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Research Article

On the Study of Global Solutions for a Nonlinear Equation

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The well-posedness of global strong solutions for a nonlinear partial differential equation including the Novikov equation is established provided that its initial value $v_0(x)$ satisfies a sign condition and $v_0(x) \in H^s(R)$ with $s > 3/2$. If the initial value $v_0(x) \in H^s(R)$ $(1 \leq s \leq 3/2)$ and the mean function of $(1-\partial_x^2)v_0(x)$ satisfies the sign condition, it is proved that there exists at least one global weak solution to the equation in the space $v(t,x) \in L^2([0,+\infty), H^s(R))$ in the sense of distribution and $v_x \in L^\infty([0,+\infty)\times R)$.

1. Introduction

Recently, Wu [1] obtained the existence of local solutions in the space $C([0,T); H^s(R)) \cap C^1([0,T); H^{s-1}(R))$ with $s > 3/2$ for the following nonlinear equation:

$$\begin{aligned} v_t - v_{txx} + k v^m v_x + (m+3) v^{m+1} v_x \\ = (m+2) v^m v_x v_{xx} + v^{m+1} v_{xxx} + \lambda (v - v_{xx}), \end{aligned} \tag{1}$$

where $m \geq 0$ is a natural number, $k \geq 0$, and λ is a constant. Letting $m = 0$ and $\lambda = 0$, (1) becomes the Camassa-Holm equation [2]. If $m = 1$, $k = 0$, and $\lambda = 0$, (1) reduces to the Novikov equation [3].

A lot of works have been carried out to study various dynamic properties for the Camassa-Holm and the Novikov equations. Xin and Zhang [4] proved that there exists a global weak solution for the Camassa-Holm equation in the space $H^1(R)$ without the assumption of sign conditions on the initial value. Coclite et al. [5] investigated the global weak solutions for a generalized hyperelastic rod wave equation or a generalized Camassa-Holm equation. It is shown in Constantin and Escher [6] that the blowup occurs in the form of breaking waves; namely, the solution remains bounded but its slope becomes unbounded in finite time. After wave breaking, the solution can be continued uniquely either as a global conservative weak solution [7] or a global dissipative solution [8–10]. The periodic and the nonperiodic Cauchy problems for the Novikov equation were discussed by Grayshan [11] in the Sobolev space. Using the Galerkin-type approximation method, Himonas and Holliman [12] established the well-posedness for the Novikov model in the Sobolev space $H^s(R)$ with $s > 3/2$ on both the line and the circle. The scattering theory was employed in Hone et al. [13] to find nonsmooth explicit soliton solutions with multiple peaks for the Novikov equation. Wu and Zhong [14] proved the existence of local strong and weak solutions for a generalized Novikov equation.

The objective of this work is to study (1) with $k = 0$. Namely, we investigate the problem

$$\begin{aligned} v_t - v_{txx} + (m+3) v^{m+1} v_x \\ = (m+2) v^m v_x v_{xx} + v^{m+1} v_{xxx} + \lambda (v - v_{xx}), \\ v(0,x) = v_0(x), \end{aligned} \tag{2}$$

where m, k, and λ are described in (1). Assuming that the initial value $v_0(x)$ satisfies a sign condition and $v_0(x) \in H^s(R)$, $s > 3/2$, we will show that there exists a unique global strong solution in the Sobolev space $C([0,\infty); H^s(R)) \cap C^1([0,\infty); H^{s-1}(R))$. If the initial value $v_0(x) \in H^s(R)$ $(1 \leq s \leq 3/2)$ and the mean function of $(1-\partial_x^2)v_0(x)$ satisfies the sign condition, it is shown that there exists at least one global weak solution to the equation in the space $v(t,x) \in$

$L^2([0,+\infty), H^s(R))$ in the sense of distribution and $v_x \in L^\infty([0,+\infty) \times R)$.

The structure of this paper is as follows. The main results are given in Section 2. Several lemmas are given in Section 3. Section 4 establishes the proof of the main results.

