



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

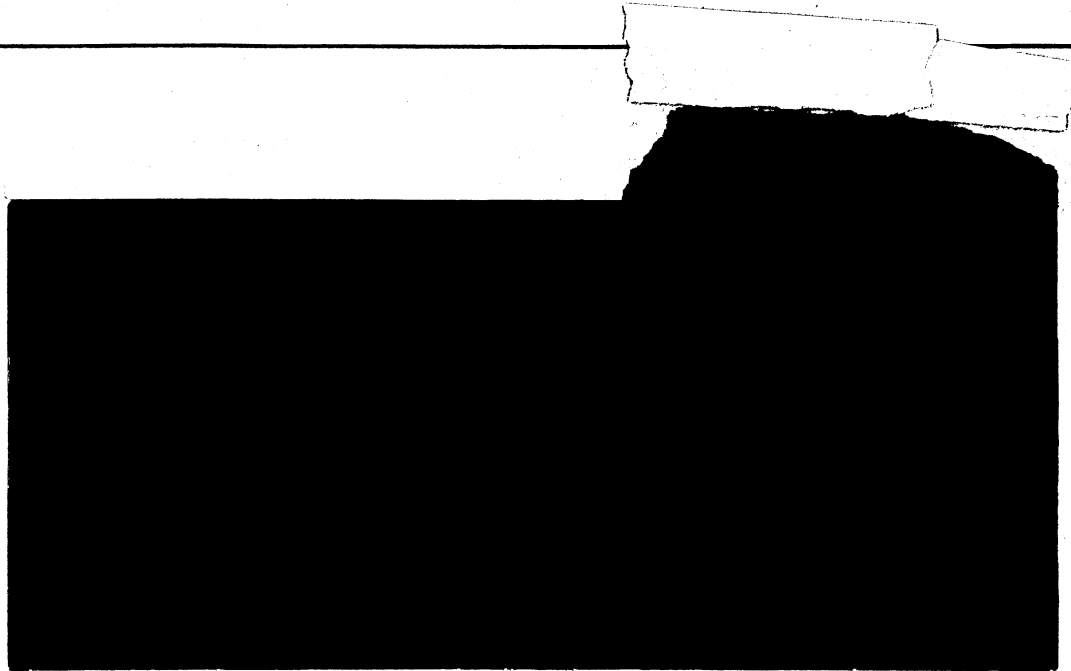
Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

378.794
G43455
WP-292
Rev.2

Working Paper Series



WAITE MEMORIAL BOOK COLLECTION
DEPT. OF AGRIC. AND APPLIED ECONOMICS

DEPARTMENT OF AGRICULTURAL AND
RESOURCE ECONOMICS

BERKELEY

CALIFORNIA AGRICULTURAL EXPERIMENT STATION

University of California



Working Paper No. 292 Rev. 2

LAND ALLOCATION UNDER UNCERTAINTY FOR
ALTERNATIVE SPECIFICATIONS OF
RETURN DISTRIBUTIONS

by

Robert N. Collender and David Zilberman

California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics

January 1985

LAND ALLOCATION UNDER UNCERTAINTY FOR
ALTERNATIVE SPECIFICATIONS OF
RETURN DISTRIBUTIONS

by

Robert N. Collender and David Zilberman

Robert N. Collender is an assistant professor of economics and business, North Carolina State University, and David Zilberman is an associate professor of agricultural and resource economics, University of California, Berkeley.

Collender, Robert N., and Zilberman, David--Land Allocation Under Uncertainty
For Alternative Specifications of Return Distributions

[Using the expected utility-moment generating function approach, a land allocation rule is derived which is both independent of the nature of the underlying distribution of each element in the choice set and accounts for all moments of that distribution. This decision rule is applied to the choice between cotton and corn in the Mississippi Delta to demonstrate that consideration of just the first two moments of the distribution can often be unjustified and damaging empirically, theoretically, and in terms of the utility of the decision-maker.]

Key words: uncertainty, moment-generating function, gamma distribution,
yield risk

Robert N. Collender is an assistant professor of economics and business, North Carolina State University, and David Zilberman is an associate professor of agricultural and resource economics, University of California, Berkeley.

LAND ALLOCATION UNDER UNCERTAINTY FOR ALTERNATIVE SPECIFICATIONS OF RETURN DISTRIBUTIONS*

Introduction

Agricultural economists have long postulated that yield and price uncertainties, combined with farmer distaste for risk, play a crucial role in land allocation decisions. While several approaches have been advanced to analyze farmer choices under uncertainty (Anderson), the expected utility approach has emerged as the dominant analytical framework for this purpose. Analysis of land allocation within this framework requires specification of a utility function, production technologies, and statistical distributions of returns.

