# Review of the Phenomenology of Noncommutative Geometry \*

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#### Abstract

We present a pedagogical review of particle physics models that are based on the noncommutativity of space-time,  $[\hat{x}_{\mu}, \hat{x}_{\nu}] = i\theta_{\mu\nu}$ , with specific attention to the phenomenology these models predict in particle experiments either in existence or under development. We summarize results obtained for high energy scattering such as would occur for example in a future  $e^+e^$ linear collider with  $\sqrt{s} = 500 \ GeV$ , as well as low energy experiments such as those pertaining to elementary electric dipole moments and other CP violating observables, and finally comment on the status of phenomenological work in cosmology and extra dimensions.

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# 4 Challenges and Conclusions

## 1 What is Noncommutative Physics?

The number of papers on noncommutative physics has been increasing rapidly in the past 15 years. This is undoubtably due to the recent ground-breaking work in string theory demonstrating how noncommutative geometry can arise in the context of D-branes [1, 2]; as the string theory community refines the fundamentals of noncommutative geometry, model-building efforts directed at experimental detection of noncommutative effects grow more numerous. This review attempts to introduce to the non-specialist the extensive work on noncommutative phenomenology in the literature and provide a summary of the work in progress and challenges ahead. This section will define the key ideas necessary to understand only the phenomenological work on noncommutative geometry. For the reader interested in the more formal side of the theory there is a large body of papers available at an accessible level (see, for example, [3, 4, 5, 6, 7, 8]).

To begin, we must define what we mean by "noncommutative space-time": Noncommutative space-time [4] is a deformation of ordinary space-time in which the space-time coordinates  $x_{\mu}$ , representable by Hermitian operators  $\hat{x}_{\mu}$ , do not commute:

$$[\widehat{x}_{\mu}, \widehat{x}_{\nu}] = i\theta_{\mu\nu} \tag{1}$$

Here  $\theta_{\mu\nu}$  is the deformation parameter: ordinary space-time is obtained in the  $\theta_{\mu\nu} \to 0$  limit. By convention it is a real tensor [9] antisymmetric under  $\mu \leftrightarrow \nu$ . Note that  $\theta_{\mu\nu}$  has dimensions of length-squared; the physical interpretation of this is that  $\theta_{\mu\nu}$  is the smallest patch of area in the  $\mu\nu$ -plane one may deem "observable," analogous to the role  $\hbar$  plays in  $[\hat{x}_i, \hat{p}_j] = i\hbar \delta_{ij}$ , defining the corresponding smallest patch of observable phase space in quantum mechanics. Noncommutative space-time as defined above is therefore one parameterization of the supposed limit to the resolution with which one may probe space-time itself. This type of space-time is often called "fuzzy" [10], as there is no definite meaning to a "point"; in noncommutative quantum field theory (see Section 2.1 below) interactions between fields at distinct points are "smeared" by a  $\theta$ -dependent weighting function. Noncommutative phenomenology in this review will involve calculating observables to  $\mathcal{O}(\theta)$  unless otherwise noted.

Noncommuting coordinates are not new in physics: a nonrelativistic charged particle in a strong magnetic field, for example, moves in a noncommuting space. Taking the particle's motion to be in the x - y plane in the presence of a *B*-field pointing in the *z*-direction, the Lagrangian is

$$L = \frac{1}{2}mv^2 + \frac{e}{c}v \cdot A - V(x, y) \tag{2}$$

where m and e are the paricle's mass and charge, respectively,  $A_{\mu} = (0, 0, 0, Bx)$  and V is some conservative potential. Taking the strong field limit  $B \to \infty$ , this Lagrangian becomes

$$L = \frac{eB}{c}x\dot{y} - V(x,y) \tag{3}$$

which is of the form  $p\dot{q} - h(p,q)$  if we identify  $(\frac{eBx}{c}, y)$  as canonical coordinates, *i.e.*  $\{x, y\} = \frac{c}{eB}$ [11, 12, 13]. This behaviour is closely related to the origin of noncommutative geometry in string theory [1]). More to the point, noncommutative geometry is expected on quite general grounds in any theory that seeks to incorporate gravity into a quantum field theory. Semi-classically, to localize a particle to within a Planck length  $\lambda_P$  in any given plane, an energy equal to the Planck mass must be available to the particle; this creates a black hole that swallows the particle. To avoid this, one must insist  $\sum_{i < j} \Delta x_i \Delta x_j \geq \lambda_P^2$ ; alternatively, one is led to think of space as a noncommutative algebra upon trying to quantize the Einstein theory [14, 15]. Noncommuting space-time coordinates are unobservable in this case as  $\lambda_P^{-1} \approx 10^{19} \text{ GeV}$ , but they may have visible effects if  $\theta$  is large or if  $\lambda_P^{-1} \approx 1 \text{ TeV}$  as is possible in theories with extra dimensions (see 3.4 below).

