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MOMENT OF INERTIA AND DAMPING OF LIQUIDS IN BAFFLED CYLINDRICAL TANKS

by Franklin T. Dodge and Daniel D. Kana

Prepared under Contract No. NAS 8-1555 *by*
SOUTHWEST RESEARCH INSTITUTE
San Antonio, Texas
for George C. Marshall Space Flight Center

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ABSTRACT

The moment of inertia and the damping of a liquid in a completely filled, closed cylindrical tank is investigated experimentally for tanks with and without baffles. The results are compared with Bauer's mechanical model, and it is shown that a simpler model, which, however, is not conceptually correct for extremely large damping, is sufficient for cases where only small damping is expected. An approximate method of computing the liquid moment of inertia in a baffled tank is given.

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I. INTRODUCTION

It has proved convenient in missile stability analyses to represent the sloshing liquid in fuel tanks by an equivalent mechanical model [1, 2, 3, 4]. Each vibration mode of the liquid is treated as a degree of freedom and represented as a spring-mass or pendulum system. The model parameters are usually calculated from the results of a potential flow, inviscid liquid analysis. This method implicitly assumes that the mode shapes, natural frequencies, sloshing masses, etc., are not altered substantially by the small amount of damping present in an actual system. The effects of damping are introduced in the model in a simple way through the use of linear viscous damping, but the damping coefficients must be determined by suitable experiments.

One of the models used to represent sloshing in a cylindrical tank is shown in Figure 1. This model, originally developed by Bauer [4], differs from most other equivalent mechanical models by the use of a massless disc I_d, located at the c. g. of the liquid and coupled to the tank motion through the linear dashpot c_d. To understand the significance of the disc, assume for the moment that the free surface motions are prevented by a rigid cap fitted to the tank, and that the liquid is completely frictionless. This implies that the masses m_n are rigidly attached to the tank (i. e., all $k_n = \infty$) and that c_d and all the c_n are zero. Then if the tank is rotated about a transverse axis through the c. g. of the liquid, the disc will not rotate ($\dot{\psi} = -\dot{\varphi}$ in Fig. 1); hence, the effective moment of inertia of the liquid is

$$I_{liquid} = I_o + m_o h_o^2 + \sum_{n=1}^{\infty} m_n h_n^2 \qquad (1)$$

Except for I_o all of the quantities in Eq. (1) may be computed by requiring that the transverse sloshing forces and the c. g. of the model and the liquid be identical [3, 4]. Thus, if I_{liquid} is known, I_o can be computed from Eq. (1); in this way the moment response of the model and the liquid during sloshing can also be made equal. Consequently, for a frictionless fluid I_d plays no part in computing the sloshing forces and moments, and in fact, it is not needed at all. This, however, is not true for a real fluid.

In Figure 2, the magnitude of I_{liquid} for an ideal frictionless fluid in an unbaffled tank is shown as a function of h/d; the plot actually gives the ratio of I_{liquid} to the moment of inertia of a rigid mass of liquid of the same shape. It can be seen that the liquid's inertia can be quite small since not all of the liquid participates in the motion; some of it is almost completely at rest. But for a real fluid the internal friction (viscosity) causes more of the fluid to follow the tank's motion. Thus, not only is a small amount of damping present but the apparent moment of inertia is somewhat greater. In the model, as the friction increases, c_d increases in proportion; but, as c_d increases, less relative motion between the tank and the disc I_d is possible. Thus, some of the tank motion is transmitted to the disc, and a part of the disc's inertia must be included in Eq. (1). As the damping increases toward infinity the moment of inertia of the liquid approaches that of a rigid mass (no relative motion in the liquid is possible); in the model this corresponds to the disc attached rigidly to the tank ($c_d = \infty$). Thus

$$I_{liquid}(c_d = \infty) = I_{rigid} = I_o + m_o h_o^2 + \sum_{n=1}^{\infty} m_n h_n^2 + I_d \tag{2}$$

and from elementary results I_{rigid} is

$$I_{rigid} = md^2 \left[\frac{1}{12} (h/d)^2 + \frac{1}{16} \right] \tag{3}$$

where m is the mass of liquid. It can thus be seen that the model is conceptually correct even for $c_d \rightarrow \infty$. In this way possible variations of the moment of inertia of the liquid with frequency or amplitude, for a given tank-baffle configuration, can be accomodated by varying the damping parameter c_d.

The purpose of the research presented in this report was to devise a method to determine I_d and c_d experimentally. Knowing their magnitudes, the differences in this model and other simpler models that do not employ "moment of inertia damping" were evaluated to determine if the differences are significant in possible applications. It was not our aim, however, to collect a large amount of data or to try to simulate any actual missile tank configuration or fuel.

II. DESCRIPTION OF APPARATUS AND EXPERIMENTAL PROCEDURE

Several kinds of test apparatus were evaluated during the program. Because there is no preferred frequency for a capped tank filled with liquid and oscillating about the liquid c. g., our first attempts were oriented toward oscillating the tank with a shaker. The moment response and the phase angle between the moment and the excitation were then to be measured over a range of frequencies. In this way the moment of inertia of the liquid and the damping could be determined. However, the dynamometers used to measure the moment needed to be very sensitive, so sensitive in fact that a number of spurious signals arising chiefly from the deflection of the massive frame could not be eliminated. Also, the damping of the liquid was sufficiently low that the "noise" in the signal overshadowed the phase angles we were attempting to measure. Thus, all direct excitation-moment measuring methods were abandoned. Instead the tank was fitted with a system of external springs and mounted as a pendulum; this arrangement proved to be satisfactory to obtain the necessary measurements.

An over-all view of the apparatus is shown in Figure 3, and a schematic in Figure 4. A view of tha tank itself, in this case fitted with five ring baffles, is shown in Figure 5. Basically, the apparatus is a rigid 19.6-cm (7.71 in.) diameter circular cylindrical plastic tank, mounted so that it pivots about two points, one on either side of the tank diameter. The pivot points consisted of steel needle points on the tank frame which rode in small conical recesses in the frame (see Fig. 5); this system, rather than bearings, was used in order to minimize the inherent damping of the system. Tension springs were mounted on the tank so that it could be given an easily controlled natural frequency. The distance from the pivot point to the line of action of the springs could be varied in a number of discrete steps. The entire arrangement, including the electromagnet exciter coil and the displacement transducer, was supported by a massive base. In all cases the test liquid was water.

The ring baffles, which were used in sets of one, three, or five (evenly spaced), were made of 0.51 mm (0.030 in.) thick, 15.2 mm (0.60 in.) wide aluminum. They were cemented directly to the tank wall with only a small clearance. The baffles were essentially rigid so that negligible flexing motion occurred.

