# PERFORMANCES ON RATIO AND INTERVAL SCHEDULES OF REINFORCEMENT: DATA AND THEORY

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Two differences between ratio and interval performance are well known: (a) Higher rates occur on ratio schedules, and (b) ratio schedules are unable to maintain responding at low rates of reinforcement (ratio "strain"). A third phenomenon, a downturn in response rate at the highest rates of reinforcement, is well documented for ratio schedules and is predicted for interval schedules. Pigeons were exposed to multiple variable-ratio variable-interval schedules in which the intervals generated in the variableratio component were programmed in the variable-interval component, thereby "yoking" or approximately matching reinforcement in the two components. The full range of ratio performances was studied, from strained to continuous reinforcement. In addition to the expected phenomena, a new phenomenon was observed: an upturn in variable-interval response rate in the midrange of rates of reinforcement that brought response rates on the two schedules to equality before the downturn at the highest rates of reinforcement. When the average response rate was corrected by eliminating pausing after reinforcement, the downturn in response rate vanished, leaving a strictly monotonic performance curve. This apparent functional independence of the postreinforcement pause and the qualitative shift in response implied by the upturn in variable-interval response rate suggest that theoretical accounts will require thinking of behavior as partitioned among at least three categories, and probably four: postreinforcement activity, other unprogrammed activity, ratio-typical operant behavior, and interval-typical operant behavior.

Key words: ratio schedules, interval schedules, postreinforcement pause, feedback functions, multiple schedules, matching law, key peck, pigeons

Two differences between performance on ratio and interval schedules are well known: (a) If a ratio schedule and an interval schedule provide the same rate of reinforcement, the ratio schedule maintains a higher response rate; and (b) the phenomenon called "ratio strain," in which interval schedules maintain responding no matter how low the rate of reinforcement, but below a certain rate of reinforcement, ratio responding becomes "strained" or irregular and eventually ceases altogether (Ferster & Skinner, 1957).

Two types of theory have been advanced to try to explain these differences: *molecular* theory, which treats responses and reinforcers as momentary events and temporal contiguity is the key explanatory principle, and *molar* theory, which treats responding and reinforcement as temporally extended variables and correlation through time is the key explanatory principle (Baum, 1973, 1989). The earliest theories were molecular. Ratio strain was thought to result from a weakening of response strength each time a response occurred without reinforcement; responses were followed by reinforcers too infrequently to offset extinction (Ferster & Skinner, 1957). On an interval schedule, in contrast, because only a single response is required after the end of the interval, responses occurring at a low rate are reinforced with high probability; hence, extinction is offset and low rates of responding are maintained.

Molar theories have accounted for ratio strain on the basis of various sorts of optimality models, all of which assume some sort of tradeoff—between programmed and unprogrammed reinforcement (e.g., Herrnstein & Vaughan, 1980), work and leisure (e.g., Rachlin & Burkhard, 1978), or cost and benefit (e.g., Baum, 1981). On these theories, ratio responding ceases when the optimal performance includes no ratio responding; with interval schedules, optimal performance always includes some responding on the schedule.

A molar theory of the ratio-interval rate difference relies on the sort of relation, known as a *feedback function*, in which a molar consequence (such as rate of reinforcement) is seen to depend on a molar aspect of behavior (such

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as response rate) (Baum, 1973, 1981, 1989). Whereas a ratio schedule enforces a direct proportionality between rate of reinforcement and response rate, an interval schedule imposes an upper limit on rate of reinforcement, providing a curved feedback function (Baum, 1992). These "diminishing returns" might explain the moderate response rates on interval schedules, and their absence might explain the high rates on ratio schedules (Baum, 1973, 1981).

Any theory of performance today must explain (or at least explain away) the matching law, in which the relative amount of behavior allocated to an alternative (i.e., an available category of behavior) matches the relative reinforcement obtained from that alternative (Herrnstein, 1961, 1970, 1974). Although the matching law has proved inadequate as a theory itself (Williams, 1988), it has been used to describe many different phenomena, including performances on both concurrent schedules and single schedules (e.g., de Villiers, 1977). If a phenomenon can be described in terms of the matching law, and a theory explains the matching law, then that theory can be said to explain that phenomenon. A useful first step, therefore, toward explaining performance differences between ratio and interval schedules might be to describe them in terms of the matching law.

Herrnstein (1970) described single-schedule performance as behavioral allocation between two categories: (a) behavior on the schedule, *B*, maintained by programmed reinforcement, *r*, and (b) other behavior,  $B_0$ , maintained by unprogrammed reinforcement,  $r_0$ . The two categories were assumed to exhaust the possibilities, with the result that their sum equaled a constant,  $K: B + B_0 = K$ . The resulting form of the matching relation is

$$\frac{B}{B+B_0} = \frac{r}{r+r_0}$$
 or  $B = \frac{Kr}{r+r_0}$ . (1)

That ratio schedules maintain higher response rates than interval schedules might be accommodated by Equation 1 in two ways. First, if  $B_0$  and  $r_0$  were smaller for ratio schedules, then B would be higher for ratio schedules. This could be explained, for example, by an optimality model in which the linear feedback function imposed by a ratio schedule requires B to be higher and  $B_0$  to be lower than for an interval schedule (e.g., Baum, 1981). Second, ratio and interval schedules could engender topographically different responses, with the result that K, which represents the maximum possible response rate, could be higher for the ratio schedule. These two possibilities could be distinguished experimentally.

