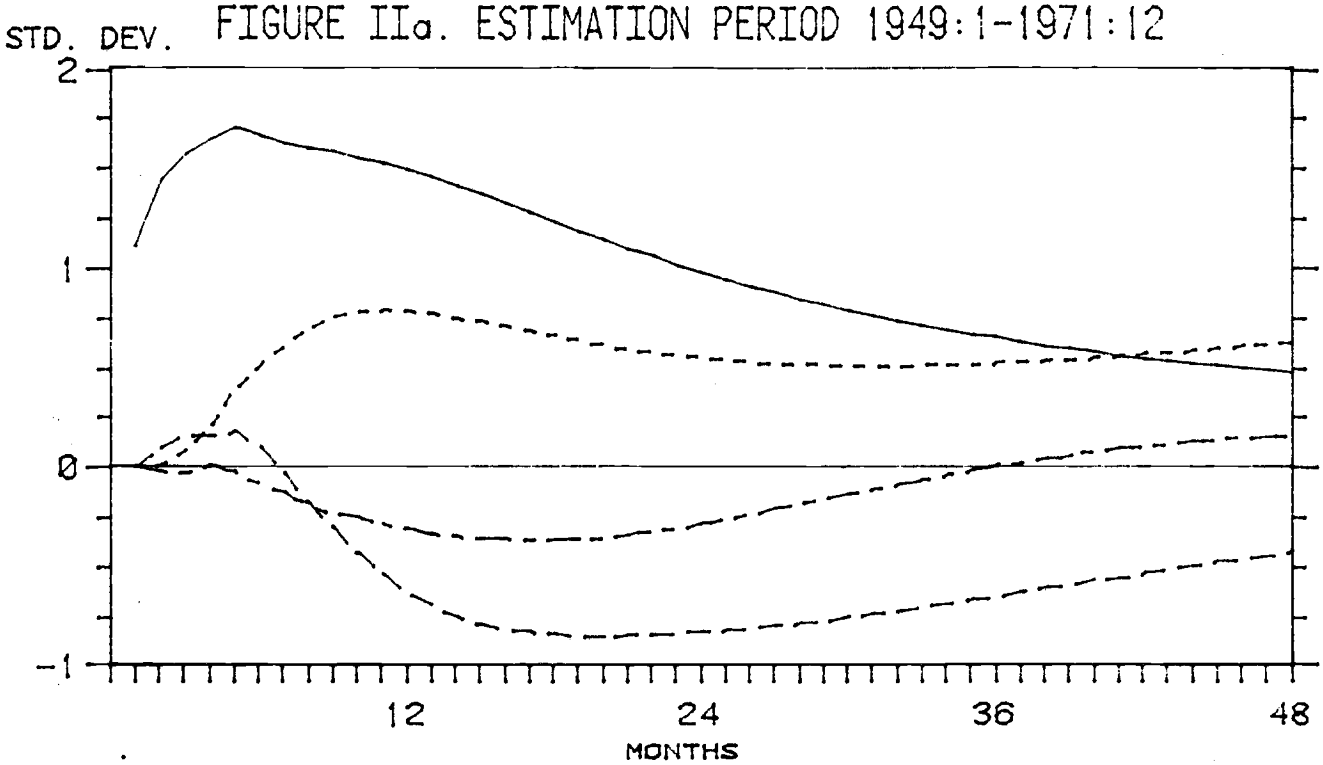
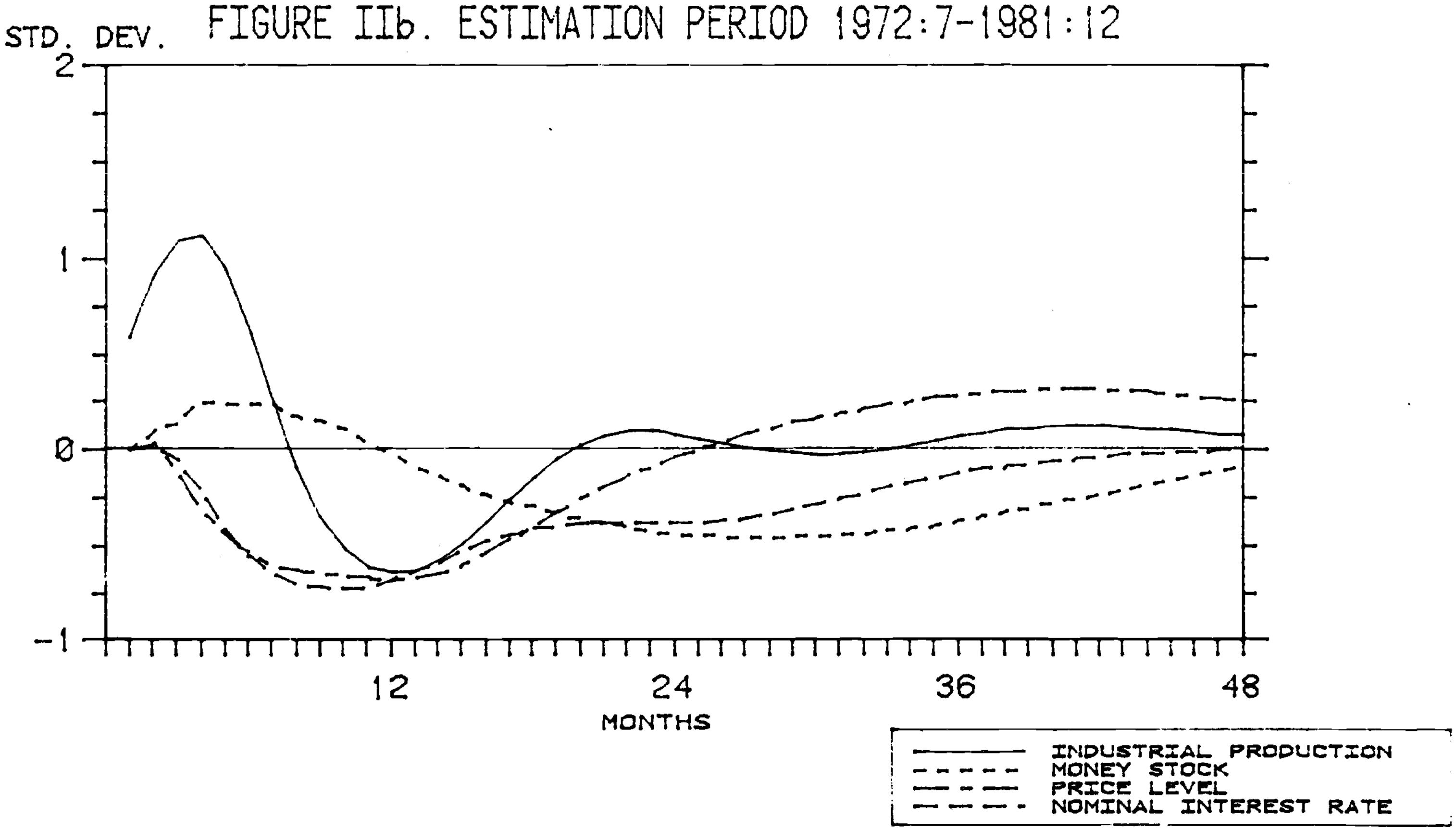


FIGURE IIb. ESTIMATION PERIOD 1972:7-1981:12



II. Interest Rates and Output--Real or Nominal?

Most macroeconomic theories suggest that real interest rates, not nominal rates, should play an important role in the determination of future output. Thus, if most variations in nominal interest rates reflect changes in anticipated inflation, then the response of output to innovations in nominal interest rates documented in the previous section is surprising. In this section we attempt to formulate proxies for the ex ante real rate to get a better idea of whether the nominal interest rate innovations isolated in the preceding section represent innovations in the real rate, or innovations in expected inflation.

The first proxy for the expected real rate is an out-of-sample forecast derived by projecting the log of prices at each point in time on a constant and three lags of data using a constant coefficient Kalman filter technique. This procedure is equivalent to reestimating an OLS regression each period.[4/] The resulting monthly expected inflation, at time t, $\hat{P}_t = E(CPI_{t+1} | M1_{t-s}, CPI_{t-s}, IP_{t-s}, BILLS1_{t-s}, s=0,1,2) - CPI_t$ is presented in Figure III. By subtracting the expected inflation from BILLS1, the one-month nominal interest rate, we generate an ex ante one-month real rate, $\hat{r}$, which is presented in Figure IV.

Movements of the one-month ex ante real rate which we have constructed are to a large extent dominated by the movements in expected inflation, which are subtracted from the more stable nominal rate. One possible concern with this measure is that if real rates with longer maturity are more important for economic decisions than short rates, and if inflation expectations over

EXPECTED AND REALIZED INFLATION RATES

FIGURE IIIa. 1 - MONTH HORIZON

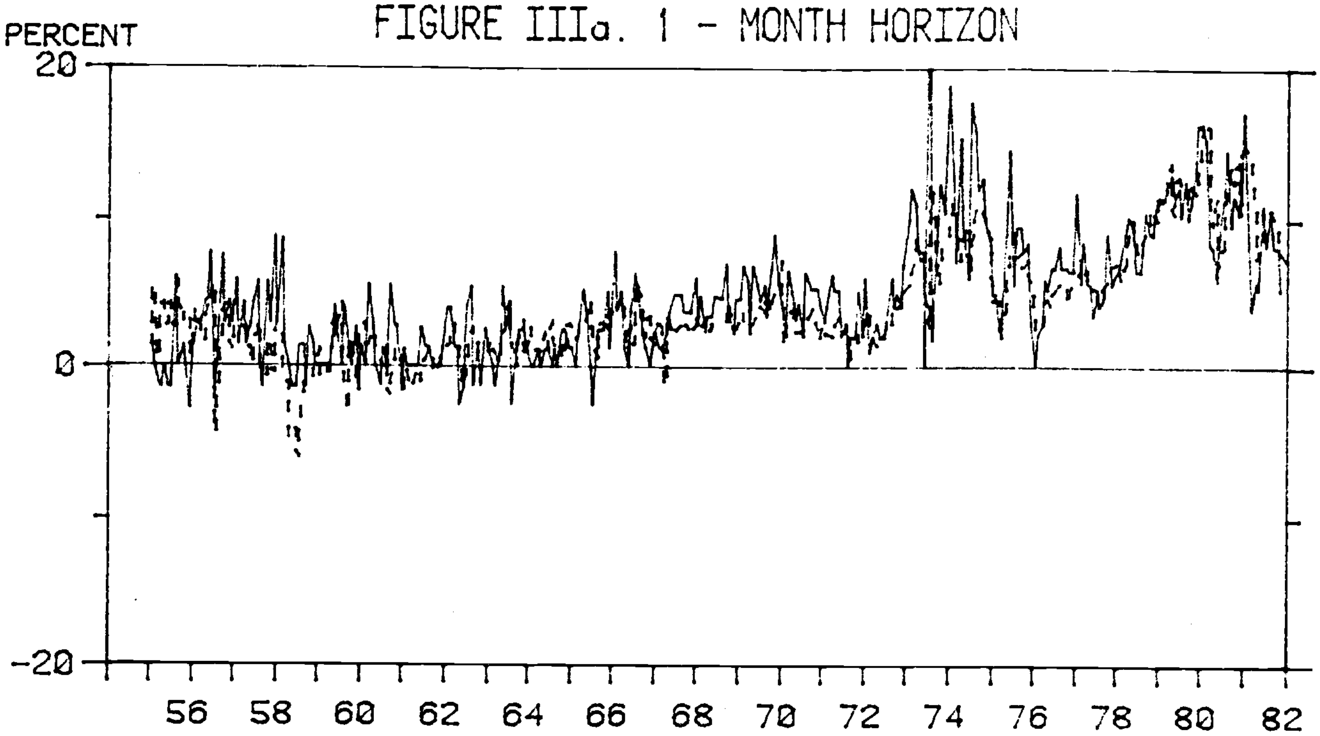
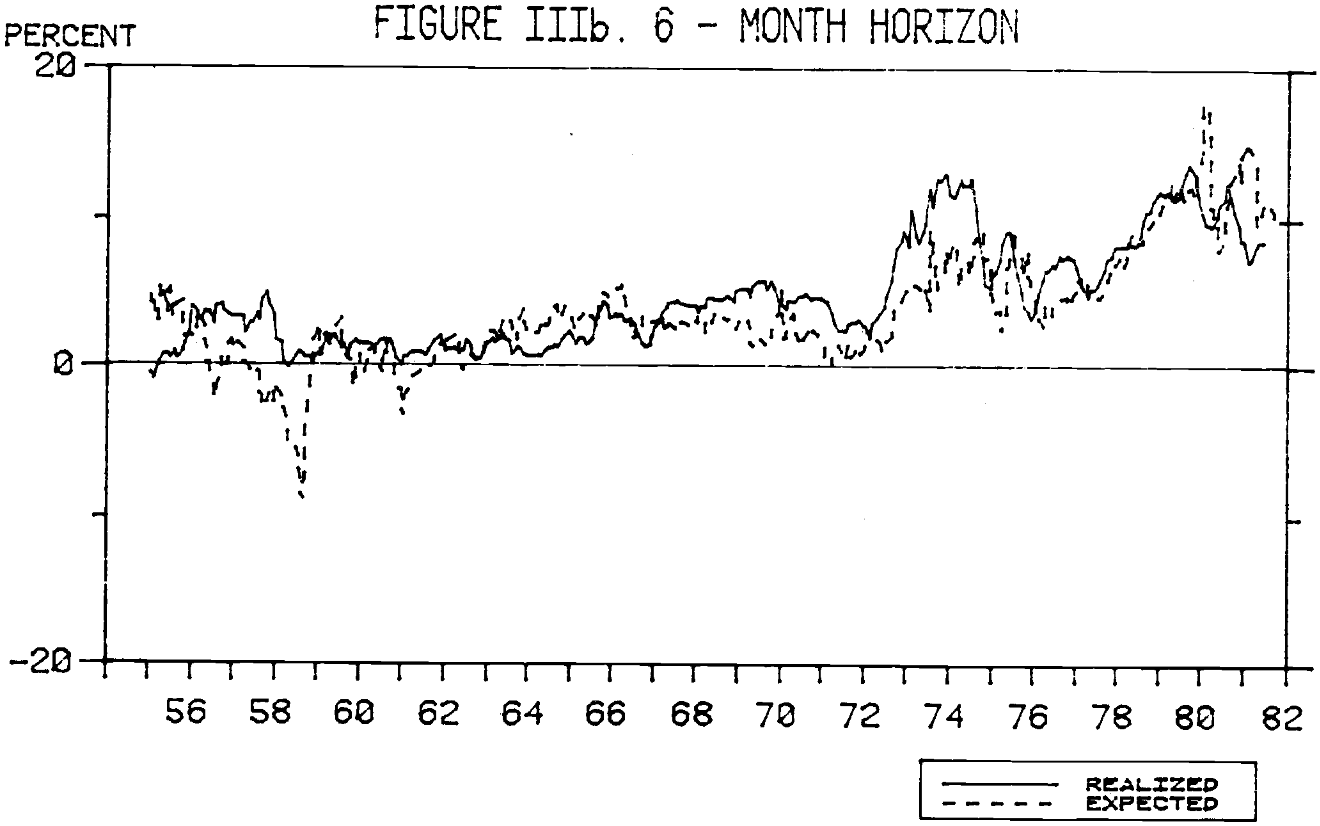