The tank was pivoted about points near the top, rather than at the liquid c. g., so that the effect of a moderate change in the moment of inertia would be maximized. This tended to minimize the error and scatter in the experimental data.

The natural frequency was determined by noting the frequency for which the maximum tank displacement was observed on the oscilloscope. The frequency was then read accurately on the frequency counter. This procedure was carried out for both the empty and the full tank for each spring arrangement and baffle configuration. In all cases, the apparent moment of inertia about the pivot point axis was computed from

$$I = \frac{k}{(2\pi f)^2} \tag{4}$$

k is the angular spring constant, and f is the natural frequency. The difference between the full and the empty tank values was considered to be the moment of inertia of the liquid alone. The liquid's moment of inertia about its c. g. axis was then obtained by the transfer axis theorem.*

Damping measurements were obtained by a logarithmic decrement technique. For each spring and baffle configuration the damping for the entire system and liquid was measured, at several initial amplitudes of oscillation, by shutting off the excitation at resonance and recording the decay curve on a strip chart. The damping of the system without liquid was then obtained by using the same process with an empty tank, but which was weighted with lead ballast so that the pivot points supported the same weight and so that essentially the same natural frequency was obtained as for the full tank. Fifty cycles of oscillation were chosen for computing the log decrements. This tended to reduce amplitude measurement errors because rather small damping values were encountered. Some arbitrariness was necessary in this choice, since a variation of damping with

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*This theorem, which usually applies only to rigid bodies, is also true for the liquid here. This is so because the actual tank motion can be thought of as a rotation about the liquid c. g. and a pure translation. In translation, however, the liquid moves as a rigid body, and hence the transfer theorem is valid.

amplitude was observed; however, the resulting values were considered to be good representative results, and their consistency and repeatibility supported this assumption. The damping of the liquid alone was assumed to be the difference of the full and empty tank values.

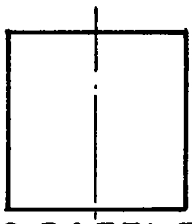
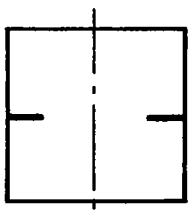
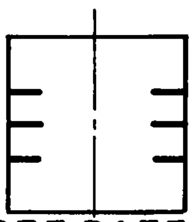
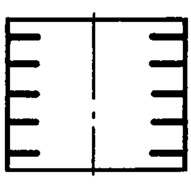
III. TEST RESULTS AND CORRELATION WITH MODEL

Results of the moment of inertia experiments are presented in Table 1. As can be seen from comparing the numbers in the central column of this table, the natural frequency of the pendulum-tank, for a given arrangement of the springs, tended to decrease very slightly as the number of baffles was increased. Although this decrease in frequency is not significant, it can be seen from the third column that the moment of inertia increased substantially as more baffles were added.

The liquid moment of inertia with no baffles in the tank was consistent with the results of a potential flow analysis (see Fig. 2 for h/d = 1.0). The moment of inertia increased with the number of baffles, and for five evenly spaced baffles, the liquid had an apparent moment of inertia equal to approximately one half that of an equivalent rigid mass of liquid. There are no theoretical analyses to compare with these results, but the trend displayed in the table seems to be reasonable. In the Appendix, it is shown by a simple calculation that the approximate moment of inertia of the liquid when baffles are spaced throughout the tank should be about one-half that of the equivalent rigid mass; this agrees with the test results.

Although the liquid moment of inertia seems to vary somewhat with frequency, it should be mentioned that our results do not conclusively

TABLE 1. LIQUID MOMENT OF INERTIA

TANK CONFIGURATION	NATURAL FREQUENCY (CPS)	$\frac{I_{liquid}}{I_{rigid}}$
 NO BAFFLES	2.10 3.39 4.98	0.17 0.17 0.19
 ONE BAFFLE	2.09 3.39 4.96	0.28 0.28 0.31
 THREE BAFFLES	2.07 3.39 4.95	0.50 0.34 0.36
 FIVE BAFFLES	2.08 3.38 4.94 6.60	0.70 0.51 0.52 0.51

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prove whether a variation actually exists or whether the apparent variation is caused by small experimental errors. This is because the moment of inertia of the liquid about its c. g. was calculated from

$$I_{liquid} = [(I_{system + liquid} - I_{system}) - m\ell^2] \qquad (5)$$

where the first two terms in parentheses are the experimentally determined moments of inertia about the pivot axis, and $m\ell^2$ is the term used to transfer the moment of inertia to the c. g. axis. The difference of the first two terms is small compared to either term separately; furthermore, the difference of these terms and the transfer term is again small compared to either one. Thus, even though each term was determined with a considerable amount of care, a small variation in any of the terms can result in a large percentage change in I_{liquid}.

Results of the damping investigations are shown in Figures 6, 7, 8, and 9. In every case the damping, i. e., the log decrement, was on the order of fifty to one hundred times smaller than the damping associated with sloshing in a similar baffled tank (see Ref. 3, Figs. 4 and 5). With no baffles the log decrement was almost constant as frequency and amplitude varied; with baffles, however, a pronounced variation of the log decrement with amplitude appeared.

In order to compare the test results with Bauer's model, it is necessary either to modify slightly the equations given in Ref. 4 or to write the equations of motion for the model shown in Figure 1, with the restriction that the masses m_n are rigidly attached to the tank. In either case, the pertinent result is

$$\left[I_{rigid} - \frac{I_d}{1 + \left(\frac{c_d}{\omega I_d}\right)^2}\right]\ddot{\varphi} + \left[\frac{c_d}{1 + \left(\frac{c_d}{\omega I_d}\right)^2}\right]\dot{\varphi} + k\varphi = F_o e^{j\omega t} \tag{6}$$

where $F_o e^{j\omega t}$ is the forcing torque applied to the tank about the liquid c. g. axis, written in complex notation. Thus, it can be seen that the apparent moment of inertia of the liquid, i. e., the experimental I_{liquid}, is

$$I_{liquid} = I_{rigid} - I_d \left[\frac{1}{1 + \left(\frac{c_d}{\omega I_d}\right)^2}\right] \tag{7}$$

and the experimental damping coefficient is

$$c_{test} = c_d \left[\frac{1}{1 + \left(\frac{c_d}{\omega I_d}\right)^2}\right] \tag{8}$$

By noting that the log decrement is $\delta = \dfrac{\pi c_{test}}{\omega I_{liquid}}$, Eqs. (6) and (7) can be rearranged to yield

$$I_d = I_{rigid} - I_{liquid} + \left[\left(\frac{\delta}{\pi}\right)^2 \frac{I_{liquid}^2}{I_{rigid} - I_{liquid}}\right] \tag{9}$$

and

$$c_d = \frac{\omega \delta I_{liquid} I_d}{\pi (I_{rigid} - I_{liquid})} \tag{10}$$

