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On Measurement of Three Dimensional Deformation Using Moire Method and Light Slicing Method*

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Methods of photographing orthographic projections of a grating and a contour on a curved surface were proposed. In photographing a contour, a sheet of light is projected on an object intermittently and the object is moved so that its surface may be swept by the sheet, while a camera is focused at the sheet. The contour obtained in this way corresponds to what would be observed from direction of movement of the object. In photographing a grating, a sheet of light is projected continuously. These methods can be applied to any object regardless of its size, whereas in the usual methods using the tele-centric system, spread of region to be photographed is restricted by the aperture of the field lens.

Some related methods of measuring shape using Young's interference fringes were modified in similar way. Further, a procedure of measurement of strain in the three dimensional deformation of sheet bulging was shown.

1. Introduction

The methods of measurement of strain using moire patterns have been steadily developed both theoretically and experimentally(1)-(4). However, most investigations concern in-plane deformation, and as for three dimensional deformation few practical methods have been reported(5)(6).

In photographing a contour or a grating on a curved surface deformed in three dimensional space, two problems arise. One is that some parts of the surface come out of focus. Usually, an iris is adjusted small for this problem. However, as a result the resolving power in photographing decreases. The other problem is that magnification factor changes in those parts. This is prevented by using the tele-centric system. However, spread of region that can be photographed is restricted by the aperture of the field lens.

In this paper, the improved methods of photographing a grating and a contour are proposed and they are found effective to above problems.

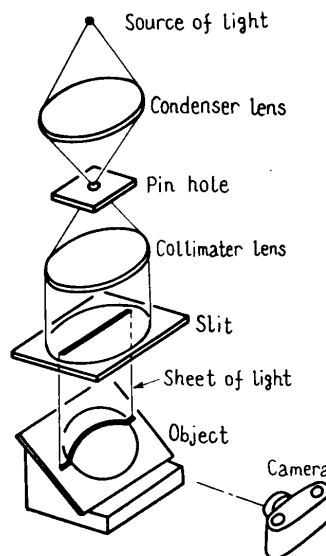
2. Methods of photographing grating and contour on curved surface

2.1 Standard method

A thin sheet of light can be produced by putting a slit in a collimated beam. Let it be projected on an object as is shown in Fig.1. Then a trace of light is obtained. It corresponds to the contour of the section of the object. This method is called the light slicing method(8). Here, if a camera is focused at the sheet of light, the whole

section is photographed in focus and magnification factor in photographing does not change all over the field.

Now, the following method of photographing a grating on a curved surface is proposed. A sheet of light is projected on a grating as in Fig.1. Let the sheet sweep on the grating and let a camera be moved at the same time, while distance between the sheet and the camera is held constant and the shutter of the camera is set open (an object may be moved and a sheet of light and a camera be fixed instead). In this way the whole grating is photographed in focus and the magnification factor does not change all over the field, because each point on the grating is illuminated only when it passes the plane in focus.



Optical axis: normal
to sheet of light

Fig.1 Standard method

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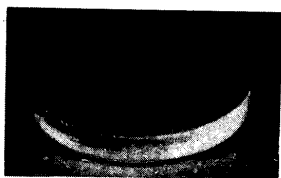
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A contour is obtained in the following way. An object is moved and a sheet of light is projected intermittently in Fig.1, while the shutter of a camera is held open. A similar method was reported in the past(9).

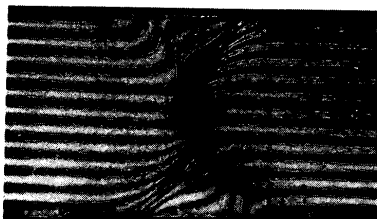
In the above methods, the sheet of light should be thin and the light be intense. Such sheet of light can be easily prepared by use of a laser and a cylindrical lens.

A cone (slope: 0.27) and a perforated sheet under uni-axial tension, shown in Figs.2(a) and (b), are used as models to be photographed. A grating is shown as a moire pattern in Fig.(b), in which rotational mismatch is given (direction of grating lines is vertical in the leaf). Photographs of gratings that follow will also be shown as moire patterns. The resolution power of the lens which was used in photographing in this paper is high and its aberration is low (Fax Nikkor, F5.6, f=210 mm, Nippon Kogaku Co., Ltd.).

