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Positive solutions to PBVPs for nonlinear first-order impulsive dynamic equations on time scales

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Abstract
By using the classical fixed point theorem for operators on a cone, in this paper, some results of one and two positive solutions to a class of nonlinear first-order periodic boundary value problems of impulsive dynamic equations on time scales are obtained.

MSC: 39A10; 34B15

Keywords: time scale; periodic boundary value problem; fixed point; impulsive dynamic equation

1 Introduction

The theory of dynamic equations on time scales has been a new important mathematical branch [1–3] since it was initiated by Hilger [4]. At the same time, the boundary value problems of impulsive dynamic equations on time scales have received considerable attention [5–21] since the theory of impulsive differential equations is a lot richer than the corresponding theory of differential equations without impulse effects [22–24].

In this paper, we concerned with the existence of positive solutions for the following PBVPs of impulsive dynamic equations on time scales

$$\begin{cases} x^{\triangle}(t) + p(t)x(\sigma(t)) = f(t, x(\sigma(t))), & t \in J := [0, T]_{\mathbb{T}}, t \neq t_k, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), & k = 1, 2, \dots, m, \\ x(0) = x(\sigma(T)), \end{cases} \tag{1.1}$$

where $\mathbb{T}$ is an arbitrary time scale, $T > 0$ is fixed, 0, $T \in \mathbb{T}$, $f \in C(J \times [0, \infty), [0, \infty))$, $I_k \in C([0, \infty), [0, \infty))$, $p : [0, T]_{\mathbb{T}} \to (0, \infty)$ is right-dense continuous, $t_k \in (0, T)_{\mathbb{T}}$, $0 < t_1 < \cdots < t_m < T$, and, for each $k = 1, 2, \dots, m$, $x(t_k^+) = \lim_{h \to 0^+} x(t_k + h)$ and $x(t_k^-) = \lim_{h \to 0^-} x(t_k + h)$ represent the right and left limits of $x(t)$ at $t = t_k$.

By using the Guo-Krasnoselskii fixed point theorem, Wang [18] considered the existence of one or two positive solutions to the problem (1.1).

In [20], by using the Schaefer fixed point theorem, Wang and Weng obtained the existence of at least one solution to the problem (1.1).

When $I_k(x) \equiv 0$, $k = 1, 2, \dots, m$, [25, 26] considered the existence of solutions to the problem (1.1) by means of the Schaefer fixed point theorem; when $p(t) = 0$, the problem (1.1) reduces to the problem studied by [12, 19].

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Motivated by the results mentioned above, in this paper, we shall obtain the existence of one and two solutions to the problem (1.1) by means of a fixed point theorem in cones. The results obtained in this paper improve the results in [18] intrinsically.

Throughout this work, we assume knowledge of time scales and the time-scale notation, first introduced by Hilger [4]. For more on time scales, please see the texts by Bohner and Peterson [2, 3].

In the remainder of this section, we state the following fixed point theorem [27].

Theorem 1.1 ([27]) *Let X be a Banach space and $K \subset X$ be a cone in X. Assume Ω_1, Ω_2 are bounded open subsets of X with $0 \in \Omega_1 \subset \overline{\Omega}_1 \subset \Omega_2$ and $\Phi : K \cap (\overline{\Omega}_2 \backslash \Omega_1) \to K$ is a completely continuous operator. If*

(i) *there exists $u_0 \in K\backslash\{0\}$ such that $u - \Phi u \neq \lambda u_0$, $u \in K \cap \partial\Omega_2$, $\lambda \geq 0$; $\Phi u \neq \tau u$, $u \in K \cap \partial\Omega_1$, $\tau \geq 1$, or*

(ii) *there exists $u_0 \in K\backslash\{0\}$ such that $u - \Phi u \neq \lambda u_0$, $u \in K \cap \partial\Omega_1$, $\lambda \geq 0$; $\Phi u \neq \tau u$, $u \in K \cap \partial\Omega_2$, $\tau \geq 1$,*

then Φ has at least one fixed point in $K \cap (\overline{\Omega}_2 \backslash \Omega_1)$.

