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Designing Integrated Fuzzy Guidance Law for Aerodynamic Homing Missiles Using Genetic Algorithms

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The Fuzzy logic controller (FLC) is well-known for robustness to parameter variations and ability to reject noise. However, its design requires definition of many parameters. This work proposes a systematic and simple procedure to develop an integrated fuzzy-based guidance law which consists of three FLC. Each is activated in a region of the interception. Another fuzzy-based switching system is introduced to allow smooth transition between these controllers. The parameters of all the fuzzy controllers, which include the distribution of the membership functions and the rules, are obtained simply by observing the function of each controller. Furthermore, these parameters are tuned by genetic algorithms by solving an optimization problem to minimize the interception time, missile acceleration commands, and miss distance. The simulation results show that the proposed procedure can generate a guidance law with satisfactory performance.

Key Words: Guidance, Missile, Fuzzy, Genetic Algorithms

Nomenclature

a_m: missile normal acceleration
a_t: target normal acceleration
C_D: drag coefficient
C_L: lift coefficient
D: missile drag force
g: gravitational acceleration
L: missile lift force
m: missile mass
M: Mach number
r: distance between the missile and the target
S_{ref}: missile reference area
T: missile thrust
V_m: missile velocity
V_t: target velocity
V_p: predicated velocity of the missile
x, h: horizontal and vertical positions
α: missile angle of attack
δ: heading error
γ_m: missile heading angle
γ_t: target heading angle
γ_p: predicated heading angle of the missile
σ: velocity error angle
ρ: air density
θ: line of sight angle

1. Introduction

Missile guidance technology is a mature field with many guidance laws already implemented in real systems. The guidance and control laws used in current tactical missiles are mainly based on classical control design techniques.[1)]

These conventional control approaches may not be sufficient to obtain tracking and interception. Therefore, advanced control theory must be applied to missile guidance and control systems to improve performance. Fuzzy logic control (FLC) has suitable properties for eliminating such difficulties. It also has the ability to reduce the effect of noise accompanying target measurements. However, few papers have addressed the issue of fuzzy missile guidance design.[2)]

Rajasekhar et al.[3)] used fuzzy logic to change the gain of the proportional navigation guidance (PNG) law. The fuzzy-based Proportional-Derivative (PD) controller was also used extensively in design of guidance laws where the line of sight (LOS) angle rate and change of LOS angle rate can be used as input linguistic variables, and the lateral acceleration command can be used as the output linguistic variable for the fuzzy guidance scheme.[4)] It has been shown that these fuzzy guidance schemes perform better than traditional proportional navigation or augmented proportional navigation schemes producing smaller miss distances and fewer acceleration commands.

In previous works, the parameters of the fuzzy guidance laws are generated by trial and error, which consumes time and effort and is not an optimum solution. Moreover, they use only one type of guidance through the whole interception range. The literature reports that each classical guidance law has an operation region where it is superior to other guidance laws.[5)] Lin et al.[6)] proposed an integrated fuzzy-based guidance law for intercepting a high-maneuvering target. The parameters of that law are obtained by engineering experience and trial and error. Becan and Kuzucu[7)] proposed a predictive fuzzy guidance law that uses classical PD-Fuzzy rules while the inputs to the fuzzy controller are varied during the interception range. The parameter that controls the transition is set manually.

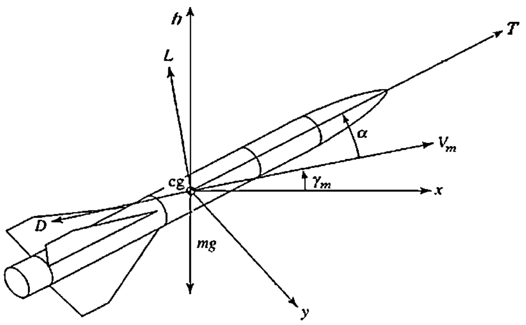

Fig. 1. Aerodynamic missile model.