In this review we consider observables in two cases: high energy collider and low energy precision experiments. The former includes  $e^+e^-$  linear and hadronic colliders, whereas the latter includes atomic experiments, anomalous magnetic moment and electric dipole moment measurements, and other CP-violation tests. After introducing the basics of noncommutative quantum field theory in Section 2, we present in Section 3 the status of the main areas of this phenomenological work in noncommutative geometry. We will also mention cosmological observables and the possibility of noncommuting extra dimensions, and in Section 4 we shall indicate the directions in which work in phenomenological noncommutative geometry needs more attention.

## 2 Status of a Noncommutative Standard Model

At present, there is no consensus in the literature as to the precise form of the Noncommutative Standard Model (NCSM) due to some theoretical difficulties which we will discuss below. However there is very promising work underway [16] which seems to explicitly show that the NCSM is realizable, albeit by some rather nontrivial adjustments to the naive model (see 2.2 below). Most of the work in noncommutative phenomenology assumes that these difficulties will be eventually addressed, perhaps motivated by the confidence that the low energy theory (the Standard Model) and the high energy theory (String Theory) are both believable candidates for descriptions of Nature, and that a NCSM consisting of aspects of both should be a theory which is valid for intermediate energies [17]. We now present some of the essential features of a noncommutative quantum field theory.

## 2.1 Quantum Field Theory Details

#### 2.1.1 Fundamentals

We define  $\theta$  to be the average magnitude of an element of  $\theta_{\mu\nu}$ ; physically  $\frac{1}{\sqrt{\theta}}$  corresponds to the energy threshold beyond which a particle moves and interacts in a distorted space-time. We expect the effects of the deformation  $\theta_{\mu\nu}$  to be somehow manifest at energies E well below  $\frac{1}{\sqrt{\theta}}$ , where an effective operator expansion in the small parameter  $E\sqrt{\theta}$  is feasible. Essentially all the studies we present subsequently approach the theory in this manner.

Working in noncommutative field theory is equivalent to working with ordinary (commutative) theory and replacing the usual product by the  $\star$  product defined as follows:

$$(f \star g)(x) \equiv e^{i\theta_{\mu\nu}\partial^y_{\mu}\partial^z_{\nu}} f(y)g(z) \mid_{y=z=x}$$
(4)

With this definition (1) holds in function space equipped with a  $\star$  product:

$$[x_{\mu}, x_{\nu}]_{\star} = i\theta_{\mu\nu} \tag{5}$$

Note that when f(x) and g(x) are fields the  $\star$  product in Equation 4 can be written in momentum space as

$$(f \star g)(x) \to f(p)g(q)e^{ip^{\mu}\theta_{\mu\nu}q^{\nu}} \equiv f(p)g(q)e^{ip\times q}$$
(6)

where we have defined  $p \times q \equiv p^{\mu} \theta_{\mu\nu} q^{\nu}$ . This  $\star$  product intuitively replaces the point-by-point multiplication of two fields by a type of "smeared" product. More detailed analysis of 1- and 2-point functions [18] bears out this intuition: spacetime is only well defined down to distances of order

 $\sqrt{\theta}$  so functions of spacetime must be appropriately averaged over the appropriate neighborhood. More precisely, in each (i, j) plane, we must replace

$$\phi(x_i, x_j) \to \int dx'_i dx'_j \phi(x'_i, x'_j) e^{-\frac{(x_i - x'_i)^2 + (x_j - x'_j)^2}{\theta_{ij}}} (\pi \theta_{ij})^{-1}$$
(7)

Making predictions with a noncommutative model therefore appears straightforward: just replace each ordinary product with a  $\star$  product. However, a number of difficulties arise with this replacement which make the task of formulating a Noncommutative Standard Model (NCSM) not entirely trivial. In this section we review these difficulties and the solutions found thus far.

As noted above, the method of computing noncommutative field theory amplitudes is effected by replacing the ordinary function product with the  $\star$  product in the Lagrangian. The theory is otherwise identical to the commuting one (i.e. the Feynman path integral formulation provides the usual setting for doing quantum field theory). In practice it is simpler to work in momentum space, where the only modification to the Feynman rules is to replace each vertex factor  $\lambda$  with a momentum-dependent factor  $\lambda V(p_1, p_2, ..., p_n)$  with

$$V(p_1, p_2, ... p_n) = e^{-\frac{i}{2}\sum_{i < j} p_i \times p_j}$$
(8)

Here  $p_i$  is the momentum flowing into the vertex from the *i*-th field. In this formulation where noncommutivity introduces pairwise contractions of momenta in Feynman diagram amplitudes (*i.e.* factors of  $e^{ip \times q}$ , see (6)) it is clear that some of these contractions are trivial, *e.g.*  $e^{iq \times q} = 1$  since  $\theta_{\mu\nu}$  is antisymmetric. In particular when the momenta associated with the internal lines of a given Feynman diagram are contracted and only the contraction of the external momenta remain, we refer to this as a SP("simply-phased") diagram, since it is equal to the ordinary diagram apart from an overall  $\theta$ -dependent phase factor; otherwise, the diagram is NSP("non-simply-phased")<sup>†</sup>. In this characterization it is the NSP Feynman diagram which typically modifies the ultraviolet behaviour of the theory in a nontrivial way (see the example in Section 2.1.2 below).