FIGURE IIIb. 6 - MONTH HORIZON



EXPECTED AND REALIZED REAL RATES

FIGURE IVa. 1 - MONTH HORIZON

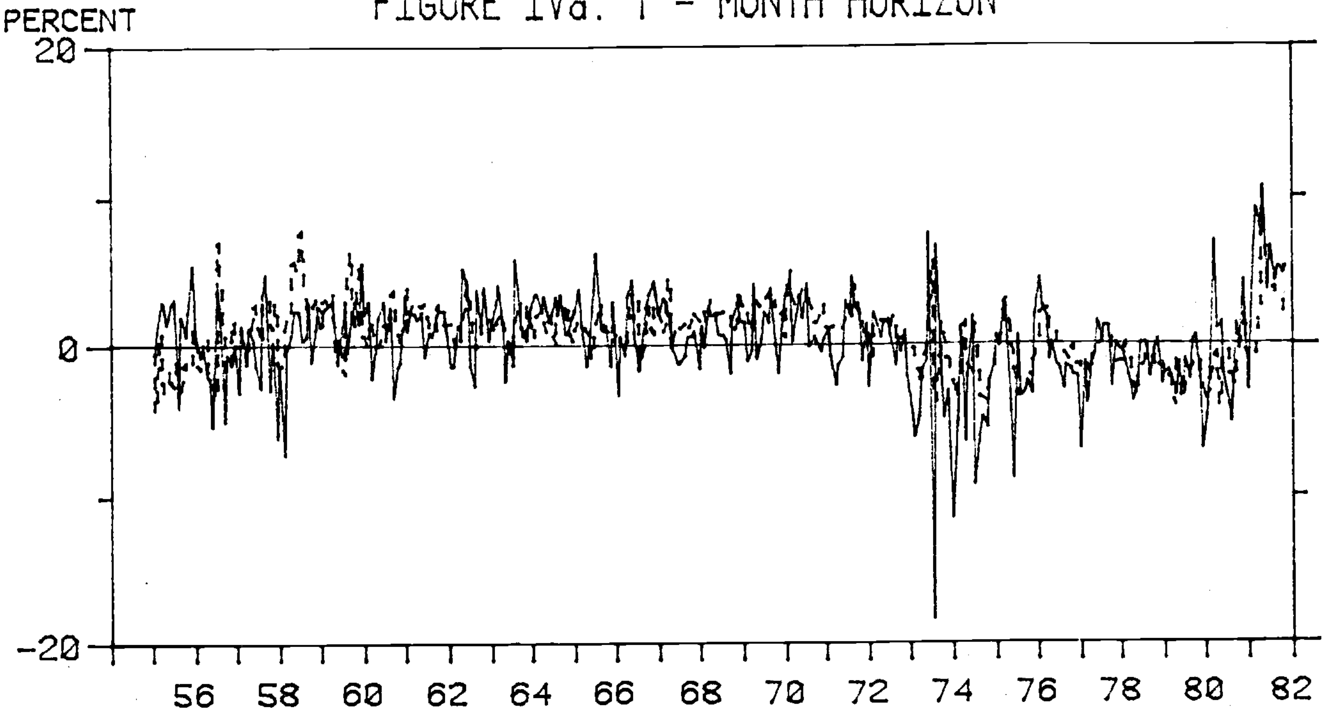
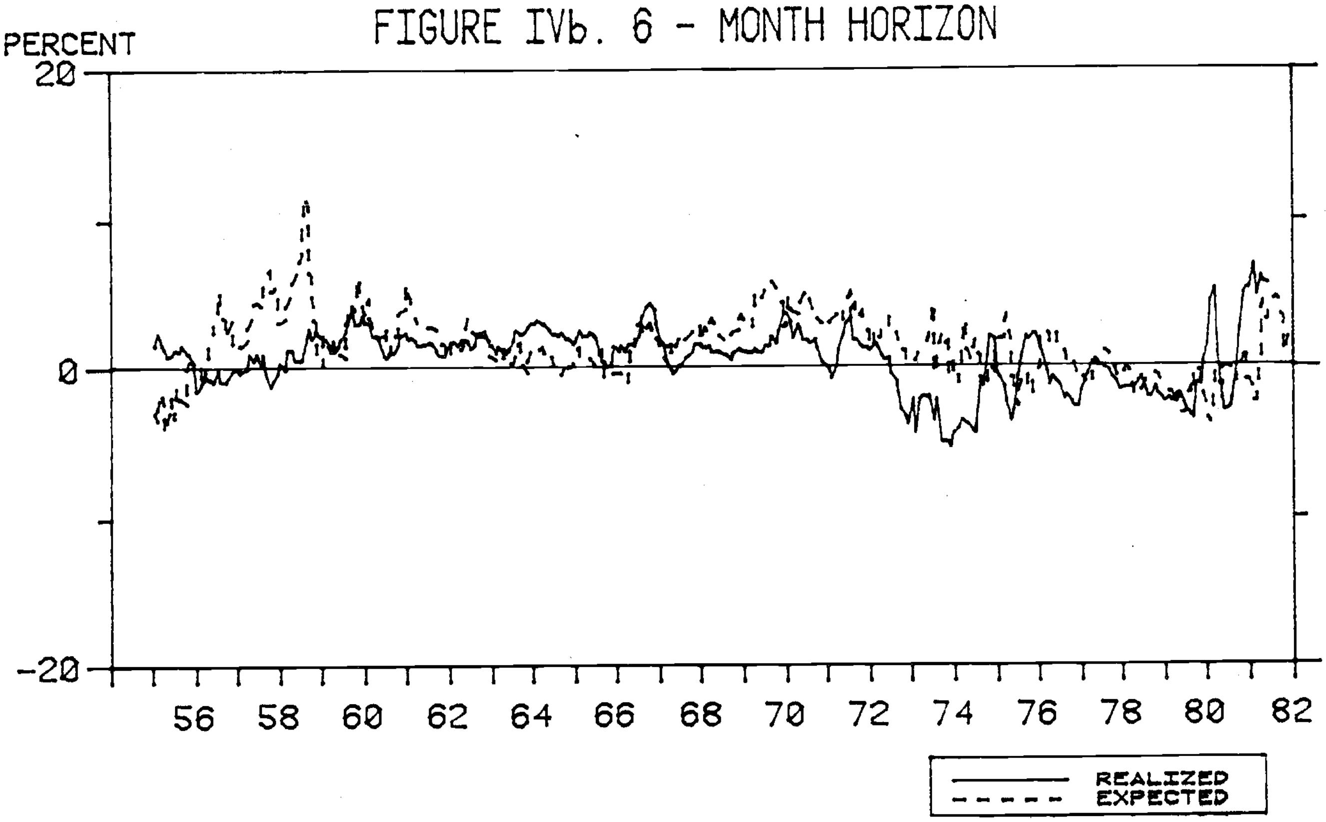


FIGURE IVb. 6 - MONTH HORIZON



longer horizons are less volatile, then a measure of the real rate over a longer maturity might explain more of the variation in output than one with a shorter maturity.

In order to test this hypothesis and to see whether the results we found were sensitive to our measure of the real rate, we have experimented with several other measures including a quarterly real rate, a six-month real rate, and a one-month ex post real rate.[5/] Although the longer maturity real rates are much smoother than the short rates, the qualitative properties of the response functions and decompositions of variance were not affected by the different definitions. In fact, we found that innovations in the one-month real rate, despite the fact that they are approximately 50 percent larger than innovations in the six-month rate, actually explained more of the forecast variance of output.

Our monthly ex ante real rate series were employed in several four variable, autoregressive systems using six lags and a constant term in each equation. The major difficulty in interpreting these systems arises from the strong contemporaneous correlation between innovations in expected inflation and innovations in the expected real rate. Table II reports the variance-covariance matrix of the innovations arising from systems which include industrial production, money, nominal rates, and expected inflation or real rates. These matrices are singular, of course.

Table II

Variance-Covariance Matrices of Innovations
(Entries Below Diagonal are Correlations)

One-Month Real Rate

(Monthly data 1955:7 to 1981:12)

	IP	M1	BILLS1	$\hat{P}$	$\hat{r}$
IP	.000081	.000003	.000636	-.003660	.004296
M1	.11	.000010	.000397	.001705	-.001308
BILLS1	.13	.22	.310370	.260240	.050122
$\hat{P}$	-.28	.36	.32	2.1582	-1.8980
$\hat{r}$	.34	-.29	.06	-.93	1.9481

Six-Month Real Rate

(Monthly data 1955:7 to 1981:12)

	IP	M1	BILLS6	$\hat{P}$	$\hat{r}$
IP	.000083	.000002	.000655	.001714	-.001059
M1	.08	.000010	.000431	.001823	-.001392
BILLS6	.16	.29	.211320	.145930	.065387
$\hat{P}$	.18	.54	.31	1.0759	-.93001
$\hat{r}$	-.12	-.43	.14	-.90	.99540

One-Quarter Real Rate

(Quarterly data 1956:1 to 1981:4)

	IP	M1	BILLS3	$\hat{P}$	$\hat{r}$
IP	.000374	.000030	.004401	.006274	-.001873
M1	.29	.000030	.000927	.003767	-.002839
BILLS3	.30	.22	.59448	.51677	.077712
$\hat{P}$	.20	.42	.40	2.7542	-2.2374
$\hat{r}$	-.06	-.34	.07	-.89	2.3151

As can be seen in Table II, there is a strongly negative correlation between innovations in expected inflation and innovations in the real rate (-.93 for the one-month rate). Also striking is the fact that the variance of both expected inflation innovations and real rate innovations in these systems is four to six times larger than that of nominal rate innovations. Apparently, most of the innovations to expected inflation are negatively correlated with innovations to real rates so as to leave the nominal rate largely unaltered. It is interesting to note the strong positive correlations among expected inflation, M1, and nominal rate innovations.6/ The positive correlation between money and nominal rate innovations suggests that demand shocks have dominated the unexpected movements in money. The strong positive correlation between nominal rates and expected inflation implies that a given movement in nominal rates is much more likely to reflect changes in expected inflation than changes in the real rate. For example, based on the correlations in Table II, in the quarterly system a 1 percent innovation in the nominal rate is most likey to reflect an increase of .86 percent in expected inflation and an increase of only .14 percent in expected real rates.

The high negative correlation between real rate innovations and expected inflation innovations implies that, unlike the system examined in Section I, the qualitative properties of the moving average response graphs and the decomposition of variance might be expected to depend on the particular orthogonalization chosen. This is confirmed in Table III, which reports the variance decomposition of output in three alternative systems which all lead to equivalent predictions of future values.

Table III

Decomposition of Variance of Industrial Production at Various Forecast Horizons With Various Orderings of Ex Ante Inflation and Real Rates[a]

Month	IP	M1	$\hat{P}$	$\hat{r}$ or BILLS1
12	82.1	1.4	9.7	6.7
24	57.1	3.0	27.6	12.2
36	42.2	4.1	41.2	12.6
48	34.8	4.1	49.3	11.8
Month	IP	M1	BILLS1	$\hat{P}$ or $\hat{r}$
12	82.1	1.4	11.3	5.1
24	57.1	3.0	24.4	15.4
36	42.2	4.1	28.6	25.1
48	34.8	4.1	29.4	31.7
Month	IP	M1	$\hat{r}$	$\hat{P}$ or BILLS1
12	82.1	1.4	3.9	12.6
24	57.1	3.0	11.6	28.2
36	42.2	4.1	19.8	34.0
48	34.8	4.1	25.5	35.6

a The table is based on one-month to maturity real rates. Data are monthly from 1955:7 to 1981:12

The linearity of the vector autoregression system and the identity, real rate ≡ nominal rate - expected inflation, implies that given one of these variables, the predictive content for output is identical whichever of the other two variables is included. Thus, in the first vector autoregression examined in Table III, the third column is orthogonalized innovations in expected inflation and the fourth column can be interpreted alternatively as orthogonalized innovations in either nominal rates or real rates.

As we saw in the earlier systems, a high proportion of the varaince in industrial production is explained by innovations in interest rates. In the system shown here, over 60 percent of the variance of industrial production at a four-year horizon is explained by orthogonalized innovations in any two of the variables, nominal rates, real rates, and expected inflation. Using our other measures of the real rate, this proportion varied from 39.4 percent for the six-month rate to 64.8 percent for the quarterly system using a bill rate with three months to maturity.

As can be seen in Table III, the proportion of this variance explained by orthogonalized innovations in expected inflation or real rates varies considerably with the ordering chosen. This sensitivity is to be expected given the high correlation between the two. Nonetheless, it is possible to detect a pattern which suggests that expected inflation may be the more important factor. Comparing the first system in Table III with the third system, it can be seen that of the 61.1 percent of variance explained, much more is attributed to expected inflation innovations when they are third in the ordering than to real rate innovations when they are third. Shown below is the breakdown of this variance for each of the four systems we looked at.

Table IV

Variance of Industrial Production Explained
by Expected Inflation or Real Rate Innovations

	Total Variance Explained	Percent of Total Explained by Third Variable When		
	By $\hat{P}$ and $\hat{r}$	$\hat{P}$ Third	$\hat{r}$ Third	Bills Third
One-Month Rate	61.1	81 %	42 %	48 %
Six-Month Rate	39.4	24	1	89
One-Quarter Rate	64.8	55	12	77
One-Month Ex Post Rate	63.3	68	49	47

In every case expected inflation innovations, when third in the ordering, explain more than real rate innovations when they are third. It is interesting to notice also that at the longer maturity rates much less of the total variance being explained by the combination is explained by either when it appears first. This aspect of the results suggests that nominal interest rate innovations alone contain much of the information useful for predicting output. Expected inflation contains a large amount of high frequency variation, which when subtracted from the nominal rate, can reduce the information content of that series.

A clue toward understanding the industrial production response can be found in the moving average response graph (Figure V) which shows that there is a qualitatively different response to output arising from a real rate innovation than from an expected inflation innovation. Figure (Va) shows that when it is third in the ordering, an expected inflation innovation has an immediate and unambiguous negative effect on output. Figure (Vb) shows that when real rates are third they exert a positive response throughout, and a nominal interest rate innovation (equivalently expected

RESPONSES OF INDUSTRIAL PRODUCTION - EX-ANTE EXPECTATIONS

ESTIMATION PERIOD 1955:1-1981:12

MONTHLY DATA, 1 - MONTH EXPECTATIONS

FIGURE Va. INCLUDING EXPECTED INFLATION

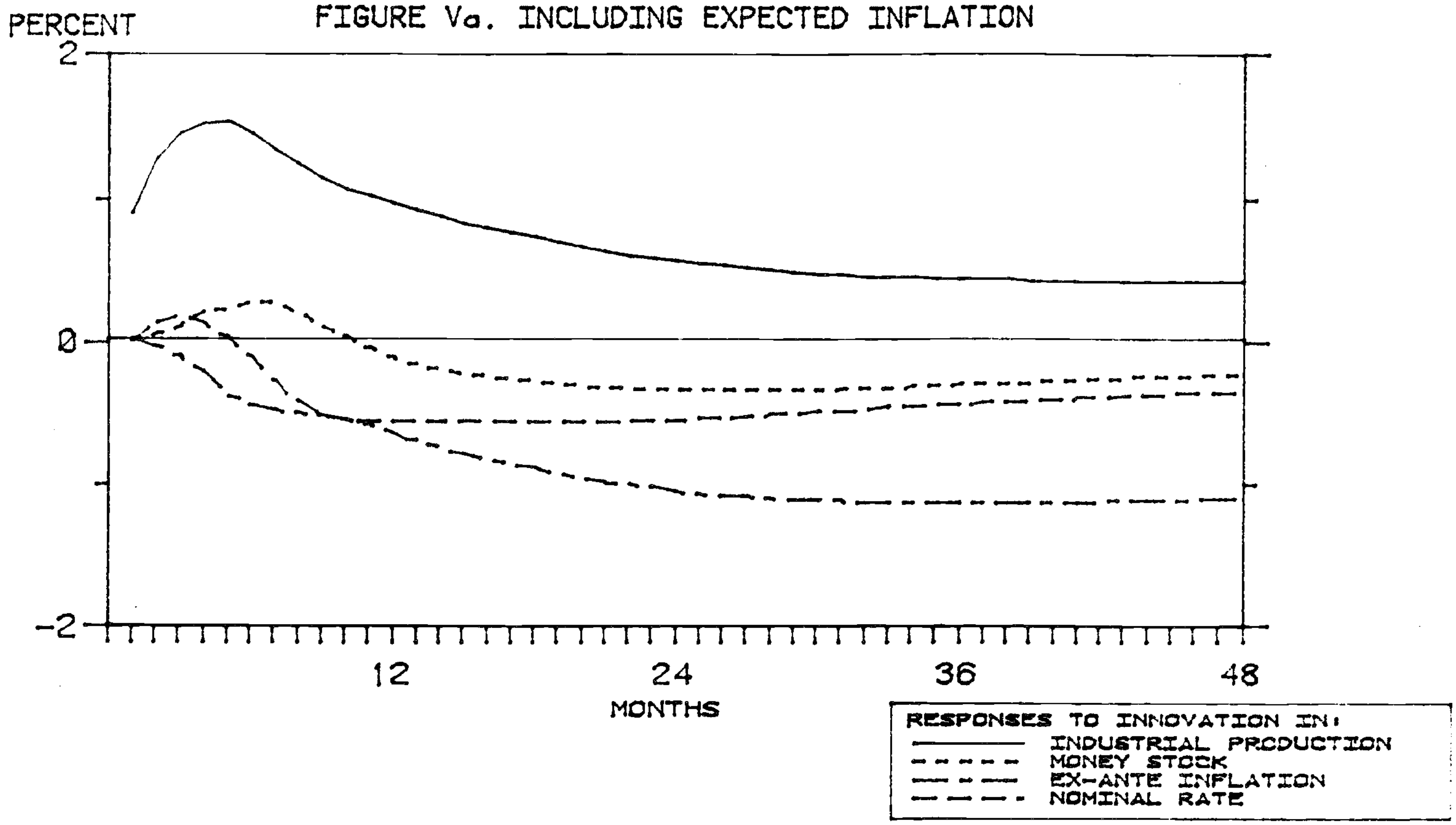
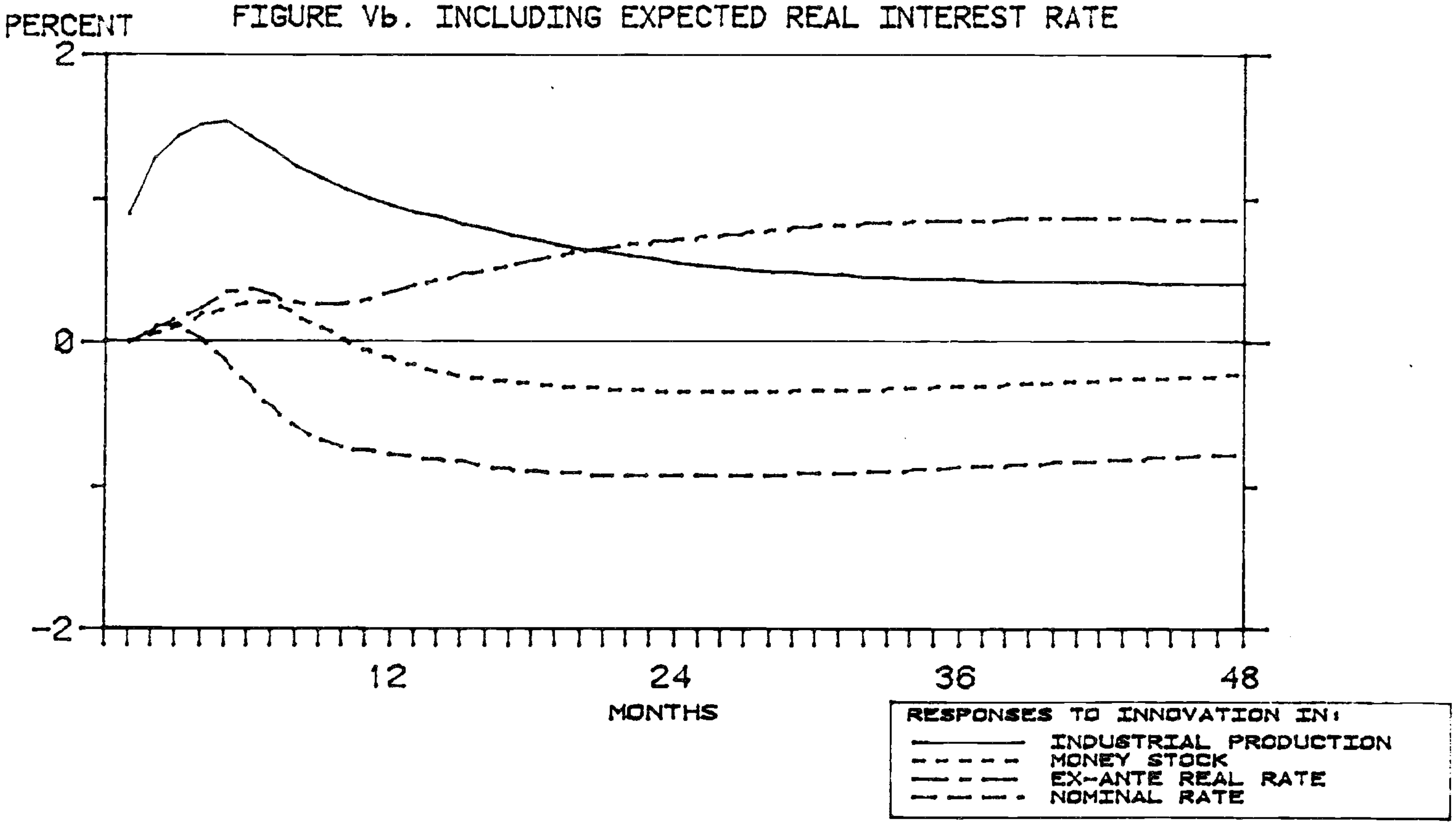