Besides the contrasts in performance—the ratio-interval rate difference and the "strain" difference-there is at least one probable similarity in performance on ratio and interval schedules: a downturn in response rate at high rates of reinforcement. It is well known that responding decreases on ratio schedules as the requirement is reduced to the extreme, fixedratio (FR) 1 (Baum, 1981; Staddon, 1983). Although there are relatively fewer data available on interval schedules near this extreme, it is likely that such a downturn would also occur for interval schedules (Baum, 1981). Allison (1980), for example, observed a decrease in response rate when the schedule was changed from a variable-interval (VI) 14-s schedule to a VI 7-s schedule. One aim of the present study was to gather data on such short interval schedules.

Several accounts of the response-rate downturn have been offered. Some have appealed to the economics of labor and wages (e.g., Allison, 1983). Some have relied on optimality models (Rachlin & Burkhard, 1978; Staddon, 1979). Others have suggested that the downturn is "regulatory" in the sense that it tends to maintain a certain rate of reinforcement (Collier & Rovee-Collier, 1981; Ettinger, Reid, & Staddon, 1987). An earlier paper (Baum, 1981) suggested a similar but simpler idea: There is an upper limit to how fast reinforcers can be consumed. Such an upper limit to rate of reinforcement would force the downturn in response rate.

There is a paucity of thorough parametric studies comparing ratio and interval performance. Catania, Matthews, Silverman, and Yohalem (1977) compared response rates in pairs of pigeons yoked together, with the ratio pigeon setting the availability of reinforcers for the interval pigeon, so as to match the rates of reinforcement. Because the yoking eliminates differences in rate and temporal distribution of reinforcement, it isolates the difference in schedules as a factor in performance, but it has the disadvantage of comparing across subjects, allowing unknown effects of individual differences. By using multiple schedules containing a variable-ratio (VR) component and a VI component, Zuriff (1970) was able to compare response rates within subjects, but his procedure failed to match the rates of reinforcement. The present study combined the within-subject comparison with the matching of reinforcement by yoking; in a multiple VR VI schedule, the VI component was yoked to the VR component, thereby both eliminating effects of individual differences and isolating the effects of the difference in schedules. The schedules were varied over the widest possible range, from "strained" performance to FR 1.

## METHOD

### Subjects

Four male adult White Carneau pigeons served. All had previous experience in a variety of experiments. They were housed individually in an animal colony lighted by natural light through windows and fluorescent lights on from approximately 7:00 a.m. to 7 p.m. They were maintained at 80% of their freefeeding body weights throughout. Water and grit were available in their home cages at all times.

### Apparatus

A standard operant conditioning chamber was used. It contained a feeder full of mixed grain, accessible through an opening immediately below a response key (Gerbrands Co.) that could be lit with white, red, or green light. A houselight in the ceiling provided additional illumination. The chamber was enclosed in a sound-attenuating outer box. A PDP-9T<sup>®</sup> computer controlled and monitored events in the apparatus.

# Procedure

Because the subjects were experienced, they were immediately exposed to the multiple schedule used throughout the experiment. When the key was lit white, a VR schedule was in effect. When the key was dark, timeout was in effect and pecks were ineffective. Although during some early conditions responseindependent reinforcers were delivered during timeout, during the conditions of this experiment, no reinforcers were available then. When the key was lit red, a VI schedule was in effect. Each peck at the lit key produced a feedback click and flicker of the keylight. The houselight was on from beginning to end of a session except during reinforcement, which consisted of 2.8 s of access to the grain hopper.

Sessions were conducted daily, with rare exceptions. A session consisted of five cycles of components in the following order: timeout, VR, timeout, VI. After the fifth cycle, the session finished with one last timeout component. The VR and VI components ended at 15 reinforcers or 2 min, whichever came first. The only exception occurred if after 2 min in the VI component a reinforcer had been scheduled but not yet received; the component continued until a peck occurred and the reinforcer was obtained. The timeout components lasted at least 2 min. If a VR or VI component terminated before 2 min, the remainder was added to the following timeout component. As a result, the session duration was practically invariant.

The VI schedule was determined by (i.e., yoked to) the VR schedule by the method illustrated in Figure 1, which shows a cumulative record of one session. The intervals generated by performance in a VR component were saved to be replayed in the same order in a VI component. They were replayed, not in the very next VI component, but in the VI component of the next cycle. The flat portions of the record in Figure 1 indicate timeout components; the VR and VI components are labeled. The arrows indicate two yoked VR-VI pairs. Separating the duplicated components by intervening VI and VR components helped to safeguard against the possibility that the yoking between VR and VI components might somehow affect performance. The intervals generated in the last VR component of a session were carried over to be replayed in the first VI component of the next session.

Table 1 shows the conditions to which the pigeons were exposed, their order, and the number of sessions conducted. All birds started on an intermediate VR. The VR was decreased across conditions to the limit, VR 1 (FR 1). It was then increased again until responding in the VR components became irregular or ceased (i.e., until "ratio strain" developed). As long as responding held up in a condition, it was included in the table. The last condition for Pigeon 258 (VR 512) was omitted, because responding virtually ceased.

The VR schedules were random-ratio schedules; at each peck, a probabilistic decision



Fig. 1. Cumulative record of a representative session. VR and VI components are labeled. Flat portions indicate timeouts between components. Arrows indicate VI components yoked to VR components.

was made whether to deliver a reinforcer or not. Conditions differed in the probability of reinforcement. Each condition continued until the response rates in both the VR and VI components appeared stable to inspection.

#### Table 1

Conditions of the experiment, in the order studied, with the number of sessions of exposure.