A contour of a cone is shown in Fig.3 (a). Here, the sheet of light intersects with the basal plane of the cone at 45° and the basal plane of the contour coincides with this sheet. Difference of height between



(a) Cone (slope: 0.27)



(b) Sheet under tension(A1)
(grating: 200 lpi)

Fig.2 Models



(a) Cone ($\Delta h = 0.5$ mm)



(b) Sheet under tension (200 lpi)

Fig.3 Measurement by the method in Sec.2.1

neighboring contour lines (pitch of contour lines) is equal to the distance in each movement of the sheet of light. It is 0.5 mm.

A grating photographed by the above method is shown in Fig.3(b) (it is shown as a moire pattern). Direction of the sheet of light is the same as that in Fig.3(a), and the photograph corresponds to the projection to the plane intersecting with the surface of the specimen at 45° . Surface of the specimen is plane, so Fig.3(b) coincides with Fig.2(b) (only the scale in the vertical direction is different).

A sheet bulging with hydrostatic pressure is adopted as a model of three dimensional deformation. The hole of a die is circular and 40 mm in diameter. The specimen is made of annealed, pure aluminium and it is 0.3 mm thick. Photographs of a contour and a grating are shown in Figs.4(a) and (b). Directions of the sheet of light is the same as those in the above examples. The pitch of contour lines $\Delta h = 0.5$ mm and density of the grating is 500 lpi (lines /in.). Deflection of the sheet at the center $h_0 = 6.3$ mm.

A shortcoming of this method is that the direction of a camera must be normal to the sheet of light and that it cannot be chosen arbitrarily. This restriction is removed in the following section.

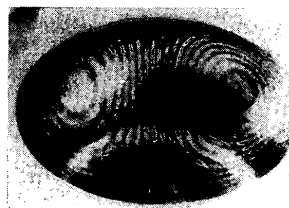
2.2 Modified method

Let an object be moved to a different direction from the optical axis of a camera, while the sheet of light and the camera are fixed in Fig.1. A contour and a grating photographed in this way correspond to what would be observed from the direction in which the object is moved. The method will be explained relating to photographing a contour.

A side view of Fig.1 is shown in Fig.5. A camera and a sheet of light SA_0 are fixed. Let an object A_0A_1 be moved in the direction A_1A_1' and the sheet of light be projected on the object at every incremental movement, then only material points which come just on the sheet of light SA_0A_1' at some incremental movement are illuminated and recorded



(a) Contour



(b) Moire pattern (500 lpi)

Fig.4 Sheet bulging

by a camera. Let locations of those points before movement be denoted by A_1, A_2, A_3, \dots and locations of those passing across the sheet of light SA_0A_3' by A_1', A_2', A_3', \dots . Here, $A_1A_1', A_2A_2', A_3A_3', \dots$ are equal to $n\Delta s$ (Δs : pitch of movement, n : integer) and A_1, A_2, A_3, \dots localize on contour lines whose basal plane is A_0A_3' . Suppose A_1, A_2, A_3, \dots are observed from the point E, and projections to the plane A_0A_3' are obtained. They coincide with $A_1'', A_2'', A_3'', \dots$ and, therefore, coincide with A_1', A_2', A_3', \dots . Here, scale to direction A_0A_3' is reduced to $\cos \alpha$ (α : shown in Fig.5).

The contour obtained by the method in Fig.5 corresponds to what would be observed from the view point E, where the basal plane is given as the sheet of light. Direction of the view point E can be chosen arbitrarily, whereas in the former method in Sec.2.1 it must be perpendicular to the sheet of light. Scale to one direction is different but it does not give a serious fault.

A grating on a curved surface can be photographed in a similar way. An object is moved in the direction EA_1 while the sheet of light is projected continuously. Then the figure of the grating which would be photographed from the point E is obtained.