While conceptual studies of the properties of optimal land allocation under uncertainty (Feder; Just and Zilberman) can afford assuming rather general functional forms, studies aimed at obtaining the exact optima require more restricted specifications. Therefore, such studies tend to abstract from technological considerations by assuming that the farmer faces a Leontief technology and reduce the farmer's objective function to a linear combination of the mean and variance of profits (Freund).

This linear E-V approach corresponds to two possible scenarios: (1) the farmer has a quadratic utility function (Markowitz) or (2) the farmer has a negative exponential utility function and the crop returns are normally distributed (Freund). However, as noted by Pratt and Arrow, the quadratic utility function is theoretically unsatisfactory because it embodies an underlying assumption of increasing absolute risk aversion. Moreover, Day's pioneering statistical analysis of Mississippi Experiment Station data on yield distributions has raised doubts regarding the realism of assuming normal distributions of returns. Thus, the above arguments suggest that the linear

E-V approach may be inappropriate in many cases, and a more general framework for land allocation is needed.

This paper develops such a framework. It is based on Hammond's observation that the negative exponential utility function, when used in the expected utility paradigm, yields a convenient closed-form solution dependent on the moment-generating function (MGF) of a random variable. The applicability of this model to decisions under uncertainty depends critically on acceptance of the exponential utility function and the constant absolute risk aversion it implies as an adequate representation of the farmer's risk preferences.

Yassour, Zilberman, and Rausser used this approach to develop a model of discrete choice among technologies and demonstrated that gamma-distributed crop yields would lead to a different rate of adoption of new technologies than would normally distributed yields. Their application relates to the special case where technologies are lumpy and technological choice discrete. Frequently, however, a farmer may allocate land to several activities, i.e., different technologies, crops, or land rental.

The current paper extends the application of the expected utility-moment generating function approach to a continuous choice to derive an optimal land allocation rule for farmers facing multivariate crop yield distributions both in the presence of and without an active rental market for land. In addition, we derive optimal solutions for the case when yields are characterized by a bivariate gamma distribution. Finally, we suggest a procedure for choosing among possible assumed "true" distributions of returns and apply this procedure to the choice between the assumption that crop yields are either normal or gamma distributed.

A Model of Optimal Land Allocation Among Crops Under Uncertainty

Consider a farmer who allocates L acres of land among N crops. Let l_i be the amount of land allocated to crop i ; v_i , the nonstochastic variable cost per acre of the i th crop; and Y_i , the revenue per acre of the i th crop. Assume that the revenues are randomly distributed with a joint distribution function $f(Y_1, Y_2, \dots, Y_N)$.

Suppose the farmer has a negative exponential utility function defined on his profits. This function is:

$$(1) \quad U(\pi) = -e^{-r\pi}$$

where $\pi = \sum_{i=1}^N l_i(Y_i - v_i)$ is the farmer's profit.

The farmer maximizes his expected utility subject to his land availability constraint. Let

$$(2) \quad M(t_1, t_2, \dots, t_N) = E \left[\exp \sum_{i=1}^N t_i Y_i \right]$$

define the MGF of the random variable Y_1, Y_2, \dots, Y_N , where E is the expectations operator. Using equations (2) and (3), the farmer's choice problem becomes:

$$(3) \quad \max_{l_i, i=1, N} - \exp \left(r \sum_{i=1}^N l_i v_i \right) \cdot M(\cdot)$$

where $M(\cdot)$ is evaluated at $-rl_1, -rl_2, \dots, -rl_N$ subject to:

$$(4.1) \quad \sum_{i=1}^N l_i \leq L$$

and

$$(4.2) \quad l_i \geq 0; \text{ for all } i.$$

Let λ be the shadow price associated with the land constraint (4) and assume an interior solution. Using the Lagrange multiplier technique, the optimization problem has $n + 1$ first-order conditions. Since the objective function is concave, the first-order conditions yield an optimal solution. They are equations (4) and

$$(5) \quad r \cdot \exp \left(r \sum_{i=1}^N l_i v_i \right) \cdot M(\cdot) \cdot \left(\frac{M_i}{M(\cdot)} - v_i \right) = \lambda$$

where M_i is the first derivative of the MGF, $M(\cdot)$, with respect to its i th element. Using (4) and (5), one can derive an alternative set of optimality conditions which excludes λ consisting of (4) and

$$(6) \quad \frac{M_i}{M} - v_i = \frac{M_1}{M} - v_1; \text{ for all } i = 2, N.$$

At the optimal solution, the marginal effects of a change in all the crop acreages on the MGF will be the same. In cases where farmers are risk neutral, the parameter, r , is equal to 0, $M = 1$, and $M_i = E(Y_i)$. Hence, equation (6) implies that the net expected profit per acre for all the crops grown by the farmer should be the same.

