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REAL BUSINESS CYCLE MODELS

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ABSTRACT

This paper attempts to provide an evaluation of both strengths and weaknesses of the real business cycle (RBC) approach to the analysis of macroeconomic fluctuations. It begins with a description of the basic analytical structure typically employed, one in which individual households make consumption and labor supply decisions while producing output from capital and labor inputs, hired on competitive markets, according to a technology that is subject to stochastic shocks. It then explores conditions on parameter values that are needed for a model of this type to yield fluctuations that provide a good quantitative match to those observed in the postwar U.S. quarterly data. The plausibility of the hypothesis that (unobservable) aggregate technology shocks have the requisite variability is considered and problems with certain cross correlations are noted. Relevant evidence obtained by formal econometric methods is summarized and a few tentative conclusions regarding business cycle research are suggested.

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I. Introduction

One of the most striking developments in macroeconomics during the early 1980s was the emergence of a substantial body of literature devoted to the "real business cycle" approach to the analysis of macroeconomic fluctuations. Particularly prominent papers have been contributed by Kydland and Prescott (1982), Long and Plosser (1983), and King and Plosser (1984) while many others of interest have been written[1] and a number of critical or skeptical pieces have begun to appear.[2] This literature's implied point of view is an outgrowth of the equilibrium strategy for business cycle analysis that was initiated by Lucas (1972) (1973) (1975) and extended by Barro (1976) (1981), but differs from that of the earlier work in two critical respects. First, the real business cycle (or "RBC") models place much more emphasis than did the previous equilibrium-approach literature on mechanisms involving cycle propagation, i.e., the spreading over time of the effects of shocks. Second, as the name implies, RBC models emphasize the extent to which shocks that initiate cycles are real--as opposed to "monetary"--in origin. In particular, the primary driving force is taken to be shocks to technology,[3] rather than the monetary and fiscal policy disturbances that are emphasized in the earlier equilibrium-approach writings.

It will be noted that these two features of the RBC approach are quite different from each other in terms of their relationship to alternative business cycle theories. Specifically, the RBC propagation analysis is entirely compatible with the Lucas-Barro monetary-misperceptions variant of the equilibrium approach and could logically be viewed as an attempt to elaborate and improve upon models of the Lucas-Barro type. Indeed, the point can be taken further by noting that the propagation phenomena stressed by RBC analysis could be relevant and important even in non-equilibrium[4] models that feature nominal wage and/or price stickiness.

With regard to initiating shocks, by contrast, the RBC viewpoint reflects much more of a departure from other theories. In this regard, two positions can usefully be identified. The weaker of the two is that technology shocks are quantitatively more important than monetary disturbances as initiators of business cycle movements, while the stronger is that monetary disturbances are of negligible consequence. The former position is compatible with monetary-misperception variants of equilibrium theory, as these have not involved denials of the role of supply shocks. The stronger RBC position, however--the hypothesis that monetary disturbances are an insignificant source of cyclical fluctuations--is clearly inconsistent with most alternative theories. In this form, the RBC approach presents a distinct challenge to mainstream macroeconomic analysis.[5]

In the discussion that follows, most of the emphasis will be implicitly given to the weaker version of the RBC hypothesis, as it is more clearly representative of the position taken in print by RBC proponents. The strong version will be accorded some attention, however, for two reasons. The more basic of these is that while the main topic of the present survey is RBC theory itself, a secondary topic is the contrast provided by RBC models with theories that rely upon monetary disturbances. Thus the sharp distinction provided by the strong RBC hypothesis is expositionally natural and convenient. But, in addition, it is this strong hypothesis that provides the RBC approach with a truly distinctive identity. As monetary-misperception variants of equilibrium theory do not deny the existence of supply shocks or propagation mechanisms, it is difficult to see how the RBC approach could be distinguished from the more general category of equilibrium analysis without reliance on the strong hypothesis. It is evidently the latter that constitutes the approach's distinguishing characteristic.

The present paper's organization is as follows. In Section II the main features of the RBC approach are introduced by means of a simple prototype model. While this discussion touches upon certain qualitative properties of the model and their relation to actual U.S. data, the main quantitative comparison between theory and evidence appears in Section III. There more elaborate versions of the model are recognized and a fairly detailed review of evidence of the type emphasized by Kydland and Prescott (1982) (1986) is provided. Then in Section IV other types of evidence and several matters of controversy are reviewed. Finally, some conclusions are tentatively put forth in Section V.

II. The Basic RBC Model

In this section the object is to describe a model that provides a simple example of the type featured in the RBC literature. In this demonstration, the intention will be to outline the workings of this model in an intuitive manner, not to develop mathematical techniques or provide formal proofs of the relevant propositions. The discussion will accordingly be less than rigorous. A few references will be included, however, to direct readers to sources that contain formal proofs and more complete descriptions of the relevant mathematical concepts and techniques.[6]

Consider an economy composed of a large number of similar, infinite-lived households, each of which acts at time t so as to maximize

$$(1) \quad E_t\left[\sum_{j=0}^{\infty} \beta^j u(c_{t+j}, \ell_{t+j})\right]$$

Here c_t and ℓ_t denote the household's consumption and leisure during period t, while β is a discount factor ($0 < \beta < 1$) that reflects a preference for current over future consumption-leisure bundles. Application of the operator $E_t(\cdot)$ yields the mathematical expectation, conditional upon complete information pertaining to period t and earlier, of the indicated argument. Leisure is time not devoted to labor, so an appropriate choice of units implies that $\ell_t = 1 - n_t$, where n_t is the household's labor supplied during t. The function u is assumed to be increasing in both arguments, differentiable, and "well-behaved;" thus for i = 1,2 we have $u_i > 0$, $u_{ii} < 0$, $u_i(0) = \infty$, $u_i(\infty) = 0$.

Each of the postulated households has access to a production function of the form

$$(2) \quad y_t = z_t\, f(n_t^d, k_t^d),$$

where y_t is output of the economy's single good during t, with n_t^d and k_t^d

denoting labor and capital inputs used during t by the household. The variable z_t is the realization in period t of a random variable that reflects the state of technology. The process generating z_t we assume to be of the stationary Markov class--the distribution of z_t depends on z_{t-1} but is otherwise constant over time. The function f is taken to be homogeneous of degree one and well-behaved with positive but diminishing marginal products. The household's output can be consumed or stored, with stored output adding to the household's stock of capital in the following period. During each period, the fraction δ of the capital in existence disappears via depreciation.

Finally, the economy under discussion is assumed to possess competitive markets for labor and capital services--markets on which the wage and rental rates are w_t and q_t, respectively.[7] Thus the budget constraint faced by our typical household in period t is as follows:[8]

$$(3)\quad c_t + k_{t+1} = z_t\, f(n_t^d, k_t^d) + (1-\delta)k_t - w_t(n_t^d - n_t) - q_t(k_t^d - k_t).$$

At t, consequently, the household acts to maximize (1) subject to a sequence of constraints of the form (3). The u and f functions have been specified so that corner solutions will be avoided, so the following first-order conditions are necessary for a maximum:[9]

$$(4a)\quad E_t\, u_1(c_{t+j},\, 1-n_{t+j}) - E_t\, \lambda_{t+j} = 0$$

$$(4b)\quad E_t\, u_2(c_{t+j},\, 1-n_{t+j}) - E_t\, \lambda_{t+j}\, w_{t+j} = 0$$

$$(4c)\quad E_t\, z_{t+j} f_1(n_{t+j}^d, k_{t+j}^d) - E_t w_{t+j} = 0$$

$$(4d)\quad E_t\, z_{t+j} f_2(n_{t+j}^d, k_{t+j}^d) - E_t q_{t+j} = 0$$

$$(4e)\quad -E_t\lambda_{t+j} + E_t\beta\lambda_{t+j+1}[z_{t+j+1} f_2(n_{t+j+1}^d, k_{t+j+1}^d) + 1-\delta] = 0.$$

Here λ_{t+j} is the shadow price in utility terms of a unit of the economy's good in period t+j--a Lagrange multiplier, if one chooses to think of the maximization in those terms. In addition to conditions (4), there is also a transversality condition pertaining to the long-range aspect of the household's plans;[10] it may be written as

(5) $$\lim_{j\to\infty} E_t \beta^{j-1} \lambda_{t+j} k_{t+j+1} = 0$$

Together, conditions (3), (4) and (5) are necessary and sufficient for an optimum. Thus they define the typical household's choice at t of c_t, n_t, n_t^d, k_t^d, and k_{t+1} in response to current values of w_t and q_t, its expectations about the future, and its accumulated stock of capital, k_t.