2. Main Results

We define

$$\phi(x) = \begin{cases} e^{1/(x^2-1)}, & |x| < 1, \\ 0, & |x| \geq 1, \end{cases} \tag{3}$$

and let $\phi_\varepsilon(x) = \varepsilon^{-1/4}\phi(\varepsilon^{-1/4}x)$ with $0 < \varepsilon < 1/4$. For the convolution $v_{\varepsilon 0} = \phi_\varepsilon \star v_0$, we know that $v_{\varepsilon 0} \in C^\infty$ for any $v_0 \in H^s$ with $s > 0$. Notation $(1-\partial_x^2)v \in N^+(R)$ (or equivalently $(1-\partial_x^2)v \in N^-(R)$) means that the mean function of $(1-\partial_x^2)v$ is nonnegative; namely, $(1-\partial_x^2)v \star \phi_\varepsilon \geq 0$ (or equivalently $(1-\partial_x^2)v \star \phi_\varepsilon \leq 0$) for an arbitrary sufficiently small $\varepsilon > 0$. For $T > 0$ and nonnegative number s, we let $C([0,T); H^s(R))$ denote the Frechet space of all continuous H^s-valued functions on $[0,T)$ and write $\Lambda = (1-\partial_x^2)^{1/2}$.

We state the result of global strong solutions for problem (2).

Theorem 1. *Let $v_0(x) \in H^s(R)$, $s > 3/2$, and $(1-\partial_x^2)v_0 \geq 0$ for all $x \in R$ or $(1-\partial_x^2)v_0 \leq 0$ for all $x \in R$. Then problem* (2) *has a unique strong solution satisfying*

$$v(t,x) \in C([0,\infty); H^s(R)) \cap C^1([0,\infty); H^{s-1}(R)). \tag{4}$$

Definition 2. A function $v(t,x) \in L^2([0,+\infty), H^s(R))$ is called a global weak solution to problem (2) if for every $T > 0$ and all $\varphi(t,x) \in C_0^\infty([0,T] \times R)$, it holds that

$$\begin{aligned} \int_0^T \int_R \big[& v_t - v_{txx} + (m+3)v^{m+1}v_x - (m+2)v^m v_x v_{xx} \\ & - v^{m+1}v_{xxx} - \lambda(v - v_{xx})\big]\varphi(t,x)\,dx\,dt = 0 \end{aligned} \tag{5}$$

with $v(0,x) = v_0(x)$.

Now we give the main result of global weak solution for problem (2).

Theorem 3. *Let $v_0(x) \in H^s(R)$, $1 \leq s \leq 3/2$, $(1-\partial_x^2)v_0 \in N^+(R)$ (or equivalently $(1-\partial_x^2)v_0 \in N^-(R)$). Then problem* (2) *has a unique global weak solution $v(t,x) \in L^2([0,+\infty), H^s(R))$ in the sense of distribution and $v_x \in L^\infty([0,+\infty) \times R)$.*

3. Several Lemmas

Lemma 4 (see [1]). *Let $v_0(x) \in H^s(R)$ with $s > 3/2$. Then the Cauchy problem* (2) *has a unique local solution*

$$v(t,x) \in C([0,T); H^s(R)) \cap C^1([0,T); H^{s-1}(R)), \tag{6}$$

where $T > 0$ depends on $\|v_0\|_{H^s(R)}$.

Using the first equation of system (2) derives

$$\frac{d}{dt}\int_R \left(v^2 + v_x^2\right)dx = 2\lambda \int_R \left(v^2 + v_x^2\right)dx, \tag{7}$$

which yields the conservation law

$$\begin{aligned} \int_R \left(v^2 + v_x^2\right)dx &= \int_R \left(v_0^2 + v_{0x}^2\right)dx \\ & \quad + 2\lambda \int_0^t \int_R \left(v^2 + v_x^2\right)dx\,dt. \end{aligned} \tag{8}$$