Optimal Land Allocation Rules

Two Land Uses--One With Certain Profits

The optimality conditions in (5) and (7) yield land allocation rules for specific distributions of revenue per acre. First, consider the case where the farmer has two land-use opportunities--one yielding certain revenue per acre of Y_1 and the other yielding random revenue per acre of Y_2 . Such a situation occurs, for example, when the farmer can lease part of his land and grow one crop on the rest of it. Feder used a similar formulation to determine optimal land allocation between an old variety (which was assumed to yield a sure profit) and a risky new variety. To find the optimality conditions for this type of problem, let

$$(7) \quad m_i(t_i) = E [\exp (t_i Y_i)]$$

be the MGF of Y_i . For the certain alternative, $m_1(t_1) = \exp (t_1 Y_1)$. Since the Y_1 and Y_2 are independent, $M(t_1, t_2) = m_1(t_1) m_2(t_2)$. Using equations (4), (6), and (7), the optimality conditions for the case of choice between risky and certain land use are

$$(8) \quad l_1 + l_2 = L$$

$$(9) \quad Y_1 - v_1 = \frac{m_2'(-r l_2)}{m_2(-r l_2)} - v_2$$

where m_2' is the first derivative of $m_2(\cdot)$. Thus, at the optimal land allocation, the farmer will set the logged derivative of the MGF less the variable costs for growing the risky crop equal to certain activity.

Renting land is an obvious example of a certain activity. Thus, conditions (8) and (9) suggest that, at the optimal land allocation, the lagged derivative of the MGF less the variable costs is equal to the rent. This conclusion can be generalized to the case of multiple land uses in the presence of a land rental market--i.e., at the optimal solution, the returns from renting out land will be set equal to the partial logged derivative of the joint MGF of the risky crops for each crop less that crop's variable costs. Given this land allocation rule, it is clear that the farmer's subjective opinion regarding the higher moments of the distribution could well determine whether he enters the land rental market and on which side of the market he participates. We show below that farmers with the same opinions regarding mean and variance and the same parameter of absolute risk aversion--but with differing opinions regarding other moments of the distribution--will not only allocate land differently but also will value land differently. This result, in turn, has important implications for the scale of farming which will be optimal for farmers depending on how they view the distribution of returns.

In the absence of an active land market, the farmer should continue to set the logged derivative of the MGF less the variable costs for growing each crop equal. If land is not a binding constraint, the farm size will be determined by setting each of these conditions equal to zero:

$$\frac{M_j}{M} - v_j = 0; \text{ for all } j = 1, N.$$

In any case, the shadow price of land will be equal to the logged derivative of the MGF less the variable costs for each crop planted.

In table 1, the optimal land allocation for the uncertain crop is computed for several revenue distributions. Note that the assumption of constant absolute risk aversion results in optimal allocations of land for the risky use which are independent of the farm size when landholdings are assumed to equal wealth. This unsatisfactory outcome might be overcome if one assumes that every farmer operates according to a constant measure of risk aversion but that the degree of risk aversion changes with the size of landholdings.¹

For all the distributions, a reduction in risk aversion increases the land in the risky use. The importance of the right specification, however, can be demonstrated by comparing optimal allocations under gamma and normal distributions. Assuming that both crops have the same variable cost per acre, optimality conditions--when revenue from the risky crop is gamma distributed--require that more land be allocated to the risky use (than would be optimal to allocate under the assumption of normality) if its expected revenue per acre is at least twice that of the alternative use ($\mu_2 > 2\mu_1$). In contrast, optimality conditions for revenues under the normal distribution require that more land be allocated to the risky use (than would be optimal to allocate under the assumption that the revenue was gamma distributed) if its expected revenue per acre is less than twice that of the alternative use ($\mu_2 < 2\mu_1$). The greater the difference between μ_2/μ_1 and two, the greater the difference will be in optimal land allocation for the two distributions.