Now consider the matter of market equilibrium. For such a state to prevail, it must be the case that $\Sigma n_t = \Sigma n_t^d$ and $\Sigma k_t = \Sigma k_t^d$, where the sums are taken over all households. But since these households are all alike, and all experience the same value for the shock z_t, those equalities imply that $n_t = n_t^d$ and $k_t = k_t^d$. Furthermore, it is assumed that expectations are rational, which means in this case that the conditional mathematical expectations in equations (4) are based on probability distributions that coincide with those implied by the economy's structure (as represented by the model). Consequently, we see that market equilibrium can be characterized by the following set of equalities, which hold for periods t = 1,2,...:

(6) $c_t + k_{t+1} = z_t f(n_t,k_t) + (1-\delta)k_t$

(7) $u_1(c_t,1-n_t) - \lambda_t = 0$

(8) $u_2(c_t, 1-n_t) = \lambda_t z_t f_1(n_t,k_t)$

(9) $\lambda_t = E_t \beta\lambda_{t+1}[z_{t+1}f_2(n_{t+1},k_{t+1}) + 1-\delta]$.

Given an initial value for k_1, these four equations define time paths of the economy's per-household values of c_t, k_t, n_t, and λ_t. There will be a multiplicity of such paths, but only one will satisfy the transversality condition (5) that is necessary for household optimality.[11]

Before continuing, we might note that precisely the same set of relations would have been obtained if we had simply treated each household as an isolated "Robinson Crusoe" unit for which the distinctions between n_t and n_t^d, and between k_t and k_t^d, are not applicable. This observation illustrates the point that Crusoe-style analysis can in certain cases be interpreted as pertaining to the behavior of quantity variables for competitive market economies. But for this type of equivalence to hold, all households must be alike and there must be no externalities. Furthermore, if there exists a government sector then the model must be elaborated so as to recognize its existence, so Crusoe-style analysis is not generally available.[12]

Let us now consider solutions to the system of equations (6)-(9). Inspection of these, plus reflection upon the nature of the economy at hand, leads to the conclusion that the state of the system at time t is fully defined by the current values of k_t and z_t.[13] Consequently, solutions to (6)-(9) will be of the form

(10) $k_{t+1} = k(k_t, z_t)$

(11) $c_t = c(k_t, z_t)$

(12) $n_t = n(k_t, z_t)$

(13) $\lambda_t = \lambda(k_t, z_t)$

where k, c, n, and λ are continuous functions. This conclusion holds, we note, not only for serially uncorrelated technology shocks but also whenever these are generated by a stationary Markov process. This is possible in the latter case because z_t provides all relevant conditioning information for the probability distribution of values occurring at time t+1.

It should be useful parenthetically to note that government purchases--denoted g_t on a per-household basis--could be incorporated in the structure under discussion by adding to the system an equation reflecting the government's budget constraint and modifying the household budget constraint to reflect taxes. (Effects of g_t on production or utility functions, if any, would also be recognized.) If the taxes were of the lump-sum variety then equations (4) would remain as shown, but these equations would have to be altered if taxes were levied on some productive activity. In either case, if government purchases conformed to a policy rule relating g_t to g_{t-1}, k_t, and z_t then solution expressions like (10)-(13) would apply but with g_t included as a third argument.

Equations (10)-(13) are simple in appearance, but this simplicity is perhaps deceptive in the following sense: there are very few functional forms for u and f that will permit derivation of explicit closed-form solutions for k_{t+1}, c_t, and n_t. There is one reasonably attractive combination that will do so, however, which consequently has been featured in several papers.[14] That combination involves a log-linear specification for u and a Cobb-Douglas form for f, as follows:

(14) $u(c_t, 1-n_t) = \theta \log c_t + (1-\theta) \log(1-n_t)$

(15) $z_t f(n_t, k_t) = z_t n_t^{\alpha} k_t^{1-\alpha}$.

In addition, this special case requires complete depreciation of capital within a single period, i.e., requires that $\delta = 1$.[15]

To lend concreteness to the discussion, let us now consider the example provided by this special case. With the functional forms in (14) and (15) and with $\delta = 1$, the system of equations (6)-(9) becomes the following:

(6') $c_t + k_{t+1} = z_t\, n_t^{\alpha}\, k_t^{1-\alpha}$

(7') $\theta/c_t = \lambda_t$

(8') $(1-\theta)/(1-n_t) = \alpha\lambda_t z_t n_t^{\alpha-1} k_t^{1-\alpha}$

(9') $\lambda_t = (1-\alpha)\ \beta E_t \lambda_{t+1}[z_{t+1} n_{t+1}^{\alpha} k_{t+1}^{-\alpha}]$.

To obtain solution equations analogous to (10)-(13) for this special system, we begin by noting that with a utility function of the form (14) and complete depreciation, the income and substitution effects of a wage rate change will just offset each other leaving the leisure choice unaffected (King, Plosser, and Rebelo, 1987). Consequently, it is reasonable to conjecture that n_t will be a constant in the solution, i.e., that $n_t = n$. Then the manner in which z_t and k_t enter the production function leads to the further conjecture that c_t and

k_{t+1} will be proportional to the product $z_t\, k_t^{1-\alpha}$. Thus our task can be reduced to the problem of evaluating Π_{10} and Π_{20} in the two expressions

(16) $c_t = \Pi_{10} z_t k_t^{1-\alpha}$

(17) $k_{t+1} = \Pi_{20} z_t k_t^{1-\alpha}$.

To do so, we first use (7') to eliminate λ_t and λ_{t+1} from (9') and then substitute in (16) and (17) as follows:

$$(18)\quad \frac{\theta}{\Pi_{10} z_t k_t^{1-\alpha}} = (1-\alpha)\ \beta E_t \left[\frac{\theta z_{t+1}\ n^{\alpha} k_{t+1}^{-\alpha}}{\Pi_{10} z_{t+1} k_{t+1}^{1-\alpha}} \right] = \frac{(1-\alpha)\ \beta\ \theta\ n^{\alpha}}{\Pi_{10}(\Pi_{20} z_t k_t^{1-\alpha})}\ .$$

Then $\theta/\Pi_{10}\ z_t k_t^{1-\alpha}$ cancels out the latter, yielding $\Pi_{20} = (1-\alpha)\ \beta\ n^{\alpha}$.

Next, substitution of (16) and (17) into (6') and cancellation of $z_t\ k_t^{1-\alpha}$ gives $\Pi_{10} + \Pi_{20} = n^{\alpha}$ from which, with the expression for Π_{20} obtained above, we find that $\Pi_{10} = [1-(1-\alpha)\beta]\ n^{\alpha}$. Finally, substitution of these two expressions into a relation, obtained by eliminating λ_t between (7') and (8'), results in the following value for n:

$$(19)\quad n = \frac{\alpha\theta}{\alpha\theta + (1-\theta)\ [1-\beta(1-\alpha)]}.$$

Thus our conjecture regarding n_t is verified and it is concluded that, in the special example at hand, consumption and capital per household fluctuate over time according to [16]

$$(20)\quad c_t = [1-(1-\alpha)\beta]\ n^{\alpha}z_t k_t^{1-\alpha}$$

$$(21)\quad k_{t+1} = (1-\alpha)\beta\ n^{\alpha}z_t k_t^{1-\alpha}.$$

Now, from (21) we can immediately observe that the logarithm of k_t obeys a stochastic process of the form

$$(22)\ \log k_{t+1} = \Phi_0 + (1-\alpha)\log k_t + \log z_t.$$

Since $|1-\alpha| < 1$, moreover, the process for k_t is dynamically stable. Furthermore, it features positive serial correlation: if $\log k_t$ is above normal, then so too will be the expected value of $\log k_{t+1}$, assuming that the process for $\log z_t$ is serially uncorrelated. If, instead, the z_t process is of the first-order autoregressive [i.e., AR(1)] form

$$(23)\ \log z_t = \rho \log z_{t-1} + \varepsilon_t,$$

with ε_t white noise, then $\log k_t$ will be second-order autoregressive [AR(2)]:

$$(24)\quad \log k_{t+1} = \Phi_0(1-\rho) + (1-\alpha+\rho)\log k_t - (1-\alpha)\rho \log k_{t-1} + \varepsilon_t.$$