There has already been extensive work on scalar field theory [21, 22, 23], NCQED (the noncommutative analog of QED) [24, 25], as well as noncommutative Yang-Mills [26, 27]; perturbation theory in  $\theta$  is applicable and the theories are renormalizable [28, 29]. For example a Yukawa theory with a scalar  $\phi$ , Dirac fermion  $\psi$ , has the action

$$S = \int d^4x \left( \overline{\psi} i \partial \!\!\!/ \star \psi + \partial_\mu \phi \star \partial^\mu \phi + \lambda \overline{\psi} \star \psi \star \phi \right)$$
(9)

which to  $\mathcal{O}(\theta)$  is

$$S = \int d^4x \left( \overline{\psi} i \partial \!\!\!/ \psi + (\partial_\mu \phi)^2 + \lambda \left( \overline{\psi} \psi \phi + \partial_\mu \overline{\psi} \theta^{\mu\nu} \partial_\nu \psi \phi + \partial_\mu \overline{\psi} \theta^{\mu\nu} \partial_\nu \phi \psi + \overline{\psi} \partial_\mu \psi \theta^{\mu\nu} \partial_\nu \phi \right) \right)$$
(10)

(Here we have used the fact that  $\int dx \xi \star \xi = \int dx \xi \xi$ , which follows straightforwardly from (4)). Likewise, the usual  $\phi^4$  theory has an action

$$S = \int d^4x \left( (\partial \phi)^2 + m^2 \phi^2 + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)$$
(11)

which to  $\mathcal{O}(\theta)$  is

$$S = \int d^4x \left( (\partial \phi)^2 + m^2 \phi^2 + \frac{\lambda}{4!} \left( \phi^4 + 6(\partial_\mu \phi \ \theta^{\mu\nu} \partial_\nu \phi) \phi^2 \right) \right)$$
(12)

We see that the interaction terms above contain derivatives coupled to powers of  $\theta_{\mu\nu}$ . It is these terms which will lead to mixing between infrared and ultraviolet limits.

<sup>&</sup>lt;sup>†</sup>in the literature these are often called "planar" and "nonplanar" diagrams [19, 20]

#### 2.1.2 IR/UV mixing

Calculating the one-loop quadratic effective action from (11), one needs to consider the contribution from both the  $\theta$ -dependent and  $\theta$ -independent interactions (see Equation 12)). Equivalently, one computes the *SP* and *NSP* graphs as done in [20], with the result

$$\Gamma_{1SP} = \frac{\lambda}{3(2\pi)^4} \int \frac{d^4k}{k^2 + m^2}$$
  

$$\Gamma_{1NSP} = \frac{\lambda}{6(2\pi)^4} \int \frac{d^4k e^{ik \times p}}{k^2 + m^2}$$
(13)

where p is the external momentum. The SP integral is quadratically divergent, but the oscillatory factor in the NSP diagram renders it ultraviolet finite. Introducing the Schwinger [30] parameter  $\alpha$  by  $\frac{1}{k^2+m^2} = \int_0^\infty d\alpha \ e^{-\alpha(k^2+m^2)}$  and a Pauli-Vilars regulator  $e^{-\frac{1}{M^2\alpha}}$  now gives

$$\Gamma_{1SP} = \frac{\lambda}{48\pi^2} \int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2 - \frac{1}{M^2 \alpha}} = \frac{\lambda}{48\pi^2} (M^2 - m^2 ln(\frac{M^2}{m^2}) + \mathcal{O}(1))$$

$$\Gamma_{1NSP} = \frac{\lambda}{96\pi^2} \int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2 - \frac{p \circ p + 1/M^2}{\alpha}} = \frac{\lambda}{96\pi^2} (M_{eff}^2 - m^2 ln(\frac{M_{eff}^2}{m^2}) + \mathcal{O}(1))$$
(14)

where  $p \circ p \equiv |p^{\mu}\theta_{\mu\nu}\theta^{\nu\rho}p_{\rho}|$  and  $M_{eff}^2 \equiv \frac{1}{1/M^2 + p \circ p}$ , from which it is clear that in the ultraviolet limit  $M \to \infty$  the nonplanar contribution is finite. The total 1PI quadratic effective action to  $\mathcal{O}(\lambda)$  is therefore

$$S_{1PI} = \int d^4 p \frac{1}{2} \phi(p) \phi(-p) \left( p^2 + \tilde{m}^2 + \lambda \frac{M_{eff}^2}{96\pi^2} - \frac{\lambda M^2}{96\pi^2} ln \left( \frac{M_{eff}^2}{M^2} \right) \right)$$
(15)