VR	Pigeons	Sessions
46	258	78
32	258	16
31	261, 348	89, 92
23.5	122	99
16	122, 258, 261, 348	48, 30, 12, 12
8	122, 258, 261, 348	17, 33, 50, 22
4	122, 258, 261, 348	28, 28, 17, 44
2	122, 258, 261, 348	28, 26, 23, 17
1 (FR 1)	122, 258, 261, 348	26, 32, 20, 23
2	122, 258, 261, 348	46, 49, 23, 13
4	122, 258, 261, 348	18, 31, 14, 19
8	122, 258, 261, 348	62, 29, 46, 44
16	122, 258, 261, 348	22, 34, 40, 68
32	122, 258, 261, 348	55, 30, 29, 34
64	122, 258, 261, 348	38, 28, 41, 54
128	122, 258, 261, 348	45, 41, 50, 61
256	122, 258, 261, 348	37, 38, 64, 60
512	261	63

# RESULTS

The data were summarized by adding counts and times across sessions of stable performance at the end of each condition. Depending on the rate of reinforcement and the day-to-day variability in response rates, fewer or more sessions were pooled. The number of sessions pooled varied from 4 to 19; the mode was 7. Most samples included 500 or more reinforcers. Only in the very last conditions, when rates of reinforcement were at the minimum, were fewer than 100 reinforcers included. For these samples, all sessions over the period of stable performance were included.

Figure 2 shows, for the VR component, response rate as a function of rate of reinforcement. Rate of reinforcement ranged from lows of 7 to 26 reinforcers per hour, equivalent to VI 8.6 min to VI 2.3 min, to highs of 3,600 to 7,975 reinforcers per hour, equivalent to VI 1 s to VI 0.45 s. As might be expected from previous research, all the relations appear to be bitonic. A substantial drop-off in response rate—to about half the maximum—occurred as rate of reinforcement approached the highest possible, on the FR 1 schedule. Response rate fell off more gradually as rate of rein-



Fig. 2. Response rate as a function of rate of reinforcement in the VR component. Note logarithmic axes.

forcement approached the minimum possible. Those conditions that were repeated (for which two points appear approximately one above the other in Figure 2) produced similar results in the two presentations.

Figure 3 shows response rate as a function of rate of reinforcement for the VI component. The yoking procedure ensured that the range of rate of reinforcement was similar to that in Figure 2: from about 10 to several thousand reinforcers per hour. Again, the relation appears to be bitonic, dropping off precipitously near the highest rate of reinforcement and more gradually toward the lowest rate. With few exceptions, the repeated conditions produced similar results in the two presentations.

Figure 4 compares the relations shown in Figures 2 and 3. The results of repeated conditions were averaged; each multiple schedule is represented by a pair of points (a square for the VR component and an octagon for the VI component). At the highest rates of reinforcement, these pairs are not right above one another. The VI rate of reinforcement tended to be lower, because the intervals saved and repeated by the computer were the obtained intervals in the VR but only the programmed intervals in the VI; the obtained intervals in the VI were generally longer.

Over most of the range of rates of reinforcement, Figure 4 reveals that the yoking procedure equalized the rates of reinforcement in the two components and that VR response rate exceeded VI response rate. This result and the abrupt downturn at the highest rates of reinforcement were to be expected on the basis of previous research.

Figures 3 and 4 also show a new and un-



Fig. 3. Response rate as a function of rate of reinforcement in the VI component. Note logarithmic axes.

expected result. Before turning down toward FR 1, in the midrange of rate of reinforcement (around 600 to 1,000 reinforcers per hour), VI response rate rose rapidly to attain levels comparable to VR response rate. In Figure 4, all 4 birds' data show positively accelerated curvature in the midrange. The curvature is less for Pigeon 348, and VI response rate never quite reached VR levels for Pigeon 122, but a pattern can be seen in all four data sets, most clearly in those of Pigeons 258 and 261. As rate of reinforcement increased, VI response rate at first tended to follow a pattern of negative acceleration (see Pigeons 258, 261, and 122, up to about 300 to 700 reinforcers per hour), as might be expected from previous research. After flattening out, however, VI response rate turned upward again, increasing up to the VR rate.

Comparison with Figure 3 reveals that the positive curvature in the VI performance curves can be seen in each bird's data both in the series of conditions up to FR 1 and in the series of conditions down from FR 1, except for the ascending series of Pigeon 122, which included only one point in the midrange, and the descending series of Pigeon 258, which produced points approximating a straight line. Thus, the positive curvature in Figure 4 did not arise as an artifact of averaging. Neither was it a result of displaying the data in logarithmic coordinates; when the data were plotted in arithmetic coordinates, the VI upturn remained visible for 3 of the 4 birds. Equation 1, the hyperbola



Fig. 4. Response rate as a function of rate of reinforcement in both components, averaged across presentations of a condition.

usually fitted to such performance curves (e.g., de Villiers, 1977; Heyman & Monaghan, 1987), accommodates no positive curvature in logarithmic coordinates.

Figure 5 shows average postreinforcement pause (PRP) plotted against average interresponse time (IRT) for each component in each condition. These averages were computed from frequency distributions with 160-ms class intervals. The biweight mean (Mosteller & Tukey, 1977) was used. This estimate of central tendency is appropriate for skewed distributions, for which the arithmetic mean tends to be inaccurate. Like the median, it is robust with respect to outliers, but it is superior to the median in retaining the sensitivity of a mean. The counting of IRTs excluded the first peck after reinforcement, which ended a PRP, and the first peck of a component. When a component ended on a reinforcer, no PRP was counted.