Since δ was always on the order of 10^{-3}, it can be seen that for all practical purposes:

$$I_d = I_{rigid} - I_{liquid} \tag{11}$$

and

$$c_d = \left(\frac{\omega\delta}{\pi}\right) I_{liquid} \tag{12}$$

[This simplification implies that $1 + \left(\frac{c_d}{\omega I_d}\right)^2 \approx 1$ in Eq. (6) or (7).] Thus, it appears, at least for the conditions used in our tests, that an equally valid model may be constructed by discarding the disc altogether. That is, from Eq. (1) with no damping

$$I_{liquid} = I_o + m_o h_o^2 + \sum_{n=1}^{\infty} m_n h_n^2$$

while from Eq. (2)

$$I_d = I_{rigid} - \left[I_o + m_o h_o^2 + \sum_{n=1}^{\infty} m_n h_n^2\right]$$

Comparing these two equations with Eq. (11) shows that the disc is redundant. In other words, the coupling of the disc to the tank for the values of c_d (or δ) measured in our tests was negligibly small. Therefore, simpler models, which do not employ the disc I_d, appear to be equally useful in predicting the moment response of the liquid.

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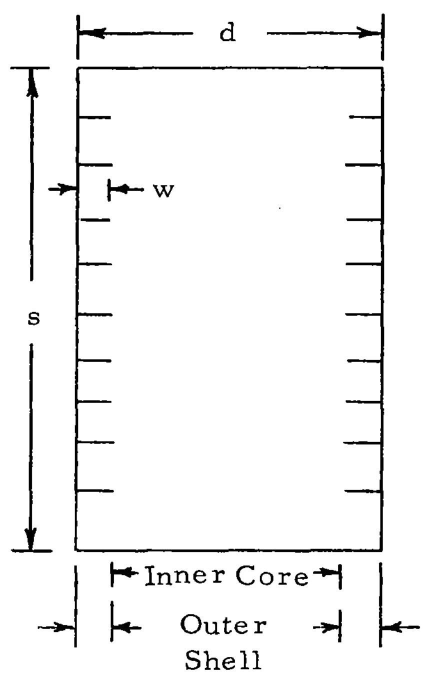
3. Abramson, H. N., Chu, W. H., and Ransleben, G. E., Jr., "Representation of Fuel Sloshing in Cylindrical Tanks by an Equivalent Mechanical Model," ARS J., 31, pp. 1697-1705, December 1961.

4. Bauer, H. F., "Fluid Oscillations in the Containers of a Space Vehicle and Their Influence on Stability," NASA TR-R-187, February 1964.

APPENDIX

When a number of baffles are spaced throughout a tank, and the ratio of the baffle spacing to baffle width is on the order of one or less, it seems reasonable to assume that the flow may be decomposed into two domains. The liquid contained in a hypothetical cylindrical shell enclosing the baffles must essentially follow the tank motion exactly, i.e., it must act as a rigid body; however, the inner core of the liquid is essentially a potential flow region similar to an unbaffled tank of diameter d-2w (see sketch).



The moment of inertia can then be computed in two parts: one part equal to the moment of inertia of a rigid outer shell and one part equal to the moment of inertia for potential flow in the inner core.

Analytically, this is represented as:

$$I_{liquid} = m_{os}\left[\frac{1}{12} h^2 + \frac{1}{16} (2d^2 - 4wd + 4w^2)\right]$$

$$+ m_{ic}\left[\frac{1}{12} h^2 + \frac{1}{16}(d - 2w)^2\right] \left(\frac{I_{liquid}}{I_{rigid}}\right)_{Fig. 2} \quad \text{(A1)}$$

In Eq. (A1), m_{os} is the liquid mass contained in the outer shell, m_{ic} is the liquid mass contained in the inner core*, and $\left(\frac{I_{liquid}}{I_{rigid}}\right)_{Fig. 2}$ is the value given in Figure 2 for a liquid depth to tank diameter ratio equal to $h/(d - 2w)$.

Using this equation with the tank dimensions used in our tests gives

$$I_{liquid} = 162.0 \text{ kg-cm}^2 \quad (55.2 \text{ lb-in.}^2)$$

From Eq. (3), $I_{rigid} = 336$ kg-cm^2 (115.2 lb-in.^{2}); thus

$$\frac{I_{liquid}}{I_{rigid}} = 0.48$$

This computed value compares very well with the test results given in Table 1 for a tank with five baffles (the only test tank in which baffles were distributed throughout the depth).

---

*$m_{os} = \frac{1}{4} \pi \rho h [d^2 - (d - 2w)^2]$; $m_{ic} = \frac{1}{4} \pi \rho h (d - 2w)^2$