The Case of Two Risky Land Uses

The Case of Normality. Consider the case where one can grow two crops, the revenue from each being a random variable. In this case, the mean yield and variance are given by μ_1 , σ_1^2 , and the correlation coefficient between the

Table 1. Optimal Allocation of Land to Risky Use Under Various Distributions of Revenue

Distribution of Revenue	Parameters	Mean	Variance	Moment-Generating Function	Optimal Allocation of Land To Risky Use (1/2)
Normal	μ, σ^2	μ	σ^2	$e^{\mu t + \sigma^2 t^2 / 2}$	$\frac{(\mu_2 - Y_1) - (v_2 - v_1)}{r\sigma_2^2}$
Gamma	α, λ	α/λ	α/λ^2	$(1 - t/\lambda)^{-\alpha}$	$\frac{\alpha/\lambda}{Y_1 - (v_1 - v_2)} - 1$
Exponential	λ	$1/\lambda$	$1/\lambda^2$	$\lambda/(\lambda - t)$	$\frac{1/\lambda - Y_1 - (v_2 - v_1)}{r(v_1 + v_2 - Y_1)}$
Poisson	μ	μ	μ	$e^{\mu(e^t - 1)}$	$\frac{1}{t} \ln \left(\frac{\mu_2}{Y_1 - v_1 + v_2} \right)$
Binomial	np	np	$np(1 - p)$	$(1 - p + pe^t)^n$	$\frac{1}{t} \ln \left[\frac{1}{(1 - p)} \left(\frac{np}{Y_1 - v_1 + v_2} \right) - p \right]$

revenues is ρ . The MGF is:

$$(10) \quad M(t_1, t_2) = \exp [t_1\mu_1 + t_2\mu_2 + .5(t_1^2\sigma_1^2 + t_2^2\sigma_2^2 + 2\rho t_1t_2 \sigma_1\sigma_2)].$$

Introducing (10) into (2) and using (6), one can derive the land allocation formula

$$(11) \quad l_1 = \frac{(\mu_1 - v_1) - (\mu_2 - v_2) + rL(\sigma_1^2 \rho \sigma_1 \sigma_2)}{r(\sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2)}.$$

The Case of the Bivariate Gamma Distribution. Actually, the use of the binormal distribution may be inferior to the use of other distributions in describing the stochastic revenues. As Day has shown, yields have skewed distributions; and, at least for cases with fixed prices (e.g., under price-support programs), one should analyze farmers' behavior using skewed revenue distributions. Following this argument, one should consider using the bivariate gamma distribution to model crop yields. The bivariate gamma distribution was introduced by Ghirtis and is presented in Johnson and Kotz. It has five parameters: $\alpha_0, \alpha_1, \alpha_2, \lambda_1, \lambda_2$. Its MGF is

$$(12) \quad M(t_1, t_2) = \left(1 - \frac{t_1}{\lambda_1} - \frac{t_2}{\lambda_2}\right)^{-\alpha_0} \cdot \left(1 - \frac{t_1}{\lambda_1}\right)^{-\alpha_1} \cdot \left(1 - \frac{t_2}{\lambda_2}\right)^{-\alpha_2}.$$

The revenue per acre of each crop has a gamma distribution with the parameters $\alpha_0 + \alpha_i \lambda_i$ for $i = 1, 2$. The mean and variance of each crop's revenue per acre is given by:

$$(13) \quad E(Y_i) = \mu_i = \frac{\alpha_0 + \alpha_i}{\lambda_i}, \quad V(Y_i) = \sigma_i^2 = \frac{\alpha_0 + \alpha_i}{\lambda_i^2}.$$

The correlation coefficient between the revenues is given by

$$(14) \quad \text{cov}(Y_1, Y_2) = \rho = \frac{\alpha_0}{\sqrt{(\alpha_0 + \alpha_1) \cdot (\alpha_0 + \alpha_2)}}.$$

From equations (13) and (14), one can derive the parameters of the bivariate gamma distribution from means, variances, and covariances of the revenues per acre of each crop. Using equation (12), the first-order condition in (6) becomes

$$(15) \quad \frac{\alpha_0(\lambda_1 - \lambda_2)}{\lambda_1\lambda_2 + \lambda_2r_{11} + \lambda_1r_{12}} - \frac{\alpha_1}{\lambda_1 + r_{11}} + \frac{\alpha_2}{\lambda_2 + r_{12}} = v_2 - v_1.$$

Because the explicit expression for the optimal land allocations for the bivariate gamma distribution is cumbersome, we will use an empirical example to compare the outcome under the bivariate gamma and normal distributions.