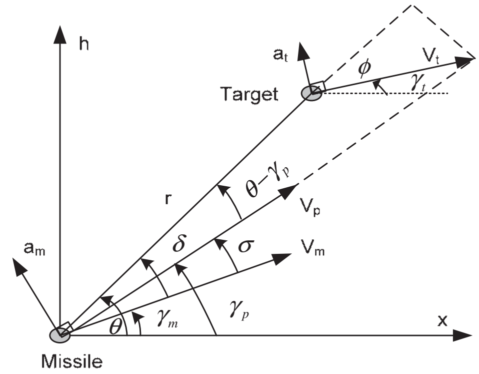

Fig. 2. Interception geometry.

Genetic Algorithms (GA) are a random search and optimization technique guided towards better performance by the selection mechanism. Unlike regular search algorithms, they do not deal with one solution, but with a set of solutions called a population. Each solution in the population is called an individual, which is an encoded representation of all parameters in the solution. GA uses so-called "genetic operators" to create new individuals from existing ones by merging (crossover) or modifying (mutation) existing individuals. New individuals replace old ones and through this process the population converges to the best solution. GA optimizes the performance index based on input/output relationships only; therefore, minimal knowledge of the plant under investigation is required. In addition, because derivative information is not needed in algorithm execution, many pitfalls of gradient search methods can be overcome.[8] Also, because GA does not need an explicit mathematical relationship between the performance of the system and the search update, GA offers a more general optimization methodology than conventional analytical techniques.

This paper proposes a simple systematic procedure to design a new integrated fuzzy-based guidance law to intercept maneuvering targets. This law consists of three fuzzy based controllers. Each is activated in a region of the interception range. Another fuzzy switching system is introduced to allow smooth transition between these controllers. At the first stage, the parameters of all the fuzzy controllers that include the distribution of the membership functions (MFs) and the rules are obtained simply by observing the function of each of these controllers. In the next step, these parameters are tuned by GA by solving an optimization problem to improve the performance of the guidance law by minimizing the interception time, missile acceleration commands and miss distance.

2. Mathematical Model

For simplicity, the missile's motion is constrained in the vertical plane. Furthermore, the missile is modeled as a point mass with aerodynamic forces applied at the center of gravity (CG). Therefore, from the missile's balanced forces shown in Fig. 1, the equations of motion can be written as:

$$\begin{aligned}
\dot{\gamma}_{\mathrm{m}} &= \frac{(L + T\sin\alpha)}{mV_{\mathrm{m}}} - \frac{g\cos\gamma_{\mathrm{m}}}{V_{\mathrm{m}}} \\
\dot{V}_{\mathrm{m}} &= \frac{(T\cos\alpha - D)}{m} - g\sin\gamma_{\mathrm{m}} \\
\dot{x}_{\mathrm{m}} &= V_{\mathrm{m}}\cos\gamma_{\mathrm{m}} \\
\dot{h}_{\mathrm{m}} &= V_{\mathrm{m}}\sin\gamma_{\mathrm{m}}
\end{aligned} \tag{1a}$$

where the drag and the lift forces can be expressed as:

$$\begin{aligned}
L &= \frac{1}{2}\rho V_{\mathrm{m}}{}^2 S_{\mathrm{ref}} C_{\mathrm{L}}, \quad C_{\mathrm{L}} = C_{\mathrm{L}\alpha}(\alpha - \alpha_0) \\
D &= \frac{1}{2}\rho V_{\mathrm{m}}{}^2 S_{\mathrm{ref}} C_{\mathrm{D}}, \quad C_{\mathrm{D}} = C_{\mathrm{D0}} + kC_{\mathrm{L}}{}^2
\end{aligned} \tag{1b}$$

The aerodynamic derivatives C_{La}, C_{D0} and k are given as functions of the Mach number M while the thrust and mass are a function of time. The angle of attack is used as the control variable and the missile normal acceleration can be determined from

$$a_{\mathrm{m}} = \dot{\gamma}_{\mathrm{m}} V_{\mathrm{m}} = \frac{(L + T\sin\alpha)}{m} - g\cos\gamma_{\mathrm{m}} \tag{2}$$

The target is assumed as a point mass with a constant velocity V_{t} and an acceleration a_{t} as shown in Fig. 2. The direction and position of the target are determined from the following relations:

$$\begin{aligned}
\dot{\gamma}_{\mathrm{t}} &= \frac{a_{\mathrm{t}}}{V_{\mathrm{t}}} \\
\dot{x}_{\mathrm{t}} &= V_{\mathrm{t}}\cos\gamma_{\mathrm{t}} \\
\dot{h}_{\mathrm{t}} &= V_{\mathrm{t}}\sin\gamma_{\mathrm{t}}
\end{aligned} \tag{3}$$