FIGURE Vb. INCLUDING EXPECTED REAL INTEREST RATE



RESPONSES OF INDUSTRIAL PRODUCTION - EX-ANTE EXPECTATIONS

ESTIMATION PERIOD 1955:1-1981:12

QUARTERLY DATA, 1 - QUARTER EXPECTATIONS

FIGURE VIa. INCLUDING EXPECTED INFLATION

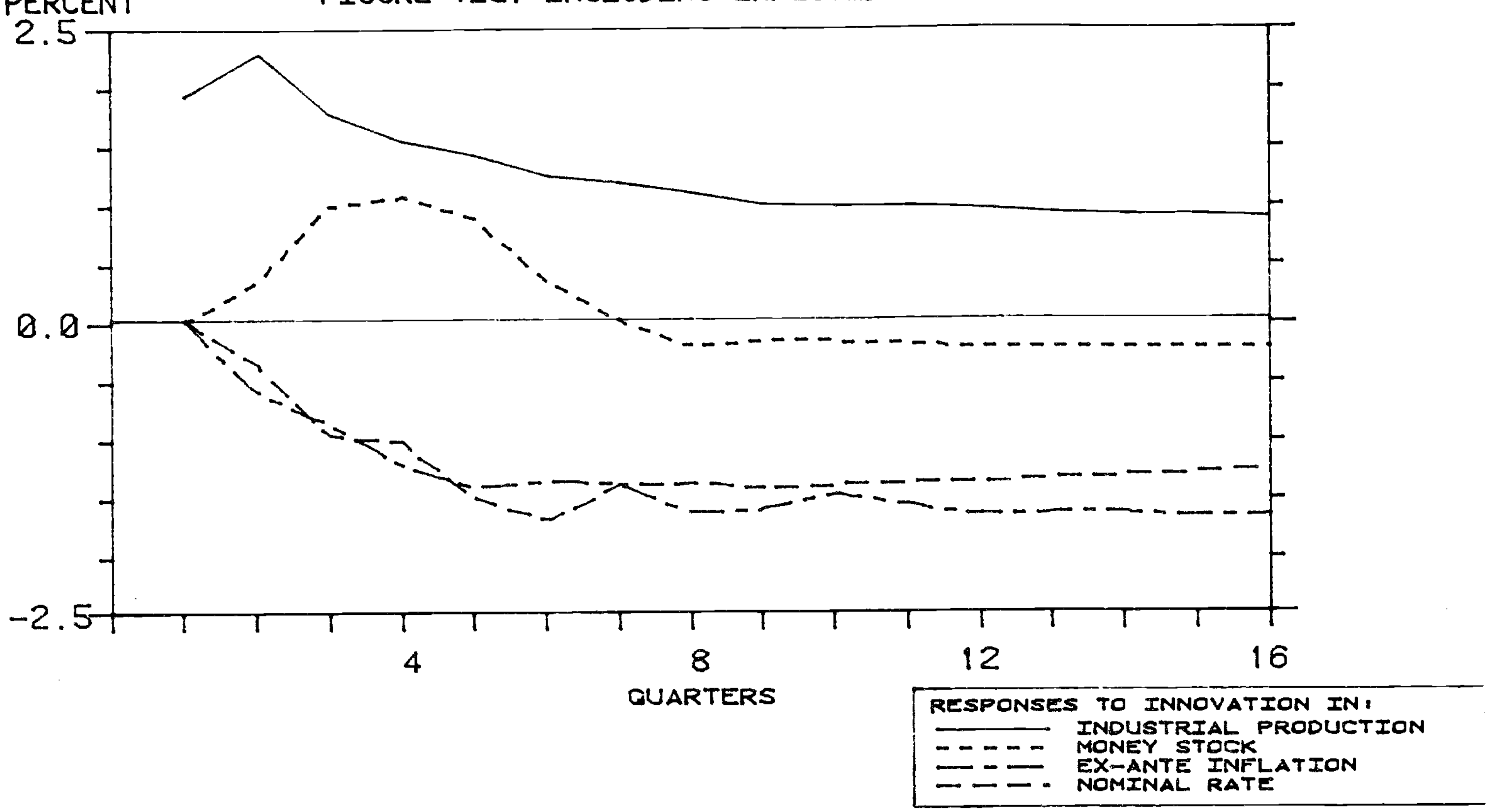
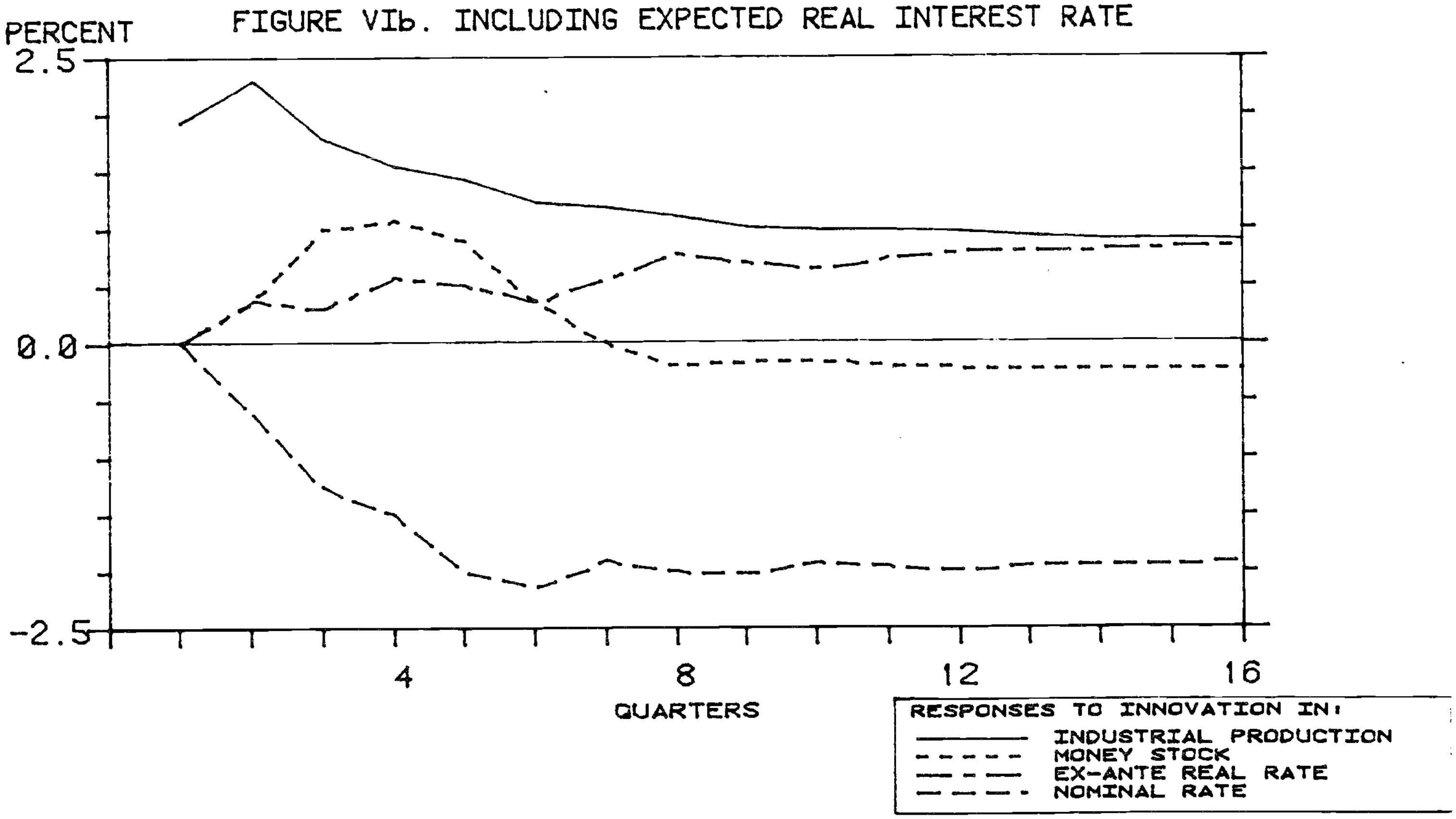


FIGURE VIb. INCLUDING EXPECTED REAL INTEREST RATE



inflation innovation) has a depressing effect past the four-month horizon. This same pattern was evident in all the systems which we looked at. Figure VI shows these responses in the quarterly data with the three-month bills rate. Table IV is designed to test the significance of this decomposition for predicting output. Specifically it calculates the standard error and marginal significance of the moving average responses of a system with real rates third and nominal rates (or equivalently expected inflation) fourth. This chart shows clearly that nominal rates have a significantly negative effect from months six through 48, while real rates have a significantly positive effect over the same horizon. A similar pattern is observed in the quarterly system.

Table V

Standard Errors and Significance Levels
For Industrial Production Response Functions
One-Month Real Rate

Percent Response of Industrial Production to a 100 Basis Point Orthogonalized Real Rate Innovation

Step	Mean	Standard Error	Significance*
1	.000	.000	.00
2	.078	.040	.03
3	.143	.070	.02
4	.195	.098	.02
5	.284	.118	.01
6	.304	.135	.02
12	.286	.192	.08
24	.590	.253	.01
36	.705	.290	.01
48	.705	.303	.01

Percent Response of Industrial Production to a
100 Basis Point Orthogonalized Nominal Rate
(or Expected Inflation) Innovation

Step	Mean	Standard Error	Significance*
1	.000	.000	.00
2	.216	.094	.01
3	.248	.169	.06
4	.053	.232	.43
5	-.217	.286	.22
6	-.527	.335	.06
12	-1.449	.370	.00
24	-1.761	.486	.00
36	-1.655	.622	.00
48	-1.495	.735	.01

One Quarter Real Rate

Percent of Response of Industrial Production to a
100 Basis Point Orthogonalized Real Rate Innovation

Step	Mean	Standard Error	Significance*
1	.000	.000	.00
2	.267	.145	.03
3	.219	.228	.16
4	.413	.280	.06
6	.255	.360	.23
8	.540	.440	.09
10	.447	.498	.17
12	.568	.559	.14
14	.580	.613	.15
16	.609	.663	.17

Percent Response of Industrial Production to a
100 Basis Point Orthogonalized Nominal Rate
(or Expected Inflation) Innovation

Step	Mean	Standard Error	Significance*
1	.000	.000	.00
2	-.847	.276	.00
3	-1.746	.468	.00
4	-2.079	.586	.00
6	-2.994	.707	.00
8	-2.806	.787	.00
10	-2.738	.867	.00
12	-2.815	.982	.00
14	-2.738	1.139	.01
16	-2.677	1.324	.01

*Significance is used here in a Bayesian sense to refer to the integral, given a noninformative prior, of the posterior distribution of the response function less than zero, or greater than zero, whichever is less.

Up to this point we have considered responses of industrial production in systems which include an inflation expectations or real rate variable generated out-of-sample, but which do not include actual inflation rates. In these systems, we have displayed the response to the orthogonalized innovations in the included variables. Another approach we consider is to estimate an unrestricted vector autoregression on the original observed variables, industrial production, money inflation and interest rates, and then to measure the expected inflation and implied real rate implicit in the system. It is then possible to construct responses of industrial production to orthogonalized innovations in expected inflation or real rates even though these variables are not entered directly as observable in the system.

We follow the same technique applied earlier of generating, in turn, responses to expected inflation and real rate innovations when they are third in the orthogonalization ordering. These shocks are defined as linear combinations of the observable innovations where the weights are obtained from the coefficients in the inflation equation. The qualitative properties of these responses, shown in Figure VII, are similar to those generated directly from out-of-sample expectations.[7/] Again, when we decompose the nominal rate symmetrically into expected inflation and expected real rates, the data suggest that it is the expected inflation component which leads to the negative response of industrial production.

Another interesting observation in these systems is that the real rates we looked at are largely exogenous. The real rate responds mostly to itself, dampening very quickly. At the 48-month horizon, 68 percent of its own variance is explained by its own innovations for the one-month out-of-sample ex ante rate and 90 percent for the one-month ex post case. The percent of variance explained by own innovations is 56 percent for the one-quarter rate and 62 percent for the six-month rate. Response functions of the real rate are shown in Figure IX.

III. Specific Tests With Vector Autoregressions

The preceding descriptive empirical findings cast doubt on the money-interest-output link suggested by Keynesian and most demand-driven models of output. Not only did the real rate fail to reflect any systematic influence from money or prices, but output appeared to respond to expected inflation more than to the real rate. In this section, we will examine these results more carefully, testing specific restrictions on the reduced form which reflect on the validity of various economic theories.