An interesting analytic property of the solution under the bivariate gamma distribution is that it reduces to the solution under the bivariate normal distribution as any of the α 's approach zero when the variable costs are the same for both alternatives.

Empirical Results of Bivariate Choice Model

To illustrate the importance of taking higher moments of the distribution into account, we apply our model to the allocation of land between corn and cotton in the lower Mississippi drainage. The derivation of the distribution of returns per acre for each crop is described in Collender. The means of per acre returns for corn and cotton are, respectively, \$53.30 and \$244.10. The standard deviations of returns to corn and cotton are, respectively, \$24.28 and

\$107.00. The correlation coefficient of returns between the two crops is 0.072.

There are not many empirical estimations of the measure of absolute risk aversion. Binswanger found that the measure of partial risk aversion $-mU''(w + m)/U'(w + m)$ where m , the certainty equivalent of profit, varies from .1 to 10 as risk aversion varies from slight to extreme. Thus, we let the measure of absolute risk aversion, r , vary from $.1/(\bar{Y}_1 - v_1) L$ to $10/(\bar{Y}_2 - v_2) L$ assuming $(\bar{Y}_1 - v_1) > (\bar{Y}_2 - v_2)$.

To compare the results of the optimal decision under normality and under gamma, we calculated the allocation of land to cotton, the certainty equivalent of the farmer for his entire crop, and the marginal value of land using rules associated with each distribution. It should be emphasized that, under both distributional assumptions, the mean and variance of the distribution are the same. The results are presented in figures 1 and 2. Figure 1 shows that the amount of land allocated to cotton will always be at least as great under the gamma distribution as under the normal. Indeed, in this case the alternative crop (corn) was not sufficiently attractive to induce allocation of land to it at any considered level of risk aversion. This is not surprising when the nature of the gamma distribution is considered. Since the gamma distribution is always positively skewed, a greater amount of the risk is above the mean for a gamma distribution as opposed to a normal distribution. This positive skewness offsets the increased variance (cf., Tsiang).

Figure 2 shows that land values generally will be much higher under the gamma distribution than under normality. Thus, if yields are gamma distributed and the major source of risk, farmers who make their choices based