That all the points in Figure 5 lie above the major diagonal, the locus of equality, indicates that the average PRP always exceeded the average IRT. There is a tendency for the points for the VR components (squares) to be displaced horizontally to the left of the points for the VI components (octagons), reflecting the generally higher response rates in the VR component. For all 4 birds, in the midrange of IRTs the PRPs in the VI component. For all 4 birds, PRP and IRT tended to covary directly to some extent.



Fig. 5. Average postreinforcement (or preresponse) pause (PRP) versus average interresponse time (IRT). Each point represents the results from one component in one condition. The diagonal line in each panel indicates the locus of equality. Note logarithmic axes.

Previous research suggests that PRP should covary directly with interreinforcer interval (Schlinger, Blakely, & Kaczor, 1990; Schneider, 1969; Wynne & Staddon, 1988). Figure 6 shows PRP as a function of interreinforcer interval, estimated from the reciprocal of rate of reinforcement. Each point represents a performance from one presentation of a condition. The lines connect averages across presentations of a condition. For all 4 pigeons, when the interreinforcer interval was small, the PRP appeared to level off, reaching a lower limit. This minimal PRP ranged from about 0.4 s to about 0.9 s across birds. Apart from this minimum, PRP increased with interreinforcer interval in a roughly linear fashion, but with a slope far less than 1.0, which in these logarithmic coordinates would indicate a power function with an exponent less than 1.0. Such a function in arithmetic coordinates would appear negatively accelerated. This result stands in contrast with the results from periodic schedules, for which the relation tends to be approximately linear (Schneider, 1969; Wynne



Fig. 6. Postreinforcement (or preresponse) pause (PRP) as a function of average interreinforcer interval. Each point represents the results from one component in one presentation of a condition. Lines connect averages across presentations: solid lines indicate VR components; broken lines indicate VI components. Note logarithmic axes.

& Staddon, 1988). There was no systematic difference between PRPs for the VR and VI schedules.

The average response rates shown in Figures 2, 3, and 4 included all pecks and all time in a component except reinforcer duration. Pauses following reinforcement and the end of timeout and time between the last peck of a component and the end of a component that terminated at the 2-min duration all would have tended to reduce the calculated response rate. Of these, the PRPs would have had the most significant effect, because they were far more frequent. Their effect would tend to increase as components got shorter and the time between reinforcers got shorter. This tendency would change the relation between response rate and rate of reinforcement (Figure 4). In



Fig. 7. Average response rate, corrected for pausing, as a function of rate of reinforcement. Note logarithmic axes. See text for further explanation.

particular, a PRP of 0.4 to 0.9 s might be negligible for relatively low rates of reinforcement but would start to reduce calculated average response rate substantially when reinforcers occurred every 3 or 4 s or less (i.e., above about 1,000 reinforcers per hour).

One way to correct calculated average response rate for the systematic effect of pauses is to take the reciprocal of the average IRT (the x axis of Figure 5). Figure 7 shows this corrected response rate as a function of rate of reinforcement. As in Figure 4, at lower rates of reinforcement VR response rates exceeded VI response rates, although in Figure 7 the differences are larger and more consistent. The upturn in VI response rate is more apparent and more rapid in Figure 7; it occurs for every pigeon at around 300 to 600 reinforcers per hour. For every pigeon the VI response rate rises to a level equal to the VR response rate. The upturn and convergence of rates were just as clear when the ascending and descending series of conditions (Table 1) were plotted separately; they are averaged here only to conserve space and facilitate comparison between VI and VR performances.

Finally, Figure 7 shows that when response rate is corrected for pausing in this way, the downturn at high rates of reinforcement disappears. The only exception to this is the point (octagon), at the highest rate of reinforcement, for the VI component yoked to FR 1, which appears without a paired ratio point (square) because in FR 1 all time is taken up by PRPs. The VI yoked to FR 1 is best viewed as unrepresentative, because in this component the great majority of responses resulted in reinforcement. This had two probable effects: (a) Because there were few IRTs, the calculated response rates are based on much smaller samples than the others, and hence may be regarded as less reliable; and (b) because almost every response produced reinforcement, the pigeons tended to move toward the grain hopper after each peck, systematically lengthening the IRT measured.

Figure 8 shows frequency distributions of IRTs for each bird in the first, fourth, and 12th conditions to which the bird was exposed (Table 1). In the first condition, except for Pigeon 122, the VR distribution (solid line) is clearly more peaked than the VI distribution. For Pigeon 122, as for the other birds, a relatively high frequency of longer IRTs in the VI component accords with the lower overall response rate. By the fourth condition (VR 8 for Pigeon 258; VR 4 for the others), the two distributions have become almost indistinguishable. In the 12th condition (VR 32 for Pigeon 258; VR 64 for the others), Pigeons 258 and 348 have clearly reverted to the usual VR-VI difference. For Pigeon 261, the VI distribution is flatter than the VR distribution and flatter than it was in the fourth condition. but it never completely reverted to the flat shape seen in the first condition. For Pigeon 122, the VI distribution remained just a little flatter than the VR throughout.

With rare exceptions, every frequency distribution contained a shorter mode, usually at about 240 to 400 ms, and a longer mode, usually at about 720 to 880 ms. When the response rates diverged, the frequency at the shorter mode was always higher for the VR. The frequency at the longer mode was invariably higher for the VI (e.g., cf. Palya, 1992). All performances contained a mixture of short and long IRTs; they varied in the relative frequency of short and long IRTs.

# DISCUSSION

In this within-subject, within-session comparison, VR response rate was found to exceed VI response rate, in accord with previous research. Because both rate of reinforcement and temporal patterning of reinforcement were matched, the results support the idea that the ratio-interval rate difference arises from the different reinforcement feedback functions enjoined by the schedules.