RESPONSES OF INDUSTRIAL PRODUCTION
IN SAMPLE EX-ANTE EXPECTATIONS
MONTHLY DATA, 1949:2-1981:8

FIGURE VIIa. INCLUDING EXPECTED INFLATION

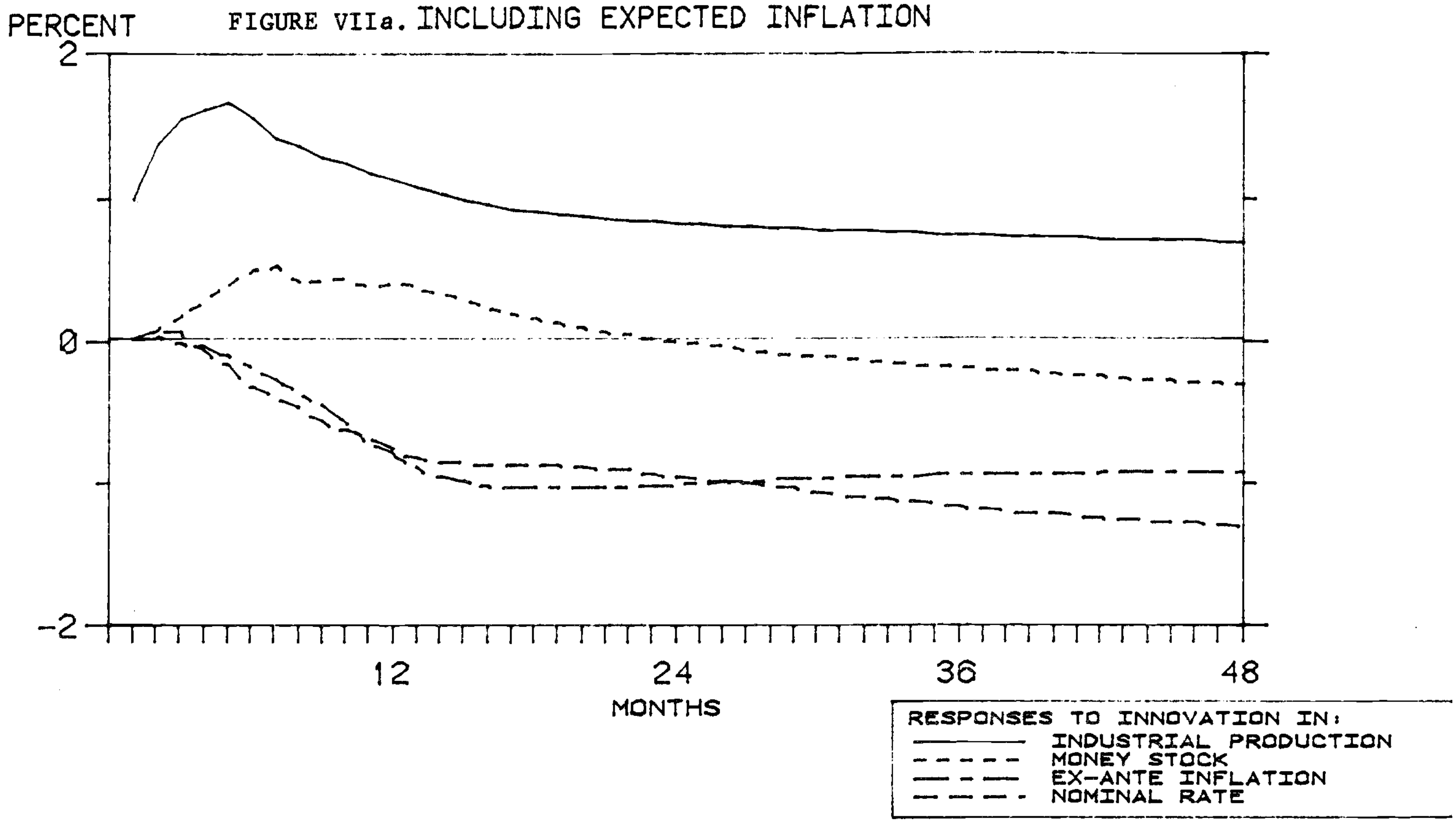
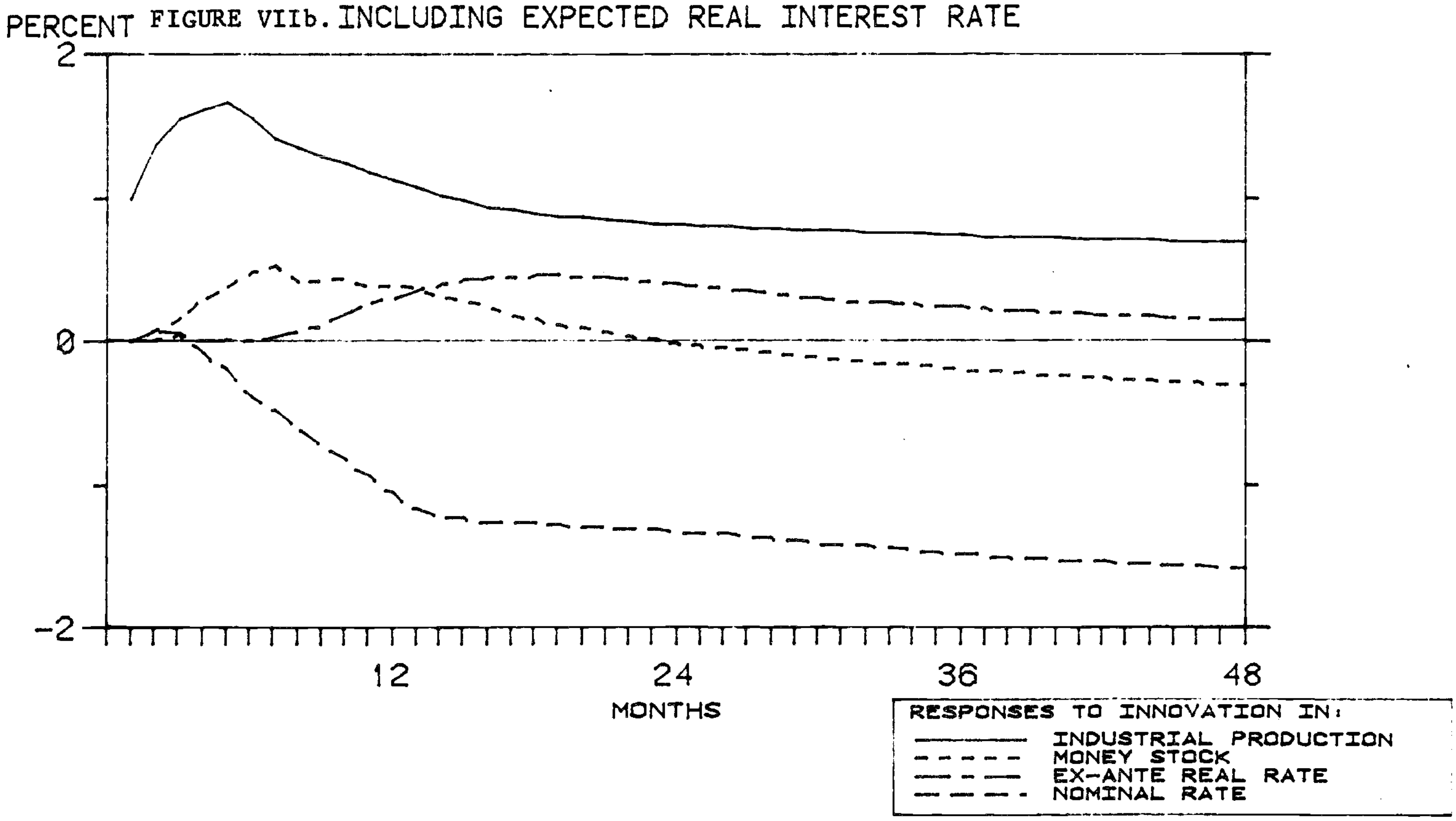


FIGURE VIIb. INCLUDING EXPECTED REAL INTEREST RATE



RESPONSES OF INDUSTRIAL PRODUCTION
IN SAMPLE EX-ANTE EXPECTATIONS
QUARTERLY DATA, 1950:2-1981:2

FIGURE VIIIa. INCLUDING EXPECTED INFLATION

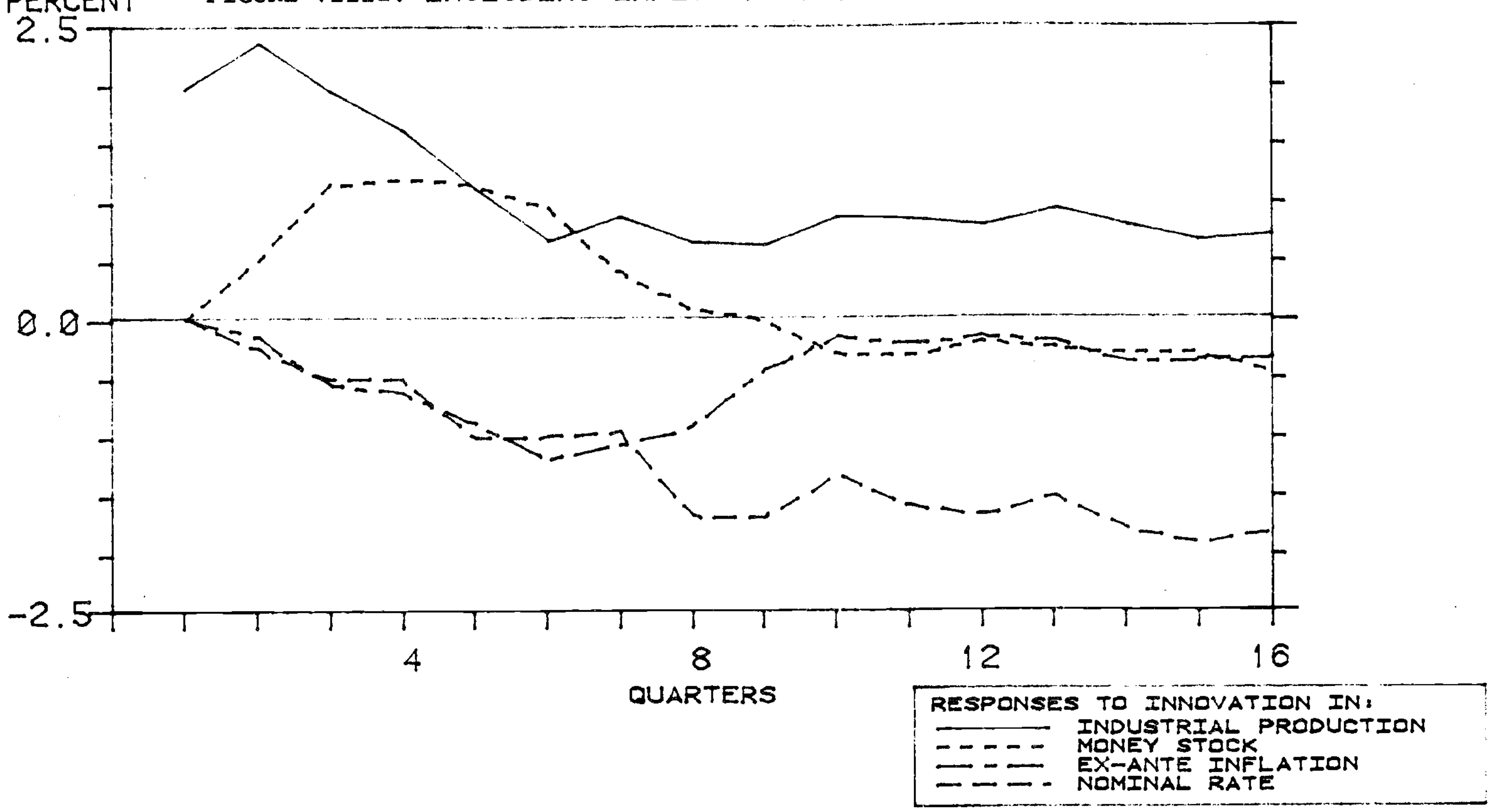
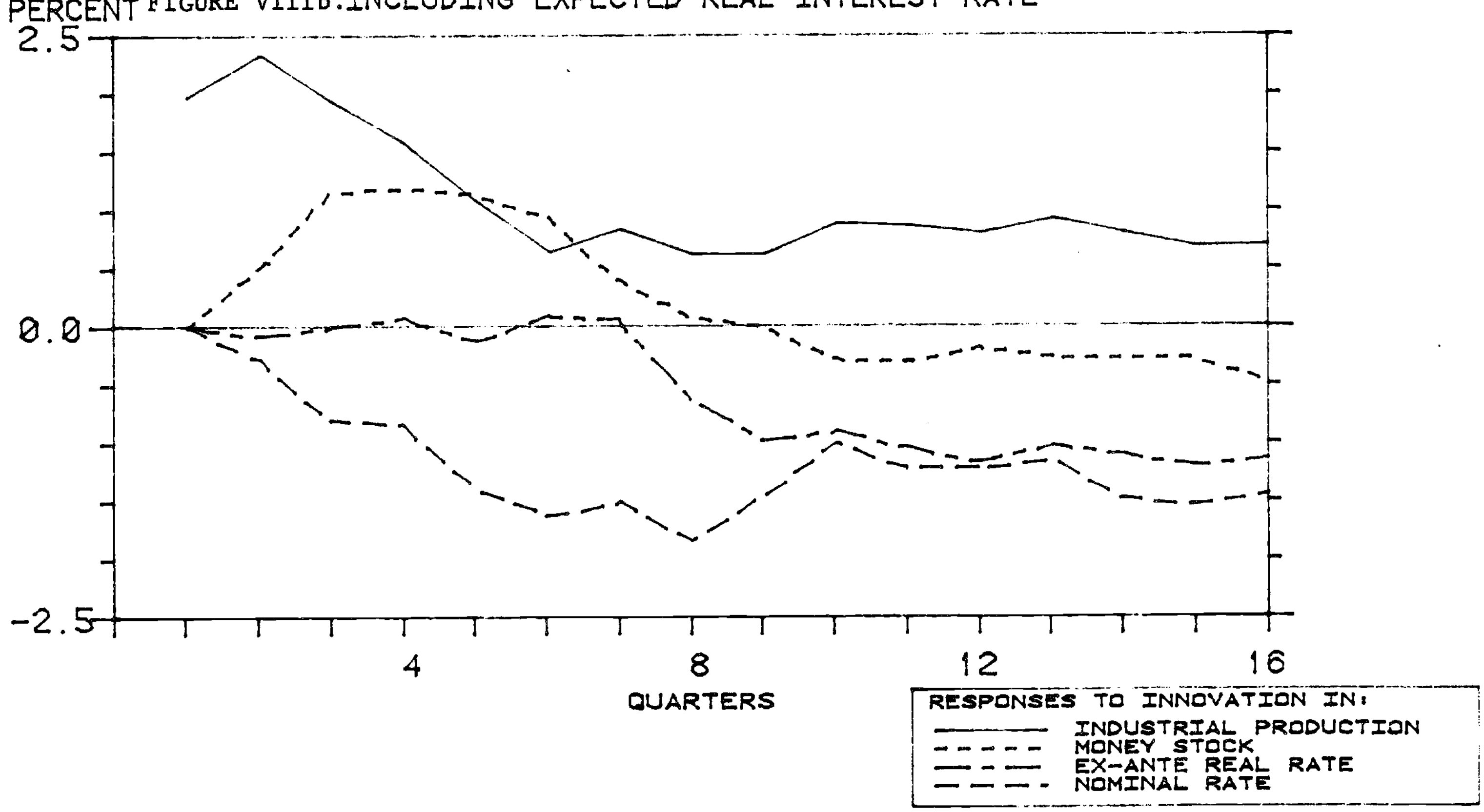