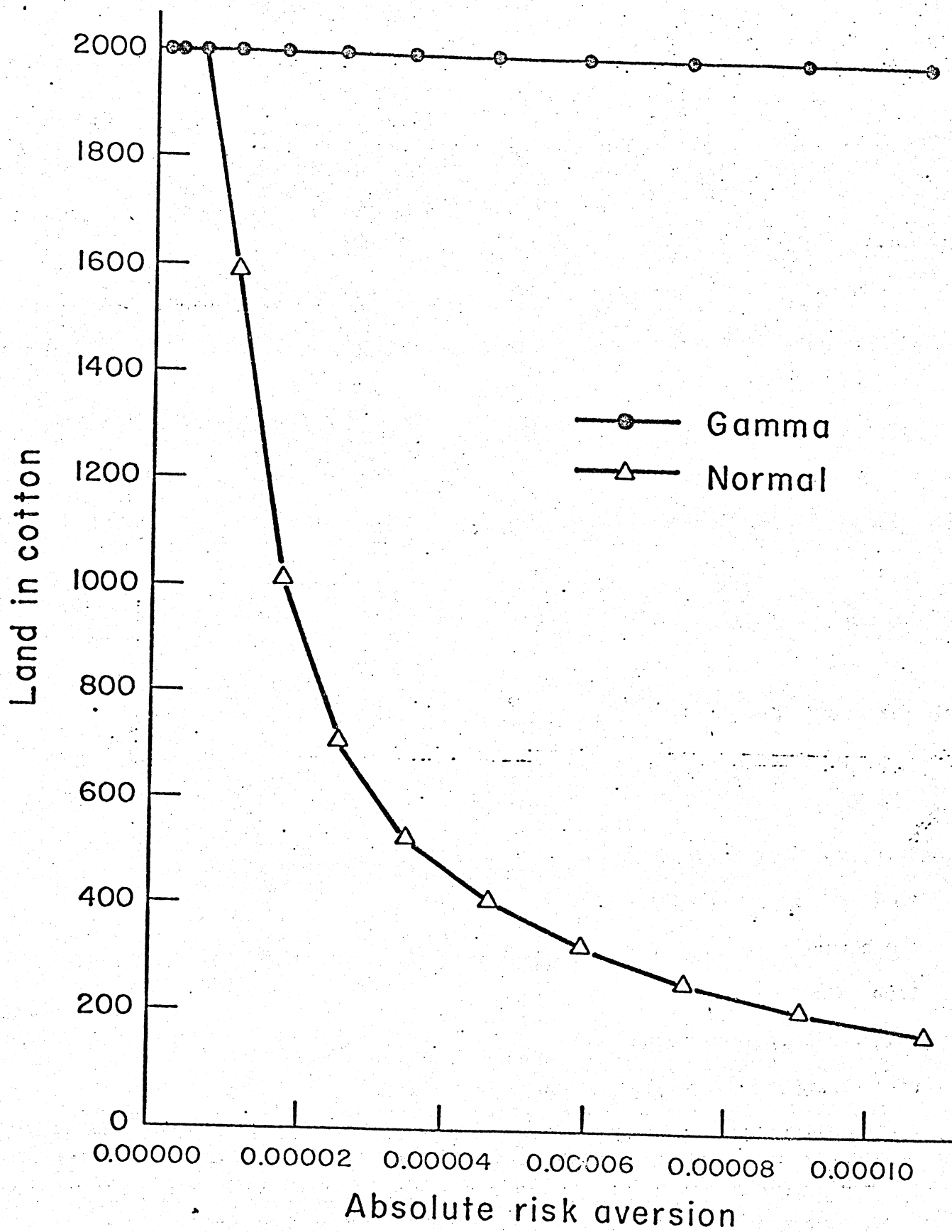


Figure 1. Land allocated to cotton under the gamma and normal decision rules

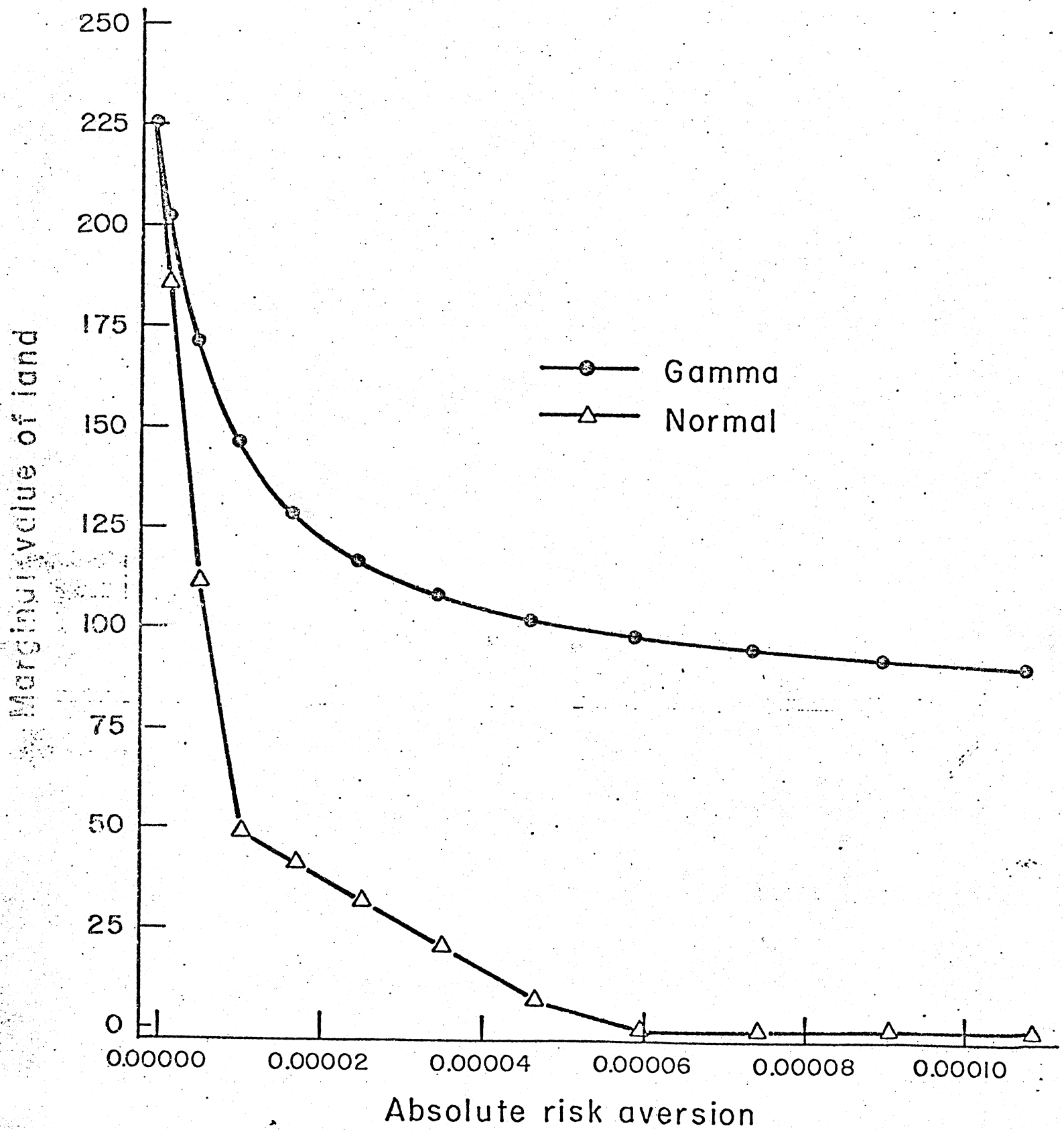


Figure 2. Marginal value of land under the gamma and normal decision rules

on an assumption of normality will undervalue their operations and the land which they farm. Thus, correct identification of the underlying distribution of crop yields rather than the degree of risk aversion could account for the success or failure of farming operations in a competitive environment since it has significant implications for resource valuation. Similarly, we calculated the certainty equivalents of one year's production given decisions based on underlying assumptions of gamma and normally distributed yields. Since the gamma distribution is positively skewed and risk averters prefer positive skewness (Tsiang), a decision-maker who believes yields are gamma distributed will outbid another decision-maker with equivalent risk preferences who believes yields are normally distributed. This will occur even though both decision-makers agree on the value of the first two moments of each yield distribution and their covariance.

The Selection of the Best Representation of the Distribution of Returns

One problem in applying the methodology suggested here is the selection of appropriate statistical distributions to characterize the joint distribution of returns to crops planted. A decision-maker may consider several alternative specifications, for example, assuming returns follow either a bivariate normal or gamma distribution. One possible way to select among them is with traditional classical statistical tests such as the chi-square or Kolmogorov-Smirnov tests. These tests, however, do not explicitly consider the actual use of the data giving the same weight to deviations at the low and high end of the assumed distribution. If the decision-maker is risk averse, however, he will want to weigh deviations in the lower tail more heavily than deviations elsewhere in the range of the distribution.