2 Preliminaries

Throughout the rest of this paper, we always assume that the points of impulse t_k are right-dense for each $k = 1, 2, \ldots, m$.

We define

$$\begin{aligned} PC = \bigl\{ & x \in \bigl[0, \sigma(T)\bigr]_{\mathbb{T}} \to R : x_k \in C(J_k, R), k = 0, 1, 2, \ldots, m \text{ and there exist} \\ & x\bigl(t_k^+\bigr) \text{ and } x\bigl(t_k^-\bigr) \text{ with } x\bigl(t_k^-\bigr) = x(t_k), k = 1, 2, \ldots, m\bigr\}, \end{aligned}$$

where x_k is the restriction of x to $J_k = (t_k, t_{k+1}]_{\mathbb{T}} \subset (0, \sigma(T)]_{\mathbb{T}}$, $k = 1, 2, \ldots, m$, and $J_0 = [0, t_1]_{\mathbb{T}}$, $t_{m+1} = \sigma(T)$.

Let

$$X = \bigl\{x : x \in PC, x(0) = x\bigl(\sigma(T)\bigr)\bigr\}$$

with the norm $\|x\| = \sup_{t \in [0,\sigma(T)]_{\mathbb{T}}} |x(t)|$, then X is a Banach space.

Lemma 2.1 *Suppose $M > 0$ and $h : [0, T]_{\mathbb{T}} \to R$ is rd-continuous, then x is a solution of*

$$x(t) = \int_0^{\sigma(T)} G(t,s)h(s)\triangle s + \sum_{k=1}^{m} G(t, t_k) I_k\bigl(x(t_k)\bigr), \quad t \in \bigl[0, \sigma(T)\bigr]_{\mathbb{T}},$$

where

$$G(t,s) = \begin{cases} \frac{e_M(s,t) e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}, & 0 \leq s \leq t \leq \sigma(T), \\ \frac{e_M(s,t)}{e_M(\sigma(T),0)-1}, & 0 \leq t < s \leq \sigma(T), \end{cases}$$

if and only if x is a solution of the boundary value problem

$$\begin{cases} x^{\triangle}(t) + Mx(\sigma(t)) = h(t), & t \in J, t \neq t_k, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), & k = 1, 2, \ldots, m, \\ x(0) = x(\sigma(T)). \end{cases}$$

Proof Since the proof is similar to that of [18], Lemma 3.1, we omit it here. □

Lemma 2.2 *Let $G(t,s)$ be defined as in Lemma* 2.1, *then*

$$\frac{1}{e_M(\sigma(T),0)-1} \le G(t,s) \le \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1} \quad \textit{for all } t,s \in [0,\sigma(T)]_{\mathbb{T}}.$$

Proof It is obvious, so we omit it here. □

Remark 2.1 Let $G(t,s)$ be defined as in Lemma 2.1, then $\int_0^{\sigma(T)} G(t,s)\triangle s = \frac{1}{M}$.

Let $m = \min_{t\in[0,T]_{\mathbb{T}}} p(t)$, $M = \max_{t\in[0,T]_{\mathbb{T}}} p(t)$, then $0 < m \le M < \infty$. For $u \in X$, we consider the following problem:

$$\begin{cases} x^{\triangle}(t) + Mx(\sigma(t)) = Mu(\sigma(t)) - p(t)u(\sigma(t)) + f(t,u(\sigma(t))), & t \in J, t \ne t_k, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), & k = 1,2,\ldots,m, \\ x(0) = x(\sigma(T)). \end{cases} \tag{2.1}$$

It follows from Lemma 2.1 that the problem (2.1) has a unique solution:

$$x(t) = \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k\bigl(x(t_k)\bigr), \quad t \in [0,\sigma(T)]_{\mathbb{T}},$$

where $h_u(s) = Mu(\sigma(s)) - p(t)u(\sigma(t)) + f(s,u(\sigma(s)))$, $s \in [0,T]_{\mathbb{T}}$.
We define the operator $\Phi : X \to X$ by

$$\Phi(u)(t) = \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k\bigl(u(t_k)\bigr), \quad t \in [0,\sigma(T)]_{\mathbb{T}}.$$

It is obvious that fixed points of Φ are solutions of the problem (1.1).