Lemma 5 (see [1]). *Let $s > 3/2$ and the function $v(t,x)$ is a solution of problem* (2) *and the initial data $v_0(x) \in H^s$. Then the following inequalities hold:*

$$\begin{aligned} \|v\|_{H^1}^2 &\leq \int_R \left(v^2 + v_x^2\right)dx \leq \int_R \left(v_0^2 + v_{0x}^2\right)dx, \quad \text{if } \lambda \leq 0. \\ \|v\|_{H^1}^2 &\leq \int_R \left(v^2 + v_x^2\right)dx \leq e^{2\lambda t}\int_R \left(v_0^2 + v_{0x}^2\right)dx, \quad \text{if } \lambda > 0. \end{aligned} \tag{9}$$

For $q \in (0, s-1]$, there is a constant c such that

$$\begin{aligned} & \int_R \left(\Lambda^{q+1}v\right)^2 dx \\ & \leq \int_R \left(\Lambda^{q+1}v_0\right)^2 dx \\ & \quad + c\int_0^t \|v\|_{H^{q+1}}^2 \Big(|\lambda| + \left(\|v\|_{L^\infty}^{m-1} + \|v\|_{L^\infty}^m\right)\|v_x\|_{L^\infty} \\ & \quad + \|v\|_{L^\infty}^{m-1}\|v_x\|_{L^\infty}^2\Big)d\tau. \end{aligned} \tag{10}$$

For $q \in [0, s-1]$, there is a constant c such that

$$\begin{aligned} \|v_t\|_{H^q} \leq c\|v\|_{H^{q+1}} \Big(& |\lambda| + \left(\|v\|_{L^\infty}^{m-1} + \|v\|_{L^\infty}^m\right)\|v\|_{H^1} \\ & + \|v\|_{L^\infty}^m\|v_x\|_{L^\infty} + \|v\|_{L^\infty}^{m-1}\|v_x\|_{L^\infty}^2\Big). \end{aligned} \tag{11}$$

Consider the differential equation

$$\begin{aligned} p_t &= v^{m+1}(t,p), \quad t \in [0,T), \\ p(0,x) &= x, \end{aligned} \tag{12}$$

where $v(t,x)$ is the solution of problem (2) and T is the maximal existence time of the solution.

Lemma 6. *Let $v_0 \in H^s(R)$, $s \geq 3$, and let $T > 0$ be the maximal existence time of the solution to problem* (2)*. Then system* (12) *has a unique solution $p(t,x) \in C^1([0,T) \times R)$. Moreover, the map $p(t,\cdot)$ is an increasing diffeomorphism of R with $p_x(t,x) > 0$ for $(t,x) \in [0,T) \times R$.*

Proof. From Lemma 4, we know that there exists a unique solution

$$v(t,x) \in C([0,T); H^s(R)) \cap C^1([0,T); H^{s-1}(R)). \tag{13}$$

The Sobolev imbedding theorem derives $H^s(R) \in C^1(R)$. This means that two functions $v(t,x)$ and $v_x(t,x)$ are bounded, Lipschitz in space and C^1 in time. Using the existence and uniqueness theorem of ordinary differential equations, we derive that problem (12) has a unique solution $p(t,x) \in C^1([0,T) \times R)$.

Differentiating (12) with respect to x gives rise to

$$\frac{d}{dt} p_x = (m+1) v^m v_x(t,p) p_x, \quad t \in [0,T), \qquad p_x(0,x) = 1, \tag{14}$$

from which we obtain

$$p_x(t,x) = \exp\left(\int_0^t (m+1) v^m v_x(\tau, p(\tau,x))\, d\tau\right). \tag{15}$$

For every $T' < T$, applying the Sobolev imbedding theorem results in

$$\sup_{(\tau,x)\in[0,T')\times R} |v_x(\tau,x)| < \infty. \tag{16}$$

Therefore, we know that there exists a constant $M > 0$ such that $p_x(t,x) \ge e^{-Mt}$ for $(t,x) \in [0,T) \times R$. The proof is completed. □

Lemma 7. *Let $v_0 \in H^s$ with $s \ge 3$, and let $T > 0$ be the maximal existence time of the problem* (2)*; it holds that*

$$y(t, p(t,x)) p_x^2(t,x) = y_0(x) e^{\int_0^t (mv^m v_x + \lambda) d\tau}, \tag{17}$$

where $(t,x) \in [0,T) \times R$ and $y := v - v_{xx}$.