where  $\tilde{m}^2 = m^2 + \frac{\lambda M^2}{48\pi^2} - \frac{\lambda m^2}{48\pi^2} ln(\frac{M^2}{m^2})$  is the renormalized mass. The surprising feature of this result is that the limits  $\theta \to 0$  or  $p \to 0$  (an IR limit) and  $M \to \infty$  (a UV limit) don't commute. Ordinarily in field theory one takes the cutoff to infinity and performs a subtraction, but here it appears that doing so leaves IR singularities (as  $p \to 0$ ). The higher the cutoff, the more sensitive the amplitude becomes to the lowest energies available. This is not in accord with a Wilsonian intuition of renormalization, in which low energy effective theory decouples from high energy dynamics. Hence this phenomenon is called the "IR/UV mixing problem". It persists in the noncommutative version of QED, NCQED.

It may well be that, as pointed out in [16, 31], IR/UV may have a novel interpretation which is not disastrous for the theory (see for example [32]), or IR/UV may simply not be an *observable* problem at all. It has been shown that supersymmetry may alleviate the problem: the authors of [33] find that dangerous IR/UV contributions to the photon propagator are proportional to the number of bosons minus the number of fermions in the theory,  $(n_F - n_B)$ , which is of course zero in unbroken supersymmetry. Reference [34] more completely discusses the theory in broken supersymmetry.

#### 2.1.3 Lorentz Violation

Because  $\theta_{\mu\nu}$  carries Lorentz indices, it does not violate Lorentz symmetries under transformations of the observer's frame, but since it is a *constant* object, it violates "particle Lorentz invariance" [35], which means that in any given frame it singles out a particular direction  $\theta^i \approx \epsilon_{ijk} \theta^{jk}$  in space. Many researchers set  $\theta_{0i} = 0$  to avoid problems with unitarity and causality [36], so we do not include the time components in the definition of this "direction". However it is not necessary to use this constraint for the purposes of low-energy phenomenology. As we will see below in 3, this orientation of  $\theta_{\mu\nu}$  provides a strong constraint on the magnitude of  $\theta_{\mu\nu}$ . However if  $\theta$  varies over intervals of space-time much smaller than those in which experiments operate, then there is no well-defined meaning to  $\theta^i$  in terrestrial experiments and the aforementioned constraints effectively drop away. Nevertheless, a nonzero  $\theta$  will affect the dispersion relation  $E^2 = p^2 + m^2$  of particles travelling through space-time. Among some of the work in Lorentz violation [35, 37, 38, 39] are predictions of a number of exotic phenomena, such as the decay of high-energy photons and charged particles producing Cerenkov radiation in vacuum.

## 2.2 Gauge Interations and Particle Spectra

Similar to the non-gauge theories above in (9,11), a noncommutative gauge theory follows from the ordinary one by inserting the  $\star$  product everywhere. For NCQED, for example, we have the action

$$S = \int d^4x \left( -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} + \overline{\psi} i \partial \!\!\!/ \psi - e \overline{\psi} \star A \star \psi - m \overline{\psi} \psi \right)$$
(16)

where

$$F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - i[A_{\mu}, A_{\nu}]_{\star}$$
<sup>(17)</sup>

Note the extra term in the field strength which is absent in ordinary QED; this nonlinearity gives NCQED a NonAbelian-like structure. There will be, for example, 3- and 4-point photon self-couplings at tree level (see Figure 1). However, if one assumes that the gauge transformation of the connection  $A_{\mu}$  is the naive one, *i.e.* 

$$A_{\mu} \to U \star A_{\mu} U^{-1} + iU \star \partial_{\mu} U^{-1} \tag{18}$$

where  $U \equiv (e^{i\lambda^a T_a})_{\star}$  for gauge parameters  $\lambda^a$  and group generators  $T^a$ , then the infinitesimal transformation of  $A_{\mu}$  is [40]

$$\delta_{\lambda}A_{\mu} = \partial_{\mu}\lambda + i\lambda \star A_{\mu} - iA_{\mu} \star \lambda$$
  
=  $T^{a}(\partial_{\mu}\lambda_{a}) + \frac{i}{2}[T^{a}, T^{b}](\lambda_{a} \star A_{b\mu} + A_{b\mu} \star \lambda_{a}) + \frac{i}{2}\{T^{a}, T^{b}\}(\lambda_{a} \star A_{b\mu} - A_{b\mu} \star \lambda_{a})$  (19)

This must be closed under the gauge group, so in particular  $\{T^a, T^b\}$  must be a generator as well, which is true only for  $T \subset U(N)$ . Therefore it appears that only U(N) gauge groups are compatible with noncommutative geometry. Moreover, a similar calculation shows that any matter content of the theory that transforms in the naive way must be in either the fundamental, anti-fundamental, or singlet representation of the gauge group. Other workers subsequently found that products of gauge groups were likewise constrained [41]. The result of recent work, however, claims that this is not a problem if one assumes a more complicated transformation [16, 42] based on the Seiberg-Witten map [1] to replace the naive one in (18).