Lemma 2.3 *$\Phi : X \to X$ is completely continuous.*

Proof Since the proof is similar to that of [18], Lemma 3.3, we omit it here. □

Let

$$K = \bigl\{u \in X : u(t) \ge \delta\|u\|, t \in [0,\sigma(T)]_{\mathbb{T}}\bigr\},$$

where $\delta = \frac{1}{e_M(\sigma(T),0)} \in (0,1)$. It is not difficult to verify that K is a cone in X.
From Lemma 2.2, it is easy to obtain the following result.

Lemma 2.4 *Φ maps K into K.*

3 Main results

For convenience, we denote

$$f^0 = \lim_{u\to 0^+} \sup \max_{t\in[0,T]_{\mathbb{T}}} \frac{f(t,u)}{u}, \qquad f^\infty = \lim_{u\to \infty} \sup \max_{t\in[0,T]_{\mathbb{T}}} \frac{f(t,u)}{u},$$

$$f_0 = \lim_{u\to 0^+} \inf \min_{t\in[0,T]_{\mathbb{T}}} \frac{f(t,u)}{u}, \qquad f_\infty = \lim_{u\to \infty} \inf \min_{t\in[0,T]_{\mathbb{T}}} \frac{f(t,u)}{u},$$

and

$$I_0 = \lim_{u\to 0^+} \frac{I_k(u)}{u}, \qquad I_\infty = \lim_{u\to\infty} \frac{I_k(u)}{u}.$$

Now we state our main results.

Theorem 3.1 *Suppose that*

(H_1) $f_0 > M, f^\infty < m, I_\infty = 0$ *for any* k; *or*
(H_2) $f_\infty > M, f^0 < m, I_0 = 0$ *for any* k.

Then the problem (1.1) *has at least one positive solution.*

Proof Firstly, we assume (H_1) holds. Then there exist $\varepsilon > 0$ and $\beta > \alpha > 0$ such that

$$f(t,u) \geq (M+\varepsilon)u, \quad t \in [0,T]_{\mathbb{T}}, u \in (0,\alpha], \tag{3.1}$$

$$I_k(u) \leq \frac{[e_M(\sigma(T),0)-1]\varepsilon}{2Mme_M(\sigma(T),0)}u, \quad u \in [\beta,\infty) \text{ for any } k, \tag{3.2}$$

and

$$f(t,u) \leq (m-\varepsilon)u, \quad t \in [0,T]_{\mathbb{T}}, u \in [\beta,\infty). \tag{3.3}$$

Let $\Omega_1 = \{u \in X : \|u\| < r_1\}$, where $r_1 = \alpha$. Choose $u_0 = 1$, then $u_0 \in K\backslash\{0\}$. We assert that

$$u - \Phi u \neq \lambda u_0, \quad u \in K \cap \partial\Omega_1, \lambda \geq 0. \tag{3.4}$$

Suppose on the contrary that there exist $\overline{u} \in K \cap \partial\Omega_1$ and $\overline{\lambda} \geq 0$ such that

$$\overline{u} - \Phi\overline{u} = \overline{\lambda}u_0.$$

Let $\zeta = \min_{t\in[0,\sigma(T)]_{\mathbb{T}}} \overline{u}(t)$, then $\zeta \geq \delta\|\overline{u}\| = \delta r_2 = \beta$, and we have from (3.1)

$$\begin{aligned}
\overline{u}(t) &= \Phi(\overline{u})(t) + \overline{\lambda} \\
&= \int_0^{\sigma(T)} G(t,s)h_{\overline{u}}(s)\Delta s + \sum_{k=1}^m G(t,t_k)I_k\bigl(\overline{u}(t_k)\bigr) + \overline{\lambda} \\
&\geq \int_0^{\sigma(T)} G(t,s)\bigl[M - p(t) + M + \varepsilon\bigr]u\bigl(\sigma(t)\bigr)\Delta s + \overline{\lambda} \\
&\geq \frac{(M+\varepsilon)}{M}\zeta + \overline{\lambda}, \quad t \in \bigl[0,\sigma(T)\bigr]_{\mathbb{T}}.
\end{aligned}$$

