2000

Marketable Pollution Permits in Oligopolistic Markets with Transaction Costs

Kathy K Dhanda
Anna Nagurney, *University of Massachusetts - Amherst*

MARKETABLE POLLUTION PERMITS IN OLIGOPOLISTIC MARKETS WITH TRANSACTION COSTS

ANNA NAGURNEY

Department of Finance and Operations Management, School of Management, University of Massachusetts, Amherst, Massachusetts 01003, nagurney@gbfin.umass.edu

KANWALROOP KATHY DHANDA

School of Business Administration, University of Portland, 5000 North Willamette Boulevard, Portland, Oregon 97203-5798

(Received November 1997; revisions received April, June, July, 1998; accepted August 1998)

In this paper, we present a variational inequality framework for the modeling, qualitative analysis, and computation of equilibrium patterns in multiproduct, multipollutant oligopolistic markets with marketable pollution permits in the presence of transaction costs. The model deals explicitly with spatial differentiation and also guarantees that the imposed environmental quality standards are met through the initial allocation of licenses. An algorithm is proposed, with convergence results, to compute the profit-maximized quantities of the oligopolistic firms' products and the quantities of emissions, along with the equilibrium allocation of licenses and their prices. Numerical examples are included to illustrate this approach.

The problem of environmental pollution occurs when emissions from firms result in ambient concentrations that are sufficiently high to cause damage to property, ecosystems, human health, and/or aesthetics. Firms may discharge pollutants when there is no attached cost to such behavior nor any incentive for the reduction of the discharges. In recent years, a variety of policy instruments has been introduced in order to curb pollution.

One policy instrument that offers an incentive to curb pollution is the marketable pollution permit system, the central concept of which can be traced to Dales (1968), whose work was on water pollution permits, and to Crocker (1966), whose work was on air pollution permits. Montgomery (1972) presented two systems of marketable pollution permits: a system of "pollution licenses" that defines allowable pollution concentrations at a set of receptor points, and a system of "emission licenses" that confers the right to emit pollutants at a certain rate. The former system is referred to as the *ambient-based* permit system (APS), whereas the latter is referred to as the *emission-based* permit system (EPS). In the ambient-based permit system, a target level of environment quality is established by the governmental authority, with the level of pollution being defined in terms of total allowable emissions. Pollution permits, the entitlement of which would enable the holder to emit a specified amount of pollution, are subsequently allocated to the firms. The firms holding permits are free to trade among themselves. In this approach, under competitive conditions (cf. Montgomery 1972), the reallocation of transferable permits can lead to substantial cost reduction while meeting environmental quality standards.

We now distinguish between two classes of pollutants, with the first and easiest class to control commonly referred to as *uniformly mixed assimilative pollutants*, and the second, and somewhat more complex class, being *nonuniformly mixed assimilative pollutants.* Assimilative pollutants are so termed because the capacity to absorb them is rather large; by uniformly mixed, it is meant that the ambient concentration depends on the total amount of emissions but not on the distribution of these emissions among the sources (cf. Tietenberg 1985). Carbon dioxide and other greenhouse gases are the archetypical uniformly mixed and, ultimately, assimilative pollutants. Volatile organic compounds are not uniformly mixed on a regional (multistate) scale, although on a smaller scale, such as within an airshed like the Los Angeles basin, they may be approximated as such. Indeed, on a multistate scale, no traditional pollutant is well mixed; but on a local scale, such an approximation may be appropriate. In this paper we consider a large region—as in the case of power utilities, for example—and hence nonuniformity is a reasonable assumption.

For uniformly mixed pollutants (cf. Tietenberg 1985), a cost-effective solution may be achieved by the EPS approach, which allows for unit-for-unit trades among any sources in the airshed—for example, in the case of air pollution. However, for nonuniformly mixed assimilative pollutants, an APS approach rather than an EPS approach may be preferable. Nevertheless, in an APS approach a pollution dispersion matrix is required, and dispersion modeling in itself is not a trivial task. Air pollution dispersion matrices, however, are not unique to APS permit systems because dispersion models were already in use by the

Subject classifications: Environment: marketable pollution permits. Economics: oligopolistic markets with transaction costs.
Area of review: ENVIRONMENT, ENERGY AND NATURAL RESOURCES.

0030-364X/00/4801-0424 $05.00
1526-5463 electronic ISSN

Environmental Protection Agency prior to the implementation of permit systems (cf. Tietenberg 1985). For additional background, we refer the reader to such standard texts on environmental economics as Tietenberg (1988), Pearce and Turner (1990), and Kahn (1998).

Another approach to curb pollution in a region is the approach whereby polluters are charged a fixed price for each unit of pollution (cf. Montgomery 1972). If the same price is charged for pollution on all firms, then the marginal costs of abatement are equated across firms, and the resulting level of pollution can be reached in a cost-minimizing manner. Yet another approach is to have polluting firms pay a price equal to the marginal external cost of their polluting activities. These price incentives, in the form of Pigouvian taxes (cf. Pigou 1920), would lead to corrective behavior on the part of the polluting firms by inducing them to internalize the full social costs of pollution. One may also use an approach discussed in Nagurney et al. (1996), in which targets are imposed on the economic variables (which in their case were supply, demand, and transportation targets) with associated penalties for failure to comply and with the taxes set accordingly. That approach utilized a generalization of goal programming through the use of variational inequalities.

In this paper we model multiproduct, multipollutant oligopolistic firms engaged in markets in ambient-based pollution permits. Besides facing production costs and emission costs, the firms also face transaction costs associated with the trade of permits, in addition to the price of purchasing licenses in excess of the initial allocations in order to fulfill the allowance of emissions.

More theoretical environmental policy analyses focus on developing policy instruments under the assumption that firms are perfectly competitive in their product markets. However, modern industrial markets may not satisfy the conditions for perfect competition, where it is assumed that there are many producers, each of which cannot affect the price of the product that they produce and takes the price as given; they might be better characterized as having an oligopolistic structure. In recent years, there has been an increasing theoretical interest in modeling oligopolies in the context of environmental policy-making (see, for example, Hahn 1984, Carraro et al. 1996, and Van Egteren and Weber 1996). Hence, in this paper we focus on oligopolistic, rather than on perfectly competitive, firms.

We assume that the firms are perfectly competitive in the permit markets. More specifically, each source of pollution takes the price of the license to pollute a particular pollutant at a certain point as given, because each source in a region is small relative to the entire economy. The model also deals explicitly with spatial differentiation through the use of a diffusion matrix that maps emissions from sources to receptor points that are dispersed in space. This is especially important because studies show that if spatial differentiation is not built into the system, then some or most of the cost savings from employing an economic-based approach can be lost (cf. Mendelsohn 1986). Nevertheless, Bohi and Burtraw (1997) showed that considerable cost savings were achieved under an EPS-based system for the U.S. SO_2 emissions. A main reason for those cost savings was the flexibility allowed electric utilities to switch to lower sulphur content coal. Under that program, some utilities mitigated the emission levels rather than engaging in trading of permits, but this is an advantage afforded by economic incentive systems, whereby the overall goal of emission reduction is attained.

Furthermore, it is clear that policy instruments must be analyzed within the context of the market structure. Here we provide a modeling and, in particular, computational framework to allow for such analyses in the case of oligopolistic firms and perfectly competitive permit markets in the presence of transaction costs. The case of perfectly competitive firms and permit markets can also be handled within the variational inequality framework as discussed in Nagurney and Dhanda (1996) and extended to include transaction costs using the modeling approach presented here.

We focus on transaction costs in this paper because they can obstruct the performance efficiency of the permit market by impeding the trading process of permits. This is especially important in the case of the APS approach that we model in this paper. For example, the information requirements needed in order to derive the pollution dispersion/diffusion matrix may raise the transaction costs in the trading process. In general terms, transaction costs can arise in any market and usually result from the transfer of any property right, because the parties to an exchange must find one another, communicate, and exchange information (cf. Stavins 1995). Empirical evidence also suggests the prevalence of transaction costs. The Fox River water pollutant trading program did not perform up to the expectations, largely because of high transaction costs in the form of administrative requirements. On the other hand, when these administrative requirements were minimal, a high level of trade took place in lead-rights trading among refineries, a program that was a part of the Environmental Protection Agency's leaded gasoline phasedown in the United States. The refineries were already experienced at striking deals with one another and, hence, the firms did not have to engage in broker fees to find trading partners (cf. Hahn and Hester 1989). In another case, the New Jersey Pinelands transferable development rights program, the transaction cost were minimized by the government body taking on a feeless brokerage role (cf. Tripp and Dudeck 1989).