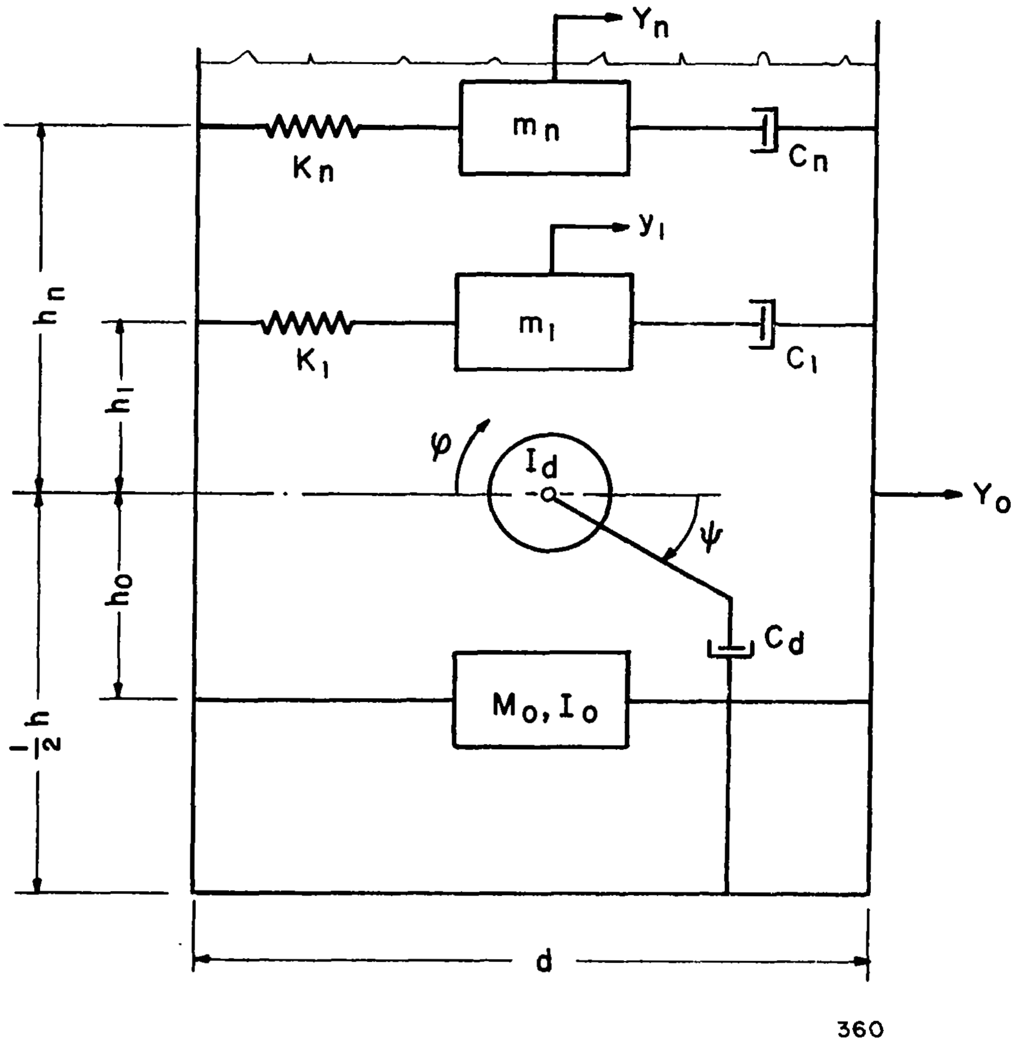


FIGURE 1. MECHANICAL MODEL FOR FUEL SLOSHING

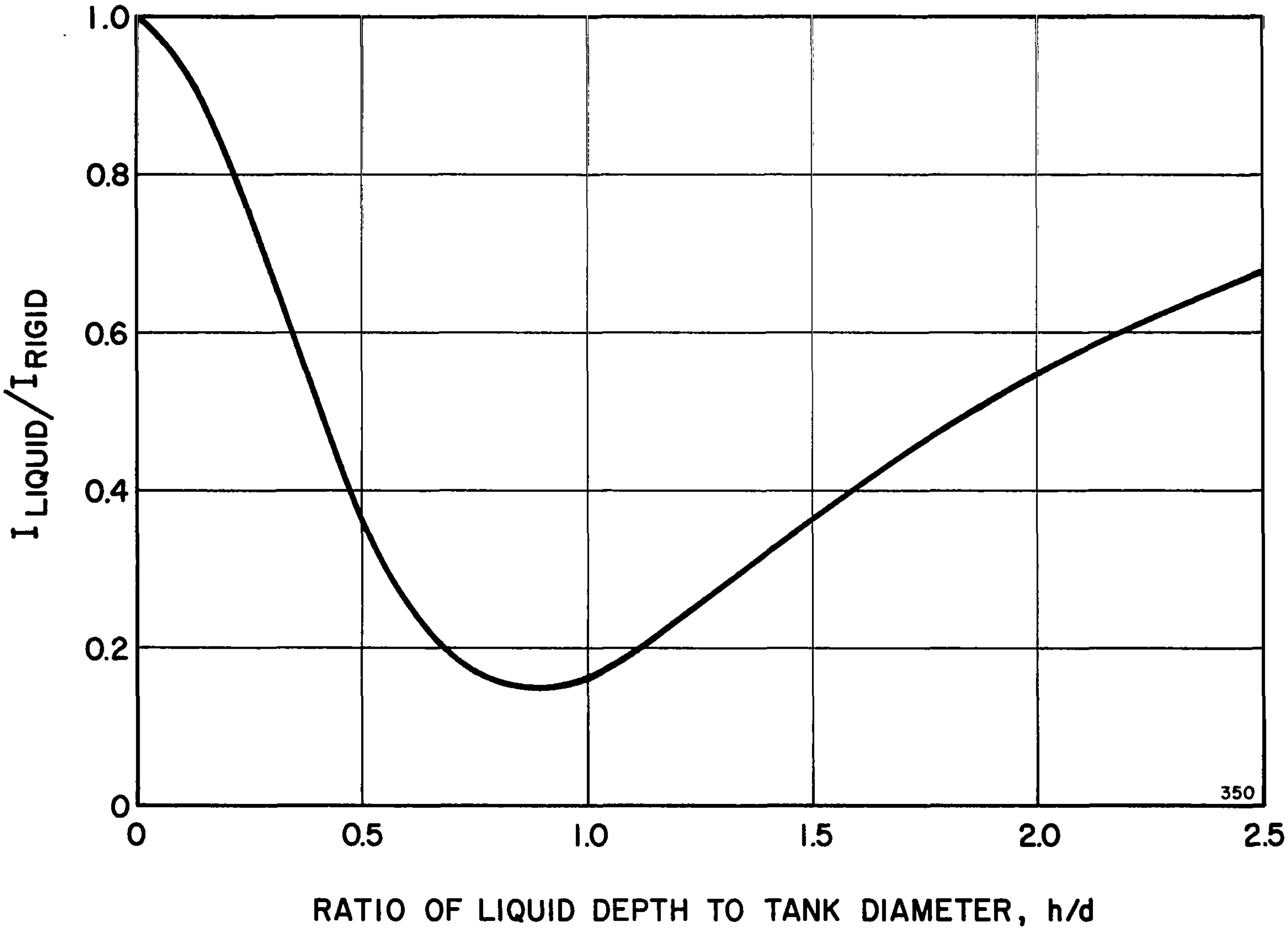


FIGURE 2. LIQUID MOMENT OF INERTIA IN A COMPLETELY FILLED CLOSED CONTAINER