Rather than taking the classical statistical approach of comparing the fit of the empirical distribution to a given analytical distribution, we suggest here a more Bayesian approach. This approach entails an examination of the impact of the decision rule derived from a given assumption regarding the underlying statistical distribution of yields on the expected utility of the decision-maker. In order to determine this impact, a decision rule is calculated for each statistical assumption (in this case bivariate normal and gamma). Observed outcomes are then used to estimate the expected utility associated with each decision rule at a range of risk-aversion levels. The expected utilities from each rule are then tested to determine if there is a statistically significant difference between them at each level of risk aversion. This process is described formally below. The results are presented for the case of cotton and corn in the lower Mississippi drainage.

Suppose we have K assumptions regarding yields distribution. Let k be an index of an assumed distribution, $k = 1, K$. Each distributional assumption has its own land allocation rule. Thus, let l_i^k denote land allocation under assumption k for activity i --for example, land allocated to cotton under the assumption that returns follow a gamma distribution. Under the k th distributional assumption, one can construct for each vector of observed yields,

$$\pi^k = \sum_{i=1}^n l_i^k (p_i Y_i - v_i)$$

and, hence,

$$U(\pi^k) = -e^{-r\pi^k}.$$

Since the Y_i 's are random variables, both π^k and $U(\pi^k)$ are themselves random variables. Given a sample of observations on yields, the decision-maker can construct a sample of observations on $U(\pi^k)$ for each k . The decision-maker can use the utility samples to infer the relative magnitudes of the expected utilities associated with each of the land allocation rules.