The mathematical framework chosen for the formulation, qualitative analysis, and computation of the equilibrium pattern in markets for pollution control in the presence of transaction costs is that of finite-dimensional variational inequality theory. Thus far, variational inequality theory has been used in environmental economic policy modeling in the context of ambient-based pollution permit markets in oligopolistic markets but only in the case of single-product, single-pollutant oligopolistic firms and in the absence of transaction costs (cf. Nagurney and Dhanda 1996). Hence, this paper extends the earlier work to a more general setting. Moreover, we emphasize that to date no general computational procedure or model has been developed to handle transaction costs in such a setting. The only prior work was that of Stavins (1995), who provided no computational procedure and whose model was very simple.

Our model applies the theory of variational inequalities to yield the profit-maximized quantities of multiple products, the optimal quantities of various emissions, the equilibrium allocation of the pollution licenses, and the prices of the licenses, all in the presence of transaction costs. We aim to make a contribution in the direction of environmental economics since the use of variational inequality theory to this field has yet to be fully explored. For a plethora of additional equilibrium problems in both operations research and in economics that have been studied as variational inequality problems, see Nagurney (1993).

The paper is organized as follows. In §1, we develop the optimization problem faced by an individual firm in the presence of transaction costs. Subsequently, we present the economic conditions governing the market model and then derive the variational inequality formulation of the equilibrium conditions. In addition, we also provide the qualitative properties of the equilibrium pattern. In §2, we propose an algorithm for the computation of the equilibrium pattern and provide conditions for convergence. This algorithm yields variational inequality subproblems of very simple structure, each of which can be solved explicitly and in closed form. This algorithm is then applied to compute solutions to several numerical examples in §3. We summarize our results and present conclusions in §4.

1. THE MULTIPRODUCT, MULTIPOLLUTANT OLIGOPOLY MODEL WITH AMBIENT-BASED POLLUTION PERMITS AND TRANSACTION COSTS

In this section, we develop the multiproduct, multipollutant oligopolistic market model with ambient-based pollution permits and transaction costs. As mentioned, transaction costs can emerge in the construction of a market in pollution permits because the firms, to trade licenses to pollute, must find one another, must communicate, and must exchange information.

We consider m firms that are sources of industrial pollution in the region, and which are fixed in location, with a typical source or firm denoted by i. There are n receptor points, with a typical receptor point denoted by j. Also, let there be r different pollutants emitted by the firms, with a typical pollutant denoted by t. Let e_i^t denote the amount of pollutant t emitted by firm i and group the firm's emissions into a vector $e_i \in R_+^r$. We assume, as given, an $r \times m \times n$ diffusion matrix H, where the component h_{ij}^t denotes the contribution that one unit of emission by source i makes to average pollutant concentration of type t at receptor point j (cf. Montgomery 1972).

Let a permit denote a license, the possession of which will allow a source to pollute a specific pollutant at some specific receptor point. Hence, each polluter will have to hold a portfolio of licenses to cover all the relevant monitored receptor points. Let l_{ij}^t denote the number of licenses for pollutant t at point j held by source i, and group the licenses for each firm i into a vector $l_i \in R_+^{rn}$. We assume throughout that some initial allocation of licenses l_{ij}^{t0}, $i = 1, \ldots, m$; $j = 1, \ldots, n$; $t = 1, \ldots, r$ has been made by the regulatory agency and, later in this section, discuss how this allocation should be made in order to ensure that environmental quality standards are met.

Furthermore, let p_j^t denote the price of the licenses for pollutant t that affects receptor point j, and group the prices of the licenses into the vector $p \in R_+^{rn}$. Also, assume that the market in pollution licenses is perfectly competitive; that is, each source of pollution takes the price of the license to pollute at a specific point as given and cannot affect the price itself because each source is small relative to the entire economy. The license trading system, as an economic-incentive approach, designs license markets in order to achieve environmental goals in a cost-effective manner. The effectiveness, therefore depends upon perfect competition in the permit market, and regulators should design policy instruments that would guarantee perfect competition in permit trading. Indeed, unlike the firms' production outputs, the supply of initial licenses is fixed and determined so that the environmental goals are achieved. The "market" power, in this sense, is entirely dependent on the initial license allocation and controlled by the regulatory agency.

Let there be s distinct products that are produced by the firms in a noncooperative manner, with a typical output denoted by d and the quantity of product d produced by firm i denoted by q_{id}. These quantities are first grouped into the vector $q \in R_+^{ms}$. We assume that each product is homogeneous; that is, the consumers are indifferent as to which firm was the producer.

The underlying idea behind the market in pollution permits is that the firms or sources of pollutants must be encouraged to trade permits. A typical firm participating in a permit market, however, has to take into account various costs, such as those of production, emission abatement, purchasing pollution licenses, and finally, the transaction costs involved in the trade of these licenses.

Each firm i in the oligopoly is faced with a cost f_i of producing the vector of quantities q_i, where

$$f_i = f_i(q_i). \tag{1}$$

Each firm i in the region is also faced with a joint-cost g_i, which is dependent both upon the product vector q_i and the emission vector e_i, where

$$g_i = g_i(e_i, q_i). \tag{2}$$

In addition, a firm encounters transaction costs to be able to participate in the permit market to trade pollution licenses. Specifically, let c_{ij}^t denote the total transaction cost that a firm i incurs to trade pollution permits for pollutant t at receptor point j in the market, where

$$c_{ij}^t = c_{ij}^t(l_{ij}^t). \tag{3}$$

Hence, we assume that the transaction cost for the purchase or sale of a license to pollute a particular pollutant at a specific receptor point depends upon the number of licenses for that particular pollutant and that receptor point held by the firm.

Because we assume that the firms are oligopolistic in their product markets, they affect the prices of the outputs. Hence, if we denote the price of a product d by ρ_d, we can write

$$\rho_d = \rho_d\left(\sum_{i=1}^{m} q_{id}\right). \tag{4}$$

Consequently, each firm i acquires a revenue

$$\sum_{d=1}^{s} \rho_d\left(\sum_{i=1}^{m} q_{id}\right) q_{id}. \tag{5}$$

Because the regulatory body bestows upon the firm i an initial allocation of licenses l_i^0, the value of a firm's initial endowment of licenses is given by $\Sigma_{j=1}^{n}\Sigma_{t=1}^{r} p_j^{t*} l_{ij}^{t0}$, where p_j^{t*} denotes the given price of a license to pollute for the specific pollutant t at receptor point j, which, under the assumption of perfect competition in the license markets, is assumed given.

Also, the cost of purchasing licenses for a specific pollutant t that affects receptor point j for source i is given by

$$\sum_{j=1}^{n} p_j^{t*}(l_{ij}^t - l_{ij}^{t0}). \tag{6}$$

We assume that each firm in the oligopoly is profit-maximizing and thus can be characterized by a function that measures its profit or net revenue. The profit function u_i faced by each such firm i; $i = 1, \ldots, m$ can hence be expressed as the difference between the total revenue acquired and the total cost incurred by the firm:

$$\begin{aligned} u_i &= u_i(q, e_i, l_i) \\ &= \sum_{d=1}^{s}\left(\rho_d\left(\sum_{i=1}^{m} q_{id}\right) q_{id}\right) - f_i(q_i) - g_i(e_i, q_i) \\ &\quad - \sum_{t=1}^{r}\sum_{j=1}^{n} c_{ij}^t(l_{ij}^t) - \sum_{t=1}^{r}\sum_{j=1}^{n} p_j^{t*}(l_{ij}^t - l_{ij}^{t0}). \end{aligned} \tag{7}$$

An oligopolistic firm's optimization problem is then expressed as:

$$\text{Maximize} \quad u_i(q, e_i, l_i) \tag{8}$$

subject to:

$$h_{ij}^t e_i^t \leqslant l_{ij}^t; \quad j = 1, \ldots, n, \ t = 1, \ldots, r, \tag{9}$$

and the nonnegativity constraints:

$$\begin{aligned} &q_{id} \geqslant 0; \quad e_i^t \geqslant 0; \quad l_{ij}^t \geqslant 0; \\ &d = 1, \ldots, s; \ t = 1, \ldots, r; \ j = 1, \ldots, n. \end{aligned} \tag{10}$$

Constraint (9) states that each firm is allowed to have an average rate of emission per pollutant that produces no more pollution at any point than the amount the firm is licensed to cause at that point. Hence, this constraint handles spatial differentiation. Note that here we allow the licenses to take on fractional values because both the diffusion matrix elements and the emissions may take on such values.