Furthermore, in this case the second-order autoregressive structure carries over to other crucial quantity variables including $\log c_t$ and $\log y_t$. To illustrate that fact, let us express (20) as

$$\log c_t = \Phi_1 + (1-\alpha)\log k_t + \log z_t.$$

But from (22) $\log k_t = [1-(1-\alpha)L]^{-1}\ [\Phi_0 + \log z_{t-1}]$ so substitution and rearrangement gives

$$(25)\quad [1-(1-\alpha)L]\log c_t = (1-\alpha)\Phi_0 + \alpha\Phi_1 + [1-(1-\alpha)L]\log z_t + (1-\alpha)\log z_{t-1}$$

which may be simplified, using $\log z_t = (1-\rho L)^{-1}\ \varepsilon_t$, to yield

$$(26)\quad \log c_t = (1-\alpha+\rho) \log c_{t-1} - (1-\alpha)\rho \log c_{t-2} + \alpha(1-\rho)\, \phi_1 + (1-\alpha)\,(1-\rho)\, \phi_0 + \varepsilon_t.$$

Thus the simple special-case example of the prototype RBC model suggests that, with AR(1) technology shocks, important quantity variables will have the time-series properties of second-order AR processes. This conclusion is of interest since detrended[17] quarterly U.S. data series for the logs of various aggregate quantities are, in fact, reasonably well described by AR(2) models.[18]

Another interesting property of the special-case model summarized by (20) and (21) is that the average product of labor is positively correlated with the level of total output. That property is, of course, an immediate implication of the constant-employment feature of the model. But it is a significant property, nevertheless, because the average product of labor is clearly procyclical in the actual U.S. quarterly data. And some of the leading orthodox theories suggest that the marginal product of labor--and thus the average product, if the production function is approximately Cobb-Douglas in form--will be countercyclical.[19]

Nevertheless, even at the qualitative level there are some prominent ways in which the special-case model fails to match important aspects of the actual U.S. time series. One of these is the constant-employment feature noted above and another is the model's implication that fluctuations in consumption and investment are of equal severity.[20] Both of these qualitative flaws can be overcome, however, by postulating that capital depreciation is incomplete within the period. With this change, the possibility of an explicit solution is lost so the claim cannot be verified analytically.[21] But simulation results reported by Gary Hansen (1985, Table 1) correspond to the case under discussion, and these involve employment variability and investment fluctuations that are several times as severe (in terms of percentage standard deviations from trend) as those of consumption. So even with the special assumptions (14) and (15) regarding utility and production functions, the prototype RBC model provides a reasonable match to important features of actual business cycle data. It is time, consequently, to turn to quantitative aspects of the match. This will be done in the next section.

III. Quantitative Aspects of RBC Models

Perhaps the strongest single stimulus to analysis with RBC models was provided by the innovative "Time to Build" paper of Kydland and Prescott (1982), which first demonstrated the possibility of obtaining a good quantitative match between RBC model implications and actual business cycle fluctuations. The model developed and simulated by Kydland and Prescott is basically of the type described in the previous section, but includes several additional features that were intended to improve its performance--i.e., its agreement with cyclical characteristics of the postwar U.S. data. Four such features are as follows.

(i) Leisure "services" in each period (i.e., quarter) are represented by a distributed lag of current and past leisure hours.

(ii) Investment projects begun in period t require additional expenditures in periods, t+1, t+2, and t+3 before becoming productive in period t+4.

(iii) Producers hold inventories of finished goods that serve as an additional factor of production.

(iv) The technology shock z_t is composed of transitory and highly-persistent components that cannot be directly distinguished by producers or consumers.

Needless to say, with these features and incomplete depreciation, the Kydland-Prescott model does not admit an analytical solution. But an approximation can be obtained, parameter values assigned, and simulations conducted. Kydland and Prescott's approach was to follow that strategy, with the average results of a number of stochastic simulations serving to characterize the model's cyclical properties. And the same kind of procedure can be applied to the basic model (with $\delta < 1$) of the previous section, or other structures of the same general type.

To see how well the basic and Kydland-Prescott structures do in mimicking actual U.S. fluctuations, we begin by examining the figures presented in Table 1. Here the first column reports, for several important quantity variables, the magnitude of cyclical fluctuations in the actual quarterly U.S. data, 1955.3-1984.1. These magnitudes are standard deviations of the quarterly observations measured relative to trend values, with the departures from trend expressed in percentage form. The trend values themselves are generated by smoothing the raw series in accordance with a procedure developed by Hodrick and Prescott (1981).[22] From this first column it is apparent that actual consumption fluctuates less, and investment much more, than total output (in percentage terms, that is). Also, the extent of fluctuations in total manhours employed in production (designated "hours") is almost as great as that of total output.

In column two, comparable figures are reported for a version of the basic model that has been specified and simulated by Hansen (1985).[23] Here the standard deviations actually reported by Hansen have all been scaled up, multiplied by the factor 1.31, so that the reported standard deviation for output is the same as in the U.S. data.[24] Thus the values in Table 1 are designed to reflect only the <u>relative</u> extent of fluctuations of other variables in comparison with output. The magnitude of output fluctuations is governed, for each model, by the variance of the technology shock z_t that is <u>assumed</u> and used in the simulations; independent evidence concerning the plausibility of these shock-variance magnitudes will be considered below.

Table 1
Standard Deviations of Percentage Departures from Trend

Variance	U.S. Economy[a]	Basic Model[b]	Kydland-Prescott[c]	Hansen Model[b]
Output	1.76	1.76[d]	1.76[d]	1.76[d]
Consumption	1.29[e]	0.55	0.44	0.51
Investment	8.60[f]	5.53	5.40	5.71
Capital Stock	0.63	0.47	0.46	0.47
Hours	1.66	0.91	1.21	1.35
Productivity	1.18	0.89	0.70	0.50

[a]Quarterly data, 1955.3-1984.1, seasonally adjusted.
Source: Hansen (1985)

[b]Source: Hansen (1985)

[c]Source: Prescott (1986)

[d]Shock variance set to provide match of output variation with actual data.

[e]This figure pertains to GNP-account consumption expenditures, which includes expenditures on durable goods. For expenditure on non-durable goods and services the figures are about 1.2 and 0.6, respectively, so the relevant number for comparison is about 0.9.

[f]For fixed investment, the figure is approximately 5.3.

From the second column it is clear that Hansen's version of the basic model implies fluctuations that have some important characteristics in common with the actual U.S. data. In particular, consumption varies less and investment more than output. The relative severity of consumption fluctuations is somewhat less in the model economy, however, and the same is true but to a greater extent for the hours (employment) variable.

Also important are the statistics given in Table 2, which pertain to contemporaneous correlations of the other variables--again measured as percentage deviations from trend--with output. Here it will be seen that the match with actual data provided by the basic model is rather good, although the hours and productivity (i.e., output per hour) variables are more highly correlated with output in the model than in actuality, and to a substantial extent.

Analogous figures for the RBC model of Kydland and Prescott, which includes the four additional features mentioned above, are reported in the third columns of Tables 1 and 2. It will be seen from these that the extent of hours variability is significantly increased by the additional features, though not enough to make the match entirely satisfactory. In terms of other variables, the extra features do not seem to add much in comparison with the basic model.[25]

Results are given in the fourth column of Tables 1 and 2 for a variant model developed by Hansen (1985)[26] that is like the basic model except that each worker is constrained to work "full time" or not at all. In particular, Hansen's indivisible-labor setup departs from the basic model as follows:

Table 2

Contemporaneous Correlations with Output
(Departures from Trend)

Variance	U.S Economy	Basic Model	Kydland-Prescott	Hansen Model
Consumption	0.85	0.89	0.85	0.87
Investment	0.92	0.99	0.88	0.99
Capital Stock	0.04	0.06	0.02	0.05
Hours	0.76	0.98	0.95	0.98
Productivity	0.42	0.98	0.86	0.87

Notes: See Table 1.

> The new commodity being introduced is a contract between the firm and a household that commits the household to work h_0 hours [full time] with probability α_t. The contract itself is being traded, so the household gets paid whether it works or not. Therefore, the firm is providing complete unemployment insurance to the workers. Since all households are identical, all will choose the same ... α_t. However, although households are ex ante identical, they will differ ex post depending on the outcome of the lottery: a fraction α_t of the continuum of households will work and the rest will not. (Hansen, 1985, p. 316).