Each grating line displaces laterally by $\Delta t \tan \alpha$ as it passes across the sheet of light, because the thickness of the sheet (Δt) is not infinitesimal. Therefore, an object must be set so that the direction of grating lines may be parallel to the leaf in Fig.5.

A contour of a cone is shown in Fig.6 (a). Directions of the sheet of light and of a camera are the same as those in Fig.3(a). Direction of movement of an object $\alpha = 45^\circ$. Accordingly, the contour corresponds to what would be obtained when the contour of Fig.3 (a) is observed from direction of the axis of symmetry of the cone.

An example of photographing a grating is shown in Fig.6(b).

Examples concerning sheet bulging are shown in Figs.7(a), (b) and (c), where photographs of gratings are shown as moire patterns (grating lines are vertical on the leaf).

2.3 Discussions about moire pattern between two contours

A moire pattern between two contours corresponds to a contour of difference between heights of those contours. This is also true as for contours obtained in Sec. 2.2, in which direction of observation is not perpendicular to the basal plane of the contours. It will be discussed in the following.

Constitution of contours is shown in Fig.8, where $\overline{AP_1}$: basal plane of contours, \overline{BC} : direction of observation and Δh : pitch of contour lines. A group of parallel planes separated by Δh from each other is assumed. Contours of the surfaces I and II are given as lines of intersection of these surfaces with the parallel planes.

Moire fringes result at C and D, where projections of

two contour lines to the plane \overline{CD} coincide with each other. At these points difference in heights is equal to $n\Delta h/\sin i$ ($i = \angle AP_1B$, n : integer) and it is held constant along each moire fringe. The pitch of contour lines $\Delta h' = \Delta h/\sin i$. Thus the proposition at the beginning has been proved to be valid concerning contours in Sec.2.2.

If the surface I is plane, the moire pattern corresponds to a contour of the surface II, whose basal plane is given as the plane I, where the pitch of contour lines $\Delta h' = \Delta h \cos \beta / \sin i$ (β : angle of intersection between the normal plane I and direction of observation). If the basal plane I is chosen so as to be perpendicular to the direction of observation, the expression of $\Delta h'$ is simplified: $\Delta h' = \Delta s$ ($\Delta h = \Delta s \cos \alpha$, $\beta = 0$, $i = \pi/2 - \alpha$). In the method in Fig.1, $\Delta h' = \Delta s \cos \beta$ ($i = \pi/2$).

In the above discussion, a contour of the plane I consists of parallel, straight

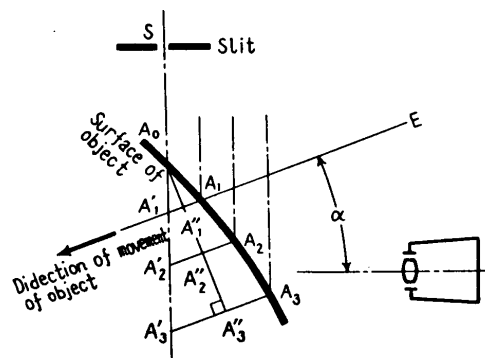
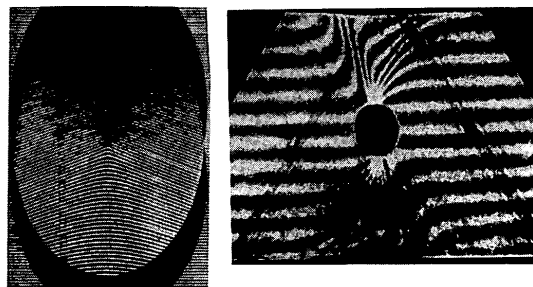
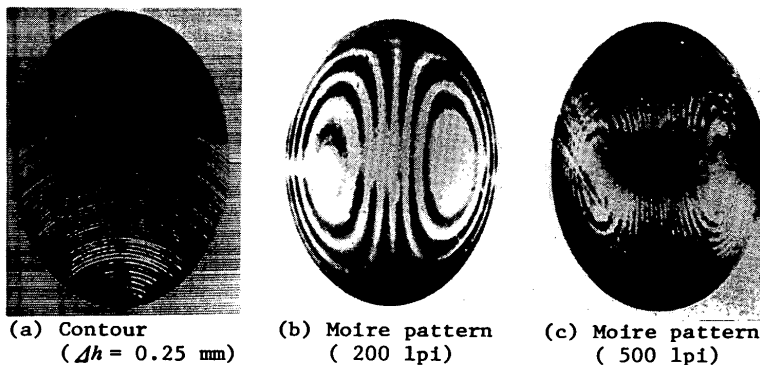