FIGURE VIIIb. INCLUDING EXPECTED REAL INTEREST RATE



RESPONSES OF THE EX-ANTE REAL INTEREST RATE

FIGURE IXa. 1 - MONTH RATE

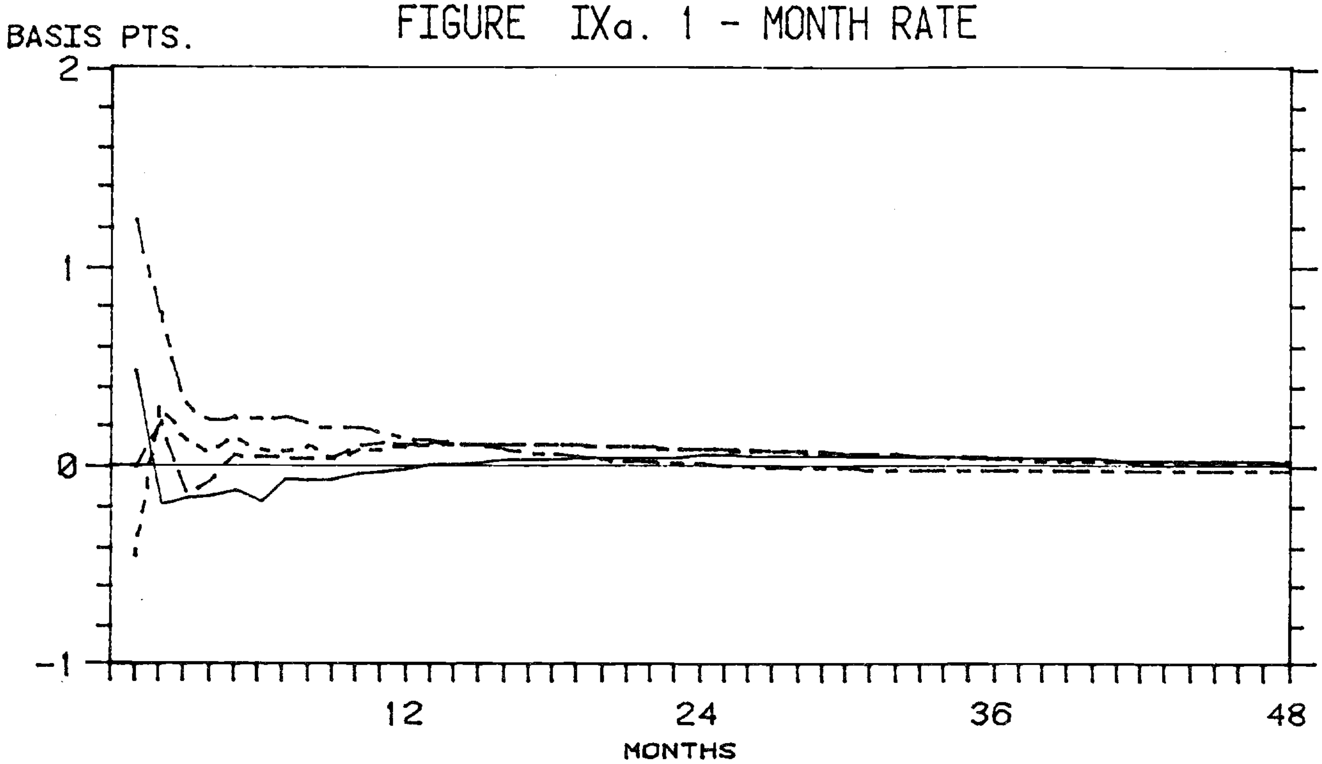
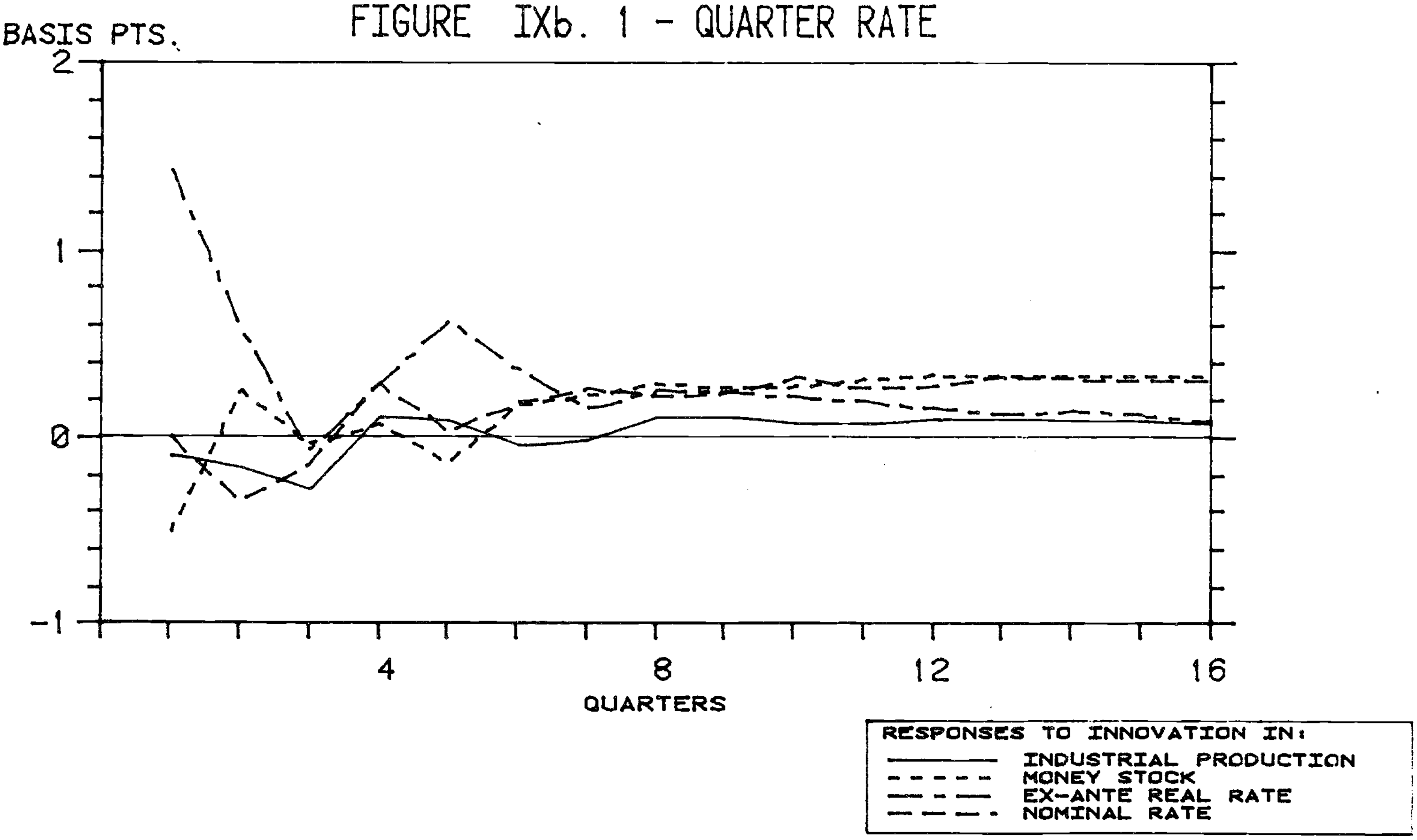


FIGURE IXb. 1 - QUARTER RATE



A. IS THE REAL RATE EXOGENOUS?

We begin by testing a restriction, suggested by our Section II results, which we feel is incompatible with theories that emphasize a role for the real rate of interest in transmitting monetary disturbances to the real economy. In particular, we test the restriction that past money, prices, and income have no additional predictive content for current real rates, given past real rates. That is, we test the hypothesis that the real rate is exogenous, or Granger causally prior, in the content of this four variable system.

Because the ex ante real rate is unobservable, testing this hypothesis requires an auxiliary hypothesis of how agents forecast future prices. We will assume that agents' expectations are rational, which in the context of our information set and in the absence of any further restrictions, identifies price expectations with the projection of future prices on current and lagged endogenous variables.

As is often the case, the imposition of the rational expectations hypothesis here leads to complicated, nonlinear cross-equation restrictions. While the imposition of these restrictions is costly in terms of computations, we find that it generates test statistics which have greater power to differentiate among hypotheses than other approaches such as Fama (1975), Fama (1982), Nelson and Schwert (1977) and Garbade and Wachtel (1978).

Interpretation of causal orderings as indicative of behavioral or structural relationships is a complicated and subtle issue (see Sims (1972, 1975)). Nevertheless, we would expect that IS-LM models would, in general, reject exogeneity of the real rate. Thus, we believe the failure to reject would raise questions about the validity of such models. We believe the test also bears on the empirical validity of the informationally constrained equilibrium models, even though our measure of the expected real rate ignores the limitations on current period information which are essential ingredients of these models. While in both cases we can imagine versions of the model which would fool us into acceptance of the hypothesis that the real rate is exogenous, we find these special cases implausible.

The compatibility of this hypothesis with the IS-LM model, the Lucas-Barro model, and the Grossman-Weiss model will each be considered in turn.

The IS-LM Model

A central feature of Keynesian IS-LM analysis is the idea that changes in the demand or supply of nominal balances can change the real interest rate. Keynesian theory achieves this connection by invoking sluggish nominal price adjustments in nonfinancial markets, particuarly the labor market.

Consider the following IS-LM model

$$\text{IS} \quad y_t = -\beta_1 r_t + \varepsilon_t \quad \beta_1 > 0$$

$$\text{LM} \quad \frac{M_t}{P_t} = \alpha_1 y_t - \alpha_2 (r_t + \Pi_t^e) + \phi_t \quad \alpha_1 > 0, \quad \alpha_2 > 0 \tag{1}$$

where r_t is the real rate and Π_t^e is expected inflation, ε_t represents all exogenous spending (including government spending and variations in desired investment not related to interest rate movements), ϕ_t represents random influences on real money demand (the state of "liquidity preference"). The reduced form equations for the endogenous variables, r_t, y_t are given by

$$r_t = \gamma_1\varepsilon_t + \gamma_2(m_t-\phi_t) + \gamma_3\Pi_t^e$$
$$y_t = \gamma_4\varepsilon_t + \gamma_5(m_t-\phi_t) + \gamma_6\Pi_t^e \qquad (2)$$

where

$$\gamma_1 = \frac{\alpha_1}{\alpha_2+\alpha_1\beta_1},\ \gamma_2 = \frac{-1}{\alpha_2+\alpha_1\beta_1},\ \gamma_3 = \frac{-1}{\alpha_2+\alpha_1\beta_1}$$
$$\gamma_4 = \frac{\alpha_2}{\alpha_2+\alpha_1\beta_1},\ \gamma_5 = \frac{\beta_1}{\alpha_2+\alpha_1\beta_1},\ \text{and } \gamma_6 = \frac{\beta_1\alpha_2}{\alpha_2+\beta_1\beta_1},\ m_t = \frac{M_t}{P_t}. \qquad (3)$$

An implication of this theory is that, unless β_1, the interest elasticity of investment demand were infinite, monetary policy can affect output only to the extent it affects the ex ante real rate.

Under what auxiliary hypothesis can this model be compatible with the finding that

$$E(r_{t+1}|r_{t-s},s>0) = E(r_{t+1}|r_{t-s},M_{t-s},P_{t-s},Y_{t-s},s>0)\ ? \qquad (4)$$

One possibility is that, over the observed sample, it was the deliberate objective of Fed policy to set expected real rates in such a way that the two hypothesis are observationally equivalent. This might arise, for example, if the policy objective were to minimize the variance of output, $E(Y_t-\bar{y})^2$, by setting $r_t = \frac{-1}{\beta_1}(\bar{y}-\varepsilon_t)$. If ε_t followed a univariate autoregressive process, then so would r_t. Although we cannot reject this possibility a priori, it is unlikely that desired interest rate targets could be expressed in terms of any single factor, let alone the past history of interest rates. It certainly appears as if policy has aimed for both price and output stability. Since prices and output exhibit some independent variation, it is implausible to take the finding that the real rate is exogenous as indicative of a particular policy reaction function.

Another possibility which could explain the lack of any influence from past money, prices, and output on current ex ante real rates is that the IS curve is horizontal. This would be true if ${}_1$, the interest sensitivity of demand, were infinite, so that variations in money supply or demand affected only output without a measurable impact on interest rates. This possibility is both highly implausible in a monthly system and is easily rejected by subsequent findings.

Still a third possibility, less easily dismissable, is that over the sample period most variations in money supply, m_t, were passive responses to ${}_t$, money demand shocks. Under this hypothesis, there would be no added explanatory power from past money to future real rates. This hypothesis requires either no deliberate attempt on behalf of the Fed for controlling real rates, except insofar as interest rate targets depend only on lagged values, or that policy-induced interest rate variations have been sufficiently small compared with exogenous money demand shifts so that our procedure cannot distinguish this variation from variation due to sample errors.

A fourth possibility which could give rise to a spurious finding of exogeneity concerns the role of omitted variables. Suppose the true reduced form for ex ante real rates is given by

$$r_t = \sum_{j=0}^{\infty} v_j m_{t-j} + \sum_{j=0}^{\infty} w_j z_{t-j} + \varepsilon_t \tag{5}$$

where z_t is a vector stochastic process of omitted variables and w_j is a vector conformable to z_t. Suppose

$$E[z_{t-K} | m_t, m_{t-1} \ldots] = \sum_{j=0}^{\infty} \alpha_{Kj} m_{t-j} \tag{6}$$

Then, in populations, the regression coefficients of r_t on lagged m's are given by

$$h_j = v_j + \sum_{K=0}^{\infty} w_K \alpha_{Kj} \qquad j = 0 \ldots \tag{7}$$

While it is certainly possible that h_j's will be zero, even though the v_j's are nonzero, this is highly unlikely as it requires an extreme coincidence between the v's, w's, and α's.

These possibilities, while being neither mutually exclusive nor exhaustive seem sufficiently implausible to us that failure to reject the hypothesis that the real rate is exogenous casts strong doubt on the Keynesian notion that monetary policy has affected output through changes in the real rate of interest.

The Lucas-Barro Model

The model presented in Lucas (1972) and modified by Barro (1976, 1980) emphasizes the effects of unperceived monetary injections on labor supply by altering perceptions of real rates of return. By positing barriers on current period information flows, these models draw a sharp distinction between expectations based on complete current period information and the expectations held by a representative trader. Nevertheless, the hypothesis that the real rate (based on complete current information) is exogenous would seem incompatible wiht most intertemporal versions of these models.

The original versions of these models assumed all random disturbances to be serially uncorrelated, and all information lags to be at most a single period. These features, while inessential, imply that both concepts of the real rate would be serially independent. Thus, in this limited sense, the models are compatible with the finding that the real rate is exogenous. However, if these models are appended to be consistent with the fact that there are substantial serial correlations in most macroeconomic time series, then it is difficult to reconcile the models.

To see this, imagine that the ex ante real rate, conditioned on aggregate information, is given by

$$r_t = \sum_{j=1}^{n_1} \lambda_j \, r_{t-j} + \sum_{j=0}^{n_2} \alpha_j \, \eta_{t-j} + \phi \, \hat{m}_t + \varepsilon_t \qquad (8)$$

where $\hat{m}_t = m_t - E[m_t | \text{information as of } t-1]$ is the unexpected component of money and η_t is a stochastic vector of real factors which affect real rates (e.g., productivity, thrift, government expenditures). Exogeneity of the real rate in the context of a system which includes a measure of real production requires either that the measure is uncorrelated with components of η_t or that the α_j are all zero. Theories which place an emphasis on a confusion between unperceived monetary injections and persistent real factors affecting the ex ante real rate would generally predict a systematic response of the real rate to changes in real production. A failure to reject exogeneity of the real rate thus raises questions about the empirical importance of this channel for monetary disturbances to have real effects.

The Grossman-Weiss Model

This model also assumes incomplete information so that the expected real rate based on complete current period information differs from the expectations held by a representative trader. The model determines the ex ante real rate based on complete current period information to be equal to $r_t = (1-\alpha) E[C_{t+1}-C_t | \text{available information at } t]$ where α is a parameter of preferences and C_t = (log) per capita consumption.

As in the Lucas-Barro model, the compatibility of this theory with an exogenous real rate depends crucially on the nature of the exogenous stochastic disturbances. In the original version of the model, it was assumed that both monetary and real disturbances were serially independent, which resulted in serially independent consumption. In this case, the ex ante real rate is a first order moving average process and the real rate is exogenous. However, if the model is modified to be consistent with serially correlated consumption, then the real rate will not appear to be exogenous. As in the models which emphasize unperceived money, when there are both real and monetary factors which determine the real rate, and these factors have lagged effects, we would not expect the real rate to be exogenous.

The three theories we have examined have in common that the real rate of interest plays a crucial role in the generation of business cycles, and that (except under special circumstances) its behavior is a function of lagged real and monetary disturbances. Any model with these two characteristics would appear to be challenged by the finding that in a system with real and monetary variables, the real rate of interest is exogenous.

B. WHAT CAUSES OUTPUT?

A central issue of most business cycle theories is the transmission mechanism between changes in the supply or demand for money and the level of economic activity. The importance of money for predicting output was demonstrated by Sims in 1972 in a bivariate system and was generally accepted as evidence of real effects of purely monetary phenomena. Explaining this relationship has occupied a central role of recent theoretical developments. Modern theories share with Keynesian theory the idea that monetary phenomena affects real variables by altering perceptions of ex ante real rates. In light of the results in Section II in which real rates appear significantly less useful for predicting output than nominal interest rates and real rates themselves appear to be exogenous, we are led to question the usefulness of both Keynesian and equilibrium theories for explaining the observed correlations between nominal and real variables.

We first confirm Sim's finding that industrial production is not exogenous in a four variable vector autoregression with nominal rates, money, and prices. We then go on to test whether the observed effects from nominal quantities can be explained through lags of the real rate alone. Given our Section II results, we would expect this hypothesis to be rejected.

Our next set of hypotheses are not implications of any completely articulated theory, but are designed to examine the descriptive results presented in the previous section. In particular, we wish to investigate the channels, if any, through which monetary disturbances affect output.

We first test whether all lagged financial effects can be filtered through lagged levels of nominal interest rates. Even if this is true, however, it does not eliminate an indirect role for money. If lagged money helps to predict nominal interest rates, then it helps in predicting future output as well. In order to examine this issue, we test whether output and nominal rates are block exogenous.

Given the identity between nominal rates, real rates and expected inflation, we also find it interesting to test the hypothesis that all financial effects on output are filtered through lags of expected inflation. By comparing the results of tests of the above hypotheses, we can measure the relative explanatory power for output of each of the components of nominal rates. Finally, we test directly whether the decomposition of nominal rates significantly helps explain output.

In examining the importance of monetary disturbances, we are led to another set of possible restrictions on the system which would eliminate a role for money in predicting output. these hypotheses are that lags of nominal rate or expected inflation innovations are sufficient to capture all lagged financial effects. Since innovations are, by construction, orthogonal to all past variables, these are tests of block exogeneity of output and nominal interest rate or expected inflation innovations in the context of the four variable autoregressive systems.

C. EMPIRICAL RESULTS

Our tests will be based on the standard likelihood ratio statistic, and in interpreting our results we will present marginal significance levels based on asymptotic distributions giving the probability, under the null hypothesis, of observing test statistics of the given magnitude. We do not, however, interpret these significance levels literally. Two problems, in particular, bias the significance levels. First, given the large number of restrictions we are testing, there is ample reason to question the validity of the asymptotic distribution approximation for samples of the size we have. Direct calculation of those distributions, while possible through numerical simulations, would be prohibitively expensive. Second, our test procedures are being applied to the same data set which suggested the hypotheses to us in the first place. Because of these problems, we put little weight on the absolute levels of the test statistics. Rather, we wish to compare the relative fit of different restrictions. We find the classical hypothesis testing framework, with a fixed unrestricted vector autoregression as the alternative, a useful device through which we can investigate specific questions by looking at the degree to which various hypotheses are consistent with the data. In this context, we interpret the calculation of a significance level of a likelihood ratio statistic as a natural way to correct the relative fits of different restrictions for differences in degrees of freedom.

The hypothesis that the ex ante real rate of interest, r_t, is a function of only its own lagged values, a constant term, and an uncorrelated random error, can be written as follows:

$$r_t = \sum_{j=1}^{m} b_j \, r_{t-j} + c^r + u_t. \qquad (9)$$

The assumption of rational expectations implies

$$r_t \equiv R_t - {}_t\Pi_{t+1} \qquad (10)$$

where R_t is the observed nominal interest rate and ${}_t\Pi_{t+1}$ is the projection of the annualized growth rate of the price level from t to t+1 on information available at time t.