Proof. We have

$$\begin{aligned}
&\frac{d}{dt}\left[y(t,p(t,x)) p_x^2(t,x)\right] \\
&\quad = y_t p_x^2 + 2y p_x p_{xt} + y_x p_t p_x^2 \\
&\quad = y_t p_x^2 + 2y(m+1) v^m v_x p_x^2 + v^{m+1} y_x p_x^2 \\
&\quad = \left[y_t + (m+2) v^m v_x y + y_x v^{m+1}\right] p_x^2 + m v^m v_x y p_x^2 \\
&\quad = \left[v_t - v_{txx} + (m+2) v^m v_x (v - v_{xx}) \right. \\
&\qquad \left. + v^{m+1}(v_x - v_{xxx}) - \lambda(v - v_{xx})\right] p_x^2 \\
&\qquad + (mv^m v_x + \lambda) y p_x^2 \\
&\quad = \left[v_t - v_{txx} + (m+3) v^{m+1} v_x - (m+2) v^m v_x v_{xx} \right. \\
&\qquad \left. - v^{m+1} v_{xxx} - \lambda(v - v_{xx})\right] p_x^2 \\
&\qquad + (mv^m v_x + \lambda) y p_x^2 \\
&\quad = (mv^m v_x + \lambda) y p_x^2,
\end{aligned} \tag{18}$$

from which we have

$$y(t,p(t,x)) p_x^2(t,x) = p_x(0,x) y_0(x) e^{\int_0^t (mv^m v_x + \lambda) d\tau}. \tag{19}$$

Using $p_x(0,x) = 1$ completes the proof. □

Lemma 8. *If $v_0 \in H^s(R)$, $s \ge 3/2$, $(1-\partial_x^2)v_0 \ge 0$ or $(1-\partial_x^2)v_0 \le 0$, then the solution of problem* (2) *satisfies*

$$\|v_x\|_{L^\infty} \le \|v\|_{L^\infty}. \tag{20}$$

Proof. We only need to prove this lemma for the case $v_0 - v_{0xx} \ge 0$ since the proof of the other case $(1-\partial_x^2)v_0 \le 0$ is similar. It follows from Lemmas 6 and 7 that $v - v_{xx} \ge 0$. Letting $\xi(t,x) = v - v_{xx}$, we have

$$v = \frac{1}{2} e^{-x} \int_{-\infty}^x e^{\eta} \xi(t,\eta)\, d\eta + \frac{1}{2} e^{x} \int_x^{\infty} e^{-\eta} \xi(t,\eta)\, d\eta, \tag{21}$$

which derives

$$\begin{aligned}
\partial_x v(t,x) &= -\frac{1}{2}\left(e^{-x} \int_{-\infty}^x e^{\eta}\xi(t,\eta)\, d\eta + e^{x} \int_x^{\infty} e^{-\eta}\xi(t,\eta)\, d\eta\right) \\
&\quad + e^x \int_x^\infty e^{-\eta}\xi(t,\eta)\, d\eta \\
&= -v(t,x) + e^x \int_x^\infty e^{-\eta}\xi(t,\eta)\, d\eta \\
&\ge -v(t,x).
\end{aligned} \tag{22}$$

On the other hand, we have

$$\begin{aligned}
\partial_x v(t,x) &= \frac{1}{2}\left(e^{-x} \int_{-\infty}^x e^{\eta}\xi(t,\eta)\, d\eta + e^{x} \int_x^{\infty} e^{-\eta}\xi(t,\eta)\, d\eta\right) \\
&\quad - e^{-x} \int_{-\infty}^x e^{\eta}\xi(t,\eta)\, d\eta \\
&= v(t,x) - e^{-x} \int_{-\infty}^x e^{\eta}\xi(t,\eta)\, d\eta \\
&\le v(t,x).
\end{aligned} \tag{23}$$