It is apparent from Table 1 that this model generates considerably more variability in manhours employment, relative to output, than the basic model (but at the cost of a poorer match for productivity). Something that is not apparent from the table is that the assumed variance of z_t, needed to generate output variability equal to that of the U.S. economy, is smaller than that of the basic model. Specifically, the standard deviation of z_t required for the column four values is only 0.767 as great as that for column two.[27]

Kydland (1984) and Kydland and Prescott (1986) have explored other specificational modifications that are designed to improve the match between model and actuality. Kydland (1984) postulates two types of labor, of differing effectiveness in production, and finds that this modification increases the variability of hours relative to output. Kydland and Prescott (1986) incorporates a variable rate of capital utilization and shows that a smaller variance for z_t is needed, with this elaboration, to yield output variability that matches actuality. In each of these two cases the magnitude of the improvement is about 20% and in each case the overall pattern of correlations is not substantially altered.

In all of these numerical investigations, it should be emphasized, the model's's parameter values are <u>not</u> obtained in a manner that provides the best fit to the quarterly time series data. Instead, values for parameters--analogous to those designated α, β, δ, θ, and ρ in Section II above--are assigned so as to agree with panel studies of individual households or with stylized facts relating to such magnitudes as labor's share of national income (suggesting $\alpha = 0.64$), the fraction of time spent in market employment ($\theta = 0.33$), etc.[28] This procedure guarantees that certain properties of the model will be "sensible," in the judgement of the model builders, a situation that might not obtain if the parameters were estimated in a more orthodox manner. In this regard it is important to note that Kydland-Prescott and Hansen choose the value 0.95 for the counterpart of the parameter ρ. In the context of their model, the high implied degree of serial correlation for the technology shock tends to impart a high degree of serial correlation to endogenous variables such as employment and output.[29]

One of the main issues considered in the literature is whether the required magnitude of technology shocks is plausible, i.e., whether the variance of z_t needed to generate fluctuations of the amplitude reported in Table 1 could plausibly apply to actual aggregate technology shocks.[30] This issue is addressed by Prescott (1986), who initially estimates the variance of ε_t (not z_t) as the sample variance of the residuals from an aggregate Cobb-Douglas production function, fitted in first differenced form.[31] Prescott then goes on to revise downward this straightforward estimate by attempting to take account of measurement error in the labor input series. His final estimate of the standard deviation in 0.00763, which can be compared with the values needed in different models to generate output fluctuations that match (as in Table 1) those of the U.S. economy. The needed value with the basic model is 0.0093, according to Hansen's (1985) results,[32] while the figure with his indivisible-labor setup is only 0.00712. In the Kydland-Prescott model there are two technology shocks, as mentioned above. But the contribution of the purely transitory shock is very small, so we can focus on the highly-persistent shock. Its standard deviation is reported in Kydland-Prescott (1986) as 0.0091. Given the relationship $z_t = 0.95z_{t-1} + \varepsilon_t$, this last figure implies a standard deviation for z_t of the value 0.0291.

These numerical results may be viewed as giving some support to the idea that technology shocks are in fact large enough to give rise to business cycle fluctuations of the magnitude experienced by the postwar U.S. economy. There are reasons, however, for skepticism. The first of these involves the type of procedure used by Prescott to estimate the shock variance. His procedure, to some extent based on Solow (1957), relies crucially on the assumption that current capital and labor are the only relevant inputs. If, however, there are in fact adjustment costs, so that previous levels of (say) labor usage are relevant for current output, then the Solow-Prescott procedure will be likely to overestimate the shock variance.[33] In this regard it is worthy of note that the literature on technical progress that followed Solow's 1957 contribution indicated that the magnitude of technical change would be strongly overstated by the Solow procedure unless steps were taken to correct for certain neglected effects. Jorgenson and Griliches (1967), for example, summarize their findings for the U.S. economy over 1945-65 as follows: "The rate of growth of [total factor] input initially explains 52.4 percent of the rate of growth of output. After elimination of aggregation errors and correction for changes in rates of utilization of labor and capital stock the rate of growth of input explains 96.7 percent of the rate of growth of output; change in total factor productivity explains the rest." Thus, in this example, the Jorgenson-Griliches adjustments reduce the average contribution to growth provided by the residual to only 7% of the initial Solow-type estimate.[34] Finally, it should be noted without prejudice that Prescott's procedure is actually not the same as Solow's. Specifically, Prescott's procedure uses the same labor-share parameter for each period while Solow's treats the labor share as a variable. Thus Prescott's procedure is likely to fit the observations less closely and leave more variation to be accounted for by the residual, thereby yielding--perhaps appropriately--a larger estimate of its variance.

A second problem is related to the nature of the unobserved random components that the RBC literature refers to as "technology shocks." If this term is taken literally to refer to shifts in the state-of-knowledge technological relationship between inputs and outputs, then it seems highly unlikely that there could exist any substantial aggregate variability. Here the point is that highly distinctive technologies involving entirely different types of machines (and other sorts of capital) are used in different industries--and for different products within any single industry. Accordingly, any specific technological discovery can impact on the production function for only a few products. According to this perspective, RBC models should be formulated so as to recognize that there are many different productive sectors whose technology shocks should presumably be nearly independent. Averaging across industries, then, the economy-wide technology shock would have a variance that is small in relation to the variance for each industry, much as the mean of a random sample of size n has a variance that is only 1/n times the variance of each observation.[35]

There is one prominent type of "supply side" disturbance that has effects across a very wide category of industries, namely, a change in the real price that must be paid for imported raw materials--especially, energy. The oil-price shocks of 1974, 1979, and 1986 clearly have had significant impact on the U.S. economy at the aggregate level (Hamilton, 1983). And since the Kydland-Prescott and Hansen models have no foreign sector, such effects are treated by their analyses as "residuals"--shifts in the production function.[36] Such a treatment is, however, avoidable since these price changes may be observed and are documented in basic aggregate data sources. It is also analytically undesirable: to lump input price changes together with production-function shifts is to blur an important distinction. Presumably future RBC studies will explicitly model these terms-of-trade effects and thereby reduce their reliance on unobserved technology shocks.

A good bit of relevant research has been prompted by the contribution of Lilien (1982), who suggested that unusually large shifts in the sectoral composition of output would necessitate unusually large employment reallocations which, being costly, tend to reduce aggregate employment and output. While Lilien emphasized relative demand shifts as a source of sectoral imbalance, sector-specific technology shocks would likewise call for reallocations. Some evidence in support of the sectoral shift hypothesis is presented by Lilien (1982), who shows that a measure of the dispersion of employment growth across two-digit industries has considerable explanatory power for the aggregate unemployment rate. A pair of subsequent studies--succinctly described by Barro (1986)--are, however, largely unsupportive of Lilien's hypothesis. In particular, Abraham and Katz (1986) find evidence to be inconsistent with the implication that job-vacancy rates should be positively related to the employment-growth dispersion measure, while Loungani (1986, p.536) finds that "once the dispersion in employment growth due to oil shocks is accounted for, the residual dispersion has no explanatory power for unemployment." More recently, Davis (1987) has challenged the Abraham-Katz conclusion, on the basis of the stock-flow distinction as applied to vacancies, and has developed a reallocation <u>timing</u> hypothesis that stresses cyclical variations in the cost of unemployment that accompanies sectoral reallocations. In addition, Davis (1987) presents various bits of evidence in support of the idea that sectoral shifts and reallocation timing are important determinants of aggregate unemployment; useful observations and caveats are provided by Oi (1987). Conclusions regarding the significance of this line of research for RBC models would seem to be as yet premature.

While considerable attention has been devoted in the literature to questions regarding (a) technology shocks and (b) the variability of hours relative to output, other important aspects of the Kydland-Prescott results have been neglected. One example is provided by the excessively strong correlation between productivity and output that is implied by the models. A related problem--noted by Kydland and Prescott (1982, p. 1366)-- that is potentially crucial concerns productivity-output correlations at various lags and leads. In this regard, Table 3 compares a few of these autocorrelations for the Kydland-Prescott model with those that pertain to the U.S. data. Evidently, the pattern implied by the model is markedly different from that existing in actuality, productivity being positively correlated with past output in the model but negatively correlated in reality. As the actual pattern is rather like that which would exist if the bivariate relationship were dominated by exogenous (demand-induced?) output movements and "labor hoarding,"[37] this discrepancy would seem to warrant particular attention by RBC proponents and critics.