Substitution of (10) into (9) leads to the following expression for the nominal interest rate:

$$R_t = {}_t\Pi_{t+1} + \sum_{j=1}^{m} b_j R_{t-j} - \sum_{j=1}^{m} b_j \; {}_{t-j}\Pi_{t-j+1} + c^r + u_t \qquad (11)$$

This equation imposes testable restrictions across the autoregressive representation for R_t, Π_t, and the other variables, Z_t, in the information set individuals use in projecting future values of Π.

Suppose a finite order autoregressive representation exists for the K-vector

$$X_t' = [R_t \;\; \Pi_t \;\; Z_t'].$$

$$X_t = \sum_{\ell=1}^{L} A_\ell \, X_{t-\ell} + C + \eta_t \qquad (12)$$

The i^{th} equation of this representation has the scalar form

$$X_t^i = \sum_{j=1}^{K} \sum_{\ell=1}^{L} a_\ell^{ij} X_{t-\ell}^j + C^i + n_t^i \tag{13}$$

where a_ℓ^{ij} is the coefficient on the ℓ^{th} lag of the j^{th} component of X. Thus, for example, the projection of inflation during period t on observables at time t-1, is given by

$$\hat{\Pi}_t = \sum_{\ell=1}^{L} a_\ell^{21} R_{t-\ell} + \sum_{\ell=1}^{L} a_\ell^{22} \Pi_{t-\ell} + \sum_{j=3}^{K} \sum_{\ell=1}^{L} a_\ell^{2j} Z_{t-\ell}^{j-2} + C^2 \tag{14}$$

The restrictions on a vector autoregression implied by (9) are generated by using (14) to replace all expected inflation terms in (11) with projections on observables, collecting terms with R_t on the left-hand side, and then projecting both sides on information available at time t-1. The resulting equation is a projection of R_t on information available at time t-1 which equates each of the coefficients in the R_t equation, a_ℓ^{1j}, with a function of the b_ℓ's, and the a_ℓ^{ij}'s for i = 2, ... K. For example, for $\ell < m$,

$$a_\ell^{11} = \frac{1}{(1-a_1^{21})} \left[b_\ell + a_{\ell+1}^{21} + \sum_{j=2}^{K} a_1^{2j} a_\ell^{j1} - \sum_{j=1}^{\ell} b_j a_{\ell-j+1}^{21} \right] \tag{15}$$

Because there are L lags in each of the projections of the observed variables, lags of the real rate become functions of observations more than L periods earlier than the current period. Thus, the reduced form projection for R must include m-1 more lags than each of the other equations. This requires us to impose (9) as a restriction on a vector autoregressive system with L+m-1 lags on all variables in the R projection and L lags of all variables in the other projections.

Equations similar to (15) express each of the coefficients in the R projection as a function of the other coefficients. Given the introduction of the m+1 new free parameters, b_1 ... b_m and c^r, these equations impose K*(L+m-1)-m nonlinear restrictions on the parameters of the vector autoregression.

In tests concerned with what causes movements in output, we consider four types of explanatory variables, observable quantities, unobservable quantities, and innovations in each. Tests with observable quantities are easiest to implement. The likelihood ratio of one equation with linear restrictions relative to the corresponding equation in an unrestricted vector autoregression is equivalent to the likelihood ratio test of the entire system with and without the equation so restricted. This result is shown by Doan (1980) using results in Revankar (1974). These results do not apply, however, when the alternative system has different numbers of lags in different equations.

Tests using unobservable quantities, expected inflation or real rates, are implemented in a manner similar to that described above for testing exogeneity of the real rate. Substitution of projections on observables for these quantities leads to nonlinear restrictions across equations.

The tests which utilize lagged innovations as explanatory variables follow essentially the same estimation procedure as for those with unobservable quantities. Innovations in nominal interest rates, for example, are defined by

$$R_t = R_t - E[R_t | Y_{t-s}, R_{t-s}, M_{t-s}, {}_{t-s}, s=1,\ldots,L]. \qquad (16)$$

The expectation is determined by the autoregressive representation for R, and the $\tilde{R}$'s can be substituted out in a manner similar to that in the earlier tests.

Innovations in expected inflation are defined by

$$\begin{aligned}\tilde{\tilde{\Pi}}_t &= E[\Pi_{t+1}|Y_{t-s},R_{t-s},\Pi_{t-s},M_{t-s},s=0,\ldots,L-1] \\ &\quad - E[\Pi_{t+1}|Y_{t-s},R_{t-s},\Pi_{t-s},M_{t-s},s=1,\ldots,L].\end{aligned} \tag{17}$$

Upon substituting the expectations implied by the autoregressive representation in (12) it is easily seen that

$$\tilde{\tilde{\Pi}}_t = a_1^{21}\,\tilde{Y}_{t-1} + a_1^{22}\,\tilde{R}_{t-1} + a_1^{23}\,\tilde{\Pi}_{t-1} + a_1^{24}\,\tilde{M}_{t-1} \tag{18}$$

In all of the systems with nonlinear constraints initial estimates were obtained by estimating an unrestricted vector autoregression, generating the implied expected inflation and ex ante real rate, and estimating the restricted equation on the basis of these observations. This procedure leads to consistent estimates of the parameters and is the one followed by Barro (1977). It is not fully efficient, however, and we prefer the full-information-maximum-likelihood (FIML) estimates obtained by minimizing the log of the determinant of the variance-covariance matrix of residuals in the constrained system. A FORTRAN program utilizing analytic gradient and Hessian was written by Litterman for this purpose.

The hypotheses are tested using the likelihood ratio statistic formed by taking

$$(T\text{-}dfc)\ \log[\det \textstyle\sum^{c} / \det \textstyle\sum^{u}] \tag{19}$$

where T is the number of observations in each equation, dfc is a degrees of freedom correction suggested by Sims (1980b) equal to the number of parameters in each equation of the unrestricted system, $\sum^{c}$ is the covariance matrix of residuals in the constrained system, and $\sum^{u}$ is the covariance matrix of residuals in the unrestricted system.

In implementing our tests we found it difficult on a priori grounds to choose a particular interest rate to focus on. Theories generally do not differentiate between rates at different maturities, but certain trade-offs clearly exist. Longer maturities are probably more relevant signals for investment decisions and in the case of real rates will be more robust to timing errors in the measurement of prices. Short rates, however, are likely to be more responsive to monetary disturbances and less subject to time aggregation problems. It will also be easier to forecast inflation rates over shorter time horizons, although those forecasts will include more short-run variation than forecasts over longer horizons. Our reaction here is similar to our response in Section II; we present all of the results for two different interest rate maturities.

The first set of data is monthly observations from 48:1 to 81:8 on M1, end of month rates on Treasury bills with one month to maturity, industrial production and the consumer price index less shelter. The second set is quarterly observations from 48:1 to 81:2 on M1, end of quarter rates on Treasury bills with three months to maturity, industrial production and the consumer price index less shelter. In the quarterly system values of money, output and prices are taken to be the values of those variables for the third month of the given quarter. All data were logged prior to estimation, and prices were converted to annualized inflation rates by differencing the logs and multiplying by the appropriate factor.

Table VI

Hypothesis Test Results
Monthly Data

	Null Hypothesis	Alternative	Log Determinant	Degress of Freedom	Marginal Significance
1.	r exogenous	A	-19.75017	29	.2302
2.	Y exogenous	B	-19.72565	18	.0242
3.	Y explained by R	B	-19.75859	12	.0793
4.	Y explained by M	B	-19.76887	12	.2115
5.	Y explained by Π	B	-19.74509	12	.0178
6.	Y exogenous	C	-19.72844	42	.1539
7.	Y explained by R	C	-19.76069	36	.3166
8.	Y explained by M	C	-19.77042	36	.4717
9.	Y explained by Π	C	-19.74560	36	.1441
10.	Y explained by r	C	-19.74557	36	.1437
11.	Y explained by $\hat{\Pi}$	C	-19.75427	36	.2319
12.	Y explained by r, $\hat{\Pi}$	C	-19.77773	30	.3114
13.	Y explained by $\tilde{R}$	C	-19.76016	36	.3089
14.	Y explained by $\tilde{\tilde{\Pi}}$	C	-19.74852	36	.1703
15.	Y, R block exogenous	C	-19.66756	48	.0102

Alternative Vector Autoregressions

Alternative	Lags in Equation R	Y	M	Π	Log Determinant	Period	Correction Factor	Effective Number of Observations
A	8	6	6	6	-19.84274	48:10-81:8	25	370
B	6	6	6	6	-19.81074	48:8-81:8	25	372
C	6	12	6	6	-19.86860	49:2-81:8	25	366

Table VII

Hypothesis Test Results
Quarterly Data

	Null Hypothesis	Alternative	Log Determinant	Degrees of Freedom	Marginal Significance Level
1.	r exogenous	A	-17.10300	15	.8347
2.	Y exogenous	A	-16.91206	12	.0019
3.	Y explained by R	A	-16.94956	8	.0007
4.	Y explained by M	A	-17.00777	8	.0088
5.	Y explained by Π	A	-16.97152	8	.0019
6.	Y exogenous	B	-16.88313	28	.0010
7.	Y explained by R	B	-16.92420	24	.0007
8.	Y explained by M	B	-16.98054	24	.0039
9.	Y explained by Π	B	-16.94673	24	.0014
10.	Y explained by r	B	-17.02255	24	.0132
11.	Y explained by $\hat{\Pi}$	B	-17.06359	24	.0391
12.	Y explained by r, $\hat{\Pi}$	B	-17.07710	20	.0153
13.	Y explained by $\tilde{R}$	B	-17.02037	24	.0124
14.	Y explained by $\overset{\approx}{\Pi}$	B	-17.04117	24	.0218
15.	Y, R block exogenous	B	-16.76604	32	.0001

Alternative Vector Autoregressions

Alternative	Lags in Equation				Log Determinant	Period	Correction Factor	Effective Number of Observations
	R	Y	M	Π				
A	4	4	4	4	-17.19012	49:2-81:2	17	112
B	4	8	4	4	-17.41069	50:2-81:2	17	108

Table VIII

Selected Coefficient Estimates
In Restricted Equations
(Standard Errors in Parentheses)

Hypothesis 1: Real Rates Exogenous

Monthly Data

$$r_t = \underset{(.049)}{.965}\, r_{t-1} - \underset{(.071)}{.431}\, r_{t-2} + \underset{(.045)}{.235}\, r_{t-3} + \underset{(.068)}{.077} + U_t$$

Quarterly Data

$$r_t = \underset{(.065)}{.702}\, r_{t-1} + \underset{(.106)}{.090} + U_t$$

Hypothesis 10: Output Explained by Real Rates

Monthly Data

$$Y_t = \underset{(.037)}{1.444}\, Y_{t-1} - \underset{(.067)}{.391}\, Y_{t-2} - \underset{(.073)}{.023}\, Y_{t-3} - \underset{(.079)}{.116}\, Y_{t-4} - \underset{(.080)}{.037}\, Y_{t-5} + \underset{(.042)}{.077}\, Y_{t-6}$$

$$- \underset{(.00035)}{.00014}\, r_{t-1} + \underset{(.00040)}{.00042}\, r_{t-2} - \underset{(.00042)}{.00122}\, r_{t-3} + \underset{(.00047)}{.00116}\, r_{t-4}$$

$$- \underset{(.00050)}{.0021}\, r_{t-5} + \underset{(.00037)}{.00022}\, r_{t-6} + \underset{(.00436)}{.00757} + U_t$$

Quarterly Data

$$Y_t = \underset{(.189)}{1.577}\, Y_{t-1} - \underset{(.384)}{.635}\, Y_{t-2} + \underset{(.359)}{.008}\, Y_{t-3} + \underset{(.159)}{.049}\, Y_{t-4}$$

$$+ \underset{(.0085)}{.0270}\, r_{t-1} - \underset{(.0106)}{.0333}\, r_{t-2} + \underset{(.0092)}{.0283}\, r_{t-3} - \underset{(.0047)}{.0072}\, r_{t-4}$$

$$+ \underset{(.0406)}{.0086} + U_t$$

Hypothesis 11: Output Explained by Expected Inflation

Monthly Data

$$Y_t = \underset{(.037)}{1.435}\, Y_{t-1} - \underset{(.070)}{.410}\, y_{t-2} + \underset{(.078)}{.046}\, Y_{t-3} - \underset{(.083)}{.096}\, Y_{t-4} - \underset{(.081)}{.063}\, Y_{t-5} + \underset{(.043)}{.090}\, Y_{t-6}$$

$$+ \underset{(.00036)}{.00056}\, \hat{\Pi}_{t-1} - \underset{(.00044)}{.00087}\, \hat{\Pi}_{t-2} - \underset{(.00049)}{.00095}\, \hat{\Pi}_{t-3} - \underset{(.00056)}{.00109}\, \hat{\Pi}_{t-4}$$

$$- \underset{(.00057)}{.00002}\, \hat{\Pi}_{t-5} - \underset{(.00039)}{.00016}\, \hat{\Pi}_{t-6} - \underset{(.00690)}{.00606} + U_t$$

Quarterly Data

$$Y_t = \underset{(.139)}{1.677}\, Y_{t-1} - \underset{(.292)}{.904}\, Y_{t-2} + \underset{(.297)}{.220}\, Y_{t-3} + \underset{(.140)}{.027}\, Y_{t-4} - \underset{(.0041)}{.0185}\, \hat{\Pi}_{t-1}$$

$$+ \underset{(.0076)}{.0316}\, \hat{\Pi}_{t-2} - \underset{(.0073)}{.0283}\, \hat{\Pi}_{t-3} + \underset{(.0040)}{.0041}\, \hat{\Pi}_{t-4} + \underset{(.0402)}{.4921} + U_t$$

We find that the hypothesis that the real rate is exogenous cannot be rejected in either the monthly or quarterly system. This finding accords with our interpretation of the Section II results. In both systems, the real rate appears as an exogenous Markov process (see Table VIII).

It is revealing to compare the response of the real rate to innovations in each of the four variables in both the unrestricted system and the first restricted system (Figures X and XI). These graphs show that no qualitative distortions are introduced by the restriction that the real rate is exogenous. The unrestricted system permits arbitrary patterns of feedback from the variables to the real interest rate. The restriction of exogeneity requires that the effects of these variables can be filtered through their contemporaneous correlations with the real rate alone.

The strongest contemporaneous correlations with real rate innovations are the positive associations with nominal rate innovations, and negative association with inflation innovations.

Notice that since both inflation and the real rate have persistent components, the strong negative contemporaneous correlation between real rates and innovations in inflation is enough to explain the negative correlation between inflation levels and future real rates observed by Summers (1980) and Mishkin (1980). Our interpretation of this phenomenon differs, however, from that of Summers who argues that money illusion is "the most plausible explanation for the nonresponse of interest rates to inflation." Since the result is compatible with an economic structure in which