Beyond the difference in response rates, these results shed new light on the downturn in responding that occurs in both schedules at the highest rates of reinforcement, and they add a new phenomenon: the upturn in VI response rate at intermediate to high rates of reinforcement. This convergence of VI and VR response rates has, to my knowledge, never been reported before. The reason appears to be that no study has incorporated such short VI schedules before.

Another possibility might be that the VI upturn resulted from presentation of the VI components within the context of a multiple schedule (see also Zuriff, 1970). By way of a test, Sandra Rutter, a graduate student at the University of New Hampshire, exposed 3 pigeons to several VI schedules, including some that were unusually short. The order of exposure was as follows: FR 1, VI 1 s, VI 2 s, VI 4 s, VI 8 s, VI 16 s, VI 32 s, and back to FR 1 in reverse order. Each schedule was maintained until response rate appeared stable. Sessions terminated after 40 reinforcers (1.75-s access to wheat). Only 1 bird, W62, completed the entire series of schedules; the other 2 died near the end. The results for W62 and the 3 birds averaged together appear in Figure 9, which shows response rate as a function of rate of reinforcement in logarithmic coordinates. The downturn at high rates of reinforcement can be seen; response rate was higher with VI 1 s than with FR 1. For W62, the downturn began at a lower rate of reinforcement in the final series of schedules than in the initial series. In the midrange of rates of reinforcement, from 110 to 1,500 reinforcers per hour, response rate steadily rose. Below 110, which would usually be considered a high rate of reinforcement, one would expect response rate to flatten out and finally fall off at the lowest rates of reinforcement, the pattern that has often been described with the hyperbolic performance curve given by Equation 1 (de Villiers, 1977; Herrnstein, 1970; Heyman & Monaghan, 1987). The tendency to flatten out can be seen more clearly in the averaged data of Figure 9, where it appears as positive curvature in the left portion of the graphs.



Fig. 8. Frequency distributions of IRTs in the VR (solid lines) and VI (dashed lines) components of the first, fourth, and 12th conditions to which each pigeon was exposed. Each row of graphs represents data from 1 pigeon. Class intervals were 160 ms wide, and frequencies appear at the centers. Note logarithmic axes.



Fig. 9. Response rate as a function of rate of reinforcement for 3 pigeons responding on simple VI schedules and FR 1 (right-most point). The left graph shows data for Pigeon W62. The right graph shows data averaged across 3 pigeons (the last three conditions are omitted because only W62 survived to them). The dotted lines show the order of conditions. The solid lines connect the averages across replications of a condition. Note logarithmic axes.

Although the response rates in Figure 9 are lower than those in Figure 2, the magnitude of the upturn, at least a doubling of response rate, is no lower.

Although Figure 9 supports the reality of the upturn, the difference in response rates between Figures 2 and 9 raises a question. Even in the conventional range of rates of reinforcement, the VI response rates observed in the multiple schedule were higher than those usually seen in single VI schedules (e.g., in the left-most points of Figure 9). Apart from the difference in subjects, the higher response rates in the multiple schedule might be explained in at least three ways. First, one might suppose some generalization from the VR component to the VI component occurred. Against this suggestion, Zuriff (1970) found conventional VI response rates in his multiple VR VI schedules. Second, one might speculate that the VI schedule yoked to the VR component comprised an unusual distribution of intervals. This explanation seems improbable because the high constant response rates in the VR component ensured that the distribution

of intervals in the VI component resembled the distribution of ratios in the VR, an ordinary exponential distribution that would be characteristic of any random-interval schedule. The third, and most likely, explanation points to the long timeouts between components. When response rate is reduced to zero for 2 min or more, it might be elevated in the following 2-min component. This would be expected, for example, if the activity  $B_0$  in Equation 1 were allocated to the timeouts, because  $B_0$  and  $r_0$ would be greatly reduced in the components of the multiple schedule compared with the continuously presented schedule (Figure 9). Although the multiple schedule may have increased response rate in general, possibly because of the long timeouts between components, Figure 9 indicates that the upturn in VI response rate can be observed in single schedules.

These observations, the upturn in VI responding in the midrange of reinforcement and the close link of the downturn at the extreme of reinforcement with pausing (PRP), have several implications for theories of operant performance.

### Performance Functions: Bitonic or Monotonic?

Because it is well established that response rate increases with rate of reinforcement in the low to moderate range, the idea that response rate decreases at high rates of reinforcement leads to the assertion that performance follows a bitonic function, with an increasing lower leg and a decreasing upper leg (Ettinger et al., 1987; Rachlin & Burkhard, 1978; Staddon & Reid, 1987). The notion that performance curves are bitonic has permeated theories of performance, particularly theories based on optimality (e.g., Rachlin & Burkhard, 1978; Staddon, 1983). The results shown in Figures 4 and 7 put this notion in a new light.

Although the familiar bitonic functions appear in Figure 4, when the response rate is corrected by removing the pauses following reinforcement (PRPs), as in Figure 7, the bitonicity gives way to a strictly monotonic relation (ignoring the problematic points generated by VI schedules yoked to FR 1). It appears that the downturn arose almost entirely from the increasing effect of PRPs at extremely high rates of reinforcement. Could this be true in general? If so, such corrections should work with other data sets.