Another significant topic that has been neglected in the main RBC literature is the cyclical behavior of relative <u>price</u> variables, including the real wage and the real interest rate. Their models' implications for the latter have been reported by Kydland and Prescott (1982) (1986), but not the

Table 3

Correlation of Output in Period t with
Labor Productivity in Period Indicated
(Departures from Trend)

	t-2	t-1	t	t+1	t+2
U.S. Data	0.60	0.51	0.34	-0.04	-0.28
Kydland-Prescott Model	0.37	0.68	0.90	0.59	0.44

Source: Kydland and Prescott (1986)

counterpart statistics for the U.S. data. One reason for this omission, presumably, is that the ex-ante real rate is the relevant variable while only ex-post values are directly observable. Attempts to estimate ex-ante real rates have been made, however, by Hamilton (1985) and Mishkin (1983). The impression that one gets from examination of their charts is that the ex-ante real rate is not nearly as strongly procyclical as the Kydland-Prescott model implies.[38] But numerical analysis is needed to be confident on this point.

In the case of the real wage, Kydland and Prescott report neither actual nor model-implied results. But with a production function that is basically Cobb-Douglas in specification, the marginal product of labor will fluctuate closely with the average product of labor--which is the "productivity" variable that appears in Tables 1-3. Many investigators including Bils (1985), and Geary and Kennan (1982) (1984) have found the real wage to be positively correlated with output. But the magnitude of the contemporaneous correlation in these studies is quite small,[39] a far cry from the 0.86-0.98 values implied by the Kydland-Prescott and Hansen models.

Let us now turn to relevant evidence obtained by other empirical methods. Incomplete models with specifications that are compatible with the RBC approach have been studied by means of econometric techniques that are quite different from those utilized by Kydland-Prescott and Hansen. Most notably, there area a number of papers that estimate, using aggregate time series observations, household optimality conditions analogous to equations (6)-(9) above--but with the typical household viewed as facing market-determined wage and interest rates rather than operating its own production facility. Leading examples in this genre are provided by Lars Hansen and Singleton (1982), Eichenbaum, Hansen, and Singleton (1986), and Mankiw, Rotemberg, and Summers (1984).

Because these studies are designed to utilize models in which the production sector is left unspecified, it is not possible to obtain implications concerning fluctuations of the type that are emphasized by Kydland and Prescott. There are certain results obtained that have some relevance, however, for the RBC type of model. In particular, the models' assumptions (including rational expectations) imply that certain variables should be orthogonal to composite "disturbances" that are involved in the instrumental-variable (or "method of moments") estimation procedure. This in turn implies that certain test statistics developed by Hansen (1982) have, under the hypothesis that the model is well-specified, asymptotic chi-square distributions with known degrees of freedom. Computed values that are improbable under this hypothesis therefore constitute formal evidence against the model at hand. In the recent study by Eichenbaum, Hansen, and Singleton (1986), five of the six such statistics computed for a model composed of equations similar to (6)-(9) above[40] call for rejection at significance levels below 0.01.[41] In addition, Eichenbaum, Hansen, and Singleton report that "estimated values of key parameters differ significantly from the values assumed in several studies," i.e., those of Kydland and Prescott. It is not clear, however, that these results are unfavorable for RBC models in general, as opposed to the Kydland-Prescott specification in particular.

Finally, mention should be made of an ambitious study conducted by Altug (1985) who has obtained maximum likelihood estimates of a model that is fairly close in specification to that of Kydland and Prescott (1982). No single test statistic for the Kydland-Prescott parameter values is reported by Altug, but the impression obtained from her various tables and charts is that agreement with the data is not very good. Also, Altug's estimated version of the model has properties that fail to match actuality in several respects. For example, spectra of individual series implied by the model have shapes that are quite different from the unrestricted estimates of these spectra.

It can be argued that the value of formal econometric test results is small in the present context, i.e., in indicating whether models like that of Kydland and Prescott (1982) do a good job of matching the actual data. Any model that is both manageable and theoretically coherent will necessarily be too simple to closely match the data in all respects, so with the number of quarterly observations available such models will inevitably be rejected at low significance levels in formal tests against generalized alternatives. There is considerable merit to this argument, in my opinion, as a general matter. It is unclear, however, that the particular models mentioned in Tables 1-3 provide good data matches according to the builders' own criteria. To some extent, this may be due to the current models' simplicity, rather than to any basic flaw in the RBC strategy. Future work on the topic should be designed with that consideration in mind.

IV. Additional Issues

There are numerous additional matters of controversy--both substantive and methodological--pertaining to RBC analysis that could be discussed. A few of the more important ones will be taken up in the present section.

The first of these topics concerns the issue of social optimality. In the models of Kydland and Prescott (1982) (1986) and Hansen (1985) there are by construction no externalities, taxes, government consumption, or monetary variables. Furthermore, all households are alike and each is effectively infinite-lived. Consequently, competitive equilibria in these particular models have the property of Pareto optimality.[42] One message that should be taken from that fact is that the mere existence of cyclical fluctuations is not sufficient for a conclusion that interventionist government policy is warranted. These models provide no basis, on the other hand, for concluding that the solutions generated by actual economies are Pareto optimal. To overturn that notion it should be sufficient to recall that in the U.S. economy, to take one example, about 20% of total output is absorbed by government purchases. Therefore, unless government services are precisely chosen to reflect individuals' preferences, some departure from Pareto optimality will result. In addition, taxes used to finance these purchases are likely to have distorting allocational effects. As the RBC models in question do not recognize the existence of government activities, they simply have nothing to say on this matter.

Another particularly prominent omission pertains to money. It is not true, as Eichenbaum and Singleton (1986) have pointed out, that the models must be interpreted as implying the literal absence of money. Indeed, it is doubtful that RBC proponents intend to advance the proposition that no less output would be produced in the U.S. (with the existing capital stock) if there were no medium of exchange--i.e., if all transactions had to be carried out by crude or sophisticated barter. But the models do imply that, to a good approximation, policy-induced fluctuations in monetary variables have no effect on the real variables listed in Table 1--at least for fluctuations of the magnitude experienced since World War II. Of course, RBC proponents do not deny that there exist correlations between monetary and real variables, but they claim that these reflect responses by the monetary system to fluctuations induced by technology shocks--"reverse causation," in the words of King and Plosser (1984).

It is somewhat ironic that the reverse-causation viewpoint should be featured in a body of analysis that is a direct outgrowth of the monetary-misperception class of equilibrium business cycle models. For the problem that Lucas set out to solve, in his papers (1972) (1973) that created this class, was to construct a competitive equilibrium model that incorporates a Phillips-type money-to-income relationship although the modelled economy is populated with rational agents--i.e., agents devoid of money illusion and expectational irrationality. In his words, Lucas's JET paper was designed to provide a model "of an economy in which equilibrium prices and quantities exhibit what may be the central feature of the modern business cycle: a systematic relation between the rate of change in nominal prices and the level of real output" (Lucas, 1972a, p. 103).[43]

It is important, then, to reflect on the process by which the equilibrium business cycle program was transformed into one in which monetary impulses play no significant role. In that regard, there are both theoretical and empirical findings that have been important. Little needs to be said here about the former, for the basic objection to monetary misperception models has become widely understood.[44] Some space must be devoted, however, to the empirical findings.

Chronologically the first major empirical development was provided by Sims's (1980) (1982) demonstration that money stock innovations, in small VAR systems, have little explanatory power for output fluctuations when a nominal interest rate variable is included in the system.[45] The interpretation offered by Sims is that money stock innovations represent surprise policy actions by the monetary authority--i.e., the Fed--so the finding is indicative of the unimportance of monetary policy actions. But this interpretation is open to objection. Suppose that the monetary authority implements its actions by manipulating, on a quarter-to-quarter or month-to-month basis, the nominal interest rate. Then interest rate innovations would reflect monetary policy surprises, with money stock innovations representing linear combinations of disturbances afflicting money demand, saving-investment, and production relations as well as policy behavior. In this case, interest rate innovations would measure monetary policy surprises better than would money stock innovations. And in fact, U.S. monetary policy has been implemented throughout the postwar period by means of interest rate instruments, even during the period 1979-82.[46] So the finding that money stock innovations provide little explanatory power for output or employment movements simply does not imply that monetary policy has been unimportant.[47] On the contrary, the considerable explanatory power provided by interest rate innovations in Sims's studies suggests just the opposite.