Assuming that there are T observations and using the decision rules associated with two different distributional assumptions $k = k, k'$, the decision-maker can construct a test statistic, z , as follows:

$$z = \frac{1}{T} \sum_{t=1}^T U(\pi_t^k) - U(\pi_t^{k'}).$$

If T is sufficiently large by the central limit theorem, z will follow a normal distribution with mean $EU(\pi^k) - EU(\pi^{k'})$ and variance $1/T * V[U(\pi^k) - U(\pi^{k'})]$. Using this statistic, the decision-maker can test the null hypothesis that the functional form chosen to represent the distribution of returns makes no difference as against the alternative hypothesis that making decisions assuming distribution k will yield a higher expected utility than making decisions assuming distribution k' . Note that it is the actual historical outcomes and how they are related to the functional form chosen to represent the distributions of returns and not the nature of the functional forms alone which determine the results of the statistical test. Thus, the fact that the gamma distribution has positive skewness [which risk averters will prefer over the nonskewed normal distribution when the mean and variance are held constant (Tsiang)] does not guarantee that the gamma distribution will be chosen over the normal distribution by this test.

We applied this test to the choice of the normal or gamma distributions to represent returns in the example problem of allocating land between corn and cotton. The values of r , the expected utility under each assumption, and the test statistic are presented in table 2. The value of the test statistic ranges from zero for low levels of risk aversion, since the optimal choice is the same for both assumptions, to 3.10 for moderate levels of risk aversion and then back to 2.10 for higher levels of risk aversion. The decline in the value of the test statistic is caused by the variance of the expected utility under the gamma assumption becoming very small relative to the variance of expected utility under the normal assumption as r increases. It is concluded that, for moderate and higher level risk aversion, assuming gamma distributions of yields is superior to assuming normal yield distributions. Thus, for risk averse farmers, the skewness of the distribution can have a statistically significant impact on expected utility and should be taken into account in normative models of land allocation.

Conclusion

In this paper, we have extended the application of the expected utility-moment generating function approach to a problem with a continuous choice variable. We have demonstrated analytic solutions to this problem under two important distributional assumptions, i.e., normality and gamma. Our approach is a practical and easily implemented alternative to currently used methods. The advantages of this method of choice under uncertainty over the popular linear form of the mean-variance paradigm include that:

1. It allows us to relax the restriction that yields are normally distributed without changing the standard assumptions with respect to the utility function.

Table 2. Test for Difference in Expected Utility Given Decisions Based on Normal or Gamma Distributions for Various Levels of Risk Aversion

Risk Aversion	Expected Utility Given Decision Based On:		Test Statistic, z
	Gamma	Normal	
9.21e-07	-0.649	-0.649	0.
2.55e-06	-0.326	-0.326	0.
5.81e-06	-0.101	-0.101	0.
1.07e-05	-2.403e-02	-3.894e-02	1.41
1.72e-05	-4.862e-03	-2.603e-02	2.99**
2.54e-05	-8.529e-04	-1.652e-02	3.02**
3.52e-05	-1.307e-04	-1.012e-02	2.65**
4.66e-05	-1.748e-05	-6.042e-03	2.26*
5.96e-05	-1.977e-06	-4.644e-03	2.10*
7.43e-05	-1.813e-07	-4.644e-03	2.10*
9.06e-05	-1.308e-08	-4.644e-03	2.10*
1.09e-04	-7.323e-10	-4.644e-03	2.10*

* Significant at the 5 percent level.

** Significant at the 1 percent level.

2. It does not require the assumption of a particular underlying distribution of returns for its validity nor does it require that all marginal distributions be from the same family. Thus, although we have not demonstrated more complex cases, the first-order conditions are applicable to a wide range of multivariate distributions for which MGF exist. The analytical tractability of such problems will depend on the nature of assumed distributions and their interactions.
3. It allows for the selection of the best statistical distribution of returns based on the improvement in the decision-makers' objective function.

In addition, we have used this method to show that the empirically unwarranted assumption of normality can lead to utility loss for the decision-maker and cause him to undervalue his land under reasonable assumptions.

Just, Richard E., and David Zilberman. "Stochastic Structure, Farm Size, and Technological Adoption in Developing Agriculture." Oxford Econ. Papers. 35(1983):307-328.

Markowitz, H. M. Portfolio Selection: Efficient Diversification of Investment, 2d ed. New York: Yale University Press, 1970.

Pratt, J. W. "Risk Aversion in the Small and in the Large." Econometrica. 32(1964):122-136.

Tsiang, S. C. "The Rationale of the Mean Standard Deviation Analysis, Skewness Preference, and the Demand for Money." Amer. Econ. Rev. 62(1972):354-371.

Yassour, Joseph, David Zilberman, and Gordon C. Rausser. "Optimal Choices Among Alternative Technologies with Stochastic Yield." Amer. J. Agri. Econ. 63(1981):718-724.