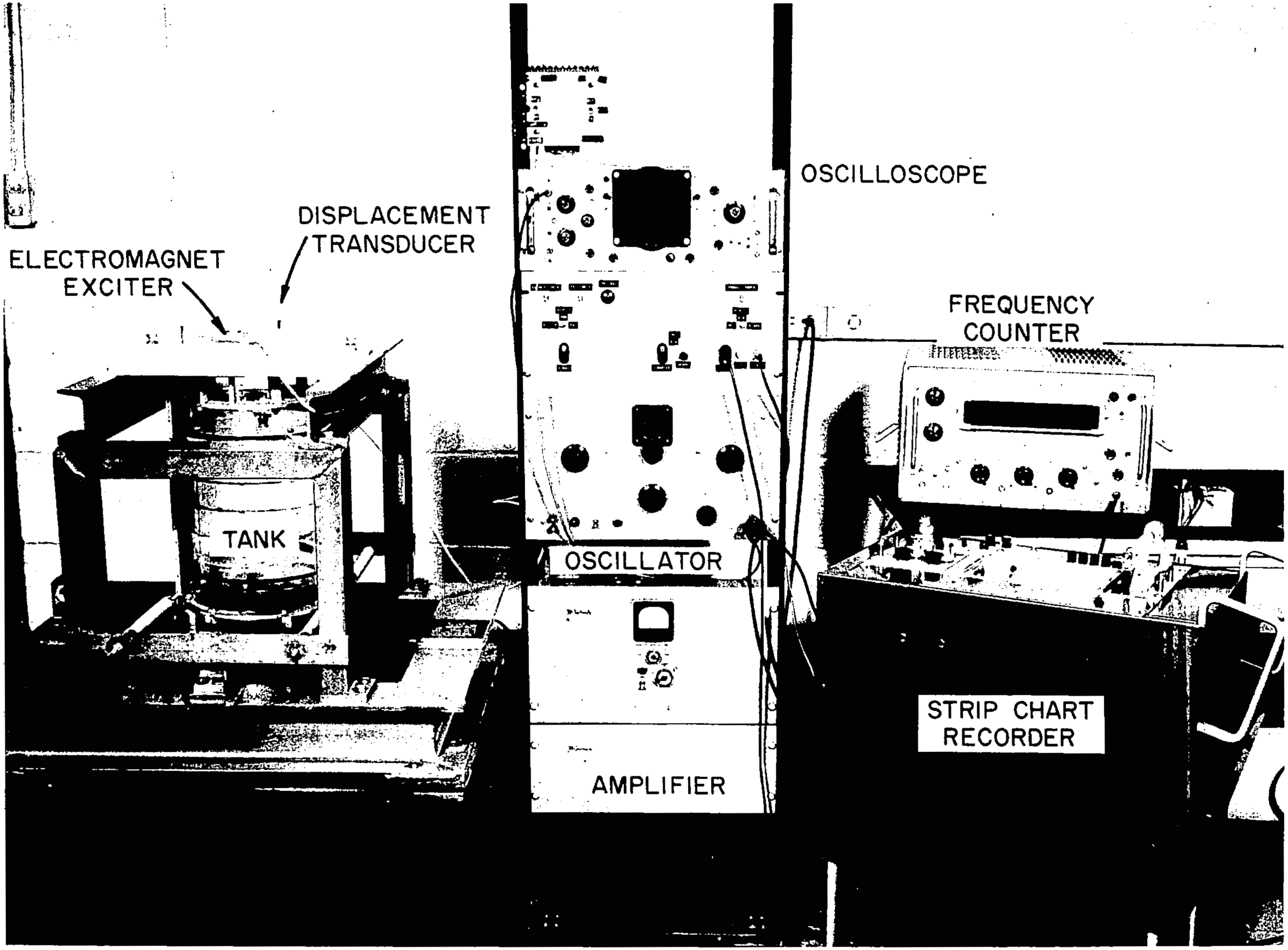


FIGURE 3. OVER-ALL VIEW OF TEST APPARATUS

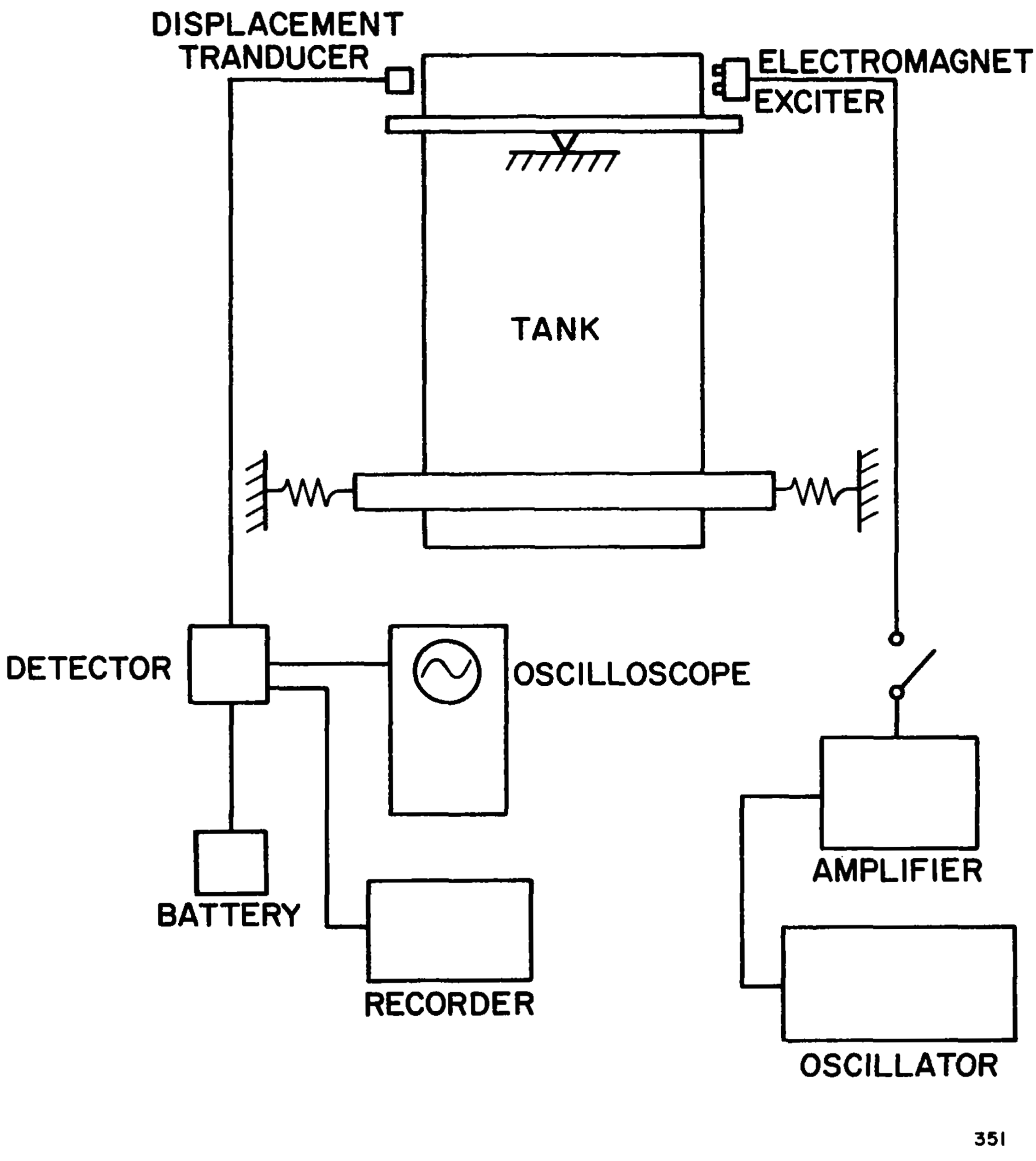


FIGURE 4. SIMPLIFIED SCHEMATIC OF TEST APPARATUS