In fixed-interval (FI) and FR schedules, in which performance usually includes longer PRPs than in VI or VR performance, one would expect that averaging PRP into response rate would have a still larger effect. That such fixed schedules have usually been used to support the bitonic performance function (Ettinger et al., 1987; Rachlin & Burkhard, 1978; Staddon, 1979, 1983) gives further reason to doubt it. In Boren's (1953) parametric study of rats' performance on FR schedules, average response rate including PRP decreased as rate of reinforcement increased, but when PRP was excluded, this decrease practically disappeared. Felton and Lyon (1966), studying FR performance in pigeons, found that when PRP is excluded from response rate, the decreasing relation between responding and rate of reinforcement turns into an increasing relation. The authors concluded that the PRP and the postpause response rate should be regarded as two separate dependent variables.

A cautionary note arises about the measurement of the PRP. The interval up to the first response following the reinforcer may underestimate the PRP. Boren (1953) noted that frequently one response would occur and then be followed by a further pause before the transition to the high response typical of ratio schedules. Similarly, in the cumulative records shown by Felton and Lyon (1966), one sees some "false starts." The best method for estimating PRP, at least in these fixed schedules, is probably Schneider's (1969) two-state analysis, which separates responding into a lowrate period followed by a high-rate period by finding the point of maximum acceleration.

An opposite error can occur when the PRP is extremely short, as in some of the conditions of the present study. Particularly when the schedule is FR 1, the time to the first response can overestimate the PRP, because it includes the time required to make the response. For example, if a pigeon requires half a second to pull back its head and strike the key, this half a second might properly be excluded from the PRP, which is occupied by activities other than key pecking. Had it been possible to do this, the points for FR 1 could have been included in Figure 7.

The PRP includes more than an obligatory time for some fixed activities following reinforcement. Boren (1953) and Felton and Lyon (1966), for example, found that the PRP grows with the FR. Similarly, Figures 5 and 6 show that PRPs in the VR and VI schedules of this experiment were far from constant. Figure 5 also reveals a tendency for the PRP to vary with the average IRT. This could occur if each PRP included the time required for a response—that is, an IRT. The tendency remained apparent, however, when the average IRT was subtracted from the PRP and this difference was plotted against the average IRT.

Figure 6 shows that PRP covaried directly with interreinforcer interval, approximating a power function with an exponent less than 1.0for long interreinforcer intervals, but leveling out at an apparent lower limit for short interreinforcer intervals. Because the relations were the same for the two types of schedule, it appears that average PRP depends only on interreinforcer interval (i.e., the reciprocal of rate of reinforcement) and not on type of schedule. These results agree with Felton and Lyon's (1966) idea that PRP and response rate should be treated as separate dependent variables, if we add the qualification that they may depend on some of the same independent variables (e.g., rate of reinforcement).

Performance curves apparently are bitonic

only if the PRP is averaged into the response rate. If the PRP is excluded, the functions become flat or monotonically increasing, because the decreasing leg at higher rates of reinforcement disappears. Staddon and Reid (1987) and Ettinger et al. (1987) termed this decreasing leg the "regulatory" segment of the function, meaning that when reinforcement is sufficiently abundant responding acts to stabilize its frequency. If so, it appears that this regulation occurs entirely by means of the activities during the PRP. Whether "regulatory" or not, the inhomogeneity of the PRP with the rest of responding points to a separate process that should be treated separately theoretically.

On the basis of their results with fixed interlocking schedules, Ettinger et al. (1987) drew two conclusions, both contradicted by the present line of reasoning. First, they concluded that molar feedback functions cannot explain changes in performance. They based this assertion on the finding that when they varied the parameters of their interlocking schedule, they always observed roughly the same decreasing relation between response rate and rate of reinforcement. Their results permit another interpretation: Variation in the slope of feedback functions that are approximately linear affects PRP more than postpause response rate. This was also true in previous studies of FR performance (Boren, 1953; Felton & Lyon, 1966). From the observation that certain variations in certain feedback functions fail to affect response rate, one cannot conclude that all variations in feedback functions are ineffective. The present results, for example, indicate that the difference between curved and linear feedback functions has major effects on responding.

The second conclusion of Ettinger et al. (1987), that the decrease in average response rate (including PRP) with increasing rate of reinforcement supports a molecular analysis, seems equally unwarranted. The changes in response rate that they observed arose almost entirely from changes in the PRP. These changes can be interpreted as confirming the familiar observation that PRP increases with increasing interreinforcer interval (Schneider, 1969; Wynne & Staddon, 1988). This relation appears to depend on a number of factors, including temporal discrimination and work requirement (Crossman, Heaps, Nunes, & Alferink, 1974). In FR performance, for example, the PRP depends not only on the preceding

ratio completed but also on the upcoming ratio to be completed. The abbreviation PRP might better stand for "preresponse pause" (Ettinger et al., 1987). However, because a present event cannot depend on a future event, to say that the PRP depends on the upcoming requirement is to say that it depends on a history of exposure to such occasions being followed by that requirement, which suggests a molar rather than a molecular explanation (Baum, 1989). Whether or not a satisfactory account of the PRP turns out to be molar or molecular, however, the evidence suggests that it is best treated as a separate behavioral process (Felton & Lyon, 1966).

### The VI Rate Upturn

Why did the VI response rate turn upward and join the VR response rate at high rates of reinforcement (Figures 4 and 7)? A molecular account would appeal to a decrease in differential reinforcement of long IRTs in short VI schedules. A molar account would point to an increase in differential reinforcement of high response rates as a result of the change in shape of the VI feedback function. Whereas ratio feedback functions are always linear throughout the range of responding, typical VI feedback functions flatten out as responding exceeds a certain low level (Baum, 1992). However, the shorter the VI schedule, the higher the response rate at which the function might be considered flat. If the VI schedule were short enough, the increasing portion of the feedback function could occupy the whole range of attainable response rates. For such short VI schedules, higher response rate would mean a higher rate of reinforcement throughout the range, just as for a ratio schedule. Short VI schedules would function just like comparable VR schedules and would, therefore, maintain equally high response rates.