Let λ_{ij}^t denote the Lagrange multiplier associated with the tjth constraint in (9), and let λ_i denote the vector of Lagrange multipliers in R_+^{rn}. Finally, group the vectors λ_i into the vector $\lambda \in R_+^{mrn}$. Note that λ_{ij}^t may be interpreted as the shadow price, and henceforth we term this Lagrange multiplier as the marginal abatement cost.

As stated earlier, the oligopolistic firms are assumed to operate in a noncooperative manner in their product markets, where the governing equilibrium concept is that of Nash-Cournot (cf. Nash 1950, Cournot 1838) and is defined as follows (see also Gabay and Moulin 1980).

DEFINITION 1 (NASH-COURNOT EQUILIBRIUM). A Nash-Cournot equilibrium is a vector of production outputs $q^* \in R_+^{ms}$, emission quantities $e^* \in R_+^{mr}$, and licenses $l^* \in R_+^{rmn}$, such that

$$u_i(q_i^*, \hat{q}_i^*, e_i^*, l_i^*) \geqslant u_i(q_i, \hat{q}_i^*, e_i, l_i)$$

$$\forall q_i, \forall e_i, \forall l_i \text{ satisfying (9) and (10), for all firms } i, \tag{11}$$

where $\hat{q}_i^* = (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*)$.

In other words, the rationality postulate here is that each firm selects its production outputs, its emissions, and its licenses, so that its profit is maximized, given the production output vector decisions of the other oligopolistic firms. Note that here we consider a Cournot oligopoly, rather than a Bertrand oligopoly, in which firms select the prices of their products. For background on Bertrand oligopolies, see Tirole (1988) and the references therein. An increasing number of energy market models (e.g., Oren 1997) are based upon a supply function conjecture, where each firm believes that other firms will not change their supply function (price vs. quantity). We have selected a Cournot oligopoly rather than a Bertrand or supply function oligopoly in order to be consistent with the firm's variables which are all quantity variables—in particular, the amounts to produce—the amounts of pollutants to emit as well as the number of licenses to purchase. We leave the modeling of Bertrand oligopolies and ambient-based pollution permits for future research.

Optimality Conditions for a Firm

If we assume that the profit function, $u_i(q, e_i, l_i)$, for each firm i is concave with respect to its arguments and is continuously differentiable, the necessary and sufficient conditions for an optimal firm-specific product, emission, license, and marginal abatement cost pattern, $(q_i^*, e_i^*, l_i^*, \lambda_i^*)$, given

p^* and $\hat{q}_i^*$, is that this pattern is nonnegative and satisfies the inequality:

$$\sum_{d=1}^{s}\left[\frac{\partial f_i(q_i^*)}{\partial q_{id}}+\frac{\partial g_i(e_i^*,q_i^*)}{\partial q_{id}} - \frac{\partial \rho_d(\sum_{i=1}^m q_{id}^*)}{\partial q_{id}}q_{id}^* - \rho_d\left(\sum_{i=1}^m q_{id}^*\right)\right]\times[q_{id}-q_{id}^*]$$
$$+\sum_{t=1}^{r}\left[\frac{\partial g_i(e_i^*,q_i^*)}{\partial e_i^t}+\sum_{j=1}^{n}\lambda_{ij}^{t*}h_{ij}^t\right]\times[e_i^t-e_i^{t*}]$$
$$+\sum_{t=1}^{r}\sum_{j=1}^{n}\left[p_j^{t*}+\frac{\partial c_{ij}^t(l_{ij}^{t*})}{\partial l_{ij}^t}-\lambda_{ij}^{t*}\right]\times[l_{ij}^t-l_{ij}^{t*}]$$
$$+\sum_{t=1}^{r}\sum_{j=1}^{n}[l_{ij}^{t*}-h_{ij}^te_i^{t*}]\times[\lambda_{ij}^t-\lambda_{ij}^{t*}]\geqslant 0,$$
$$\forall q_{id}\geqslant 0,\quad e_i^t\geqslant 0,\quad l_{ij}^t\geqslant 0;\quad \lambda_{ij}^t\geqslant 0;$$
$$j=1,\ldots,n;\ d=1,\ldots,s;\ t=1,\ldots,r. \tag{12}$$

Note that (12) expresses the optimality conditions, that is, the Kuhn-Tucker conditions in variational inequality format. Indeed, convex optimization problems can be formulated as variational inequality problems, although the converse is not true, except under certain assumptions (cf. Kinderlehrer and Stampacchia 1980, Nagurney 1993). Inequality (12) can be interpreted as follows: The optimality of a point implies that, for every feasible point, the rate of change of the function starting from that point is nonnegative (see also Bertsekas and Tsitsiklis 1989).

In the case of a Nash-Cournot equilibrium in which each of the firms optimizes unilaterally, an inequality similar to (12) needs to hold for each of the other oligopolistic firms. Indeed, Gabay and Moulin (1980) have shown that Nash equilibria admit variational inequality formulations.

If, on the other hand, we consider the case where the firms are perfectly competitive and the price of each product d is now fixed and given by $\bar{\rho}_d$, for $d=1,\ldots,s$, then the fixed price would replace the price function $\rho_d(\cdot)$ in the profit function (7) and the optimality conditions for a perfectly competitive firm would still take the form of (12), but with

$$\frac{\partial \rho_d(\sum_{i=1}^m q_{id}^*)}{\partial q_{id}}q_{id}^* - \rho_d\left(\sum_{i=1}^m q_{id}^*\right)$$

in (12) replaced by $\bar{\rho}_d$.

Market Clearing Conditions for Licenses

We now describe the system of equalities and inequalities governing the quantities and prices of licenses in the region at equilibrium.

Mathematically, the economic system conditions governing market clearance in pollution permits are: For each receptor point j; $j=1,\ldots,n$, and for each pollutant t; $t=1,\ldots,r$:

$$\sum_{i=1}^{m}[l_{ij}^{t0}-l_{ij}^{t*}]\begin{cases}=0, & \text{if } p_j^{t*}>0,\\ \geqslant 0, & \text{if } p_j^{t*}=0.\end{cases} \tag{13}$$

The system (13) states that if the price of a license for pollutant t at a point j is positive, then in equilibrium the market for licenses at that point must clear; if there is an excess supply of licenses for a particular pollutant t at a receptor point, then the price of a license at that point must be zero.

We first give the governing equilibrium conditions for the entire problem and then derive the variational inequality formulation, which is a unified framework within which all the inequalities and equalities can be expressed as a single inequality.

DEFINITION 2 (MARKET EQUILIBRIUM). A vector of production outputs, emissions, licenses, associated marginal costs of abatement, and license prices, $(q^*,e^*,l^*,\lambda^*,p^*)\in R_+^{ms+mr+2rmn+nr}$, is an equilibrium of the multiproduct, multipollutant oligopoly with ambient-based pollution permits developed above if and only if it satisfies inequality (12) for all firms i; $i=1,\ldots,m$, and the system of equalities and inequalities (13) for all receptor points: j; $j=1,\ldots,n$, and for all pollutants: t; $t=1,\ldots,r$.

We now derive the variational inequality formulation of the equilibrium conditions for the market model.

THEOREM 1 (VARIATIONAL INEQUALITY FORMULATION). *A vector of firm production outputs, emissions, licenses, and associated marginal abatement costs, and license prices,*

$$(q^*,e^*,l^*,\lambda^*,p^*)\in R^{ms+mr+3rmn+rn},$$

is an equilibrium if and only if it satisfies the variational inequality problem:

$$\sum_{i=1}^{m}\sum_{d=1}^{s}\left[\frac{\partial f_i(q_i^*)}{\partial q_{id}}+\frac{\partial g_i(e_i^*,q_i^*)}{\partial q_{id}} - \frac{\partial \rho_d(\sum_{i=1}^m q_{id}^*)}{\partial q_{id}}q_{id}^* - \rho_d\left(\sum_{i=1}^m q_{id}^*\right)\right]\times[q_{id}-q_{id}^*]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\left[\frac{\partial g_i(e_i^*,q_i^*)}{\partial e_i^t}+\sum_{j=1}^{n}\lambda_{ij}^{t*}h_{ij}^t\right]\times[e_i^t-e_i^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\left[p_j^{t*}+\frac{\partial c_{ij}^t(l_{ij}^{t*})}{\partial l_{ij}^t}-\lambda_{ij}^{t*}\right]\times[l_{ij}^t-l_{ij}^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[l_{ij}^{t*}-h_{ij}^te_i^{t*}]\times[\lambda_{ij}^t-\lambda_{ij}^{t*}]$$
$$+\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\sum_{i=1}^{m}(l_{ij}^{t0}-l_{ij}^{t*})\right]\times[p_j^t-p_j^{t*}]\geqslant 0,$$
$$\forall(q,e,l,\lambda,p)\in R_+^{ms+mr+2rmn+rn}. \tag{14}$$

PROOF. See the appendix.