#### 2.3 A Working Model

Following the above discussion a viable NCSM is possible, though some points need further clarification. The Feynman rules are shown in Figure 1. The essential difference from the usual SM Feynman rules are the momentum-dependent oscillatory factors present at each vertex. Otherwise, there is no change from the SM (for a detailed presentation of the model and related work, please see [16, 41, 43, 44, 45]. For the rest of this review we assume that it is permissible to compute in the NCSM operating under the assumption that any subsequent refinements of the basic theory won't drastically affect the  $\mathcal{O}(\theta)$  expansion of the above Feynman rules and the phenomenological work presented below.



Figure 1: Feynman rules for fermions (solid lines), gauge particles (wavy lines), and ghosts (dotted lines). Notation: p, q, r, s Momenta  $\mu, \nu, \rho, \sigma$  Lorentz indices a, b, c, d gauge indices  $T_{bc}^{a}$  gauge generator  $f_{abc}$  structure constants for SU(N):  $[T_{a}, T_{b}] = f_{abc}T^{c}$   $d_{abc}$  structure constants for SU(N):  $[T_{a}, T_{b}] = f_{abc}T^{c}$   $d_{abc}$  structure constants for SU(N):  $\{T_{a}, T_{b}\} = d_{abc}T^{c} + \frac{1}{N}\delta_{ab}$   $L_{abcd} \equiv d_{abe}d_{cde} + d_{ade}d_{cbe} - f_{abe}f_{cde} - f_{ade}f_{cbe}$   $M_{abcd} \equiv d_{abe}f_{cde} - d_{ade}f_{cbe} + f_{abe}d_{cde} - f_{ade}d_{bce}$   $T_{\mu\nu\rho\sigma} \equiv g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\nu\rho} - 2g_{\mu\rho}g_{\nu\sigma}$  For QED/Weak vertices, index 0 corresponds to a photon:  $d_{0,i,j} = \delta_{ij}$ ,  $d_{0,0,i} = 0$ , and  $d_{0,0,0} = 1$ ,  $f_{0,a,b} = 0$ .



Figure 2: The one-loop contributions to the electron vertex function  $\Gamma_{\mu}$  in NCQED. Each  $\gamma ee$  vertex contributes a momentum-dependent phase factor which affects the loop integral. Whereas (a) is present in ordinary QED, (b) is a novel contribution due to noncommutative geometry.

As an example of the application of this working model, consider the diagrams in Figure 2 which contribute to the one-loop electron vertex function  $\Gamma^{(1)}{}_{\mu}$  in NCQED.

These arise from

$$\Gamma^{(1)}{}_{\mu} = \Gamma^{(1a)}{}_{\mu} + \Gamma^{(1b)}{}_{\mu} \\
\Gamma^{(1a)}{}_{\mu} = i(-ie)^{2}e^{\frac{i}{2}p\times p'} \int \frac{d^{4}k}{(2\pi)^{4}} e^{-ik\times q} \frac{\gamma_{\sigma}}{k^{2}-\mu^{2}+i\epsilon} \frac{p'-l+m}{(p'-k)^{2}-m^{2}+i\epsilon} \gamma_{\mu} \frac{p-l+m}{(p-k)^{2}-m^{2}+i\epsilon} \gamma^{\sigma} \\
\Gamma^{(1b)}{}_{\mu} = -ie^{2}e^{\frac{i}{2}p\times p'} \int \frac{d^{4}k}{(2\pi)^{4}} \frac{1-e^{iq\times k+ip'\times p}}{(k^{2}-m^{2})((p'-k)^{2}-\mu^{2})((p-k)^{2}-\mu^{2})} \\
\cdot (\gamma_{\nu}(l+m)\gamma_{\rho}[g^{\mu\nu}(2p'-p-k)^{\rho}+g^{\nu\rho}(2k-p'-p)^{\mu}+g^{\rho\mu}(2p-p'-k)^{\nu}])$$
(20)

Here  $\Gamma^{(1a)}{}_{\mu}$  is the contribution from the diagram in Figure 2(a), similar to the one in ordinary QED, while  $\Gamma^{(1b)}{}_{\mu}$  is from the diagram in Figure 2(b), a new contribution with no counterpart in QED (note the tri-photon coupling), p, p' are external electron momenta, k is the photon loop momentum, q = (p' - p) and  $\mu$  is a small nonzero mass for the photon to regulate IR divergences [46]. When we introduce the Schwinger parameters and insert a UV regulator as done above Equation 14 (for more details, see the computation in [46]),