The inequalities (22) and (23) derive that inequality (20) is valid. □

Lemma 9. *For $s > 0$, $u \in H^s(R)$, and $u_\varepsilon = \phi_\varepsilon \star u$, it holds that*

$$\begin{gathered}
\|u_{\varepsilon x}\|_{L^\infty} \le c\|u_x\|_{L^\infty}, \\
\|u_\varepsilon\|_{H^q} \le c, \quad \text{if } q \le s, \\
\|u_\varepsilon\|_{H^q} \le c\varepsilon^{(s-q)/4}, \quad \text{if } q > s, \\
\|u_\varepsilon - u\|_{H^q} \le c\varepsilon^{(s-q)/4}, \quad \text{if } q \le s, \\
\|u_\varepsilon - u\|_{H^s} = o(1),
\end{gathered} \tag{24}$$

where c is a constant independent of ε.

The proof of this lemma can be found in [15, 16].

From Lemma 4, it derives that the Cauchy problem

$$\begin{aligned} v_t - v_{txx} &= -(m+3)\, v^{m+1} v_x + (m+2)\, v^m v_x v_{xx} \\ &\quad + v^{m+1} v_{xxx} + \lambda (v - v_{xx}) \\ &= -\frac{m+3}{m+2}\left(v^{m+2}\right)_x + \frac{1}{m+2}\partial_x^3\left(v^{m+2}\right) \\ &\quad - (m+1)\,\partial_x\left(v^m v_x^2\right) + v^m v_x v_{xx} + \lambda (v - v_{xx}), \\ v(0,x) &= v_{\varepsilon 0}(x), \end{aligned} \tag{25}$$

has a unique solution v depending on the parameter ε. We write $v_\varepsilon(t,x)$ to represent the solution of problem (25). Using Lemma 4 derives that $v_\varepsilon(t,x) \in C^\infty([0,T), H^\infty(R))$ since $v_{\varepsilon 0}(x) \in C_0^\infty(R)$.

Lemma 10. *Provided that $v_0 \in H^s(R)$, $1 \le s \le 3/2$, and $(1-\partial_x^2)v_0 \in N^+(R)$ (or equivalently $(1-\partial_x^2)v_0 \in N^-(R)$), then there exists a constant $c > 0$ independent of ε and t such that the solution of problem* (25) *satisfies*

$$\|v_{\varepsilon x}\|_{L^\infty} \le c e^{ct}. \tag{26}$$

Proof. Using Lemmas 5 and 9, if $v_0 \in H^s(R)$ with $1 \le s \le 3/2$, we have

$$\|v_\varepsilon\|_{L^\infty(R)} \le c\|v_\varepsilon\|_{H^1(R)} \le c e^{ct}\|v_{\varepsilon 0}\|_{H^1(R)} \le c e^{ct}, \tag{27}$$

where c is independent of ε and t.

From Lemma 8, we have

$$\|v_{\varepsilon x}\|_{L^\infty(R)} \le \|v_\varepsilon\|_{L^\infty(R)}, \tag{28}$$

which completes the proof. □

4. Proof of Main Results

Proof of Theorem 1. Since $\|v\|_{L^\infty(R)} \le c\|v\|_{H^1(R)} \le c e^{ct}$ and taking $q+1 = s$ in inequality (10), we have

$$\|v\|_{H^s}^2 \le \|v_0\|_{H^s}^2 + c\int_0^t e^{c\tau}\|v\|_{H^s}^2\left(\|v_x\|_{L^\infty} + \|v_x\|_{L^\infty}^2\right) d\tau, \tag{29}$$

from which we obtain

$$\|v\|_{H^s} \le \|v_0\|_{H^s} e^{c\int_0^t e^{c\tau}(\|v_x\|_{L^\infty} + \|v_x\|_{L^\infty}^2) d\tau}. \tag{30}$$