For analogous reasons, moreover, it is incorrect to presume that movements in the monetary base are accurate reflections of U.S. monetary policy behavior. While the Fed could use the base as its operating instrument if it chose to do so, in fact it never has. Consequently, empirical analyses that treat the base as the Fed's instrument involve a serious misspecification that must be accounted for in interpreting their findings.

These particular difficulties concerning the monetary authority's operating procedures can in principle be partially circumvented by noting that, in a system that includes all variables of macroeconomic importance to agents, output, employment, and other real variables will be block exogenous to all nominal variables--prices and interest rates, for example, as well as monetary aggregates. It would then appear that this property of RBC models could be used as the basis of a statistical test; if the RBC hypothesis were true, then output, employment, etc. should not be Granger-caused by any nominal variable or variables.[48] Unfortunately, further analysis indicates that in practice the presence of Granger causality from nominal to real variables is neither necessary nor sufficient for rejection of the RBC hypothesis. It is not necessary because a Lucas-style monetary misperceptions model, in which monetary policy actions affect output but only if they are unanticipated, will not imply Granger-causality from nominal to real variables.[49] Conversly, a finding of such causality is not sufficient for RBC rejection if variables that are important to private agents are not observed by the econometrician. In such cases, as Litterman and Weiss (1985), King (1986), and Eichenbaum and Singleton (1986) have shown, monetary variables may move in response to real shocks in a manner that has predictive content for real variables even though the latter would be block exogenous in a wider system that included the (practically unobserved) variables.

Furthermore, it is unclear whether or not significant nominal-to-real Granger causality prevails in the available data. Some studies report test statistics that imply strong rejections of non-causality null hypotheses; see, e.g., Geary and Kennan (1984, Table 2) or Litterman and Weiss (1985), (Table VII, line 18). But Eichenbaum and Singleton (1986) have documented a marked tendency for nominal-to-real causality to become insignificant when the data series are rendered stationary, prior to analysis, be means of first differencing rather than removal of deterministic trend components. Despite much recent research, it is not entirely clear what conclusion should be drawn regarding the presence of causality when results differ in this fashion.[50]

The subject of first-differencing leads naturally to a relevant argument put forth by Nelson and Plosser (1982) that has recently attracted considerable attention. This argument begins with the presumption that monetary impulses can have no effects on the trend component of output (or employment) and continues with the claim that, in fact, output fluctuations are dominated by trend (as opposed to cyclical) movements. The argument's conclusion, then, is that output movements must be primarily induced by real rather than monetary impulses--a notion that is clearly pertinent for the RBC hypothesis.

The Nelson-Plosser argument is related to the first differencing of data because its contention that output fluctuations are trend-dominated relies critically upon the analysis of output (etc.) measures that have been first differenced to remove the non-stationary trend component. For most aggregate U.S. data series it is the case that the variability that is left to be studied, or classified as "cyclical," is much smaller after differencing than after removal of deterministic trends.

With regard to the Nelson-Plosser line of argument, McCallum (1986) has objected that the empirical evidence does not warrant the conclusion that first-differencing is appropriate (and deterministic trend removal inappropriate) as a method for rendering the series stationary. Such would be the case if the series in question were generated by ARMA processes in which the AR polynomial possesses a root precisely equal to 1.0. That is, if the process for the variably y_t is expressed as

$$(27) \quad y_t = \frac{\theta(L)}{\psi(L)} \varepsilon_t = \frac{\theta_0+\theta_1 L+\ldots+\theta_q L^q}{\psi_0+\psi_1 L+\ldots+\psi_p L^P} \varepsilon_t,$$

with ε_t the white-noise innovation, then differencing would be required if the solutions to the equation $\psi_0 + \psi_1 z + \psi_2 z^2 + \ldots + \psi_p z^P = 0$ include one root that equals 1.0 precisely. The Nelson-Plosser evidence only shows, however, that the hypothesis that such a root exists cannot be rejected at conventional significance levels. But such test results are entirely consistent with the possibility that the root in question is close to but not precisely equal to unity; power against alternative hypotheses that the true value is 0.98 or 0.96 (for example) is extremely low. Thus the variables' ARMA processes may easily be of a form such that deterministic detrending would be entirely appropriate.

The foregoing objection the Nelson-Plosser position does not constitute a claim that output (employment, etc.) series can be firmly shown to be trend-stationary, but rather that the Nelson-Plosser tests are incapable of establishing their contention, i.e., that the data series are such that differencing is necessary to induce stationarity. But without that contention, the Nelson-Plosser evidence does not provide support for the RBC hypothesis.[51]

The final matter to be taken up in this section concerns the connection between business cycles and economic growth. Traditionally, of course, macroeconomic analysis has typically proceeded under the maintained assumption that these two types of phenomena can without serious loss be studied separately--that the latter basically involves capital accumulation and technical progress while the former concerns the extent to which existing capital and labor are utilized. More recently, however, support has grown for the idea that if technology shocks are the principle driving force behind cyclical movements, as the RBC models presume, then the two types of phenomena may be simply different manifestations of the same basic process.

Partly for the reason, perhaps, Prescott (1986, pp. 12-13) has emphasized the desirability of simultaneously accounting for both growth and cycles. In practice, however, the Kydland and Prescott (1982) (1986) studies have not provided an integration at either the theoretical or empirical level. Thus their theoretical specification pertains to an economy in which there is no growth, either in total or per-capita terms.[52] And the Kydland-Prescott empirical procedure involves detrending of the data series prior to analysis, so the economy's growth is abstracted from before the study of cycles is begun. This latter aspect of the Kydland-Prescott procedure has been criticised by Singleton (1986), who emphasizes that maximizing behavior implies certain restrictions relating to the trend or growth components of <u>different</u> variables. When such restrictions are ignored, parameter estimation will be inefficient and some opportunities for model testing will go unexploited.

It is possible, depending on the extent to which technical progress can be appropriately viewed as exogenous, that the current emphasis on integration of growth and cycle analysis may be misplaced or excessive. The point is this: if technical progress were exogenous, then even if RBC views were correct there would be little necessary relation between the magnitude of growth and the extent of cycles, as they depend upon two different aspects of the technical-progress process. Specifically, if the stochastic process for z_t is $\log z_t = \gamma_0 + \gamma_1 t + \gamma_2 \log z_{t-1} + \varepsilon_t$, with $|\gamma_2| < 1$, then growth will depend upon γ_1 while cyclical properties will be related to the independent parameter σ_ε^2.[53] If $\gamma_1 = 0$, the system would be one that features cycles but no growth.[54] In this case the Kydland-Prescott practice--as opposed to rhetoric--would be largely appropriate.

On the other hand, it may be that growth results not from exogenous technical progress but from endogenous forces. Quite recently, King and Rebelo (1986) have--following some leads provided by Romer (1986)--begun an investigation of the idea that growth may be the consequence of human-capital accumulation with the latter determined endogenously. In the particular example described by King, Plosser, and Rebelo (1987), the economy's rate of steady growth per capita depends upon the ratio of human to physical capital and the fraction of resources that are devoted to human capital formation and maintenance, all of which are determined endogenously. There is in this model no parameter, analogous to γ_1 in the previous paragraph, for which a special value would imply zero growth without affecting cyclical properties. The link between growth and cycles is, therefore, more intimate.

Some versions of the endogenous growth model have been formulated in a manner that could lead readers to believe that steady-state growth, as opposed to growth at ever increasing or decreasing rates, is possible only with a production function for human capital that is excessively special. King, Plosser, and Rebelo (1987) show, however, that the situation is essentially the same as in the standard competitive growth model: steady growth requires constant returns to scale in production, labor-argumenting technical change, and utility functions of a certain class. Thus criticism on this basis is unwarranted; these requirements for steady growth apply quite generally.

The discussion in King, Plosser, and Rebelo (1987) suggests that the endogenous growth approach can account for the presence of autoregressive unit roots in the univariate time series processes for capital stock and other real variables. But while their analysis is promising in this regard, it apparently implies roots <u>precisely</u> equal to unity only when production functions are homogenous of degree precisely one. While many analysts would accept constant returns as a good approximation, the implication for autoregressive roots is only that values close to unity will be found.

V. Concluding Remarks

As an identifiable topic of study, RBC analysis is only a few years old. Accordingly, it is perhaps too early for any reliable conclusions to be drawn. But the arguments presented above can be summarized and a few judgements attempted, even though the latter should be regarded as highly preliminary and more of the nature of predictions than settled opinions.