RESPONSES OF THE REAL INTEREST RATE

FIGURE Xa. Unrestricted Vector Autoregression

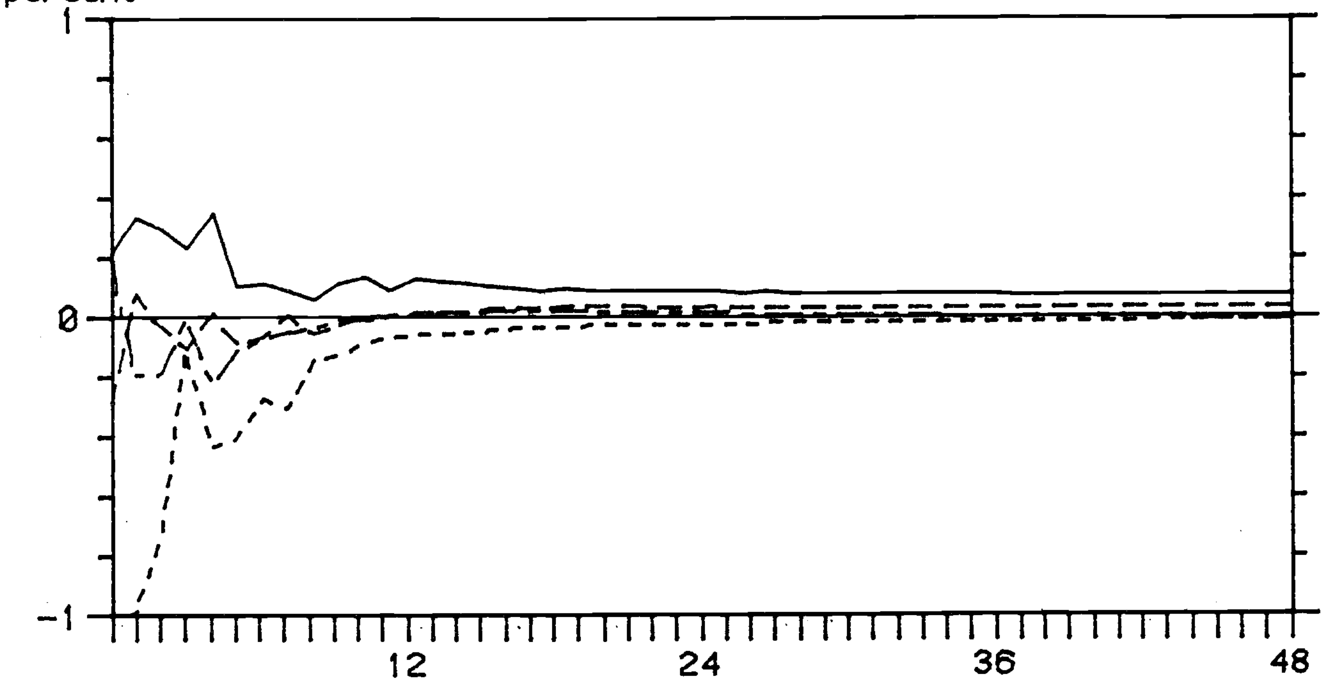
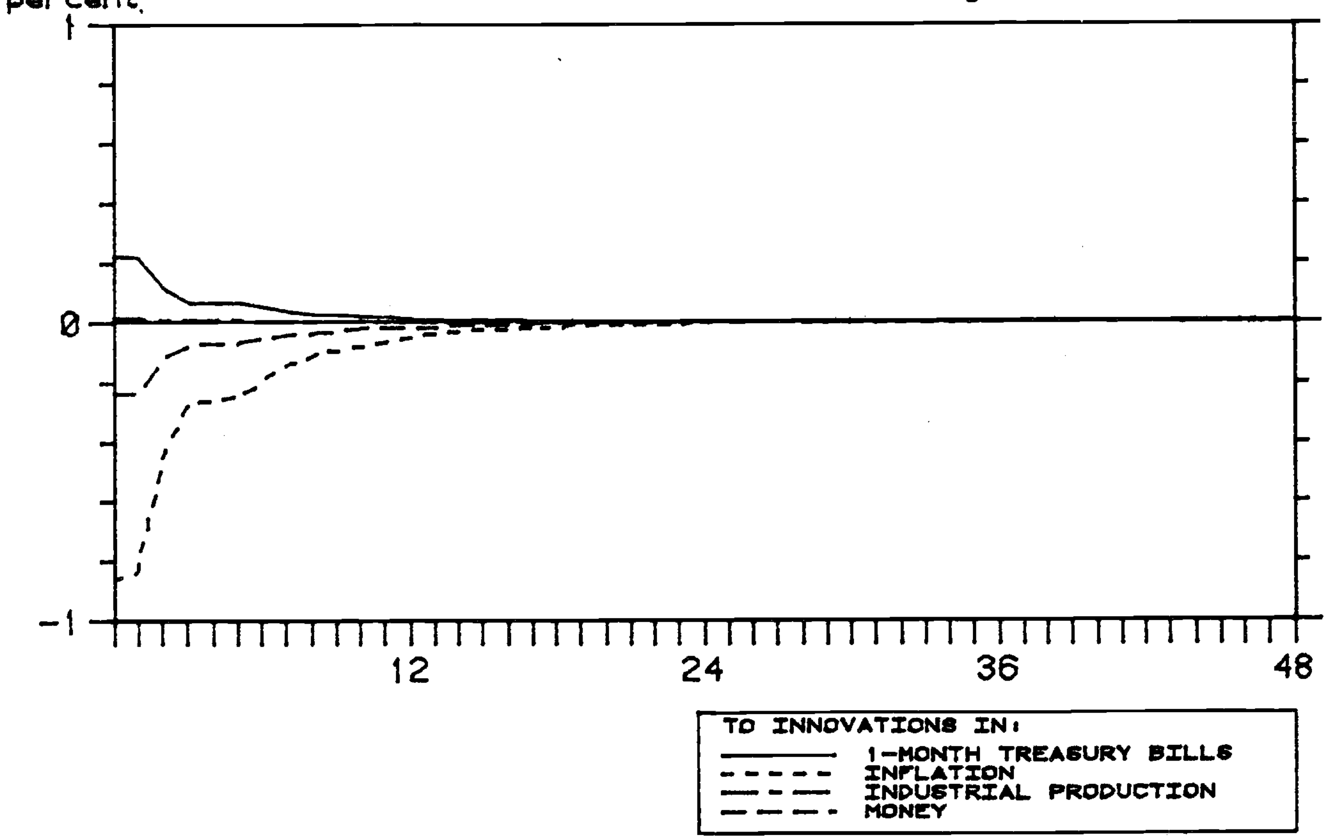


FIGURE Xb. Restricted Vector Autoregression



RESPONSES OF THE REAL INTEREST RATE

FIGURE XIa. Unrestricted Vector Autoregression

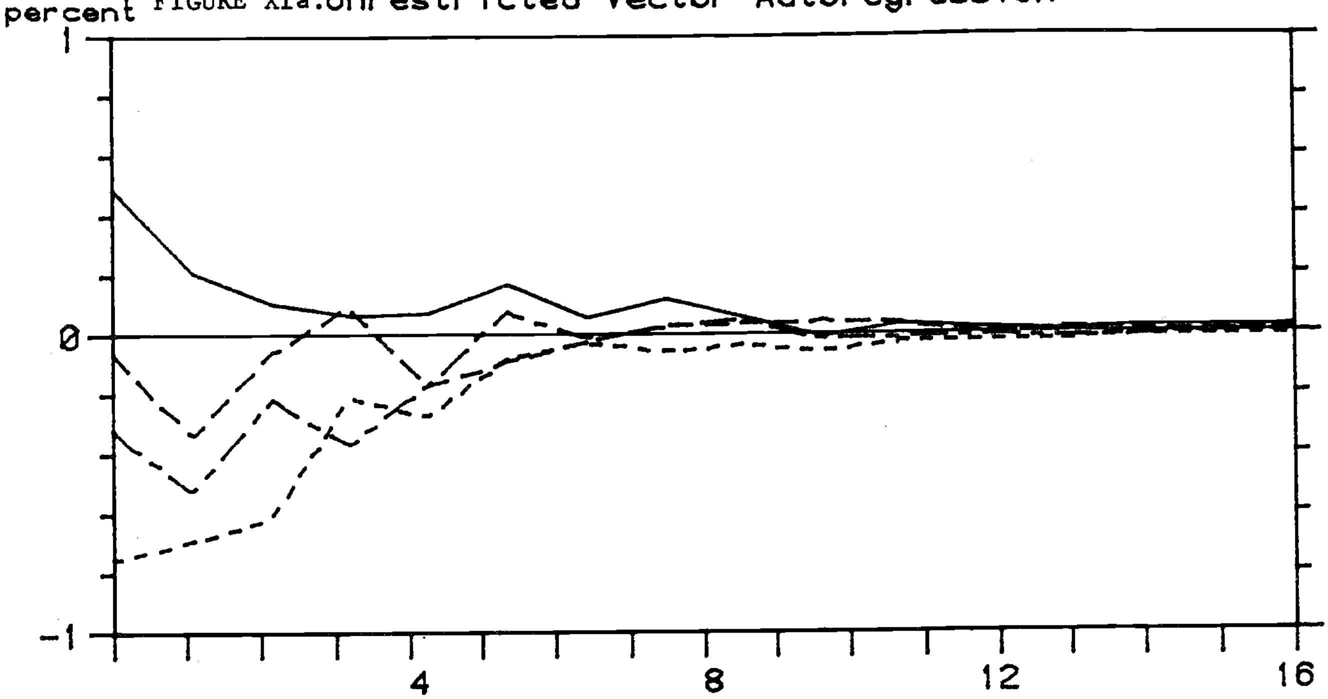
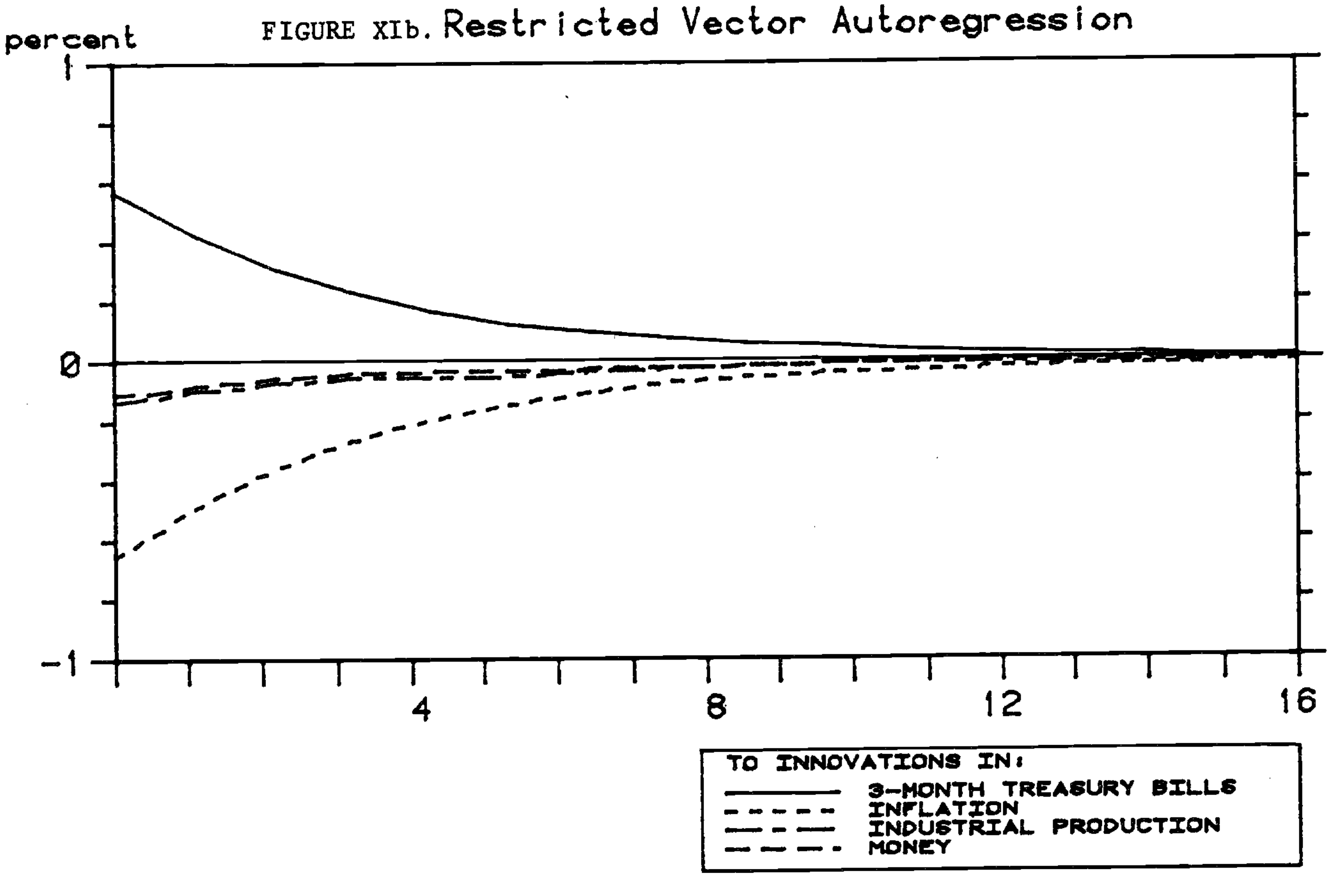


FIGURE XIb. Restricted Vector Autoregression



there is no feedback from prices and money to real interest rates, we conclude that short-run changes in both real rates and inflation can be attributed to the same, as yet unidentified, random factor.

The results of our hypothesis tests investigating the transmission mechanism between money and output raise a number of puzzling questions. Overall, the results are less clear-cut, leaving room for differing interpretations. For example, the results from the monthly system differ from those of the quarterly system. In our discussion, we attempt to focus on those results which are least equivocal and to point out where uncertainty remains.

We first confirm that output is not exogenous in the context of these four variable autoregressions. While the rejection of output exogeneity is unambiguous in the quarterly system, it is sensitive to lag length in the monthly system. In the monthly system with six lags, we can reject output exogeneity at the conventional 5 percent significance level. However, our other hypothesis are not restrictions of these vector autoregressions. Our restrictions involve projections on unobservables which are themselves projections on lagged observables. Thus, for example, when six lags of the unobservable real rate are included, the reduced form will contain 11 lags of the observables. Our most general alternative for each hypothesis in the context of the monthly system contains 12 lags in the output equation and six lags in the other equations, and the quaterly system contains eight lags in the output equation and four lags in the other equations. All hypotheses involve restrictions on these two systems. In the monthly system, none of our hypotheses, including output exogeneity, can be rejected relative to this alternative. Nevertheless, the significance level can be interpreted as a way of ranking these alternative hypotheses.

The hypothesis that the effects from nominal quantities to output can be filtered through lags of the real rate does not appear to fit the data well. By comparison, lagged values of expected inflation have far more explanatory power for output than does the real rate in both the monthly and quarterly system. However, somewhat puzzling is the finding that nominal rates and money do better than either of these variables for predicting output in the monthly system, but noticeably worse in the quarterly system. This anomalous result is confirmed by testing the hypothesis that for the purpose of predicting future output is useful to decompose the nominal interest into its expected inflation and expected real rate components. Consistent with the above findings the decomposition helps only in the quarterly system. A likelihood ratio test of the restriction that coefficients on real rates and expected inflation are the same in the output equation, that is that only nominal rates matter, has a marginal significance level of .4178 in the monthly system and .0093 in the quarterly system.

Although we find that nominal interest rates are sufficient for predicting output in a monthly system, we can reject the hypothesis that output and nominal rates are block exogenous in a four-variable system. This arises because money and prices have predictive content for nominal interest rates. Thus, we cannot conclude from these tests alone that money has no predictive content for output past a one-period forecast horizon. The next hypothesis is designed to assess the importance of money for predicting output at any forecast horizon. Specifically, we test whether three lags of nominal interest rate innovations are sufficient to capture all lagged effects. Since nominal interest rate innovations are, by construction, orthogonal to all past variables, this is a test of block-exogeneity of output and nominal interest rate innovations in a four-variable system. In the monthly system, this restriction fits surprisingly well, and the predictive content of six lags of the innovation of the nominal rate is virtually identical to that of six lagged levels. In the quarterly system, the innovations have more predictive content than the levels, although not as much as expected inflation levels.

IV. A Possible Explanation

A central result of this paper is that there is information in the level of nominal rates for predicting future output which is not contained in the history of past output or past and future expected real rates. We suspect that this statistical link between expected inflation and output arises because agents in the economy have some information about the level of future output, not directly observable to the econometric investigator, which is first reflected in nominal quantities. To see how this could arise, consider the following structural model in which output is independent of the money supply process.

$$
\begin{aligned}
Y_{t+1} &= Y_t + Z_t + U_{t+1} \\
M_t - P_t &= M_1 Y_t - M_2 R_t \\
R_t &\equiv \hat{P}_{t+1} - P_t + r_t \\
r_t &= \lambda r_{t-1} + \varepsilon_t
\end{aligned}
\tag{20}
$$

The crucial feature is that there is some information in Z_t which is known to agents in the economy and is useful for predicting future output, but is not directly observable to the econometric investigator.

Suppose the model is closed by specifying a money supply process

$$M_t \equiv 0 \tag{21}$$

and the exogenous disturbances ε_t, Z_t, U_t are serially independent. It is straightforward to show the reduced form equations for expected inflation and nominal rates given by

$$
\begin{aligned}
\hat{\Pi}_t &= \left(\frac{-M_2}{1+M_2}\right) Z_t + \left(\frac{-M_2(1-\lambda)}{1+M_2(1-\lambda)}\right) r_t \\
R_t &= \left(\frac{-M_2}{1+M_2}\right) Z_t + \left(\frac{1}{1+M_2(1-\lambda)}\right) r_t
\end{aligned}
\tag{22}
$$

and the solutions for the innovations of these variables are

$$
\begin{aligned}
\tilde{\hat{\Pi}}_t &= \left(\frac{-M_2}{1+M_2}\right) Z_t + \left(\frac{-M_2(1-\lambda)}{1+M_2(1-\lambda)}\right) \varepsilon_t \\
\tilde{R}_t &= \left(\frac{-M_2}{1+M_2}\right) Z_t + \left(\frac{1}{1+M_2(1-\lambda)}\right) \varepsilon_t
\end{aligned}
\tag{23}
$$

This model shows most simply that nominal interest rate innovations or expected inflation innovations will be correlated with "Z" innovations, and thereby will be useful for predicting output when Z_t is not observed directly. This occurs despite the lack of structural feedback from past, current, or future money and prices to output.