Although this line of reasoning might suggest the beginnings of an explanation, much additional work remains to be done before any of the current theories of performance are able to explain the VI upturn. For example, the type of optimality theory outlined in an earlier paper (Baum, 1981) probably requires modification. Performance there was considered to be the outcome of a balancing of reinforcement against cost of responding. The cost function was presented as positively accelerated, increasing ever more rapidly as response rate increased. The absence of an inflection point in either the VI feedback function or the cost function appears to imply the absence of any inflection point in the predicted VI performance function. The empirical VI performance functions in Figures 4 and 7, however, clearly call for such an inflection point. The solution to this problem remains to be seen.

## Partitioning Free-Operant Behavior

Applying the matching law to operant performances depends crucially on the categories among which behavior is partitioned. The hyperbola represented in Equation 1 rests on the simple dichotomy between behavior allocated to the schedule and behavior allocated to unprogrammed "background" reinforcement. One could imagine more complicated partitioning among three or more activities. For example, in a concurrent schedule, behavior is partitioned among at least three categories: the two schedules and the background activity.

Likewise, the background activity might be subdivided between postreinforcement activities versus other background activities that can occur at any time. During the PRP, postreinforcement activities dominate, mixed with some of the other background activities, and when the pause is over, these activities give way to operant responding.

Such qualitative distinctions among behavioral categories form the basis for partitioning in the matching law (Herrnstein, 1974). A general way to state the law, allowing for any number of categories, is

$$\frac{r_{s,i}}{B_{s,i}} = \frac{r_{s,j}}{B_{s,j}} \qquad \text{(for all } i \neq j \text{ in all } s\text{)}, \quad (2)$$

where r stands for reinforcement, B stands for behavior (responses or time), the subscripts *i* and *j* denote alternative activities, and the subscript s denotes the different possible situations or experimental conditions (Baum, 1974). In a situation with only one programmed source of reinforcement (i.e., one explicit schedule) and a simple partitioning of behavior between B and  $B_0$ , as in Equation 1, Equation 2 simplifies to

$$\frac{r}{B} = \frac{r_0}{B_0} = \frac{r_0}{K - B},$$
(3)

which is equivalent to Equation 1 and is understood to hold across various situations studied in an experiment. In the present experiment, one would suppose it to hold across the various VR and VI schedules.

Can Equation 3 accommodate the downturn in responding at high rates of reinforcement? One possibility, an increase in  $r_0$ , cannot plausibly explain the downturn, because  $r_0$  would need to attain unprecedented magnitudes, on the order of thousands of reinforcers per hour (Equation 1; Baum, 1981). Alternatively, in accordance with its apparent dependence on the PRP (Figure 4 vs. Figure 7), one can attribute the downturn to the increasing importance of activities that occur only during the PRP. This, however, requires Equation 2 and a new partitioning of behavior, one that places activity that occurs exclusively after reinforcement into a category separate from other background behavior.

At least a tripartite division is required, distinguishing postreinforcement activity  $(B_{\nu})$  as a third category; thus,  $B + B_p + B_0 = K$ . Because  $B_p$  is strictly tied to the occurrence of reinforcement, a certain time x, roughly constant, is spent in  $B_p$  each time a reinforcer is delivered, and the proportion of time allocated to  $B_p$  equals xr. This time x would correspond to the apparent lower limit to PRP in Figure The increases in PRP as interreinforcer interval increases would result from the inclusion of some background activity  $B_0$  in the **PRP.** Expressed as a portion of the total behavior K,  $B_p$  equals xKr. From this, we can write  $B + xKr + B_0 = K$ . Utilizing Equations 1, 2, and 3, we write

$$\frac{r}{B} = \frac{r+r_0}{B+B_0} = \frac{r+r_0}{K-xKr}.$$
 (4)

Rearranging Equation 4, we arrive at

$$B = \frac{Kr - xKr^2}{r + r_0}.$$
 (5)

The parameter x in Equations 4 and 5 can be thought of as the outcome of pause activity that culminates in a response. If the PRP is thought of as resembling an IRT but filled with different sorts of activity, then, just as the total of behavior can be represented by K in the units of the operant response rate B, so the total of behavior could be represented by K'in units of postreinforcement activity (PRP) plus a response (the response at the end of the PRP). Just as K equals the reciprocal of the minimal IRT, excluding any  $B_0$ , so K' equals the reciprocal of the minimal PRP, excluding any  $B_0$ . This total of behavior K' in PRP units can readily be shown to equal the reciprocal of x.

This line of reasoning allows us to conceive of the first response following reinforcement as an integral part of the postreinforcement activity, in keeping with the method of measuring the PRP as the interval from the reinforcer to the first response. Further, the average response rate, including the PRP, includes one response for each reinforcer that is actually tied to the PRP. Hence, the response rate B in Equations 4 and 5 would be lower than the measured average response rate  $B^*$ by the rate of reinforcement  $r: B^* = B + r$ . This correction may be negligible at low rates of reinforcement but becomes crucial at high rates of reinforcement, when almost all of the behavior between pecks is allocated to  $B_{\rho}$  and B in Equations 4 and 5 approaches zero.