We now put variational inequality (14) into standard form (cf. Nagurney 1993). Define the column vector $X \equiv (q, e, l, \lambda, p) \in R_+^{ms+mr+2rmn+rn}$ and $F(X)$ as the row vector consisting of the row vectors $(G(X), E(X), L(X), \Lambda(X), P(X))$, where $G(X)$ is the ms-dimensional vector with component id given by: $-\partial u_i/\partial q_{id}$, $E(X)$ is the mr-dimensional vector with component it given by $-\partial u_i/\partial e_i^t + \Sigma_{j=1}^n \lambda_{ij}^t h_{ij}^t$; $L(X)$ is the mrn-dimensional vector with component itj given by $p_j^t + \partial c_{ij}^t(l_{ij}^t)/\partial l_{ij}^t - \lambda_{ij}^t$; $\Lambda(X)$ is the mrn-dimensional vector with component itj given by $l_{ij}^t - h_{ij}^t e_i^t$; and $P(X)$ is the rn-dimensional vector with tjth component given by $\Sigma_{i=1}^m (l_{ij}^{t0} - l_{ij}^t)$.

Variational inequality (14) can now be expressed as

$$F(X^*) \cdot (X - X^*) \geqslant 0, \quad \forall X \in K, \tag{15}$$

where $K \equiv \{X = (q, e, l, \lambda, p) \in R_+^{ms+mr+2rmn+rn}\}$, and $\cdot$ denotes the inner product in the Euclidean space R^N. Note that in the case of perfectly competitive firms, the governing variational inequality would be as in (14), with the term

$$\frac{\partial \rho_d(\sum_{i=1}^m q_{id}^*)}{\partial q_{id}} - \rho_d\left(\sum_{i=1}^m q_{id}^*\right)$$

in (14) replaced by $\bar{\rho}_d$.

Note that the variational inequality (14) provides a formulation of the equilibrium conditions that consist of both equalities and inequalities, and one does not know a priori which equilibrium variables will have positive values. Moreover, it provides a concise formulation, consisting of a single inequality. In addition, numerous equilibrium problems in both economics and in operations research have now been formulated and studied as variational inequality problems (cf. Nagurney 1993 and the references therein).

In the next corollary we prove that the equilibrium pattern is independent of the initial license allocation, provided that the sum of licenses for each pollutant and each receptor point is fixed. We then provide what those sums should be equal to in order to guarantee that the imposed environmental quality standards are met. In particular, let Q_j^t denote the imposed environmental standard for receptor point j and pollutant t.

COROLLARY 1 (INDEPENDENCE OF EQUILIBRIUM PATTERN FROM INITIAL LICENSE ALLOCATION). *If $l_{ij}^{t0} \geqslant 0$, for all $i = 1, \ldots, m$; $j = 1, \ldots, n$; $t = 1, \ldots, r$, and $\Sigma_{i=1}^m l_{ij}^{t0} = Q_j^t$, for $j = 1, \ldots, n$, and $t = 1, \ldots, r$, with each Q_j^t fixed and positive, then the equilibrium pattern $(q^*, e^*, l^*, \lambda^*, p^*)$ is independent of the l_{ij}^{t0}.*

PROOF. See the appendix.

Corollary 1 shows that any initial allocation of licenses that adheres to the imposed sum Q_j^t for each pollutant and receptor point will not affect the equilibrium pattern.

In the next theorem, we give a mechanism for the determination of the sums of the initial license allocation in order to guarantee that the environmental standards are met.

THEOREM 2 (ACHIEVEMENT OF ENVIRONMENTAL STANDARDS). *An equilibrium vector achieves environmental quality standards represented by the vector $Q = (Q_1, \ldots, Q_n)$, where $Q_j = \{Q_j^1, \ldots, Q_j^r\}$, provided that $\Sigma_{i=1}^m l_{ij}^{t0} = Q_j^t$, for all $j = 1, \ldots, n$ and all $t = 1, \ldots, r$.*

PROOF. See the appendix.

We now discuss the above model in relationship to another model that has appeared in the literature. In particular, if there is only a *single* pollutant and *no* transaction costs, then the above model (and the variational inequality formulation) collapses to the single-product, single-pollutant oligopolistic model with no transaction costs, developed in Nagurney and Dhanda (1996).

1.1. Qualitative Properties

In this subsection we investigate certain qualitative properties of the equilibrium. In particular, we establish properties of the function $F(X)$ that are needed for convergence of the algorithm in §2.

LEMMA 1 (MONOTONICITY). *If the profit functions u_i are concave for each firm i, then $F(X)$ is monotone; that is,*

$$[F(X^1) - F(X^2)] \cdot [X^1 - X^2] \geqslant 0, \quad \forall X^1, X^2 \in K. \tag{16}$$

PROOF. See the appendix.

LEMMA 2 (LIPSCHITZ CONTINUITY). *The function $F(X)$ is Lipschitz continuous; that is, there exists a positive constant L, such that*

$$\|F(X^1) - F(X^2)\| \leqslant L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in K, \tag{17}$$

under the assumption that the profit functions have bounded second-order derivatives.

PROOF. See the appendix.

DEFINITION 3 (COERCIVE FUNCTION). A function $F(X)$, from a feasible set K to R^N, is said to be coercive if

$$\frac{(F(X) - F(X^1)) \cdot (X - X^1)}{\|X - X^1\|} \to \infty, \tag{18}$$

as $\|X\| \to \infty$, for $X \in K$, and for some $X^1 \in K$.

We now state the existence result.

THEOREM 3 (EXISTENCE). *If $(q^*, e^*, l^*, \lambda^*, p^*) \in R^{ms+mr+2rmn+rn}$ satisfies variational inequality (14) then the equilibrium production, emission, and license vector is a solution to the variational inequality problem:*

$$\sum_{i=1}^m \sum_{d=1}^s -\frac{\partial u_i(q^*, e_i^*, l_i^*)}{\partial q_{id}} \times [q_{id} - q_{id}^*]$$
$$+ \sum_{i=1}^m \sum_{t=1}^r \frac{\partial g_i(e_i^*, q_i^*)}{\partial e_i^t} \times [e_i^t - e_i^{t*}]$$

$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\frac{\partial c_{ij}^t(l_{ij}^{t*})}{\partial l_{ij}^t}\times[l_{ij}^t-l_{ij}^{t*}]\geqslant 0,$$

$$\forall(q,e,l)\in K^1, \tag{19}$$

where

$$K^1\equiv\left\{(q,e,l)\in R^{ms+mr+rmn};\right.$$

$$\left.h_{ij}^t e_i^t\leqslant l_{ij}^t;\ \sum_{i=1}^{m}(l_{ij}^{t0}-l_{ij}^t)\geqslant 0;\ \forall i,t,j\right\}. \tag{20}$$

A solution to (19) *is guaranteed to exist provided that* $-\nabla u(\cdot)$ *is coercive, where* ∇u *is the gradient of* u. *Moreover, if* (q^*,e^*,l^*) *is a solution to* (19), *there exist* $\lambda^*\in R_+^{rmn}$ *and* $p^*\in R_+^{rn}$, *such that* $(q^*,e^*,l^*,\lambda^*,p^*)$ *is a solution to variational inequality* (14) *and, hence, an equilibrium.*

PROOF. See the appendix.

We now present the uniqueness result.

THEOREM 4 (UNIQUENESS). *Assume that* $-\nabla u(\cdot)$ *is strictly monotone over* $R_+^{ms+mr+mnr}$, *that is,*

$$-\sum_{i=1}^{m}\sum_{d=1}^{s}\left[\frac{\partial u_i(q^1,e_i^1,l_i^1)}{\partial q_{id}}-\frac{\partial u_i(q^2,e_i^2,l_i^2)}{\partial q_{id}}\right]\times[q_{id}^1-q_{id}^2]$$

$$-\sum_{i=1}^{m}\sum_{t=1}^{r}\left[\frac{\partial u_i(q^1,e_i^1,l_i^1)}{\partial e_i^t}-\frac{\partial u_i(q^2,e_i^2,l_i^2)}{\partial e_i^t}\right]\times[e_i^{t1}-e_i^{t2}]$$

$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\frac{\partial c_{ij}^t(l_{ij}^{t1})}{\partial l_{ij}^t}-\frac{\partial c_{ij}^t(l_{ij}^{t2})}{\partial l_{ij}^t}\right]\times[l_{ij}^{t1}-l_{ij}^{t2}]>0,$$

$$\forall(q^1,e^1,l^1),\ (q^2,e^2,l^2)\in R_+^{ms+mr+mnr},$$

$$(q^1,e^1,l^2)\neq(q^2,e^2,l^2). \tag{21}$$

Then the equilibrium production, emission, and license pattern (q^*,e^*,l^*) *is unique.*

PROOF. See the appendix.