From the interception geometry shown in Fig. 2, the line of sight angle rate and the derivative of the relative distance between the missile and the target can be written as:

$$\begin{aligned}
\dot{\theta} &= (V_{\mathrm{m}}\sin(\theta - \gamma_{\mathrm{m}}) - V_{\mathrm{t}}\sin(\theta - \gamma_{\mathrm{t}}))/r \\
\dot{r} &= -V_{\mathrm{m}}\cos(\theta - \gamma_{\mathrm{m}}) + V_{\mathrm{t}}\cos(\theta - \gamma_{\mathrm{t}})
\end{aligned} \tag{4}$$

3. Design of Integrated Guidance Law

3.1. Interception phases

All surface-to-air missile have three guidance phases. The first is called the launch or boost phase, which occurs for a short time. The function of the launch phase is to take the missile away from the launcher base. After this phase, midcourse guidance is started. The function of the midcourse guidance phase is to bring the missile near to the target quickly. The last few seconds of the engagement constitutes the terminal guidance phase, which is most crucial because its success or failure determines the success or failure of the entire mission.

There are two basic guidance laws for homing missiles: Pursuit Guidance (PG) and Proportional Navigation Guidance (PNG). PG guides the missile to the current position of the target whereas PNG orientates it to the estimated interception point. Therefore PNG has a smaller interception time than PG, but may have unstable behavior for excessive values of the navigation constant. Therefore, it is recommended to use PNG in the launching phase to get a fast heading to the target because stability is not a big problem at this stage and then to use PG in the terminal guidance phase.[7)]

Since PNG is used during the boost phase to direct the missile velocity to the predicted interception location, the missile velocity should be aligned with the predicted interception velocity. Therefore, the missile command should be a function of the velocity error angle (σ) and its derivative. In the terminal phase, position error dominates the final miss distance so it is better to use PG. Therefore, the missile command must be a function of the heading error to achieve a stable system with minimum miss distance. During the midcourse phase, the missile should reach the terminal phase at high speed as far as possible but with reduced heading error. Thus, the missile acceleration is a function of both variables.[6)]

To implement the proposed guidance law, the direction of the predicted interception velocity, V_p, must be calculated. The estimated value of the angle of this direction can be obtained directly from the interception geometry (Fig. 2) as:

$$\gamma_p = \theta - \tan^{-1}\left(\frac{V_T t_p \sin(\phi)}{r + V_T t_p \cos(\phi)}\right) \tag{5}$$

The derivative of this angle is:

$$\dot{\gamma}_p = \dot{\theta} - \frac{V_T t_p[-\dot{r}\sin\phi + \dot{\phi}(V_T t_p + r\cos\phi)]}{(V_T t_p)^2 + 2rV_T t_p \cos\phi + r^2} \tag{6}$$

where t_p is the predicted time to intercept the target, which can be obtained from:

$$t_p \approx -\frac{r}{\dot{r}} \tag{7}$$

3.2. Structure of proposed guidance law

The configuration of the proposed guidance law is shown in Fig. 3. It consists of three fuzzy-based guidance laws for the launch, midcourse and terminal phases. For smooth transition between the three guidance laws, a fuzzy switching controller with two gains is introduced. These switching gains are determined from the following fuzzy rules:

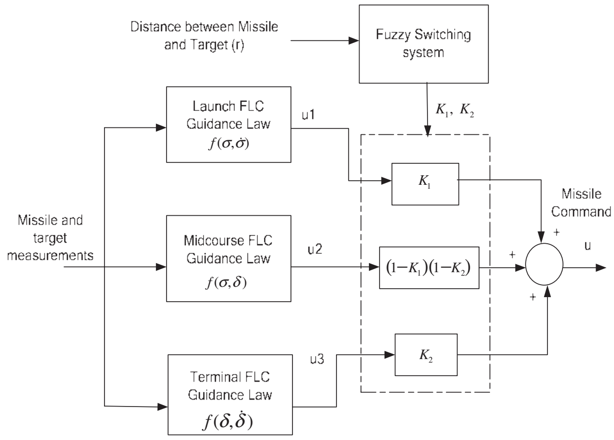

Fig. 3. Configuration of proposed guidance law.