Fig.5 Modified method



(a) Cone ($\Delta h = 0.25$ mm) (b) Sheet under tension (200 lpi)

Fig.6 Measurement by the method in Sec.2.2 ($\alpha = 45^\circ$)



(a) Contour ($\Delta h = 0.25$ mm) (b) Moire pattern (200 lpi) (c) Moire pattern (500 lpi)

Fig.7 Sheet bulging ($\alpha = 45^\circ$)

lines, so it can be substituted by some grating. Then, if one contour of an object, such as in Fig.6(a), is prepared, a contour with arbitrary basal plane can be obtained as a moire pattern by superposing a suitably enlarged reference grating on that contour.

Application to the contours of Fig.3(a) and Fig.6(a) are shown in Figs.9(a) and (b). In Fig.(b) the scale in vertical direction is $\sqrt{2}$ times that in horizontal direction. It can be made equal by applying the optical system Fig.1 to the pattern Fig.9 again. The result is shown in Fig.9(c).

A contour of a slope can be obtained by applying the shifting method to a contour, such as Fig.6(a). An example is shown in Fig.11. A contour of a slope can also be obtained by applying the shifting method to the usual moire pattern by the shadow moire method. However, contrast is lower than that in Fig.11.

3. Modification of methods of measurement using Young's interference fringes

- (i) Standard method of measurement using Young's interference fringes
Let two collimated beams, inclined to

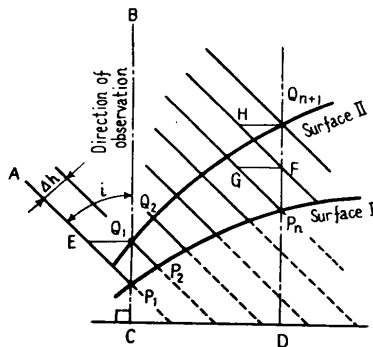
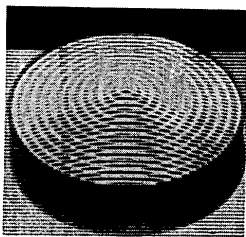
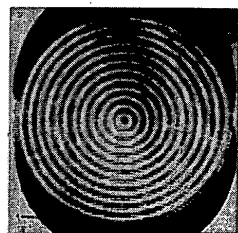


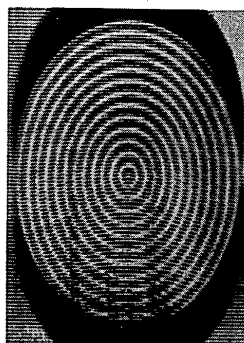
Fig.8 Constitution of moire fringes



(a) Application to Fig.3(a)
($\Delta h' = 0.35$ mm)



(c) Scale in Fig.(b) is restored
($\Delta h' = 0.35$ mm)



(b) Application to Fig.6(a)
($\Delta h' = 0.35$ mm)

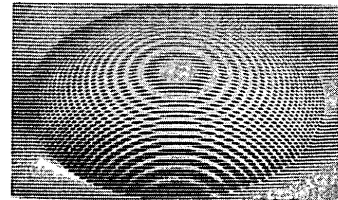
Fig.9 Contours of cone constructed by use of moire patterns

each other, be superposed, then innumerable parallel sheets of light are constructed. They are projected on an object, and contour lines are obtained(7). In photographing this contour, problems as are mentioned in Chap.1 arise relating to focusing and to unevenness of magnification factor.