Therefore,

$$\zeta = \min_{t\in[0,\sigma(T)]_{\mathbb{T}}} \overline{u}(t) \geq \frac{(M+\varepsilon)}{M}\zeta + \overline{\lambda} > \zeta,$$

which is a contradiction.

On the other hand, let $\Omega_2 = \{u \in X : \|u\| < r_2\}$, where $r_2 = \frac{\beta}{\delta}$.

Then $u \in K \cap \partial\Omega_2$, $0 < \delta\beta = \delta\|u\| \leq u(t) \leq \beta$, and in view of (3.2) and (3.3) we have

$$\begin{aligned}
\Phi(u)(t) &= \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k\bigl(u(t_k)\bigr) \\
&\leq \int_0^{\sigma(T)} G(t,s)\bigl[M - p(t) + m - \varepsilon\bigr]u\bigl(\sigma(s)\bigr)\triangle s \\
&\quad + \sum_{k=1}^{m} G(t,t_k)\frac{[e_M(\sigma(T),0) - 1]\varepsilon}{2Mme_M(\sigma(T),0)}u(t_k) \\
&\leq \frac{(M-\varepsilon)}{M}\|u\| + \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0) - 1}\sum_{k=1}^{m}\frac{[e_M(\sigma(T),0) - 1]\varepsilon}{2Mme_M(\sigma(T),0)}\|u\| \\
&= \frac{(M - \frac{\varepsilon}{2})}{M}\|u\| \\
&< \|u\|, \quad t \in \bigl[0,\sigma(T)\bigr]_{\mathbb{T}},
\end{aligned}$$

which yields $\|\Phi(u)\| < \|u\|$.

Therefore

$$\Phi u \neq \tau u, \quad u \in K \cap \partial\Omega_1, \tau \geq 1. \tag{3.5}$$

It follows from (3.4), (3.5), and Theorem 1.1 that Φ has a fixed point $u^* \in K \cap (\overline{\Omega}_2 \backslash \Omega_1)$, and u^* is the desired positive solution of the problem (1.1).

Next, suppose that (H_2) holds. Then we can choose $\varepsilon' > 0$ and $\beta' > \alpha' > 0$ such that

$$f(t,u) \geq \bigl(M + \varepsilon'\bigr)u, \quad t \in [0,T]_{\mathbb{T}}, u \in \bigl[\beta',\infty\bigr), \tag{3.6}$$

$$I_k(u) \leq \frac{[e_M(\sigma(T),0) - 1]\varepsilon'}{2Mme_M(\sigma(T),0)}u, \quad u \in \bigl(0,\alpha'\bigr] \text{ for any } k, \tag{3.7}$$

and

$$f(t,u) \leq \bigl(m - \varepsilon'\bigr)u, \quad t \in [0,T]_{\mathbb{T}}, u \in \bigl(0,\alpha'\bigr]. \tag{3.8}$$

Let $\Omega_3 = \{u \in X : \|u\| < r_3\}$, where $r_3 = \alpha'$. Then for any $u \in K \cap \partial\Omega_3$, $0 < \delta\|u\| \leq u(t) \leq \|u\| = \alpha'$.

It is similar to the proof of (3.5), by (3.7) and (3.8) we have

$$\Phi u \neq \tau u, \quad u \in K \cap \partial\Omega_4, \tau \geq 1. \tag{3.9}$$

Let $\Omega_4 = \{u \in X : \|u\| < r_4\}$, where $r_4 = \frac{\beta'}{\delta}$. Then for any $u \in K \cap \partial\Omega_4$, $u(t) \geq \delta\|u\| = \delta r_4 = \beta'$, by (3.6) , it is easy to obtain

$$u - \Phi u \neq \lambda u_0, \quad u \in K \cap \partial\Omega_3, \lambda \geq 0. \tag{3.10}$$

It follows from (3.9), (3.10), and Theorem 1.1 that Φ has a fixed point $u^* \in K \cap (\overline{\Omega}_4 \backslash \Omega_3)$, and u^* is the desired positive solution of the problem (1.1). □

In particular, we have the following results, which are main results of [18].