$$\Gamma^{(1a)}{}_{\mu} = -\frac{\alpha}{\pi} e^{\frac{i}{2}p \times p'} \int_{0}^{1} d\alpha_{1} d\alpha_{2} d\alpha_{3} \delta(1 - \alpha_{1} - \alpha_{2} - \alpha_{3}) e^{-i(\alpha_{2} + \alpha_{3})q \times p} \\
\cdot \left( \frac{2A_{\mu}K_{1}(2\sqrt{X})}{\sqrt{X}M_{eff}^{2}} + 2B_{\mu}K_{0}(2\sqrt{X}) + 2\sqrt{X}M_{eff}^{2}C_{\mu}K_{1}(2\sqrt{X}) \right) \\
\Gamma^{(1b)}{}_{\mu} = -\frac{\alpha}{\pi} e^{\frac{i}{2}p \times p'} \int_{0}^{1} d\alpha_{1} d\alpha_{2} d\alpha_{3} \delta(1 - \alpha_{1} - \alpha_{2} - \alpha_{3}) \\
\cdot \left[ e^{-i(\alpha_{2} + \alpha_{3})q \times p} \left( \frac{2\tilde{A}_{\mu}K_{1}(2\sqrt{Y})}{\sqrt{Y}M_{eff}^{2}} + 2\tilde{B}_{\mu}K_{0}(2\sqrt{Y}) + \sqrt{Y}M_{eff}^{2}\tilde{C}_{\mu}K_{1}(2\sqrt{Y}) \right) \\
+ \left( \frac{2\overline{A}_{\mu}K_{1}(2\sqrt{Z})}{\sqrt{Z}M^{2}} + 2\overline{B}_{\mu}K_{0}(2\sqrt{Z}) \right) \right]$$
(21)

where

$$\begin{aligned} A_{\mu} &= \gamma_{\mu} \left( p' \cdot p - \frac{1}{2} (\alpha_{2} + \alpha_{3}) (p' + p)^{2} + \frac{1}{2} m^{2} (\alpha_{2} + \alpha_{3})^{2} - \frac{1}{2} \alpha_{2} \alpha_{3} q^{2} \right) + \frac{m}{2} \alpha_{1} (\alpha_{2} + \alpha_{3}) (p' + p)_{\mu} \\ \tilde{A}_{\mu} &= \frac{1}{2} \left( (\alpha_{2} + \alpha_{3}) (p' + p)^{2} - 3m^{2} - m^{2} (\alpha_{2} + \alpha_{3})^{2} + \alpha_{2} \alpha_{3} q^{2} \right) + \frac{m\alpha_{1}}{2} (\alpha_{2} + \alpha_{3}) (p' + p)_{\mu} \\ \bar{A}_{\mu} &= \frac{1}{2} \gamma_{\mu} \left( (\alpha_{2} + \alpha_{3}) (p' + p)^{2} - 3m^{2} - m^{2} (\alpha_{2} + \alpha_{3})^{2} \right) + \frac{m\alpha_{1}}{2} (\alpha_{2} + \alpha_{3}) (p' + p)_{\mu} \\ iB_{\mu} &= \frac{\gamma_{\mu}}{2i} + \frac{\gamma_{\mu} q \times p}{2} (2 - \alpha_{2} - \alpha_{3}) + \frac{m}{2} (1 + \alpha_{2} + \alpha_{3}) q^{\nu} \theta_{\nu\mu} + \frac{1}{4} (1 + \alpha_{1}) (p' + p)_{\mu} q \times \gamma \\ iB_{\mu} &= \frac{3i}{2} \gamma_{\mu} + \frac{1}{2} (mq^{\nu} \theta_{\nu\mu} + \gamma_{\mu} q \times p) (2 - \alpha_{2} - \alpha_{3}) - \frac{q \times \gamma}{4} (p' + p)_{\mu} (1 + \alpha_{2} + \alpha_{3}) \\ \overline{B}_{\mu} &= \frac{3}{2} \gamma_{\mu} \\ C_{\mu} &= -\frac{\gamma_{\mu} q \circ q}{4} + \frac{\gamma_{\mu} q \circ q}{4} \\ X &= \frac{\alpha_{1} \mu^{2} + (\alpha_{2} + \alpha_{3}) \mu^{2} + (\alpha_{2} + \alpha_{3})^{2m^{2} - \alpha_{2} \alpha_{3} q^{2}}}{M_{eff}^{2}} \\ Y &= \frac{(\alpha_{1} - \alpha_{2} - \alpha_{3}) m^{2} + (\alpha_{2} + \alpha_{3}) \mu^{2} + (\alpha_{2} + \alpha_{3})^{2m^{2} - \alpha_{2} \alpha_{3} q^{2}}}{M^{2}} \\ Z &= \frac{(\alpha_{1} - \alpha_{2} - \alpha_{3}) m^{2} + (\alpha_{2} + \alpha_{3}) \mu^{2} + (\alpha_{2} + \alpha_{3})^{2m^{2} - \alpha_{2} \alpha_{3} q^{2}}}{M^{2}} \end{aligned}$$

$$(22)$$

and  $M_{eff}^2 \equiv 1/(M^{-2} - q \circ q/4)$  and  $K_0, K_1$  are modified Bessel functions of the first and second kind, respectively. As  $M_{eff}^2 \to \infty$ , all of X, Y, Z tend to zero, and the Bessel functions approach the asymptotic forms

$$\begin{aligned}
K_0(\xi) &\to -ln(\xi) \\
K_1(\xi) &\to 1/\xi
\end{aligned} \tag{23}$$