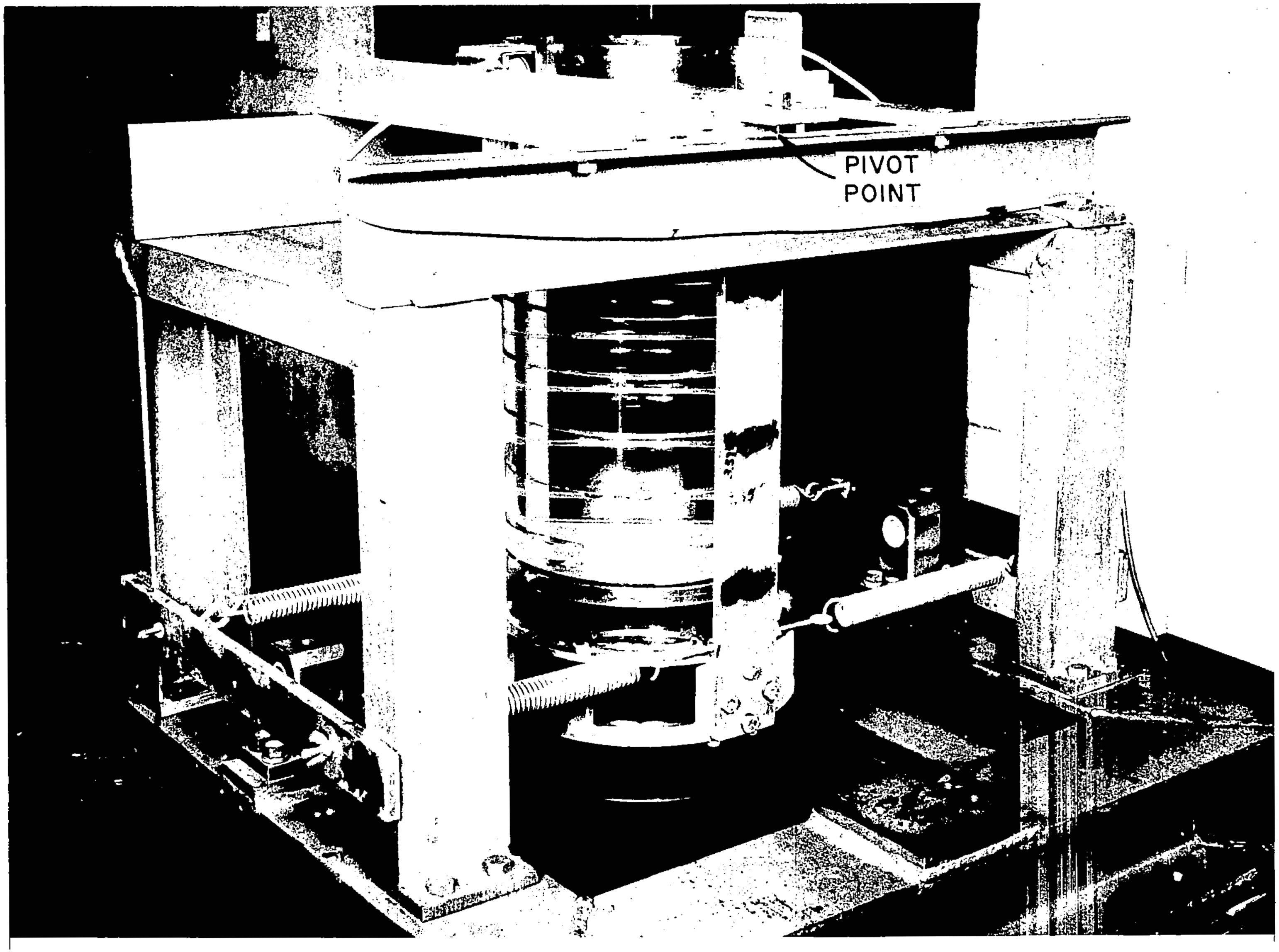


FIGURE 5. VIEW OF TEST TANK AND MOUNTING

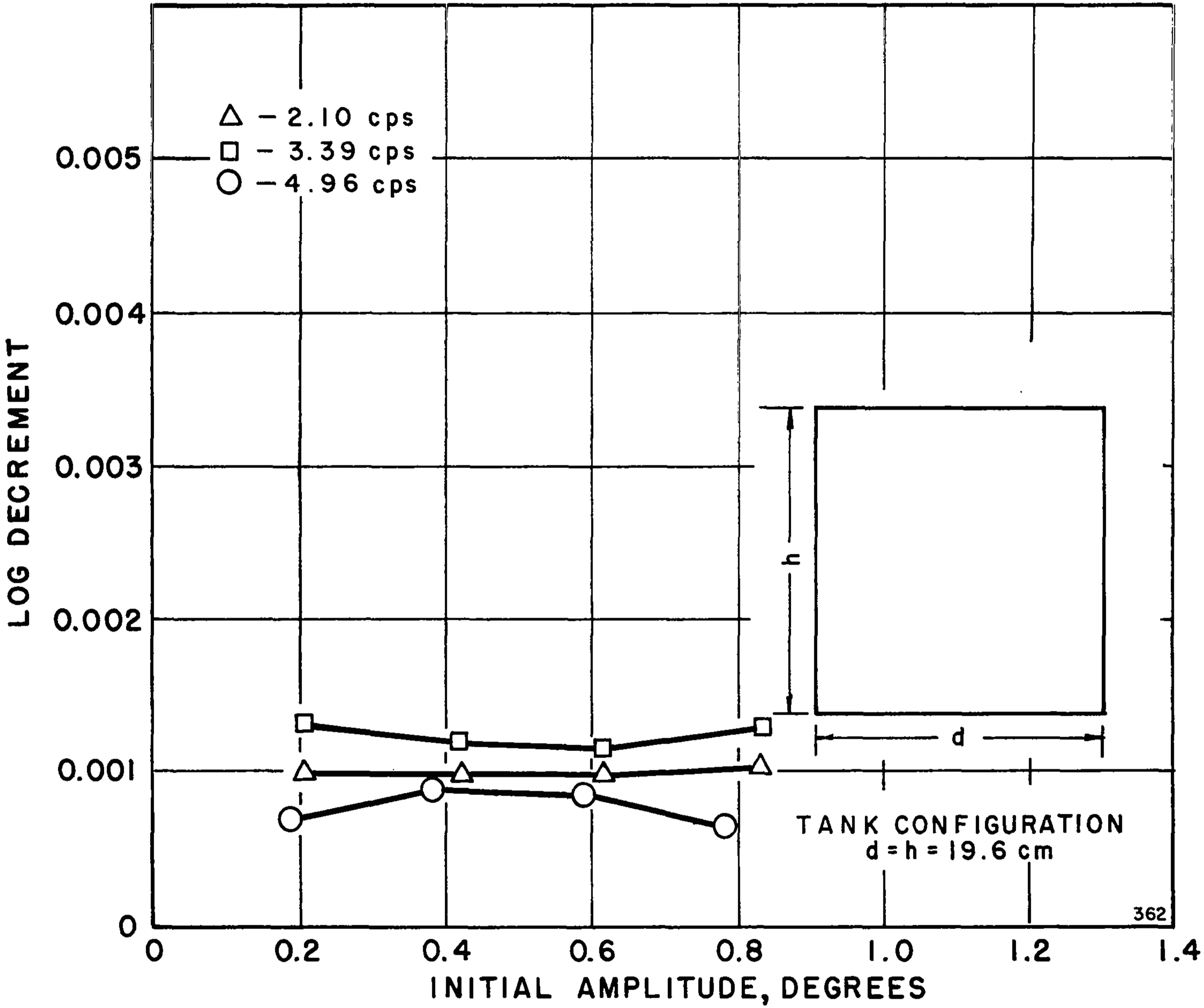


FIGURE 6. LIQUID DAMPING FOR TANK WITH NO BAFFLES