2. THE ALGORITHM

In this section we present an algorithm for the solution of variational inequality (14) governing the market equilibrium model for pollution permits. The algorithm resolves the variational inequality problem into very simple subproblems, each of which can be solved explicitly and in closed form.

The algorithm we propose for the computation of the equilibrium pattern is the modified projection method for Korpelevich (1976). The algorithm is guaranteed to converge, provided that F satisfies only the monotonicity condition and the Lipschitz continuity condition, assuming that a solution exists. The algorithm has been applied by Nagurney and Dhanda (1996) previously to compute the equilibrium pattern in single-pollutant, single-product oligopolistic and perfectly competitive market equilibrium problems with ambient-based permits, but without transaction costs. It has also been applied to compute the equilibrium pattern in spatial markets with ad valorem tariffs (cf. Nagurney et al. 1996), as well as the equilibrium pattern in a variety of financial problems (cf. Nagurney and Siokos 1997).

The statement of the modified projection method is as follows.

Step 0: *Initialization.* Set $X^0\in K$. Let $\mathcal{T}=1$ and let α be a scalar such that $0<\alpha<\frac{1}{L}$, where L is the Lipschitz continuity constant (cf. (17)).

Step 1: *Computation.* Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$[\bar{X}^{\mathcal{T}}+\alpha F(X^{\mathcal{T}-1})^T-X^{\mathcal{T}-1}]^T\cdot[X-\bar{X}^{\mathcal{T}}]\geqslant 0,$$

$$\text{for all } X\in K. \tag{22}$$

Step 2: *Adaptation.* Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$[X^{\mathcal{T}}+\alpha F(\bar{X}^{\mathcal{T}})^T-X^{\mathcal{T}-1}]^T\cdot[X-X^{\mathcal{T}}]\geqslant 0,$$

$$\text{for all } X\in K. \tag{23}$$

Step 3: *Convergence Verification.* If $\max|X_l^{\mathcal{T}}-X_l^{\mathcal{T}-1}|\leqslant\varepsilon$, for all l, with $\varepsilon>0$, a prespecified tolerance, then stop; else, set $\mathcal{T}=\mathcal{T}+1$, and go to Step 1.

We now discuss the modified projection method more fully. We first recall the definition of the projection of X, on the closed convex set K, with respect to the Euclidean norm, and denoted by $P_K X$, as

$$y=P_K X=\arg\min_{z\in K}\|X-z\|. \tag{24}$$

In particular, we note that (cf. Nagurney 1993, Theorem 1.2) $\bar{X}^{\mathcal{T}}$ generated by the modified projection method as the solution to the variational inequality subproblem (22) is actually the projection of $(X^{\mathcal{T}-1}-\alpha F(X^{\mathcal{T}-1})^T)$ on the closed convex set K, where K here is simply the nonnegative orthant. In other words,

$$\bar{X}^{\mathcal{T}}=P_K[X^{\mathcal{T}-1}-\alpha F(X^{\mathcal{T}-1})^T]. \tag{25}$$

Similarly, $X^{\mathcal{T}}$ generated by the solution to variational inequality subproblem (23) is the projection of $X^{\mathcal{T}-1}-\alpha F(\bar{X}^{\mathcal{T}})^T$ on the nonnegative orthant, that is,

$$X^{\mathcal{T}}=P_K[X^{\mathcal{T}-1}-\alpha F(\bar{X}^{\mathcal{T}})^T]. \tag{26}$$

Because the feasible set here is of box type, the above projections immediately decompose across the coordinates of the feasible set. In fact, the solution of each of the variables encountered in (22) and (23) amounts to projecting onto R_+ separately. Consequently, we can provide closed-form expressions for the solution of problems (22) and (23). In particular, we have that (22) can be solved as:

For all firms i; $i=1,\dots,m$, and all products d; $d=1,\dots,s$, set:

$$\bar{q}_{id}^{\mathcal{T}} = \max\left\{0, \alpha\left(\frac{\partial u_i(q^{\mathcal{T}-1}, e_i^{\mathcal{T}-1}, l_i^{\mathcal{T}-1})}{\partial q_{id}}\right) + q_{id}^{\mathcal{T}-1}\right\}; \tag{27}$$

and for all firms i; $i=1,\dots,m$, and for all pollutants t; $t=1,\dots,r$, set:

$$\bar{e}_i^{t\mathcal{T}} = \max\left\{0, \alpha\left(\frac{\partial u_i(q^{\mathcal{T}-1}, e_i^{\mathcal{T}-1}, l_i^{\mathcal{T}-1})}{\partial e_i^t} - \sum_{j=1}^{n} \lambda_{ij}^{t\mathcal{T}-1} h_{ij}\right) + e_i^{t\mathcal{T}-1}\right\}. \tag{28}$$

For all firms i; $i=1,\dots,m$, all receptor points j; $j=1,\dots,n$, and all pollutants t; $t=1,\dots,r$, set:

$$\bar{l}_{ij}^{t\mathcal{T}} = \max\left\{0, \alpha\left(-p_j^{t\mathcal{T}-1} - \frac{\partial c_{ij}^t(l_{ij}^{t\mathcal{T}})}{\partial l_{ij}^{t\mathcal{T}}} + \lambda_{ij}^{t\mathcal{T}-1}\right) + l_{ij}^{t\mathcal{T}-1}\right\}; \tag{29}$$

and

$$\bar{\lambda}_{ij}^{t\mathcal{T}} = \max\{0, \alpha(-l_{ij}^{t\mathcal{T}-1} + h_{ij} e_i^{t\mathcal{T}-1}) + \lambda_{ij}^{t\mathcal{T}-1}\}. \tag{30}$$

Finally, for all receptor points j; $j=1,\dots,n$, and for all pollutants t; $t=1,\dots,r$, set

$$\bar{p}_j^{t\mathcal{T}} = \max\left\{0, \alpha\left(-\sum_{i=1}^{m} l_{ij}^{t0} + \sum_{i=1}^{m} l_{ij}^{t\mathcal{T}-1}\right) + p_j^{t\mathcal{T}-1}\right\}. \tag{31}$$

Variational inequality subproblem (27) can be solved explicitly in closed form in a similar manner.

Convergence is given in the following theorem.

THEOREM 5 (CONVERGENCE). *The modified projection method described above converges to the solution of variational inequality* (14) *under the assumptions that the profit functions have bounded second-order derivatives and are concave.*

PROOF. See the appendix.

3. NUMERICAL EXAMPLES

In this section we present numerical examples illustrating the model presented in §1, along with the performance of the algorithm of §2. The data access information is presented in tables in the appendix.

We present three oligopoly examples of increasing complexity, and then for each of these examples we subsequently increase and then decrease the transaction costs to evaluate the effect on the trades of licenses.

We assume that each firm faces a production cost function of the form

$$f_i(q_i) = \sum_{d=1}^{s}\left[c_{id} q_{id} + \frac{\beta_{id}}{(\beta_{id}+1)} K_{id}^{-1/\beta_{id}} q_{id}^{(\beta_{id}+1)/\beta_{id}}\right], \tag{32}$$

with the specific terms for the parameters reported along with the examples. The demand price function for product d, in turn, in each example is assumed to be given by

$$\rho_d\left(\sum_{i=1}^{m} q_{id}\right) = 5000^{1/1.1}\left(\sum_{i=1}^{m} q_{id}\right)^{-1/1.1}. \tag{33}$$

We note that similar production cost and demand price functions have been used by Murphy et al. (1982).

Each firm i in each example also faces an emission cost function of the form

$$g_i(e_i, q_i) = \sum_{t=1}^{r}[g1_{it}(e_i^t)^2 + g2_{it} e_i^t + g4_{it}] + \sum_{d=1}^{s} g3_{id} q_{id}, \tag{34}$$

with the specific terms for the parameters reported along with the examples.

The transaction cost function employed by firm i for pollutant t at receptor point j in each example was of the form

$$c_{ij}^t(l_{ij}^t) = \phi 1_{ijt}(l_{ij}^t)^2 + \phi 2_{ijt} l_{ij}^t + \alpha_{ijt}, \tag{35}$$

with the specific terms for the parameters reported along with the examples.