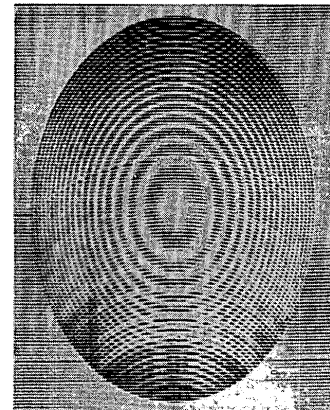
The optical system is modified as is shown in Fig.12. Here, the mirror M and an object are moved in the horizontal direction, while a camera and a slit are fixed. An example is shown in Fig.13(a). The contour is shown as a moire pattern as in the example of Fig.9(a). The pitch of contour lines $\Delta h' = 0.17$ mm. Original, fine contour lines, whose basal plane inclines as in Fig.4(a), are magnified and are shown in Fig.13(b). Here, an aperture of a camera is set at $F = 5.6$.

- (ii) Method of measurement using row of beams

A row of fine beams is obtained if a



(a) Application to Fig.4(a)
($\Delta h' = 0.35$ mm)



(b) Application to Fig.7(a)
($\Delta h' = 0.35$ mm)

Fig.10 Sheet bulging



Fig.11 Moire pattern by shifting method

sheet of light is put in the interferometer shown in Fig.12. Let it sweep an object, and loci of those beams on the object construct contour lines.

In Fig.14(a), a row of beams is projected on an object along z axis. Let it be moved along x axis, then a contour is obtained. Here, a camera is set in the normal direction to the plane containing the row of beams. The mirrors A and C and an object are set on a carriage and are moved in one body along the bisector between x and y axes. Details in arrangement are shown in Fig.14(b).

A contour of a cone is shown as moire pattern in Fig.15(a)* and original contour lines are in Fig.15(b). Here, the pitch of beams constructing a row is 0.27 mm. The pitch of each contour line is shown in the figures.

4. Application to measurement of three dimensional deformation of sheet metal

In the usual moire method, moire fringes correspond to contour lines of

- * The moire pattern shown in Fig.15(a) corresponds to the contour constructed by sheets of light shown by dash lines in Fig.14(b). Fine lines show loci of fine beams.
- ** The moire pattern made of two specimen gratings is equivalent to the second order moire pattern which is to be made of usual moire patterns concerning those specimen gratings. Contrast of the former moire pattern is higher, if the resolution power in photographing gratings is high. Such moire pattern is used in this chapter.

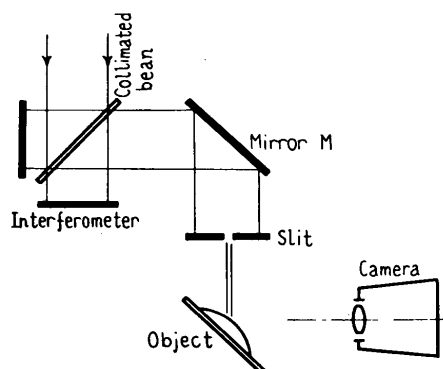
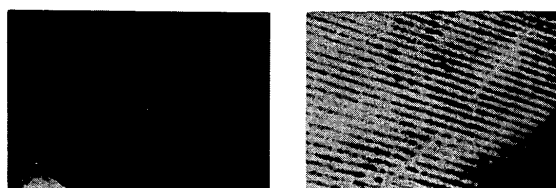


Fig.12 Method of measuring shape by use of Young's interference fringes



(a) Moire pattern corresponding to contour ($\Delta h' = 0.12$ mm)
(b) Original contour lines ($\Delta h = 0.17$ mm)