Basically, we have seen in our discussion that a model of a competitive market economy, in which an aggregate technology shock affects the quantity of output producible from capital and labor inputs, will experience fluctuations in per-capita quantities of consumption, investment, and total product. Under most functional specifications for preferences and technology, the quantity of labor employed will also fluctuate and investment variability will exceed that of consumption. If the technology shock process is strongly autoregressive--close to a random walk--so too will be the model's quantity variables, and the contemporaneous correlations among these variables will in several ways be notably similar to those that appear in detrended postwar quarterly data for the U.S. economy. Furthermore, if the technology shock has a standard deviation of about 3% of its mean value--about 1% for the surprise component of this shock--then the magnitude of the model's output fluctuations will match those of quarterly postwar real GNP, with a fairly good variability match for other key variables as well.

Of course there are ways in which the model's stochastic properties do not closely match actual data. For example, the correlations with output of hours worked and productivity are much higher in the model than in reality, and the correlation of productivity with lagged output provides a serious mismatch. It is possible that such discrepancies are simply due to the model's simplicity, however, rather than to any fundamental flaw in the RBC approach. Until studies have been conducted that convincingly distinguish between sources of fluctuations, it will be possible for responsible researchers to maintain sharply differing beliefs in this regard. An important gap in the RBC analysis has been a clear and convincing story concerning the nature of the postulated aggregate shock variable. If the latter were actually a proxy for observable variables that have been omitted from the models--fiscal policy and/or import prices, for example--then the interpretation and conclusions of the work would be quite different than if the shocks were of a strictly technological nature.

As for the notion that monetary policy irregularity has been an unimportant source of output fluctuations, at least during the postwar era, it is too early to reach any firm conclusion. RBC proponents have pointed to empirical findings that are suggestive of that hypothesis, but there are significant problems with the evidence that has been developed to date.

It would seem to be virtually indisputable that the RBC literature has provided a substantial number of innovative and constructive technical developments that will be of lasting benefit in macroeconomic analysis. In particular, the example of Kydland and Prescott (1982) has suggested one route toward the goal of a dynamic equilibrium model with optimizing agents that can be used for quantitative macroeconomic analysis. The type of model employed does not, it should be emphasized, require a belief that the workings of the economy are socially optimal. In addition, the literature has spurred interest in several purely methodological topics, including alternative methods for the investigation of the dynamic properties of non-linear general equilibrium models and for the detrending of time series data. Also, issues concerning the practical reliability of Granger-causality tests have been highlighted by controversies initiated by RBC analysis.

From a substantive perspective, the RBC studies have provided a healthy reminder that a sizeable portion of the output and employment variability that is observed in actual economies is probably the consequence of various unavoidable shocks, i.e., disturbances not generated by erratic monetary or fiscal policy makers. Thus it is unlikely that many scholars today would subscribe to the proposition that all or most of the postwar fluctuation in U.S. output has been attributable to actions of the Federal Open Market Committee. Whether a substantial fraction can be so attributed remains a topic of interest and importance.

Footnotes

1. An incomplete list of other notable items includes Black (1982), Kydland and Prescott (1980) (1986), Kydland (1984), King (1986), King, Plosser, and Rebelo (1987), King, Plosser, Stock, and Watson (1987), Hansen (1985), and Prescott (1986).

2. To date, these include critical pieces by Fischer (1987), McCallum (1986), and Summers (1986) plus sympathetic but skeptical discussions by Barro (1986), Eichenbaum and Singleton (1986), King and Dotsey (1987), Lucas (1985), and Mankiw (1986). A guide to the Prescott-Summers exchange is provided by Manuelli (1986) and expository articles have been prepared by Rush (1987) and Walsh (1986).

3. Clearly, real shocks could in principle pertain as well to individuals' preferences, but in fact the literature has emphasized technology disturbances.

4. Here the term "equilibrium" is being used as a shorthand for the more precise term, "flexible-price equilibrium." McCallum (1982) suggests that there is no inherent reason why equilibrium models could not accommodate sluggish price adjustments.

5. That such a challenge is intended by some of the leaders of RBC analysis is suggested by King and Plosser's (1984, p. 363) statement that "there are good reasons for dissatisfaction with existing macro economic theories" in conjunction with a reference to "alternative hypotheses."

6. A basic reference in this regard is Brock and Mirman (1972), which provides formal analysis of a social planner's problem in the context of a related stochastic growth model. Brock (1974) indicated how the mathematics could be re-interpreted so as to constitute a descriptive model of a competitive economy with a large number of similar households. The only

significant differences between the re-interpreted Brock-Mirman model and the one with which we begin is the non-recognition of leisure in the former--a difference that is of very little importance on the design of proofs--and the former's assumption of serially-independent technology shocks. This latter limitation was removed by Donaldson and Mehra (1983).

7. Alternatively, we could proceed under the assumption that there is a market for one-period bonds instead of one for capital services. Suppose that a bond purchased in t for $1/(1+r_t)$ is a claim to one unit of output in t+1. Then the household's net expenditure on bonds during t could be denoted $(1+r_t)^{-1} b_{t+1}-b_t$, with b_{t+1} the number of bonds purchased in t. The price $(1+r_t)^{-1}$ would be determined by the market-clearing condition, implied by the similarity of all households, that $b_{t+1} = 0$. Equilibrium values of c_t, n_t, and k_{t+1} would be precisely the same as with the market structure implied by equation (3).

8. Obviously, the equality in (6) should properly be written as the inequality "$\leq$." But our assumptions are such that the equality will in fact hold. This type of notational shortcut is used extensively in what follows.

9. If we neglect the variable-leisure aspect of our model, then proofs can be found in Brock (1982) and Donaldson and Mehra (1983).

10. Since λ_{t+j} is the utility value of a unit of capital acquired in period t+j, $\beta^{j-1} \lambda_{t+j} k_{t+j+1}$ is the present value in t of capital held by the household at the end of period t+j. Condition (5) serves to rule out the possibility that the household would forever accumulate capital at an excessive rate, something that is not precluded by any of conditions (4).

11. On this, see Brock (1982).

12. Some readers have objected to the way in which the optimization analysis is here described, suggesting that instead it be expressed in terms of the

identity between Pareto-optimality and competitive equilibrium that has been used in several of the key RBC papers. I have quite deliberately chosen not to proceed in that manner so as to avoid portraying RBC analysis in a overly restrictive manner. An approach that is applicable only to economies known to be Pareto optimal would be of quite limited usefulness, but such is in fact not the case for RBC analysis. The latter point is recognized by King, Plosser, and Rebelo (1987).

13. For a more complete discussion, see Lucas (1987). Strictly speaking, our statement presumes that interest is limited to minimal sets of state variables, a limitation that rules out "bubbles." But since any bubble path would in the present model be inconsistent with the transversality condition (5), this limitation is not significant.

14. These include Long and Plosser (1983).

15. Very recently Hercovitz and Sampson (1986) have developed a different special case that does not require complete depreciation yet results in explicit log-linear solutions that feature variable employment. These attractive features are obtained at the price of adopting unfamiliar specifications for utility and depreciation functions. Specifically, the within-period utility function is $u(c_t, 1-n_t) = \log(c_t - a\, n_t^{\gamma})$ with $a > 0$ and $\gamma > 1$, a form that makes the marginal rate of substitution between consumption and leisure independent of c_t. Ignoring complications involving human capital that are stressed by Hercovitz and Sampson, let technology continue to be $y_t = z_t\, n_t^{\alpha}\, k_t^{1-\alpha}$ as in (15) and maintain the usual concept of gross investment, $i_t = y_t - c_t$. Regarding depreciation, however, assume that $k_{t+1} = k_t^{1-\delta}\, i_t^{\phi}$ where $\phi > 0$ and $1 > \delta > 0$. One can think of this log-linear expression as departing from the usual linear form for adjustment-cost reasons as in Lucas and Prescott (1971), or alternatively as an approximation adopted for

analytical convenience. In any event, with this specification the model yields solutions of the form

$c_t = \phi_{11} z_t^{\phi_{12}} k_t^{\phi_{13}}$, $n_t = \phi_{21} z_t^{\phi_{22}} k_t^{\phi_{33}}$, $k_{t+1} = \phi_{31} z_t^{\phi_{32}} k_t^{\phi_{33}}$.

16. The shadow price λ_t obviously obeys $\lambda_t = \theta[1-(1-\alpha)\beta]^{-1} n^{-\alpha} z_t^{-1} k_t^{\alpha-1}$.

17. Whether it is appropriate to use detrended data is an important issue that will be briefly discussed below, in Section IV.