Applying Lemma 8 yields

$$\|v\|_{H^s} \le \|v_0\|_{H^s} c e^{e^{ct}}, \tag{31}$$

from which we complete the proof of Theorem 1. □

Provided that $1 \le s \le 3/2$, for problem (25), applying Lemmas 5, 8, and 10, and the Gronwall's inequality, we obtain the inequalities

$$\begin{aligned} \|v_\varepsilon\|_{H^1} &\le \|v_{\varepsilon 0}\|_{H^1} \le c e^{ct}, \\ \|v_\varepsilon\|_{H^q} &\le c\|v_{\varepsilon 0}\|_{H^q} \exp\left[\int_0^t \left(\|v_{\varepsilon x}\| + \|v_{\varepsilon x}\|_{L^\infty}^2\right) d\tau\right] \le c e^{e^{ct}}, \\ \|u_{\varepsilon t}\|_{H^r} &\le c\|u_\varepsilon\|_{H^{r+1}}\left(1 + e^{ct}\right) \le c\left(1 + e^{ct}\right), \end{aligned} \tag{32}$$

where $q \in (0,s]$, $r \in [0, s-1]$, and c is a constant independent of t and ε. Using the Aubin compactness theorem, we know that that there is a subsequence $\{v_{\varepsilon_n}\}$ of $\{v_\varepsilon\}$ such that $\{v_{\varepsilon_n}\}$ and their temporal derivatives $\{v_{\varepsilon_n t}\}$ converge weakly to a function $v(t,x)$ and its derivative v_t in the space $L^2([0,T], H^s(R))$ and $L^2([0,T], H^{s-1}(R))$, respectively, where T is an arbitrary fixed positive number. In addition, for any real number $M_1 > 0$, $\{v_{\varepsilon_n}\}$ converges strongly to the function v in the space $L^2([0,T], H^q(-M_1, M_1))$ for $q \in (0,s]$ and $\{v_{\varepsilon_n t}\}$ converges strongly to v_t in the space $L^2([0,T], H^r(-M_1, M_1))$ for $r \in [0, s-1]$.

Proof of Theorem 3. For an arbitrary fixed $T > 0$, using Lemma 10, we know that $\{v_{\varepsilon_n x}\}$ ($\varepsilon_n \to 0$) is bounded in the space L^∞. Therefore, we derive that the sequences $\{v_{\varepsilon_n}\}$, $\{v_{\varepsilon_n x}\}$, $\{v_{\varepsilon_n x}^2\}$, and $\{v_{\varepsilon_n x}^3\}$ converge weakly to v, v_x, v_x^2, and v_x^3 in $L^2([0,T], H^r(-R_1, R_1))$ for any $r \in [0, s-1)$, separately. Applying the identity $v^m(v_x^2)_x = (v^m v_x^2)_x - (v^m)_x v_x^2$, we conclude that v satisfies the equation

$$\begin{aligned} &-\int_0^T\int_R v(\varphi_t - \varphi_{xxt})\, dx\, dt \\ &= \int_0^T\int_R \Bigg[\left(\frac{m+3}{m+2}v^{m+2} + (m+1)\, v^m v_x^2\right)\varphi_x \\ &\qquad - \frac{1}{m+2}v^{m+2}\varphi_{xxx} - \frac{1}{2}v^m v_x^2 \varphi_x \\ &\qquad - \frac{m}{2}v^{m-1}v_x^3\varphi + \lambda v(\varphi - \varphi_{xx})\Bigg] dx\, dt, \end{aligned} \tag{33}$$

where $\varphi(t,x) \in C_0^\infty([0,T]\times R)$. We know that $Y = L^1([0,T]\times R)$ is a separable Banach space and $\{v_{\varepsilon_n x}\}$ is a bounded sequence in the dual space $Y^* = L^\infty([0,T]\times R)$ of Y. Thus, there exists a subsequence of $\{v_{\varepsilon_n x}\}$, still denoted by $\{v_{\varepsilon_n x}\}$, weakly star convergent to a function u in $L^\infty([0,T]\times R)$. Since $\{v_{\varepsilon_n x}\}$ weakly converges to v_x in $L^2([0,T]\times R)$, it derives that $v_x = u$ almost everywhere. Therefore, we obtain $v_x \in L^\infty([0,T]\times R)$. Since $T > 0$ is an arbitrary number, we complete the proof of existence of global weak solutions to problem (2). □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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