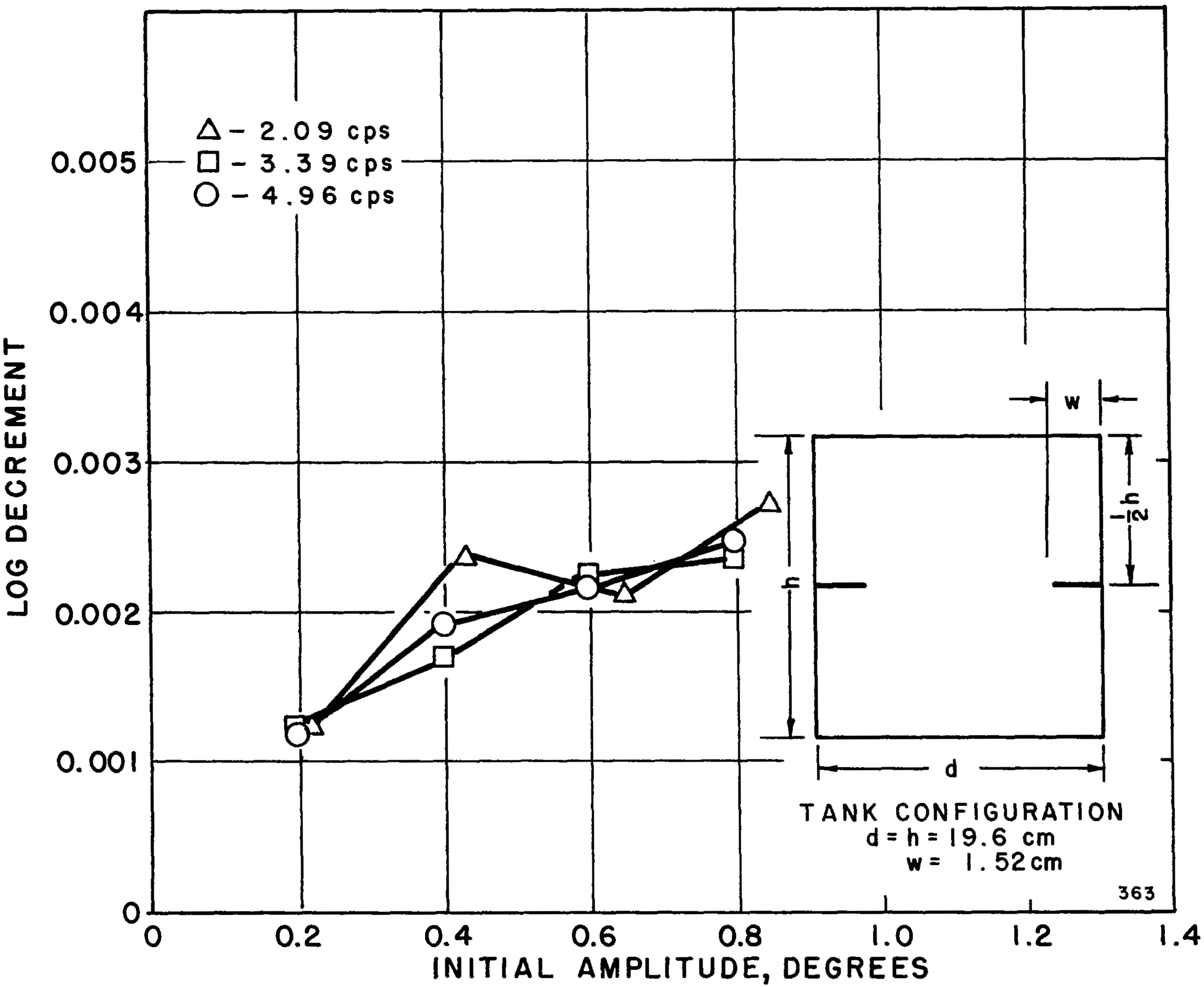


FIGURE 7. LIQUID DAMPING FOR TANK WITH ONE BAFFLE

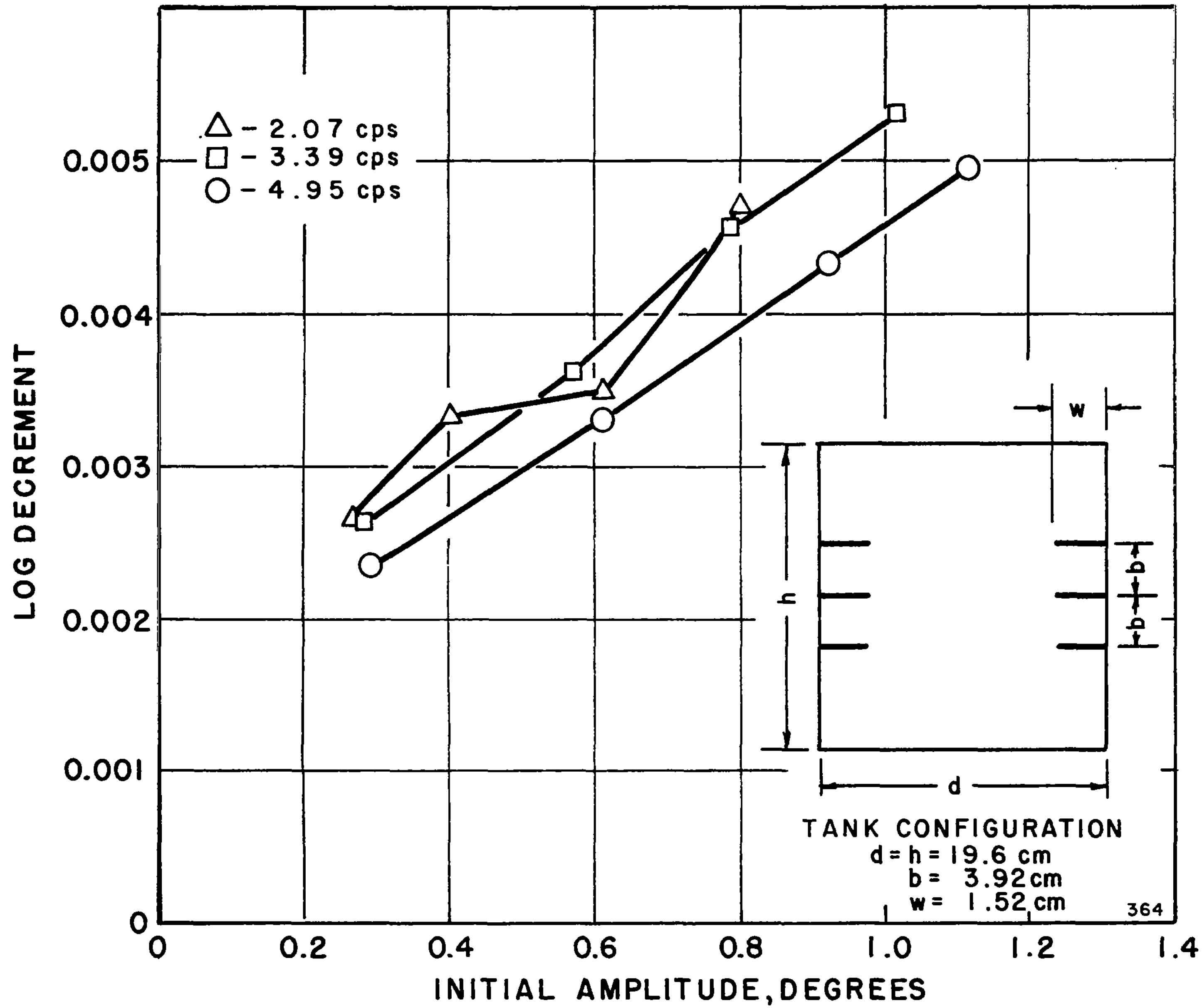