Note in Equation 21 above this implies that all the terms containing  $K_1$  are finite. However, those terms containing  $K_0$  (all proportional to  $B_{\mu}$ ,  $\tilde{B}_{\mu}$ , or  $\overline{B}_{\mu}$ ) logarithmically diverge; we can absorb these divergences into counterterms proportional to each  $B_{\mu}$ ,  $\tilde{B}_{\mu}$ , or  $\overline{B}$  term, just as in the usual

QED. The renormalized vertex function is then found to contain well-behaved finite terms plus the following:

$$\Gamma^{(1)}{}_{\mu} \supset -\frac{\alpha e^{\frac{i}{2}p \times p'}}{\pi} \int_{0}^{1} d\alpha_{1} d\alpha_{2} d\alpha_{3} \delta(1 - \alpha_{1} - \alpha_{2} - \alpha_{3}) \\ \cdot M_{eff}^{2} \left( C_{\mu} e^{-i(\alpha_{2} + \alpha_{3})q \times p} - \tilde{C}_{\mu} e^{i(\alpha_{2} + \alpha_{3})q \times p} e^{-ip \times p'} \right)$$
(24)

Now the dependence on the cutoff scale M is retained to illustrate IR/UV mixing explicitly: if we take  $\theta \to 0$  or the photon momentum  $q \to 0$  before taking  $M \to \infty$ , the terms in this equation vanish  $(M_{eff} \to M, \text{ and both } C \text{ and } \tilde{C} \text{ are proportional to } q^2\theta^2)$ . But if we take the ultraviolet limit  $M \to \infty$  first, then  $M_{eff}^2 \to 1/q \circ q$  (cancelling the  $q^2\theta^2$ -dependence in C and  $\tilde{C}$ ), and these terms are now finite and nonzero as  $\theta \to 0$  or  $q \to 0$ . Although this problem does not necessarily hinder phenomenological analysis in a noncommutative theory since in either order of limits one obtains finite  $\mathcal{O}(\theta)$  corrections to the commutative theory, it is nonetheless a theoretical problem that needs further attention.

We note that not every situation requires renormalization with  $\theta$ -dependent counterterms. Take for example  $K - \overline{K}$ -mixing, where the relevant loop integral is approximately [48]

$$i\lambda^{3}\rho \int d^{4}k \; \frac{\left(\frac{k^{2}m_{q}^{4}}{4m_{W}^{4}} + k^{2} - \frac{2m_{q}^{4}}{m_{W}^{2}}\right)|k| \; |p_{1} \cdot \theta \cdot p_{2}|}{(k^{2} - m_{q}^{2})^{2}(k^{2} - m_{W}^{2})^{2}} \tag{25}$$

This is finite for all  $\theta$  and external momenta.

## 3 Phenomenology

## 3.1 New Frames of Reference for Particle Experiments

Since  $\theta$  may have a preferred direction, experiments sensitive to noncommutative geometry may therefore be measuring the components of  $\vec{\theta}$ , and it is necessary to take into account the motion of the lab frame in this measurement [49]. Since noncommutative effects are measured in powers of  $p^{\mu}\theta_{\mu\nu}p'^{\nu}$ , where p, p' are some momenta involved in the measurement, it is possible that odd powers of  $\theta$  will partially average to zero if the time scale of the measurement is long enough. Effects of first order in  $\theta$  vanish at a symmetric  $e^+e^-$ -collider, for example, if the measurement averages over the entire  $4\pi$  solid angle of decay products. If the data is binned by angle then it is possible to restore the sensitivity at  $\mathcal{O}(\theta)$ . In addition to any other averaging process over short time scales, terrestrial experiments performed over several days will only be sensitive to the projection of  $\vec{\theta}$  on the axis of the Earth's rotation. Of course binning the data hourly or at least by day/night, taking into account the time of year, can partially mitigate this effect. This axis, as well as the motion of the solar system, galaxy, *etc.*, does not vary over time scales relevant to terrestrial experiments. In the subsequent review of the phenomenology, this consideration of frame of reference should be held in mind.

## 3.2 High-Energy Scattering Experiments

One would expect effective operators involving  $\theta$  (mass dimension -2) to be most relevant at high energies. Leptonic experiments may be possible at a future collider with  $\sqrt{s} = 500 GeV$  [50, 51, 52] and hadronic experiments will occur at  $\sqrt{s} \approx 10 TeV$  [53]. These will constrain  $\theta$ .