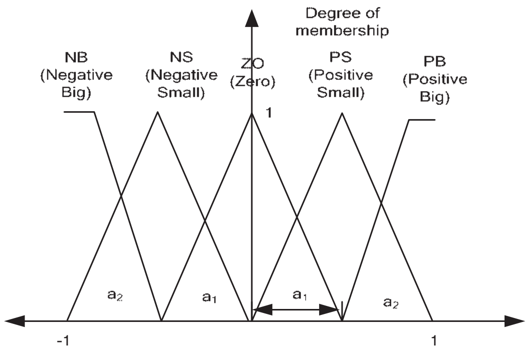

Fig. 4. Typical normalized membership functions.

$$\begin{aligned}
&\text{If } r \text{ is Big (B) then } K_1 = 1 \text{ and } K_2 = 0 \\
&\qquad\qquad \text{(launch phase)} \\
&\text{If } r \text{ is Medium (M) then } K_1 = 0 \text{ and } K_2 = 0 \\
&\qquad\qquad \text{(Midcourse phase)} \\
&\text{If } r \text{ is Small (S) then } K_1 = 0 \text{ and } K_2 = 1 \\
&\qquad\qquad \text{(Terminal Phase)}
\end{aligned} \tag{8}$$

The fuzzy controller has three main components: scaling factors, membership functions, and rules. The starting point in design of the fuzzy guidance law is to choose numbers and shapes of MFs for inputs and outputs variables. In this work, MFs with triangular shapes are chosen for all inputs and outputs variables as shown in Fig. 4. All the variables have positive and negative values except the range which is always positive. Therefore, only three MFs are used for the range and five MFs are used for the other variables.

The second step is to determine the scaling factors that convert the physical ranges of the fuzzy variables into the normalized ranges between -1 and 1. The scaling factors can be determined from the expected maximum values of the controllers' variables, which can be obtained from engineering experience of missile dynamics.

To complete the definition of the fuzzy guidance law, the rules defining the relationship between the control action and missile-target measurements should be determined.

At the first stage of the design process, equally distributed MFs (i.e. $a_1 = a_2 = 1/2$) with the rules obtained from imi-

Table 1. PD-fuzzy rules.

u	$\dot{e}$				
e	NB	NS	ZO	PS	PB
NB	NB	NB	NS	NS	ZO
NS	NB	NS	NS	ZO	PS
ZO	NS	NS	ZO	PS	PS
PS	NS	ZO	PS	PS	PB
PB	ZO	PS	PS	PB	PB

tating the behavior of the PD controller can be used for the three guidance laws since all these laws have the same form as the PD controller which can be written as:

$$u = K_p e + K_d \dot{e} \tag{9}$$

These rules can be derived easily by observing that the output is the summation of error (e) and error derivative ($\dot{e}$) which are shown in Table 1.

The next step is to tune these MFs and rules obtained from the first stage to get the final results. GA is used to achieve this task by solving the optimization problem using the procedure of Omar.[9)]

To include the linguistic rules in the optimization process, an integer encoding system is used to refer to the output fuzzy variables as shown in Table 2.

3.3. Formulation of optimization problem

The goal is to find or tune the existing consequents of the FLC rules and MFs parameters to minimize the performance index, which is a function of interception time, missile acceleration commands, and miss distance. The optimization problem can be formulated as:

$$\min f(z) = w_1 t_f + w_2 \int_0^{t_f} {a_m}^2 dt + w_3 |r(t_f)| \tag{10}$$

subject to:

$$|r(t_f)| < R_{\text{miss-allowed}}$$

where t_f is the interception time, w's are weighting factors, $R_{\text{miss-allowed}}$ is the allowed miss distance, and z is a vector that contains the unknown parameters of the fuzzy system. In this problem, the number of these unknowns is 20 for the MFs parameters and 45 for the rules after assuming that the missile acceleration is an odd function.