FIGURE 8. LIQUID DAMPING FOR TANK WITH THREE BAFFLES

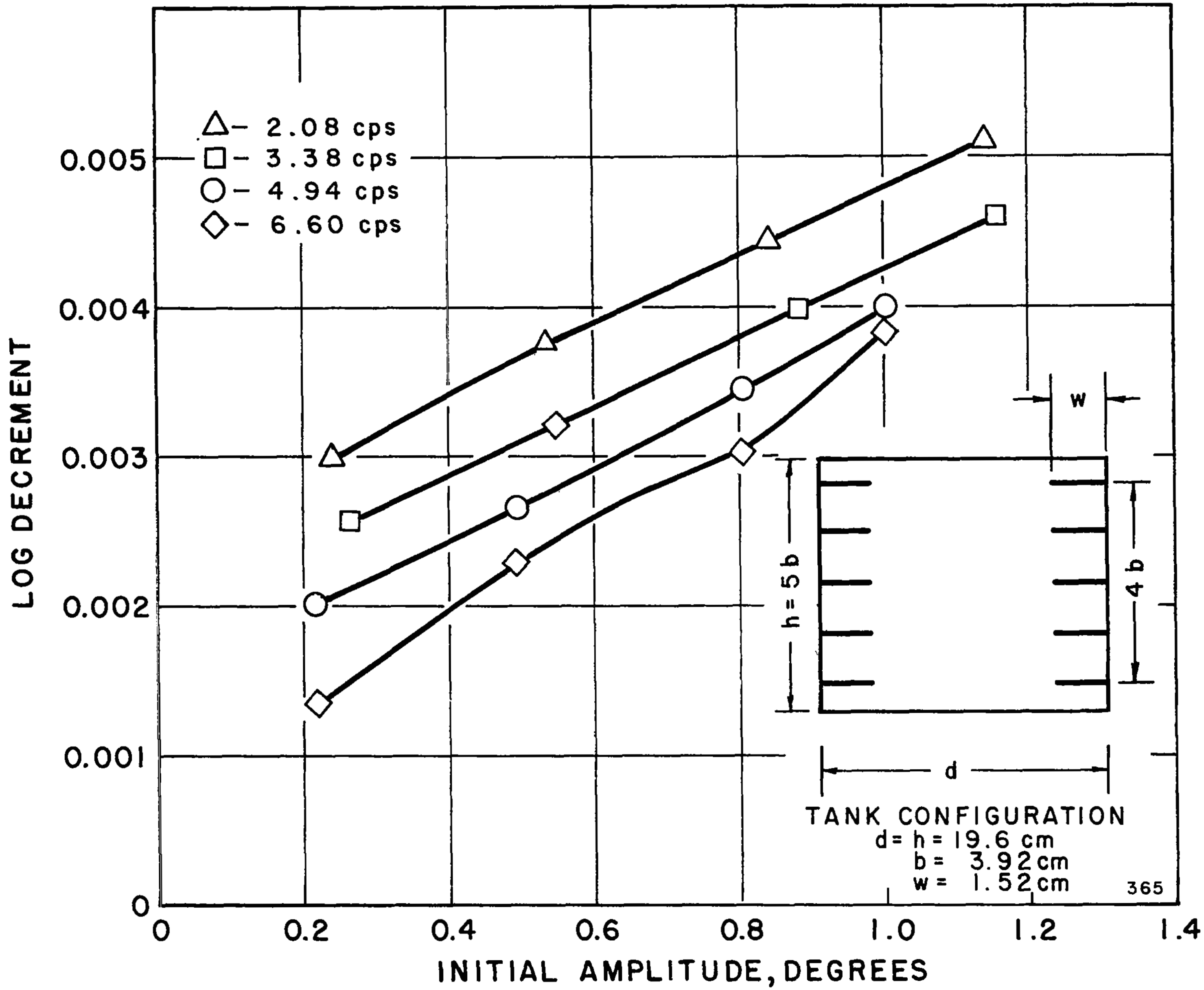


FIGURE 9. LIQUID DAMPING FOR TANK WITH FIVE BAFFLES

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