Of course, this model could not account for the predictive content of money in a bivariate system. However, it would not be difficult to change the specification of the money supply process to be consistent with this finding, as well as other characteristic features of the data. Consider the money supply process

$$\Delta M_t = \Delta M_{t-1} + \delta_1(\Pi_t^e - \Pi_{t-1}^e) + \delta_2 U_t + \gamma_t. \quad (24)$$

We would expect δ_1 to be negative; the monetary authority reacts to an increase in inflationary expectations by contracting. We would expect δ_2 to be positive as the money supply responds positively to an unexpected increase in output. With this specification, the reduced form equation for expected inflation and changes in money supply are given by

$$\hat{\Pi}_t = \Delta M_t - \left(\frac{M_2(1-\delta_1)}{1+M_2}\right) Z_t - \left(\frac{M_2(1-\lambda)}{1+M_2(1-\lambda)-\delta_1\lambda}\right) r_t \quad (25)$$

$$\Delta M_t = \Delta M_{t-1} - \frac{\delta_1 M_2}{1+M_2}(Z_t - Z_{t-1}) - \left(\frac{\delta_1 M_2(1-\lambda)}{1+M_2(1-\lambda)-\delta_1\lambda}\right)$$

$$\times\ ((\lambda-1)r_{t-1}+\varepsilon_t) + \frac{\delta_1}{1-\delta_1} U_t + \frac{1}{1-\delta_1}\gamma_t \quad (26)$$

and for the innovations of these variables

$$\tilde{\tilde{\Pi}}_t = - \frac{M_2}{1+M_2} Z_t - \left(\frac{M_2(1-\lambda)(1+\delta_1)}{1+M_2(1-\lambda)-\delta_1\lambda}\right) \varepsilon_t + \frac{\delta_2}{1-\delta_1} U_t + \frac{1}{1-\delta_1} \gamma_t$$

$$\tilde{M}_t = - \frac{\delta_1 M_2}{1+M_2} Z_t - \left(\frac{\delta_1 M_2(1-\lambda)}{1+M_2(1-\lambda)-\delta_1\lambda}\right) \varepsilon_t + \frac{\delta_2}{1-\delta_1} U_t + \frac{1}{1-\delta_1} + \gamma_t \qquad (27)$$

This modification shows how monetary innovations could be positively associated with "Z" innovations and thus be useful for predicting real output in a bivariate system in a way which is consistent with the block exogeneity of income and either nominal rate innovations or expected inflation innovations in the context of a larger system. A "Phillips Curve" relationship--a positive correlation between ex post inflation and lagged output growth--could arise from this system if δ_2 is positive, meaning the money growth rate rises with unexpected output shocks.

This model suggests an empirical test of the hypothesis that the inflation-output link is spurious because inflation is proxying for other information relevant to predicting future output. If this view is correct, we would expect that innovations in expected inflation (i.e., that component of expected inflation which was unforecastable in earlier periods) would be more useful for predicting output than the level of expected inflation. This may be seen by comparing the reduced form equations (25) and (26). The primary component of expected inflation is doubtless the growth in nominal money, which is largely predictable on the basis of lagged information. However, the structural model shows that it is only that component of expected inflation not related to expected money growth which is useful for predicting future output. Expected inflation innovations are purged of lagged money growth and thus more highly correlated with "Z" innovations, and thus, more useful for predicting output changes.

This test can distinguish between the aforementioned model, which inflation-output link is spurious from the competing hypothesis that there are numerous structural institutional features of the American economy which imply perfectly foreseen inflation can have real and depressing output affects. Among the leading examples cited in support of this view are the nonindexation of the tax system, the nonindexation of some administered prices, the effects of nominal interest rate ceilings, and the distortionary effects of taxation of liquidity services. If this structuralist interpretation is valid, we would expect the effects of inflation on output is independent of the sources of the inflation. In particular, we would expect the level of expected inflation, which includes that component of inflation related to money growth, to be more useful for predicting output than expected inflation innovations, which are orthogonal to past money growth.

The test results designed to pass on the validity of the view that the inflation-output link is spurious are ambiguous. In the monthly system, innovations to inflation have virtually the same predictive content for output as do the levels of expected inflation, although neither is as powerful as either levels or innovations of nominal rates alone. In the quarterly system, innovations to expected inflation have lower predictive ability than do levels, but either levels or innovations explain more of the movement of output than do any of the other specifications we tried.

We interpret these ambiguous results to be surprisingly consistent with our story. If it is the innovations which contain the useful information, most of that information can be filtered from lagged levels, and thus we would expect to see approximately equal explanatory power. In fact, if we have truncated the true lag distribution on innovations, we might expect to see some improvement through the use of the same number of lagged levels which would incorporate some of that lost information. On the other hand, if there were indeed some structural links between levels of expected inflation and output, then virtually no information would be found in recent innovations in expected inflation and one would expect to see very much poorer fit using them.

The effects of an innovation in expected inflation on the time paths of the observable variables and the responses of industrial production can be seen in Figures XII and XIII. An expected inflation innovation is defined to be a weighted sum of unit standard deviation shocks to the observable variables, where the weights are given by the regression coefficients on the first lag in a projection of one-period ahead inflation on lagged values of these variables. The figure makes clear that an increase in expected inflation has an immediate and persistent negative effect on both output and money. We take this finding to be consistent with a value of δ_1 between -1 and 0.

FIGURE XII. RESPONSES OF INDUSTRIAL PRODUCTION
RESTRICTED SYSTEM: HYPOTHESIS 14

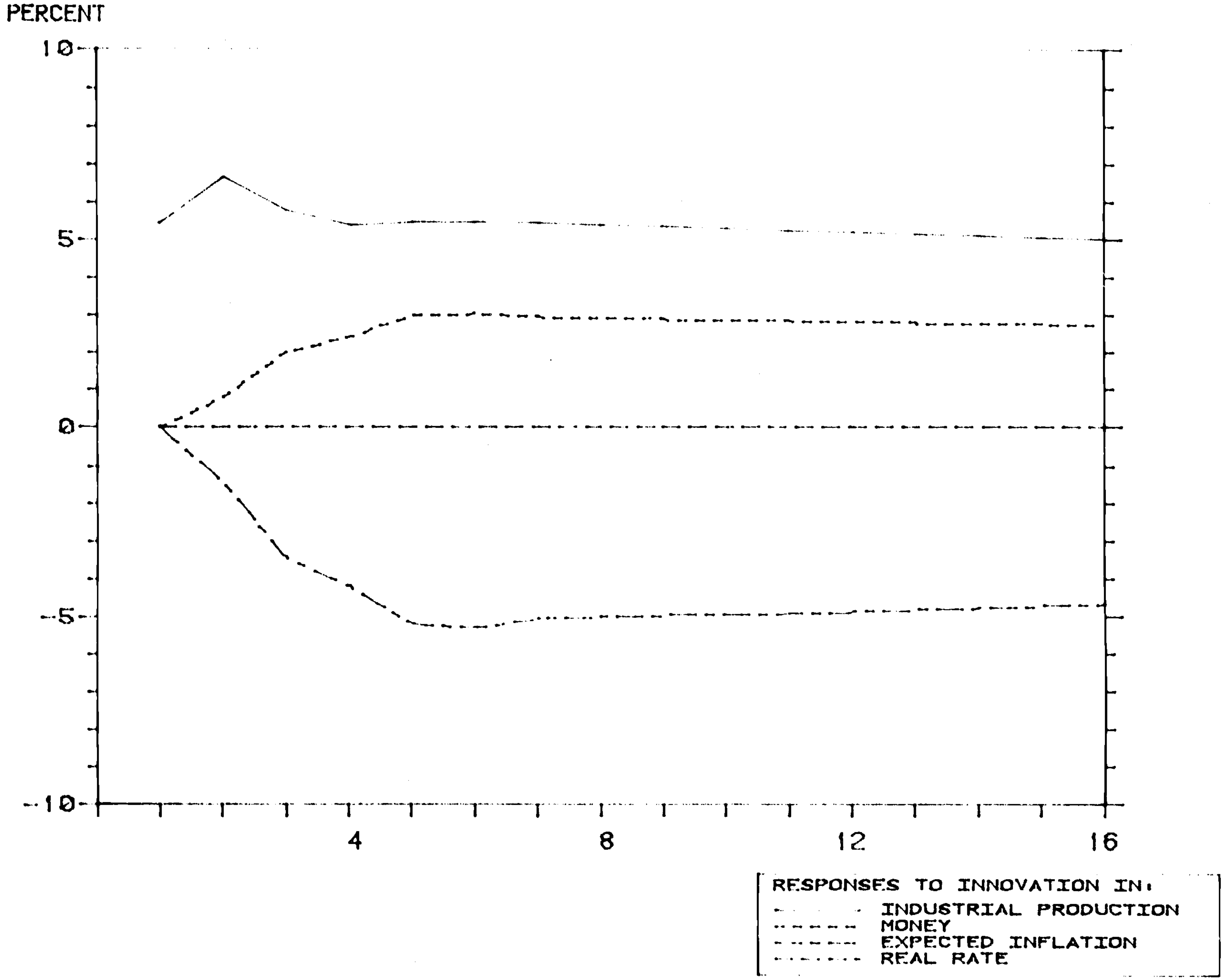
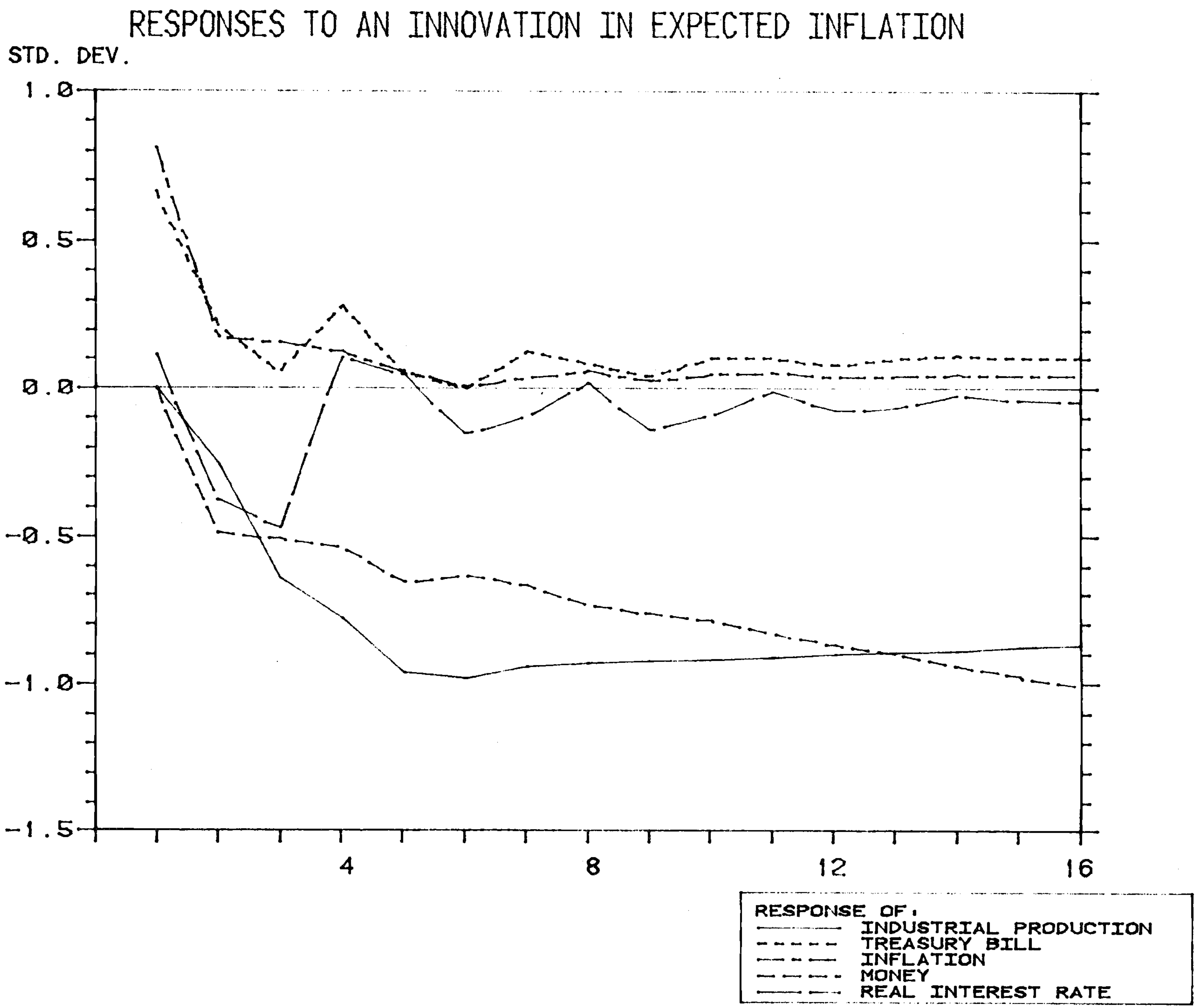


FIGURE XIII


RESPONSES TO AN INNOVATION IN EXPECTED INFLATION
STD. DEV.
1.0
0.5
0.0
-0.5
-1.0
-1.5
4
8
12
16
RESPONSE OF:
INDUSTRIAL PRODUCTION
TREASURY BILL
INFLATION
MONEY
REAL INTEREST RATE

V. Conclusion

This paper has examined the empirical support for a number of hypotheses about the money-interest-output link. Because the relevant real rate is unobservable, an appropriate empirical counterpart suggested by a particular class of structural models was formulated. This class might be considered "dynamic IS-LM" with rational expectations. Although this class does not include those models which explicitly posit barriers to information flows, some of our results bear on their empirical validity.

The first set of tests sought to identify the determinants of the real interest rate. Specifically, we could not reject the hypothesis that this variable is governed only by its own past history, with no separate influence coming from money, output, nominal rates, or prices. Although this hypothesis is not an implication of any particular alternative to the Keynesian theory, it is incompatible with models of this sort, except for some very restrictive and economically uninteresting special cases. Taken literally, our results imply that monetary policy has not discernably affected the real rate, although it has causally influenced nominal interest rates. Our results also show a strongly negative correlation between expected real rates and inflation innovations. Since both inflation and expected real rates have some persistent component, this can explain the well-documented negative correlation between the level of current period inflation and real rates, even in the absence of any structural link between past inflation and future real rates.

Our second set of tests found that it is not possible to filter the observed influence of money and nominal interest rates on output through expected real rates. This result casts doubt on the Keynesian transmission mechanism between money and output. A central new result of this paper is that there is information in the level of expected inflation for predicting future output which is not contained in the history of past output or past and future expected real interest rates. Two explanations for the apparent predictive content of expected inflation on output have been advanced. The structuralist interpretation focuses on nonneutralities of various (nonoptimal) institutional nominal rigidities, but leaves unanswered the causes of changes in inflation. The other hypothesis argues that output is structurally exogenous to money and prices, but that new information is first reflected in expected inflation and interest rates. We find that data to be surprisingly consistent with this latter hypothesis.

FOOTNOTES

[1]The data are from Salomon Brothers, yields are recorded on the first trading day of the following month. However, prior to 1964 Salomon Brothers reports midmonth yields, and after 1977 the data are taken from the U.S. Treasury Bulletin, which reports yields on the last trading day of the month.

[2]We also estimated a number of larger systems including (not all at one time) inventories, retail sales, real wages, wage settlements, the monetary base, a stock price index, the unemployment rate, 10-year bond yields, and a trade-weighted index of the value of the dollar. The qualitative behavior of the output response to interest rate innovations described above appeared in every system estimated.

[3]Our test is based on a likelihood ratio statistic generated from the restriction that separate coefficients in vector autoregressions on two subsamples are equal. The systems use the variables described in the text and include six lags of each plus a constant. The two subsamples are 1949:1 through 1971:12, and 1972:7 through 1981:8. The variables in the latter half are scaled so that residual error variances are the same in both subsamples. The test follows the procedure described by Sims (1980b), page 17, to correct for degrees of freedom. The results of the test were affected little by varying the sample break between 1966 and 1973.

We interpret this test statistic with extreme skepticism, however, because of its low observation-to-parameter ratio in this application and the asymptotic nature of the distribution theory on which it is based. To judge the applicability of this distribution theory to our test, we created artifical data by simulating the system with fixed coefficients (and normally distributed errors with covariances) equal to those estimated using the entire sample. That is, we generated data under conditions in which the null hypothesis is known to be true. Under these conditions our test procedures led to statistics which rejected the null hypothesis eight times out of ten at the .01 significance level. One of the ten statistics was 185.8, larger than the 180.92 we observed on the actual data.

[4]/The expected inflation projections use data beginning in 1948:1. The first projection is made in 1955:1 and projections are made each period through the end of 1981. The projections are made out-of-sample in order to better measure the expectations of agents who do not benefit from the hindsight afforded by in-sample projections. We take these expectations as data and then fit a vector autoregressive representation. We follow this procedure in order to avoid the prescient expectations which are embodied in the in-sample projections of a vector autoregression fit directly to the observable data. For equations such as ours with many free parameters, the forecast errors generated by in-sample fit may be substantially smaller than out-of-sample errors.

[5]/The quarterly real rate is based on the IP, CPI, and M1 observations for the third month of each quarter. The nominal rate is the end-of-quarter value for Treasury bills with three months to maturity, BILLS3. Inflation expectation projections are generated in the same manner as described previously except only two lags of the quarterly data were used.

The six-month real rate is based on monthly data and the same projection equations except that the one-month nominal rate is replaced with a six-month to maturity Treasury bill rate, BILLS6. The chain rule of forecasting is then used to project inflation expectations six months forward.

The ex post one-month real rate is the nominal rate, BILLS1, minus the actual inflation at annualized rates from the current to the next month.

[6]We should caution the reader that the innovations discussed in this section are defined relative to the four variables in the vector autoregression. In particular, these innovations are based on projections on lagged expected inflation, (or equivalently real rates) but not on lagged actual inflation. Asymptotically this does not matter since if the coefficients are fixed and known, then for given initial conditions the observations on expected inflation and other variables are sufficient to determine the actual inflation rates.

[7]We view the similarity of these results with the earlier results as evidence that the use of in-sample expectations is not likely to be misleading, and we continue with this approach in the next section. The unrestricted vector autoregressions used here are described fully in the next section as Alternative C for monthly data and Alternative B for quarterly data.

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