The initial allocation of the licenses, the l_{ij}^{t0}s, were set as $l_{ij}^{t0}=3$, for all i,j,t. The diffusion matrix H terms, the h_{ij}^ts were set as follows: For $t=1$: $h_{ij}^1 = 5\frac{i}{j}$, if $i \Leftarrow j$; and $h_{ij}^1 = 0.5\frac{i}{j}$, otherwise, for all i,j. For $t=2$: $h_{ij}^2 = 10\frac{i}{j}$, if $i \Leftarrow j$; and $h_{ij}^2 = 5.\frac{i}{j}$, otherwise, for all i,j. The initial values for the quantities were set as follows: $q_{id}^0 = 15$, if $i \Leftarrow d$; and $q_{id}^0 = 25$, otherwise. The initial values for the emissions were set as follows: $e_i^t = 10$, for all i,t. All other initial variables were initialized to 1.

The convergence tolerance ε was set to 0.0001, and the parameter α was set to 0.1 in the algorithm for all the examples. We also computed the maximum error for this convergence tolerance and the average error, where the error was defined as the absolute value of the respective left-hand-side term in the variational inequality (14) for each variable that had a computed positive value. Note that this value should be as close to zero as possible.

The algorithm was coded in FORTRAN 77. The system used was the IBM SP2 at the University of Massachusetts at Amherst.

In addition, to measure the effect of changes in the transaction costs on the trade of licenses, we defined the volume of trade measure Δ as follows:

$$\Delta = \sum_{t=1}^{r}\sum_{i=1}^{m}\sum_{t=1}^{r}\frac{|l_{ij}^{t0} - l_{ij}^{t*}|}{2}.$$

EXAMPLE 1. In this example, the oligopoly consisted of two firms that produce two products and emit two pollutants, which in turn affect two receptor points. Refer to Table 1 in the appendix for the input data for Example 1.

The modified projection method converged in 1702 iterations and yielded the equilibrium output vectors reported in Table 2 in the appendix, with a maximum error of 0.0097 and an average error of 0.00016. The positive equilibrium license prices, in this example, signal that the market clears for the licenses for each pollutant and receptor point. The volume of trade measure $\Delta = 10.114$.

We then perturbed the transaction cost data by increasing the transaction costs by multiplying the $\phi 1_{ijt}$ parameter by 5 for all ijt. For brevity, we do not report the new equilibrium pattern but note that the equilibrium prices for licenses affecting receptor point 1 and receptor point 2 dropped to the new prices as reported at the bottom of Table 2. The volume of trade Δ now decreased to 4.937. Hence, the transaction costs, as expected, had a negative effect on the trades that took place. We then decreased the original $\phi 1_{ijt}$ terms by a factor of 5 for all i, j, t and obtained the new computed $\Delta = 11.92$. Such a decrease in transaction costs resulted in an increase in the trade volume.

EXAMPLE 2. To evaluate the effect of the number of firms on the equilibrium pattern, we increased the number of firms from two to three in the second example. The three firms in the oligopoly produce two products and emit two pollutants that affect two receptor points. Refer to Table 3 in the appendix for the input data for this example.

The algorithm required 1,414 iterations for convergence to the equilibrium output vectors reported in Table 4 in the appendix. Note that in this example, as in Example 1, the markets in licenses for each pollutant and receptor point also cleared and, hence, the prices were positive. In this example, $\Delta = 17.568$, with the maximum error equal to 0.00018 and the average error equal to 0.001.

We subsequently increased the transaction costs in the same manner as in Example 1 by multiplying the coefficient $\phi 1_{ijt}$ by 5 for all ijt. As expected, Δ then decreased to 13.385, reflecting that transaction costs are a barrier to trade. The equilibrium prices for the licenses also dropped.

Finally, we decreased the transaction cost terms in the identical manner as Example 1 and the new computed $\Delta = 19.325$, providing further evidence that transaction costs act as a barrier to trade.

EXAMPLE 3. In the third example, we increased the number of receptor points from two to three and the three firms in the oligopoly still produce two products and emit two pollutants, as in Examples 1 and 2. Refer to Table 5 in the appendix for the input data for this example.

The modified projection method converged in 1,718 iterations and yielded the equilibrium vectors reported in Table 6 in the appendix. The maximum and average errors were, respectively, 0.00094 and 0.00017. In Example 3, the markets in licenses for each pollutant and receptor point cleared and the prices were positive. In this example, the value of Δ is 28.234. We note that the equilibrium quantity vector remains the same as in the previous example indicating that a change in the number of receptor points does not directly impact the quantities a firm produces, at least in this particular example.

We next increased the transaction costs as we had in the preceding two examples by multiplying the coefficient $\phi 1_{ijt}$ by 5 for all ijt, with the consequence that the volume of trade measure Δ then decreased to 23.869. In addition, the prices of the licenses dropped, reflecting the decrease in the volume of licenses traded.

As in Examples 1 and 2, we then decreased the transaction costs by dividing the original $\phi 1_{ijt}$ values by 5 for all i, j, t. As expected, the volume of trade increased, with the new $\Delta = 29.775$.

These examples illustrate the effect of certain parameters, such as changes in transaction costs, on the volume of licenses traded. Specifically, we note that in these examples an increase in transaction costs leads to a decrease in the volume of licenses traded, whereas a decrease leads to an increase in the volume of licenses traded.

The above numerical examples highlight the variety of multiproduct, multipollutant oligopoly problems with marketable pollution permits that can be solved. Although the algorithm requires a large number of iterations for convergence, each iteration of the algorithm is remarkably simple and computationally very efficient because closed-form expressions are used. Indeed, each of the above examples was solved in a negligible amount of CPU time—less than 0.001 CPU seconds. Moreover, the computed solutions are very accurate. The algorithm appears suitable for the evaluation of alternative transaction cost scenarios.

4. SUMMARY AND CONCLUSIONS

In this paper we have presented a variational inequality framework for the formulation, qualitative analysis, and computation of equilibria in multiproduct, multipollutant oligopolistic markets with marketable pollution permits and in the presence of transaction costs. The model explicitly handles spatial differentiation for the pollutants as well as the transfers between firms that take place.

The model significantly extends those that have been presented in the literature todate. Moreover, we proposed an algorithm, along with convergence results, that resolves what we expect to be large-scale problems into very simple subproblems, which can then be solved in closed form. Finally, to illustrate both the model and the algorithm, we presented numerical results.

Additional research in the future will include empirical analysis, as well as the incorporation of dynamics into the modeling scheme.

APPENDIX

Input and output data can be found tables in an additional appendix at the *Operations Research* Home Page:

http://www.informs.org/pubs

in the Online Collection.

PROOF OF THEOREM 1. Assume that $(q^*, e^*, l^*, \lambda^*, p^*) \in R^{ms+mr+2rmn+rn}$ is an equilibrium. Note that inequality (12) holds for all firms $i = 1, \ldots, m$. Hence, summing (12) over all firms, we obtain

$$\sum_{i=1}^{m}\sum_{d=1}^{s}\left[\frac{\partial f_i(q_i^*)}{\partial q_{id}} + \frac{\partial g_i(e_i^*, q_i^*)}{\partial q_{id}} - \frac{\partial \rho_d(\sum_{i=1}^{m} q_{id}^*)}{\partial q_{id}} q_{id}^* - \rho_d\left(\sum_{i=1}^{m} q_{id}^*\right)\right] \times [q_{id} - q_{id}^*]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\left[\frac{\partial g_i(e_i^*, q_i^*)}{\partial e_i^t} + \sum_{j=1}^{n}\lambda_{ij}^{t*}h_{ij}^t\right] \times [e_i^t - e_i^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\left[p_j^{t*} + \frac{\partial c_{ij}^t(l_{ij}^{t*})}{\partial l_{ij}^t} - \lambda_{ij}^{t*}\right] \times [l_{ij}^t - l_{ij}^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[l_{ij}^{t*} - h_{ij}^t e_i^{t*}] \times [\lambda_{ij}^t - \lambda_{ij}^{t*}] \geqslant 0$$
$$\forall (q, e, l, \lambda) \in R_+^{ms+mr+2rmn}. \tag{A.1}$$

Also, from the system (13) we can conclude that the equilibrium must satisfy

$$\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\sum_{i=1}^{m}(l_{ij}^{t0} - l_{ij}^{t*})\right] \times [p_j^t - p_j^{t*}] \geqslant 0, \quad \forall p \in R_+^{rn}. \tag{A.2}$$

Finally, summing (A.1) and (A.2), one obtains variational inequality (14).