The scaling factors are chosen such that w_1 and w_2 have fixed values while w_3 is defined with two levels as shown in Eq. (11) to convert the constraint problem into a non-constraint one.

$$w_3 = \begin{cases} w_m & 0 \le r(t_f) \le R_{\text{miss-allowed}} \\ 100 w_m & r(t_f) > R_{\text{miss-allowed}} \end{cases} \tag{11}$$

GA is used to solve the above large scale constrained optimization problem to find the best solution. In this problem, the solution contains two types of data: integer numbers for the rules consequents and real numbers for the parameters that describe the distribution of MFs (Fig. 5).[9)]

Table 2. Encoding system for FLC output.

MF	NB	NS	ZO	PS	PB
Code	1	2	3	4	5

$$z=\{\overbrace{\overbrace{r_1,r_2,..,r_{45}}^{\text{Rules}}}^{\text{Integer}}, \overbrace{a_1,a_2,........,a_{20}}^{\text{Real}}\}$$

Fig. 5. Structure of GA individual.

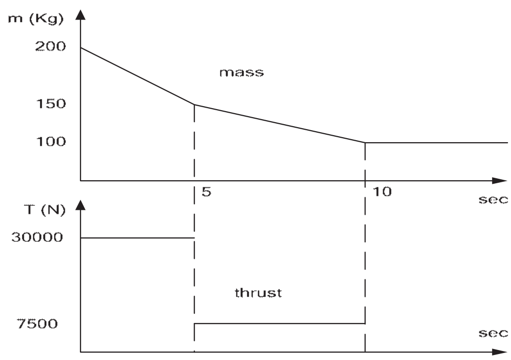

Fig. 6. Time history of missile mass and thrust.

3.4. Initial population of GA

Usually the initial population of GA is generated randomly. However, the literature reports that using a strong individual in the initial population improves the performance of GA rather than generating all the individuals randomly.[10)] This strong individual can consist of equally distributed MFs with the PD fuzzy rules obtained in Table 1.

4. Example

Assumed that the missile has the thrust and mass that vary with time as shown in Fig. 6, while the other parameters are given as:

$$\begin{aligned} &C_{L\alpha} = 2.9 + 0.3M + 0.25M^2 + 0.01M^3, \quad \alpha_0 = 0 \\ &C_{D0} = 0.45 - 0.01M, \quad k = 0.06, \quad S_{\text{ref}} = 0.08 \end{aligned} \tag{12}$$

The target is assumed to have a constant speed of 400 m/s with a constant acceleration of $3G$ ($G = 9.8\,\text{m/s}^2$). The weighting factors in the optimization function are chosen as

$$w_1 = 1, \quad w_2 = 10^{-4}, \quad w_m = 10 \tag{13}$$

The allowed miss distance is set to 2.0 m and the initial values for the missile and target variables are

$$\begin{aligned} &v_m = 10\,\text{m/s}, \quad \gamma_m = 30^\circ, \quad r = 5000\,\text{m} \\ &\theta = 50^\circ, \quad \gamma_t = 0 \end{aligned} \tag{14}$$

The maximum allowed ranges for the fuzzy input and output variables can be estimated as

$$\begin{aligned} &\alpha_{\max} = 20^\circ, \quad \delta_{\max} = \sigma_{\max} = 20^\circ \\ &\dot{\delta}_{\max} = \dot{\sigma}_{\max} = \frac{a_{\max}}{600} \approx 28.6\,\text{deg/s} \end{aligned} \tag{15}$$