18. For output this is well known. For consumption we have $\log c_t = .06 + 1.21 \log c_{t-1} - .24 \log c_{t-1} + .00024t$, $\sigma = .007$, DW = 2.08.

19. In these models, positive demand shocks result in high output because they induce high employment in response to low real wages.

20. To obtain this implication, note that with $\delta = 1$ investment during period t is identically the same as k_{t+1}. Then recall that the stochastic processes for k_t and c_t have the same autoregressive components and the same forcing variable, ε_t.

21. More precisely, the claim cannot be verified without resort to some sort of approximation procedure. Using one, King, Plosser, and Rebelo (1987) are able to show that, in a version with no labor-leisure decision, consumption variability is greater than for investment.

22. The smoothing procedure for measuring the trend component of the various series is somewhat unorthodox. If x_t is the variable to be smoothed and s_t the smoothed value for period t, the method entails minimizing the sum of

$T^{-1} \sum_{t=1}^{T} (x_t - s_t)^2$ and $\mu T^{-1} \sum_{t=2}^{T} [(s_{t+1} - s_t) - (s_t - s_{t-1})]^2$, where μ is a weighting factor that is chosen to provide the desired degree of smoothness. Use of this particular method of detrending the data has not attracted much discussion or criticism to date, but inspection of material in King, Plosser, and Rebelo (1987) suggests that cross-variable correlations may be sensitive to the method. Some issues concerning detrending (by any method) will be briefly considered in Section IV.

23. The parameter values utilized by Hansen are $\theta = 0.33$, $\alpha = 0.64$, $\beta = 0.99$, and $\delta = 0.025$. Also, the counterpart of ρ is taken to be 0.95. It is necessary to refer to "the counterpart of ρ" rather than to ρ itself because Hansen's specification (like that of Kydland and Prescott) is of the form $z_t = 0.95 z_{t-1} + \xi_t$, with ξ_t log-normal, rather than $\log z_t = \rho \log z_{t-1} + \varepsilon_t$.

24. Since the model is not linear in logs of the variables, this scaling is not strictly legitimate; an increase in the variance of the technology shock would change the relative standard deviations to a small extent. It is my belief that such an effect would be negligible in the case at hand.

25. It is possible that serial correlation properties are enhanced, however. Statistics relating to this dimension of performance are not reported by Hansen (1985).

26. As Hansen (1985) emphasizes, his specification is based on theoretical analysis originally developed by Rogerson (1985).

27. Again, this magnitude is (because of ignored nonlinearities) only approximate. The correct statement is that in Hansen's simulations the basic model has output variability only 0.134/0.176 = 0.767 as large as that for the indivisible labor model when the same z_t variance is used for both.

28. As previously mentioned, other typical values are $\beta = 0.99$ and $\delta = 0.025$ (quarterly data). There is room for dispute regarding some of these magnitudes. Significantly, Summers (1986) has questioned the Kydland-Prescott-Hansen choice for β, which implies a steady-state real rate of return of 4% per year, and the θ value that implies that about a third of the typical household's time is spent in employment. Summers notes that the actual figure is closer to one-sixth, a magnitude that is borne out by estimates obtained by

Eichenbaum, Hansen, and Singleton (1986).

29. A sensitivity of this type has been mentioned by Christiano (1987a, p. 341). Some related facts are brought out by King, Plosser, and Rebelo (1987).

30. The issue has been raised by Barro (1986), Lucas (1987), and Summers (1986), among others.

31. Thus this estimate pretends that $\rho = 1$. While this is inconsistent with 0.95 value used by Kydland and Prescott (1982) (1986), the resulting effect is probably small in the present context (i.e., obtaining an estimate of the variance of ε_t). Prescott's procedure involves a weighted manhours series and imposes the restriction $\alpha = 0.75$.

32. Actually, this number is here inferred, in the manner described in footnote 27, from Hansen's reported level of output fluctuations.

33. By failing to take account of quarter-to-quarter dynamics, the procedure throws into the residual fluctuations that which would properly be attributed to this source of dynamics (if the adjustment-cost hypothesis were true). This problem, it might be said, concerns misspecification of the (dynamic) production-employment relation, not measurement error. The omission of relevant lagged variables is only likely (rather than certain) to lead to over-estimation of the shock variance because the procedure uses factor shares as parameters rather than estimating the latter in a regression designed to yield Δz_t residuals.

34. This calculation, it should be said, pertains to the <u>mean</u> level of productivity change, not its variance.

35. Long and Plosser (1983) includes an interesting discussion of a multisector RBC framework. From their Figure 3 it would appear that the magnitude of aggregate fluctuations is smaller than for individual sectors, but numerical statistics relating to this particular point are not included.

The discrepancy between sectoral and aggregate magnitudes would be heightened, of course, by recognition of more sectors; the Long and Plosser example has only six.

36. An increase in the price of imported materials will lead to a decline in their quantity utilized, relative to the quantities of domestic factors. That will reduce the quantity of output for any given usage of domestic factors, i.e., will shift downward the production schedule relating output to domestic factors alone.

37. Labor hoarding is mentioned by Summers (1986). What I understand by the term is that costs of hiring and firing lead firms to adjust their workforce much more slowly than if no such costs existed. To see that the lag correlation pattern could easily be as claimed, consider an example in which $y_t = by_{t-1} + e_t$ and $n_t = ay_t + (1-a)y_{t-1}$, where y_t and n_t denote logs of output and hours so that the log of productivity is $x_t = y_t - n_t$. Then it is easy to show that $Ey_tx_{t+1} < 0$ while $Ey_tx_{t-1} > 0$ and $Ey_tx_t > 0$. An interesting recent attempt to discriminate between the supply-shock and labor-hoarding hypotheses has been made by Shapiro (1987), whose results are supportive of the former. Some apparent inconsistencies with other wage-productivity studies may be due to Shapiro's use of annual data.

38. The Kydland-Prescott correlations with output analogous to those of Table 3 are as follows: 0.46, 0.68, 0.84, 0.56, 0.42.

39. The U.S. correlation in Geary and Kennan (1984), for example, is 0.20.

40. The model is <u>less</u> restrictive than (6)-(9) as it permits current utility to be affected by a distributed lag of leisure hours.

41. Eichenbaum, Hansen, and Singleton (1986) develop evidence which suggests that the model's rejection stems primarily from a failure to satisfy the intratemporal, as opposed to intertemporal, optimality conditions.

42. In fact, the competitive equilibria are computed in these papers by solving Pareto optimality problems for a fictitious social planner.

43. From its context, it is clear that Lucas presumed the causal relation to be from money to income.

44. The point is that in actual developed economies data is available on monetary aggregates too promptly to be consistent with the assumption--critical in the Lucas-Barro models--that individuals are ignorant of contemporaneous monetary magnitudes. A key item in the literature is King's (1981) demonstration that this problem is not circumvented by measurement error on the "true" aggregate.

45. This statement pertains to the postwar U.S. data, quarterly and monthly.

46. Relevant discussion of the 1979-82 episode appears in McCallum (1985).

47. For an elaboration on this argument, see McCallum (1986) and references cited therein.

48. Block exogeneity of x_t of course implies that x_t is not Granger-caused by any variable not included in the vector x.

49. See Sargent (1976) and the comments on this paper that appeared in April 1979 issue of the <u>Journal of Political Economy</u>.

50. Christiano (1987b) has provided some simulation results that suggest (but do not establish) that the results with undifferenced data are more reliable in this particular context. On the other hand, it might be noted that only a small fraction of the predictive power for output movements can be attributed to nominal variables even in those studies that feature strong rejections of the no-causality hypothesis; most of the explanatory power lies in past movements of output itself. But this type of finding appears so frequently in Granger-causality studies of other, entirely unrelated, issues that one is led to suspect that the basic procedure is seriously flawed in some

respect--possibly involving measurement error--that has thus far eluded formal analysis.

51. Some support for the foregoing argument is provided by interesting recent studies by Cochrane (1986), Watson (1986), and Evans (1986). A contrary position is taken in a notable paper by Campbell and Mankiw (1986).

52. Exogenous technical progress can be added to the model, of course, but that step does not constitute much of an integration.

53. This fact is not invalidated by the dependence of both growth and cycle magnitudes on γ_2.

54. If the log z_t process were a random walk with drift, the definition of "cycles" would become problematical. But it would remain reasonable to say that zero drift implies zero growth while leaving scope for cycles.

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