We now establish the converse of the proof, that is, the solution to (14) also satisfies (12) and (13).

Let $(q^*, e^*, l^*, \lambda^*, p^*) \in R^{ms+mr+2rmn+rn}$ be a solution of (14). If one lets $q_{id} = q_{id}^*$, $e_i^t = e_i^{t*}$, $l_{ij}^t = l_{ij}^{t*}$, $\lambda_{ij}^t = \lambda_{ij}^{t*}$ for all i, d, j, t, and substitutes these values into (14), one obtains

$$\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\sum_{i=1}^{m}(l_{ij}^{t0} - l_{ij}^{t*})\right] \times [p_j^t - p_j^{t*}] \geqslant 0, \quad \forall p \in R_+^{rn}, \tag{A.3}$$

which implies (13).

Similarly, if one lets $p_j^t = p_j^{t*}$ for all j, t, and substitutes these values into (14), one obtains

$$\sum_{i=1}^{m}\sum_{d=1}^{s}\left[\frac{\partial f_i(q_i^*)}{\partial q_{id}} + \frac{\partial g_i(e_i^*, q_i^*)}{\partial q_{id}} - \frac{\partial \rho_d(\sum_{i=1}^{m} q_{id}^*)}{\partial q_{id}} q_{id}^* - \rho_d\left(\sum_{i=1}^{m} q_{id}^*\right)\right] \times [q_{id} - q_{id}^*]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\left[\frac{\partial g_i(e_i^*, q_i^*)}{\partial e_i^t} + \sum_{j=1}^{n}\lambda_{ij}^{t*}h_{ij}^t\right] \times [e_i^t - e_i^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\left[p_j^{t*} + \frac{\partial c_{ij}^t(l_{ij}^{t*})}{\partial l_{ij}^t} - \lambda_{ij}^{t*}\right] \times [l_{ij}^t - l_{ij}^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[l_{ij}^{t*} - h_{ij}^t e_i^{t*}] \times [\lambda_{ij}^t - \lambda_{ij}^{t*}] \geqslant 0$$
$$\forall (q, e, l, \lambda) \in R_+^{ms+mr+2rmn}, \tag{A.4}$$

which implies that (12) must hold for all the firms. The proof is complete.

PROOF OF COROLLARY 1. Note that the first four terms in variational inequality (14) are independent of l_{ij}^{t0}. The last term in (14), on the other hand, depends only on the sum $\sum_{i=1}^{m} l_{ij}^{t0}$, for $j = 1, \ldots, n$ and a fixed pollutant t. The proof is complete.

PROOF OF THEOREM 2. From constraint (9) we have that

$$h_{ij}^t e_i^{t*} \leqslant l_{ij}^{t*}, \quad \forall j, \forall t. \tag{A.5}$$

It then follows from equilibrium conditions (13) that

$$\sum_{i=1}^{m} h_{ij}^t e_i^{t*} \leqslant \sum_{i=1}^{m} l_{ij}^{t*} \leqslant \sum_{i=1}^{m} l_{ij}^{t0} = Q_j^t, \quad \forall t, \forall j. \tag{A.6}$$

The proof is complete.

PROOF OF LEMMA 1. In view of the definition of $F(X)$ in the above model, the left-hand side of inequality (16) takes the form

$$\sum_{i=1}^{m}\sum_{d=1}^{s} - \left[\frac{\partial u_i(q^1, e_i^1, l_i^1)}{\partial q_{id}} - \frac{\partial u_i(q^2, e_i^2, l_i^2)}{\partial q_{id}}\right] \times [q_{id}^1 - q_{id}^2]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\left[\left(-\frac{\partial u_i(q^1, e_i^1, l_i^1)}{\partial e_i^t} + \sum_{j=1}^{n}\lambda_{ij}^{t1}h_{ij}^t\right) - \left(-\frac{\partial u_i(q^2, e_i^2, l_i^2)}{\partial e_i^t} + \sum_{j=1}^{n}\lambda_{ij}^{t2}h_{ij}^t\right)\right] \times [e_i^{t1} - e_i^{t2}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\left(p_j^{t1} + \frac{\partial c_{ij}^t(l_{ij}^{t1})}{\partial l_{ij}^t} - \lambda_{ij}^{t1}\right) - \left(p_j^{t2} + \frac{\partial c_{ij}^t(l_{ij}^{t2})}{\partial l_{ij}^t} - \lambda_{ij}^{t2}\right)\right] \times [l_{ij}^{t1} - l_{ij}^{t2}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[(l_{ij}^{t1} - h_{ij}^t e_i^{t1}) - (l_{ij}^{t2} - h_{ij}^t e_i^{t2})] \times [\lambda_{ij}^{t1} - \lambda_{ij}^{t2}]$$
$$+\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\sum_{i=1}^{m}(l_{ij}^{t0} - l_{ij}^{t1}) - (l_{ij}^{t0} - l_{ij}^{t2})\right] \times [p_j^{t1} - p_j^{t2}]. \tag{A.7}$$

After combining and simplifying terms, the expression (A.7) reduces to

$$-\sum_{i=1}^{m}\sum_{d=1}^{s}\left[\frac{\partial u_i(q^1, e_i^1, l_i^1)}{\partial q_{id}} - \frac{\partial u_i(q^2, e_i^2, l_i^2)}{\partial q_{id}}\right] \times [q_{id}^1 - q_{id}^2]$$

$$-\sum_{i=1}^{m}\sum_{t=1}^{r}\left[\frac{\partial u_i(q^1,e_i^1,l_i^1)}{\partial e_i^t}-\frac{\partial u_i(q^2,e_i^2,l_i^2)}{\partial e_i^t}\right]\times[e_i^{t1}-e_i^{t2}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\frac{\partial c_{ij}^t(l_{ij}^{t1})}{\partial l_{ij}^t}-\frac{\partial c_{ij}^t(l_{ij}^{t2})}{\partial l_{ij}^t}\right]\times[l_{ij}^{t1}-l_{ij}^{t2}]. \quad (A.8)$$

But under the assumption that the profit functions are concave in their respective arguments, we know that the negative of the gradient of the utility functions is monotone, and hence the expression in (A.8) must be greater than or equal to zero. Monotonicity of $F(X)$ has thus been established. The proof is complete.

PROOF OF LEMMA 2. Follows from the same arguments as the proof of Lemma 3 in Nagurney and Dhanda (1996).

PROOF OF THEOREM 3. Assume, on the contrary, that

$$\sum_{i=1}^{m}\sum_{d=1}^{s}-\frac{\partial u_i(q^*,e_i^*,l_i^*)}{\partial q_{id}}\times[q_{id}-q_{id}^*]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\frac{\partial g_i(e_i^*,q_i^*)}{\partial e_i^t}\times[e_i^t-e_i^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\frac{\partial c_{ij}^t(l_{ij}^{t*})}{\partial l_{ij}^t}\times[l_{ij}^t-l_{ij}^{t*}]<0,$$
$$\text{for some } (q,e,l)\in K^1. \quad (A.9)$$

But according to variational inequality (14) and (A.9) it then follows that

$$\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\lambda_{ij}^{t*}h_{ij}^t\times[e_i^t-e_i^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[p_j^{t*}-\lambda_{ij}^{t*}]\times[l_{ij}^t-l_{ij}^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[l_{ij}^{t*}-h_{ij}^te_i^{t*}]\times[\lambda_{ij}^t-\lambda_{ij}^{t*}]$$
$$+\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\sum_{i=1}^{m}(l_{ij}^{t0}-l_{ij}^{t*})\right]\times[p_j^t-p_j^{t*}]$$
$$\geqslant\sum_{i=1}^{m}\sum_{d=1}^{s}\frac{\partial u_i(q^*,e_i^*,l_i^*)}{\partial q_{id}}\times[q_{id}-q_{id}^*]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}-\frac{\partial g_i(e_i^*,q_i^*)}{\partial e_i^t}\times[e_i^t-e_i^{t*}]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}-\frac{\partial c_{ij}^t(l_{ij}^{t*})}{\partial l_{ij}^t}\times[l_{ij}^t-l_{ij}^{t*}]>0. \quad (A.10)$$

Letting now $\lambda_{ij}^t=0$, and $p_j^t=0$, for all t,i,j, and substituting these terms into the left-hand side of the inequality in (A.10), after some simplifications, yields

$$\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\lambda_{ij}^{t*}h_{ij}^t\times[e_i^t]-\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[p_j^{t*}-\lambda_{ij}^{t*}]$$
$$\times[l_{ij}^t]+\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\sum_{i=1}^{m}l_{ij}^{t0}\right]\times[-p_j^{t*}], \quad (A.11)$$

which can be expressed as

$$\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[h_{ij}^te_i^t-l_{ij}^t]\times\lambda_{ij}^{t*}$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}[l_{ij}^{t0}-l_{ij}^t]\times[-p_j^{t*}]. \quad (A.12)$$

But each term in (A.12) must be less than or equal to zero in view of the feasible set given in (20). Therefore, (A.10) cannot hold true, and we have established a contradiction to (A.9). Therefore, variational inequality (19) must be satisfied.