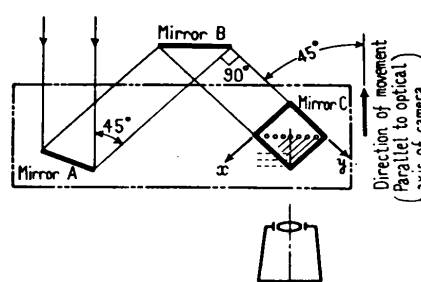
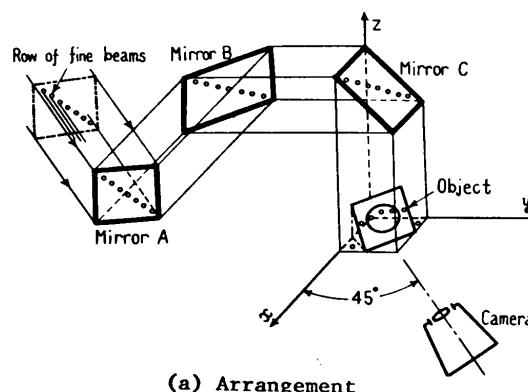
Fig.13 Measurement by the method in Fig.12

displacement. Accordingly, measured values must be differentiated for calculation of strain or of incremental strain. It results in an increase of relative error. In the above methods first order derivatives are obtained directly as moire patterns**. So, times of numerical differentiation decrease by one and degradation of precision is reduced.

In deformations such as sheet bulging, rotation relating to deflection is great, and Green's finite strain must be used. It is defined with respect to the coordinate system before deformation. Therefore, expressions of Green's strain must be modified.

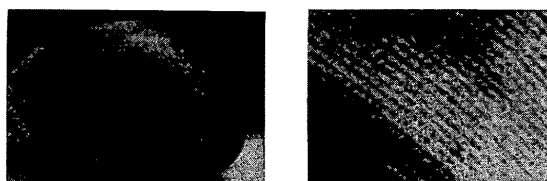
(i) Expression of strain

Cartesian coordinate system x, y, z will be used, where x and y axes are contained in the plane of a sheet (or a specimen). Let x, y - and z -components of displacement be designated by u, v, w , coordinates before and after deformation by $(x_0, y_0, 0)$ and (x, y, w) , respectively. Here, u, v, w depend on x_0, y_0 or x, y . Thickness of sheet is neglected.



(b) Plane figure

Fig.14 Method using row of fine beams



(a) Moire pattern corresponding to contour ($\Delta h' = 0.19$ mm)
(b) Original contour lines ($\Delta h = 0.19$ mm)

Fig.15 Measurement by the method in Fig.14

Suppose that deformation is comparatively small* and that the third order derivatives of displacement can be neglected. Using expressions such as

$$\frac{\partial u}{\partial x_0} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial x_0} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial x_0} = \frac{\partial u}{\partial x} \left(1 + \frac{\partial u}{\partial x}\right) + \frac{\partial u}{\partial y} \frac{\partial y}{\partial x} \quad \dots\dots\dots(1)$$

expressions of Green's strain are modified. A typical expression of E_{11} is shown in the following.

$$E_{11} = \frac{\partial u}{\partial x_0} + \frac{1}{2} \left\{ \left(\frac{\partial u}{\partial x_0} \right)^2 + \left(\frac{\partial v}{\partial x_0} \right)^2 + \left(\frac{\partial w}{\partial x_0} \right)^2 \right\} \\ \approx \frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x} \right)^2 + \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{1}{2} \left\{ \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 \right\} \quad \dots\dots\dots(2)$$

In the sheet bulging all second order terms except $(\partial w/\partial x)^2$ are small, so

$$E_{11} \approx \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \quad \dots\dots\dots(3)$$

Now

$$\frac{u(x+\Delta x, y) - u(x, y)}{\Delta x} = \frac{\partial u}{\partial x} + \frac{\Delta x}{2} \frac{\partial^2 u}{\partial x^2} + \dots \quad \dots\dots\dots(4)$$

The left-hand side of Eq.(4) is obtained using the shifting method. This approximates $\partial u/\partial x$ (error due to the second order derivative can be removed by iterative calculation). $\partial w/\partial x$ is obtained in a similar way. Substituting these into Eq.(3), we obtain E_{11} .