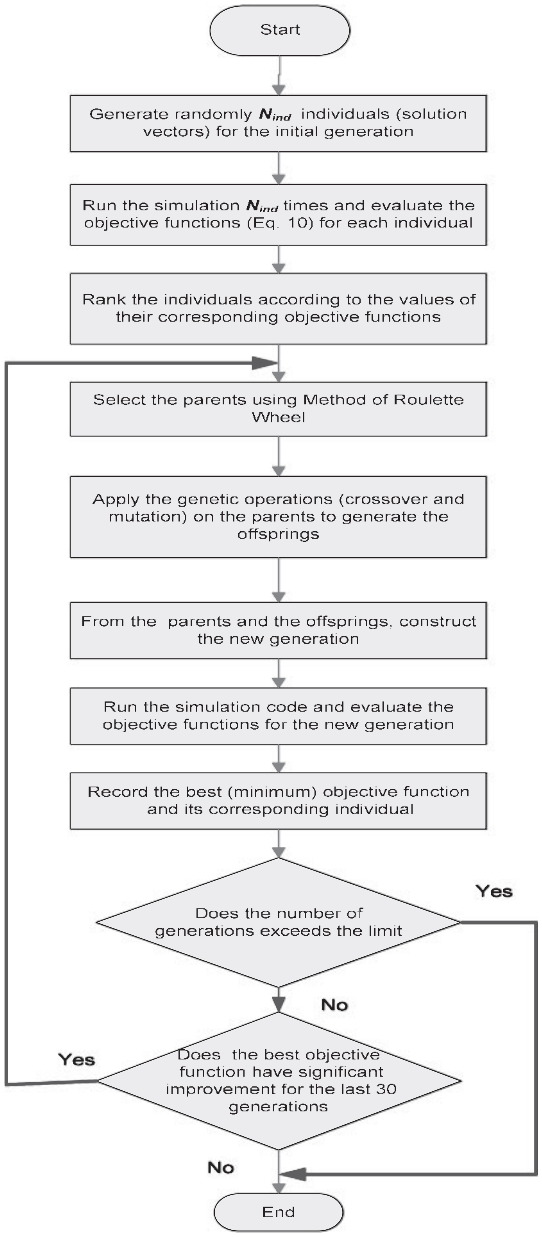

Fig. 7. Flow chart of GA algorithm.

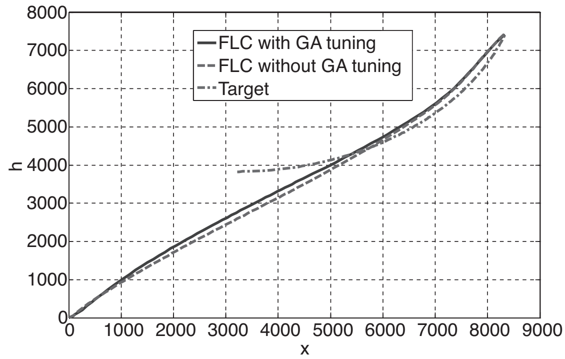

Fig. 8. Time history of interception.

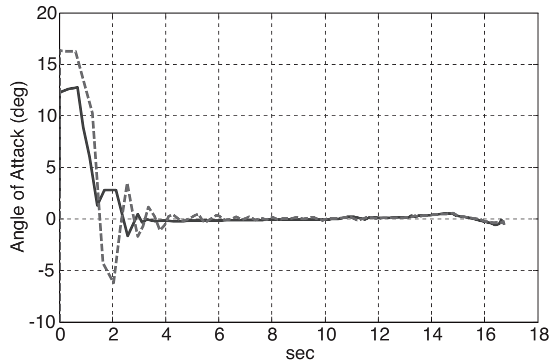

Fig. 9. Time history of control action.

Table 3. Performance indices.

Index	With GA tuning	Without GA tuning
Interception time (t_f)	16.6427	16.7395
Miss distance r (t_f)	0	6.5 m
$\int_0^{t_f} a_m{}^2 \mathrm{d}t$	58958	66347

Simulation was performed using a variable step solver. The simulation stops when the closing velocity becomes positive. The time and relative distance at that instant are the final interception time and miss distance, respectively. The GA algorithm is executed according to the flow diagram shown in Fig. 7. The values used for the GA parameters are: 30 for population size (i.e. number of the individuals), 0.7 for crossover rate, and 0.01 for mutation rate. Linear ranking with the method of roulette wheel is used to select individuals which produce offsprings that join the next generation.

The time history of interception and the resulting control action using the proposed integrated guidance law with and without GA tuning is shown in Fig. 8 and Fig. 9, respectively. The untuned guidance law could intercept the target with a miss distance of 6.5 m while the tuned law intercepts the target with nearly zero miss distance. Also, the tuned guidance law has a smaller interception time and fewer missile acceleration commands compared to the untuned law as shown in Table 3. These results indicate that with simple reasoning and engineering experience, a fuzzy-based guidance law can be designed with satisfactory performance. Moreover, better performance can be obtained by tuning the parameters using GA. Although, our previous work showed that GA can produce FLC with satisfactory performance when all individuals of the initial generation are generated randomly without embedding a strong individual.[9,11] However, the embedding improves GA convergence.