Corollary 3.1 *Suppose that*

(H$_1$) $f_0 = \infty, f^\infty = 0, I_\infty = 0$ *for any* k; *or*
(H$_2$) $f_\infty = \infty, f^0 = 0, I_0 = 0$ *for any* k.

Then the problem (1.1) *has at least one positive solution.*

Theorem 3.2 *Suppose that*

(H$_3$) $f^0 < m, f^\infty < m, I_0 = 0, I_\infty = 0$;
(H$_4$) *there exists* $\rho > 0$ *such that*

$$\min\bigl\{f(t,u) - p(t)u \mid t \in [0,T]_{\mathbb{T}}, \delta\rho \le u \le \rho\bigr\} > 0. \tag{3.11}$$

Then the problem (1.1) *has at least two positive solutions.*

Proof By (H$_3$), from the proof of Theorem 3.1, we see that there exist $\beta'' > \rho > \alpha'' > 0$ such that

$$\Phi u \neq \tau u, \quad u \in K \cap \partial\Omega_5, \tau \ge 1, \tag{3.12}$$

$$\Phi u \neq \tau u, \quad u \in K \cap \partial\Omega_6, \tau \ge 1, \tag{3.13}$$

where $\Omega_5 = \{u \in X : \|u\| < r_5\}$, $\Omega_6 = \{u \in X : \|u\| < r_6\}$, $r_5 = \alpha''$, $r_6 = \frac{\beta''}{\delta}$.
By (3.11) of (H$_4$), we can choose $\varepsilon > 0$ such that

$$f(t,u) - p(t)u \ge \varepsilon u, \quad t \in [0,T]_{\mathbb{T}}, \delta\rho \le u \le \rho. \tag{3.14}$$

Let $\Omega_7 = \{u \in X : \|u\| < \rho\}$, for any $u \in K \cap \partial\Omega_7$, $\delta\rho = \delta\|u\| \le u(t) \le \|u\| = \rho$, from (3.14), it is similar to the proof of (3.4), and we have

$$u - \Phi u \neq \lambda u_0, \quad u \in K \cap \partial\Omega_7, \lambda \ge 0. \tag{3.15}$$

By Theorem 1.1, from (3.12), (3.13), and (3.15) we conclude that Φ has two fixed points $u^{**} \in K \cap (\overline{\Omega}_6 \backslash \Omega_7)$ and $u^{***} \in K \cap (\overline{\Omega}_7 \backslash \Omega_5)$, and u^{**} and u^{***} are two positive solutions of the problem (1.1). □

Similar to Theorem 3.2, we have the following.

Theorem 3.3 *Suppose that*

(H$_5$) $f_0 > M, f_\infty > M$;
(H$_6$) *there exists* $\rho > 0$ *such that*

$$\max\bigl\{f(t,u) - p(t)u \mid t \in [0,T]_{\mathbb{T}}, \delta\rho \le u \le \rho\bigr\} < 0;$$

$$I_k(u) \le \frac{[e_M(\sigma(T),0) - 1]}{Mme_M(\sigma(T),0)}u, \quad \delta\rho \le u \le \rho \textit{ for any } k.$$

Then the problem (1.1) *has at least two positive solutions.*

Remark 3.1 If (H_3) in Theorem 3.2 is replaced by $f^0 = 0, f^\infty = 0$, or if (H_5) in Theorem 3.3 is replaced by $f_0 = \infty, f_\infty = \infty$, then the results of Theorem 3.2 and Theorem 3.3 are also hold.

Competing interests
The author declares that she has no competing interests.

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