Moreover, under the coercivity condition on $-\nabla u$, the existence of a solution to (19) and (20) is guaranteed from the standard theory of variational inequalities (cf. Nagurney 1993). Finally, according to the Lagrange multiplier theorem, there exist nonnegative multipliers λ^* and p^* corresponding to the linear inequality constraints in K^1, which must satisfy (14). The proof is complete.

Note that a coercivity condition on the profit functions of the firms in an oligopolistic market was also imposed by Gabay and Moulin (1980) in order to obtain an existence result.

PROOF OF THEOREM 4. Assume that there are two solutions, X^* and $\bar{X}$, to variational inequality (14); that is, we have that, for $X^*\in K$ and $\bar{X}\in K$,

$$F(X^*)\cdot(X-X^*)\geqslant 0,\quad \forall X\in K, \quad (A.13)$$

and

$$F(\bar{X})\cdot(X-\bar{X})\geqslant 0,\quad \forall\bar{X}\in K. \quad (A.14)$$

Let $X=\bar{X}$ and make this substitution into (A.13); similarly, let $X=X^*$, and make the substitution into (A.14). Adding then the two resulting inequalities yields:

$$[F(X^*)-F(\bar{X})]\cdot[\bar{X}-X^*]$$
$$=-\sum_{i=1}^{m}\sum_{d=1}^{s}\left[\frac{\partial u_i(q^*,e_i^*,l_i^*)}{\partial q_{id}}-\frac{\partial u_i(\bar{q},\bar{e}_i,\bar{l}_i)}{\partial q_{id}}\right]\times[\bar{q}_{id}-q_{id}^*]$$
$$-\sum_{i=1}^{m}\sum_{t=1}^{r}\left[\frac{\partial u_i(q^*,e_i^*,l_i^*)}{\partial e_i^t}-\frac{\partial u_i(\bar{q},\bar{e}_i,\bar{l}_i)}{\partial e_i^t}\right]\times[e_i^{t*}-\bar{e}_i^t]$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{r}\sum_{j=1}^{n}\left[\frac{\partial c_{ij}^t(l_{ij}^{t*})}{\partial l_{ij}^t}-\frac{\partial c_{ij}^t(\bar{l}_{ij}^t)}{\partial l_{ij}^t}\right]\times[l_{ij}^{t*}-\bar{l}_{ij}^t]\geqslant 0.$$
$$(A.15)$$

But the left-hand side of (A.15) must be less than zero, under the assumption of strict monotonicity, and thus we must have that $q_{id}^* = \bar{q}_{id}$, for all $i, d, e_i^{t*} = \bar{e}_i^t$, for all i, t, and $l_{ij}^{t*} = \bar{l}_{ij}^t$, for all i, j, t. The proof is complete.

PROOF OF THEOREM 5. It follows from Lemmas 1 and 2 that the function $F(X)$ is both monotone and Lopschitz continuous, under the stated assumptions. Hence, as established in Theorem 2 of Korpelevich (1976), the modified projection method is guaranteed to converge under these conditions.

ACKNOWLEDGMENTS

This research was supported by Grant No. DMS-9024071 from the National Science Foundation. This support is gratefully appreciated. The authors are indebted to three anonymous referees for their many helpful suggestions and comments and to the area editor, Ben Hobbs, for his thorough report.

REFERENCES

Bertsekas, D. P., J. N. Tsitsklis. 1989. *Parallel and Distributed Computation – Numerical Methods.* Prentice-Hall, Englewood Cliffs, NJ.

Bohi, D. R., D. Burtraw, 1997. SO_2 *allowance trading*: how experience and expectations measure up. Resources for the Future Discussion Paper 97-24, Washington, DC, 1–18.

Carraro, C., Y. Katsoulacos, A. Xepapadeas (eds.). 1996. *Environmental Policy and Market Structure.* Kluwer Academic Publishers, Dordrecht, The Netherlands.

Cournot, A. A. 1838. *Researches into the Mathematical Principles of the Theory of Wealth.* English translation. 1897, MacMillan, London, England.

Crocker, T. D. 1966. The structuring of atmospheric pollution control systems. *The Economics of Air Pollution.* H. Wolozin, ed. W. W. Norton, New York.

Dales, J. H. 1968. *Pollution, Property, and Prices.* University of Toronto Press, Toronto, Ontario, Canada.

Gabay, D., H. Moulin. 1980. On the uniqueness and stability of Nash-equilibria in non-cooperative games. A. Bensoussan, P. Kleindorfer, C. S. Tapiero, eds. *Applied Stochastic Control in Econometrics and Management Science.* North-Holland, Amsterdam, The Netherlands.

Hahn, R. W. 1984. Market power and transferable property rights. *Quart. J. Econom.* **3** 753–765.

——, G. L. Hester. 1989. Marketable permits: lessons for theory and practice. *Ecol. Law Quart.* **16** 361–406.

Kahn, J. R. 1998. *The Economic Approach to Environmental & Natural Resources.* 2nd ed. Dryden Press, Orlando, FL.

Kinderlehrer, D., G. Stampacchia. 1980. *An Introduction to Variational Inequalities and Their Applications.* Academic Press, New York.

Kohn, R. E. 1991. Transaction costs and the optimal instrument and intensity of air pollution control. *Policy Sci.* **24** 315–332.

Korpelevich, G. M. 1976. The extragradient method for finding saddle points and other problems. *Ekon. i Mat. Metody.* Translated as *Matecon* **12** 747–756.

Mendelsohn, R. 1986. Regulating heterogeneous emissions. *J. Environmental and Econ. Management* **4** 301–312.

Montgomery, W. D. 1972. Markets in licenses and efficient pollution control programs. *J. Econom. Theory* **5** 395–418.

Murphy, F. H., H. D. Sherali, A. L. Soyster. 1982. A mathematical programming approach for determining oligopolistic market equilibrium. *Math. Programming* **24** 92–106.

Nagurney, A. 1993. *Network Economics: A Variational Inequality Approach.* Kluwer Academic Publishers, Boston, MA.

——, K. Dhanda. 1996. A variational inequality approach for marketable pollution permits. *Comput. Econom.* **9** 363–384.

——, C. F. Nicholson, P. M. Bishop. 1996. Massively parallel computation of large-scale spatial price equilibrium models with discriminatory ad valorem tariffs. *Ann. Oper. Res.* **68** 281–300.

——, S. Siokos. 1997. *Financial Networks: Statics and Dynamics.* Springer-Verlag, Berlin, Germany.

——, S. Thore, J. Pan. 1996. Spatial market policy modeling with goal targets. *Oper. Res.* **44** 393–406.

Nash, J. 1950. Equilibrium points in N-person games. *Proc. National Acad. Sci.* **36** 48–49.

Oren, S. S. 1997. Economic inefficiency of passive transmission rights in congested electricity systems with competitive generation. *Energy J.* **18**(1) 63–83.

Pearce, D. W., K. R. Turner. 1990. *Economics of Natural Resources and the Environment.* The Johns Hopkins University Press, Baltimore, MD.

Pigou, A. C. 1920. *The Economics of Welfare.* MacMillan, London, England.

Stavins, R. N. 1995. Transaction costs and tradeable permits. *J. Environmental Econ. Management* **29** 133–148.

Tietenberg, T. H. 1985. *Emissions Trading: An Exercise in Reforming Pollution Policy.* Resources for the Future, Inc., Washington, DC.

——. 1988. *Environmental and Natural Resource Economics.* Second edition. Scott, Foresman, Glenview, IL.

Tirole, J. 1988. *The Theory of Industrial Organization.* MIT Press, Cambridge, MA.

Tripp, J. T. B., D. J. Dudek. 1989. Institutional guidelines for designing successful transferable rights programs. *Yale. J. Regulation* **6** 369–391.

Van Egteren, H., M. Weber. 1996. Marketable permits, market power, and cheating. *J. Environmental Econ. Management* **30** 161–173.