The best distribution for the membership functions of the relative distance between the missile and the target is shown in Fig. 10 and the variations of switching gains that result from this distribution are shown in Fig. 11. These figures indicate that the duration of the launching guidance law (i.e. $K1 = 1$) is longer than the duration of the terminal guidance law (i.e. $K2 = 1$) because the range (r) exceeds its maximum limit at the initial phase of the interception as shown in Fig. 12. The midcourse phase has the largest

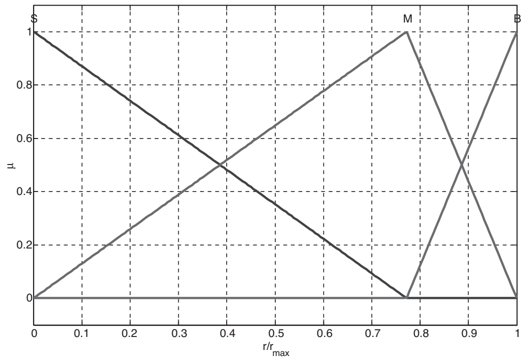

Fig. 10. Best obtained MFs for relative distance (r).

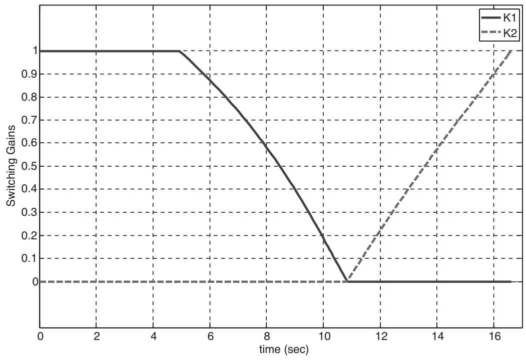

Fig. 11. Time history of variation of switching gains.

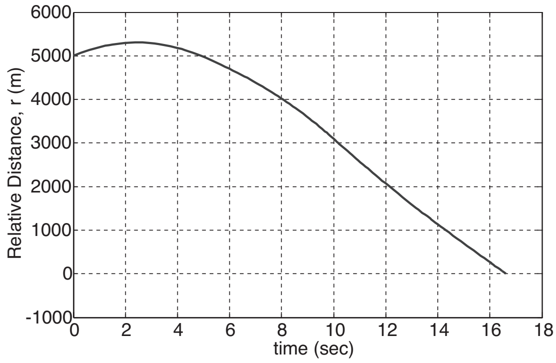

Fig. 12. Time history of relative distance (r).

contribution to the guidance period among the other two laws. However, it is usually accompanied by one of other two laws since the switching gains become zeros for a very short period. Due to the continuity in these gains, the transition from one law to another is smooth. To achieve a more realistic guidance law, each of these laws should be completely activated alone for some period. These periods should be short for the launch and the terminal guidance laws and long for the midcourse guidance law. This can be achieved using a trapezoid membership function to describe the interception range.

It worth mentioning that the obtained fuzzy guidance law using GA is tuned around one scenario. This scenario is chosen for illustrative purposes only to show the effectiveness of the proposed technique to design an integrated fuzzy guidance law without the need for the usual trial and error method. To get satisfactory performance over the expected operating range of the missile, the designer should include all the expected scenarios in the evaluation of the objective function. In this case, the total value of the objective function is the sum of the objective functions from these scenarios.[9] This will certainly ensure the robustness of the designed guidance law but will increase the computational time.

5. Conclusion

Using simple reasoning and engineering experience, an integrated fuzzy based guidance law is designed to intercept maneuvering targets with satisfactory performance. GA is used to tune the fuzzy parameters of the guidance law to further improve performance. The proposed procedure can be used as a tool to help guidance engineers to design guidance laws to achieve more complex interception missions. For future research, we recommend combining the proposed technique with the multi-objectives technique to give the designer a complete picture of the relationship among the three optimization indices without imposing certain weighting factors.

Acknowledgments

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