Rewriting Equation 5 in terms of K' and  $B^*$ , we arrive at

$$B^* = \frac{KK'r - Kr^2}{K'(r + r_0)} + r.$$
 (6)

This equation has two important properties. It passes through a maximum, as is required by the performance curves in Figure 4; the level of r at which  $B^*$  is maximal depends on the values of the parameters K, K', and  $r_0$ . As  $B^*$  declines at high rates of reinforcement and r converges on K', the lefthand term on the right side of Equation 6 approaches zero, and  $B^*$  approaches r as the situation converges on FR 1.

Figure 10 shows the results of applying Equation 6 to the VR response rates shown in Figure 4. The curves indicate the best fit, by the method of least squares, to the points. The values of the parameters, K, K', and  $r_0$ appear above each graph. Despite small deviations, the fits are tolerably good;  $r^2$  ranged from 0.84 to 0.95. The previously reported "bitonic" relation appears, but, because of the assumptions behind Equation 6, it is entirely accounted for by postreinforcement activity that produces a pause after each reinforcer.

Figure 11 shows the results of applying Equation 6 to the VI performances from Figure 4. As in Figure 10, the curves indicate the best fits to the points, and the values of the parameters appear above each graph. The fits are poor;  $r^2$  ranged from 0.57 to 0.73. The fits are poor, not because of random error, but because the data deviate systematically from the curves.

Equation 6 fails with the VI performances in Figure 11 because it cannot accommodate the upturn in VI response rate. The data points call for an inflection point in the midrange of rates of reinforcement, because response rate tended to level off and then to increase again as rate of reinforcement increased further. Equation 6 fits the VR performances (Figure 10) because they show little or nothing of such a pattern. To accommodate the upturn in VI responding, something additional will be required.

The tripartite division of behavior into operant responding (B), unprogrammed alternatives ( $B_0$ ), and postreinforcement activity ( $B_p$ ) underlying Equation 6 suffices for VR performance but not for VI. Explaining the VI upturn will require modification of existing theories. Several possibilities present themselves. One might discard the matching law, substituting some other principle. Alternatively, one might retain the matching law, as embodied in Equation 2, but make additional assumptions about the partitioning of behavior along the lines suggested above.

For example, according to Staddon's (1982) idea that background activity  $B_0$  could be unequally allocated to the different components of a multiple schedule with a resulting inequality in the values of  $r_0$ , it is possible that  $B_0$  and  $r_0$  could differ from the VR component to the VI component. The level of  $B_0$  in the VR component would be considered minimal, whereas the level in the VI component would be substantial at low rates of reinforcement and would decrease as VI responding turned upward at high rates of reinforcement, decreasing  $r_0$  in Equation 6. This line of theory is rendered improbable, however, by two features of the present experiment. First, if background activity is allocated unevenly, most of it ought to have been allocated to the long timeouts between components, because no operant responding occurred then. This would have minimized the VI and VR components' levels of  $r_0$  and, hence, their inequality. Second, the account is rendered improbable by the magnitude that  $r_0$  would have to assume in Equation 1. If r equaled 200 reinforcers per hour, then a reduction of response rate from



Fig. 10. Fitting the tripartite model (Equation 6) to the performances in the VR component. The parameter  $r_0$  appears in the titles as "RO." See text for further explanation.

VR to VI of 150 to 100 responses per minute would require  $r_0$  to increase from VR to VI up to at least 100 reinforcers per hour, a much greater level than is usually found (e.g., de Villiers, 1977; Herrnstein, 1970; Heyman & Monaghan, 1987).

Another approach, supported by the IRT distributions in Figure 8 and Palya's (1992) results, follows the line of Equation 6, further partitioning behavior. The VI upturn shown in Figures 4 and 7 entails a qualitative shift from the type of key pecking characteristic of longer VI schedules to that characteristic of ratio schedules (Palya, 1992). Whereas typical VI responding includes many single, well-defined pecks, typical VR responding often in-

cludes variants that result in multiple operations of the key as a result of a single "peck" (Smith, 1974). The multiple operations produce extremely short IRTs. Because the VI upturn appears to be more gradual than a step function, some of the performances in the VI component apparently included various mixtures of both VI-typical and VR-typical responding. The shifts in the VI component from VI-typical IRT distributions to VR-typical distributions and back (Figure 8) suggest such a mixing.

Reasoning along this line, one would enlarge the three-way division to a four-way division by distinguishing between two types of operant behavior, VR-typical responding and



Fig. 11. Fitting the tripartite model (Equation 6) to the performances in the VI component. The parameter  $r_0$  appears in the titles as "RO." See text for further explanation.

VI-typical responding. One would work from the same sort of assumption as before, that all categories sum to a constant total:

$$B_{\rm vr} + B_{\rm vi} + B_p + B_0 = K, \tag{7}$$

where  $B_{vr}$  and  $B_{vi}$  are frequencies of VR-type and VI-type operant responding. One would then go on to specify the relation by which  $B_{vr}$ waxes and  $B_{vi}$  wanes as rate of reinforcement increases in the midrange. One possibility, for example, would link the two to a dimension called *intensity* of responding. Then one could conceive of the VI upturn and the gain in  $B_{vr}$ at the expense of  $B_{vi}$  as an increase in intensity.

Further research will decide among these theoretical alternatives. At present, it appears

that a fruitful approach to understanding ratio and interval performances will be to consider behavior as partitioned among at least three categories and possibly four. The functional independence of the PRP implies at least three; a qualitative difference between ratio-typical responses and interval-typical responses might imply a fourth. When changes in feedback functions affect response rate, they do so by reallocating behavior among these three or four categories.

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