(ii) Expression of incremental strain

Let the strain at some stage of deformation be given by Eq.(3) and let primes be added to variables after incremental deformation. An incremental strain is

$$E_{11}' - E_{11} = \frac{\partial}{\partial x} (u' - u) + \frac{1}{2} \left\{ \left(\frac{\partial w'}{\partial x} \right)^2 - \left(\frac{\partial w}{\partial x} \right)^2 \right\} \\ \approx \frac{\partial}{\partial x} (u' - u) + \frac{\partial w}{\partial x} \frac{\partial}{\partial x} (w' - w) \quad \dots\dots\dots(5)$$

where $u' - u$ is obtained from a moire pattern between two gratings on a specimen before

* In the deformation of sheet bulging $(\partial w/\partial x)^2$ cannot be neglected even if strain ϵ is comparatively small ($\epsilon \lesssim 0.05$).

In case that slope at some part becomes large, photographs of a grating and a contour from the normal direction to that part can be used.

In case that strain becomes large, higher order terms must be considered, where expressions can be introduced in a similar way.

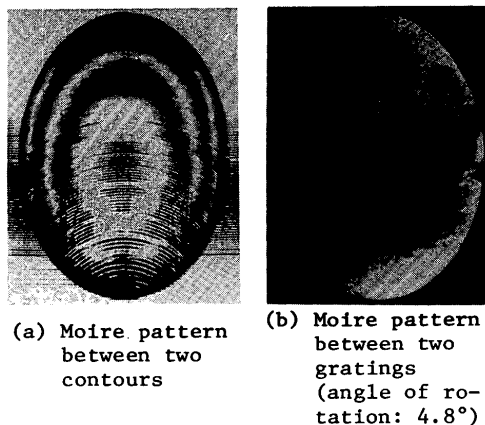


Fig.16 Incremental deformation

and after deformation and $w' - w$ is obtained from a moire pattern between two contours (refer to Figs.16(a) and (b)).

A result of measurement is shown in Fig.17. Here, lateral displacement (x -component) and deflection (z -component) of primary deformation are given in Fig.7(b) and Fig.10(b) and those of incremental deformation are given in Figs.16(a) and (b).

5. Properties of the methods of measurement and comparison with other methods

Properties of the methods in this paper will be summarized and compared with those of usual methods.

(i) Measurement of contour

The first property of the method in Sec.2.2 is that the basal plane of a contour is inclined and that density of contour lines is high. Therefore, if it is used for constructing a moire pattern as in the shifting method, contrast of a moire pattern becomes high (refer to Fig.11 and Fig.16(a)). A contour with arbitrary basal plane can be obtained as a moire pattern easily (refer to Fig.9(b) and Fig.10(b)).

The second property is that it can be applied to a large object, whereas spread of region to be measured is restricted to within the aperture of the field lens in the method using the tele-centric system.

Low sensitivity to deformation is a shortcoming. Sensitivity depends on the pitch of the sheets of light. In examples of Fig.9(b) and Fig.10(b), $\Delta h' = 0.35$ mm is reached.

In the method in Sec.3.1, trouble in measurement is comparatively small and sensitivity is high. Sensitivity is restricted by speckle noise. The limit is given as $\Delta h' = 0.12$ mm (refer to Fig.13). Shortcomings of this method are that spread of region to be measured is restricted by size of a collimator lens and mirrors, that exposure time is long and that direction of observation cannot be chosen arbitrarily.

In the method in Sec.3.2, loss of laser is reduced and exposure time is lessened by 1/5 or so. However, sensitivity decreases ($\Delta h' = 0.19$ mm in Fig.15).

The most popular ones among usual methods of photographing contours are the shadow moire method(10)-(12) and the holographic immersion method(13). In these methods orthographic projections of contours are obtained using the tele-centric systems.

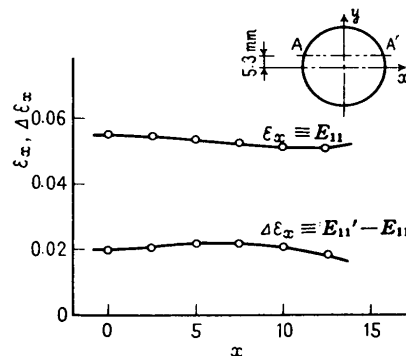


Fig.17 Result of measurement of strain

Accordingly, problems about the tele-centric system are left. In these methods usual contours are obtained (basal plane is orthogonal to direction of observation). So, in case that moire fringes between these contour lines are used as in the shifting method, its contrast becomes low.

As for the shadow moire method, procedure of measurement is simple and it is used widely. However, height of an object that can be measured is restricted* (see Fig.18). In the authors' methods, procedure is more troublesome but restriction concerning height of an object is removed.

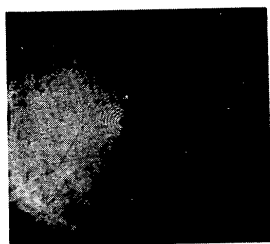
As for the holographic immersion method, sensitivity (or pitch of contour lines) can be chosen arbitrarily. However, procedure is more complicated than the authors' methods.

(ii) Photographing a grating

A property of the methods in Sec.2.1 and Sec.2.2 is that spread of region to be photographed is not restricted as in the method of photographing a contour. Sensitivity depends on density of grating lines. The limit is 500 lpi when a moire pattern between photographed grating and a reference (or master) grating is used and 200 lpi when a moire pattern between two photographed gratings is used as in the shifting method (refer to Fig.7(c) and Fig.16(b)).

There are two other methods of measuring deformation of a grating on a curved surface. In one method, a grating is photographed directly using the tele-centric system. In the other method, Young's interference pattern is projected on a grating and a moire pattern between them is used(6). In the former method, it is necessary that the aperture of the field lens be large and aberration (distortion of image) be small.

* The shadow moire method is suited to rough measurement of a large object(12) (pitch of contour lines is rather great).



Apex is 1.0 mm distant from grating.
Tele-centric system is used.
Grating: 200 lpi, $\theta = 45^\circ$.

Fig.18 Shadow moire method

In the latter method, it is necessary that size of a collimator lens and mirrors for preparation of Young's interference pattern be large. By contrast, in the methods in Sec.2.1 and Sec.2.2 the aperture of a lens gives no restriction, so any lens of high resolution power in the market can be used.

6. Conclusions

Methods of photographing orthographic projections of a grating and a contour on a curved surface were proposed. These methods can be applied to any object regardless of its size, whereas in the usual method using the tele-centric system the spread of an object to be photographed is restricted by the aperture of the field lens. As for a contour obtained above, the basal plane is inclined and it consists of fine lines. This property is effective when a moire pattern between two contours is used as in the shifting method.

Some related methods of measuring shape using Young's interference fringes were modified in similar ways.

Further, a procedure of measurement of strain in the three dimensional deformation of sheet bulging was shown.

References

- (1) Durelli, A.J. and Parks, V.J., Moire Analysis of Strain, (1970), Prentice-Hall.
- (2) Post, D., Appl. Opt., 6-11 (1976-11), 1938.
- (3) Sciammarella, C.A. and Lurowist, N., Trans. ASME, Ser.E, 34-2 (1967-6), 425.
- (4) Schaeffer, B.J. and others, Exp. Mech., 11-4 (1971-4), 172.
- (5) Sciammarella, C.A. and Ciang, F.P., Angew. Math. Phys., 19-2 (1968-2), 326.
- (6) Kato, K. and others, Bull. JSME, 15-90 (1972-12), 1494.
- (7) e.g. Tsujiuchi, J. and Tsuruta, T., Oyobuturi, 36-3 (1967-3), 232 (in Japanese).
- (8) e.g. Handbook of Techniques in Optics, (1968), 905, Asakura Shoten (in Japanese).
- (9) Tsunekane, S., The 8th Japanese Sample Fair of Optical Instruments (Technical lecture), (1971), 1, Japanese Industrial Society of Optical Instruments.
- (10) Theocaris, P.S., Exp. Mech., 7-7 (1967-7), 289.
- (11) Meadows, D.M. and others, Appl. Opt., 9-4 (1970-4), 942.
- (12) Takasaki, H., Appl. Opt., 9-6 (1970-6), 1457.
- (13) Shiotake, N. and others, Japan. J. Appl. Phys., 7-8 (1968-8), 904.