Figure 3: Prediction of electron scattering differential cross sections in the azimuthal angle  $\phi$  in [54] for (a) Bhabha scattering with  $\theta_{02} = (3 \ TeV)^{-2}$  and (b) Möller scattering with  $\theta_{12} = (3 \ TeV)^{-2}$  at a  $\sqrt{s} = 3 \ TeV$  linear collider with  $1 \ ab^{-1}$  integrated luminosity. Cuts on the polar angle  $|cos(\theta)| = 0.9, 0.7, 0.5$  (top to bottom) are shown. (c) Pair annihilation at  $\sqrt{s} = 3 \ TeV$  or  $\sqrt{s} = 5 \ TeV$ ,  $\theta_{02} = 1/s$ . In all cases the solid lines are the SM prediction.

#### 3.2.1 Electron Scattering

There are already a number of predictions of what one might observe in a high energy  $e^+e^$ scattering experiment [54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65]. Such studies carry some degree of robustness since NCQED is the simplest noncommutative extension of the Standard Model; there is no need to define electroweak gauge interactions (see 2.2 above) as long as one only works with electrons, positrons, and photons (quarks aren't permissible in the simplest scheme since they carry *fractional* charge, which wouldn't be a fundamental representation of U(1) in the same theory electrons existed in). The conclusion of the work is that  $e^+e^-$  cross sections depend most sensitively on the projection of the particles' momenta on the plane perpendicular to  $\vec{\theta}$ . For example, if the beam is in the 1-direction, then cross sections will not be sensitive to  $\theta_{23}$  (and  $\theta_{32}$ ). Sensitivity to the other components of  $\theta_{\mu\nu}$  depends on the process under consideration. The



Figure 4:  $e - \gamma$  scattering at a  $\sqrt{s} = 500 \ GeV = 1/\sqrt{\theta}$  linear collider considered in [57] for space-space (I) and space-time (II) noncommutivity and polarization states of the (initial electron, photon, and final electron). Here the dotted lines are the SM prediction, and  $\Lambda_{NC} \equiv 1/\sqrt{\theta}$ .

authors in [56, 54, 61] find that differential cross sections for Möller scattering depend chiefly on  $\theta_{13}$  and  $\theta_{12}$  (the "space-space" components, due to t- and u- channel interference) and show a statistically significant azimuthal dependence in a 500 GeV linear  $e^+e^-$  collider with 300  $fb^{-1}$ integrated luminosity, assuming the scale of noncommutivity is a few TeV. There is moreover a periodic behavior in the total cross section as the  $\sqrt{s}$  of the machine changes. Bhabha scattering and pair annihilation processes on the other hand test  $\theta_{0i}$  (now the s-channel plays a role in the interference), *i.e.* it is the "space-time" components of  $\theta_{\mu\nu}$ . Since it is likely that noncommutative effects lie above 1 TeV(see Sec 3.3 below), we reproduce in Figure 3 more recent predictions in [54] for the azimuthal dependence of various cross sections in a linear  $e^+e^-$  collider operating at 3 TeV or more with 1  $ab^{-1}$  integrated luminosity, assuming  $1/\sqrt{\theta} \approx \sqrt{s}$ .

In a high energy  $e^+e^-$  linear collider, it is possible to backscatter laser beams off the electron and positron beams, making  $e\gamma$  or  $\gamma\gamma$  collisions. The author of [57], for example, finds  $e\gamma$  scattering at a 500 GeV linear collider with 500  $fb^{-1}$  integrated luminosity can measure or exclude  $1/\sqrt{\theta} < 1 \text{ TeV}$ at 95% C.L. In Figure 4 we show an azimuthal angle distribution computed from that work. Similarly,  $\gamma\gamma$  scattering can provide constraints of comparable strength, as considered in [58]. Figure 5 shows the result of the analysis in that paper. Higgs production as per  $\gamma\gamma \to H^0H^0$ , strictly forbidden in the SM, can yield up to a 100 fb cross-section at a 1.5 TeV linear collider if the Higgs mass is less than 200 GeV [59].

### 3.2.2 Hadronic Scattering

At present no work has been done in hadronic noncommutative phenomenology, although it is possible to measure the effects of a noncommutative space-time in hadronic processes. From the



Figure 5: Limits on  $1/\sqrt{\theta}$  (95% C.L.) from  $\gamma - \gamma$  scattering at an  $e^+ - e^-$  linear collider considered in [58], at various luminosities. Solid lines are for monochromatic photons, which are difficult to obtain in practice. Dotted lines are for using backscattered photons. The  $\sqrt{s}$  under consideration is listed next to each line.

Feynman rules in Figure 1 we see that gluon couplings are generically  $\theta$ -dependent, so one would expect some observable effect in jet production rates and angular distributions in a hadron collider, for instance. If the theory were managable (see Sec. 2.2 above with respect to SU(N) groups) this would provide a non-trivial consistency check through the precise dependence of the gluon couplings on  $\theta$  [66]. Uncertainties in calculating QCD rates in hadron colliders would make the small effects of noncommutative physics harder to detect than those in  $e^+e^-$  colliders. This would be compensated by the larger available energies.

#### **3.3** Low-Energy and Precision Experiments

Low-energy experiments (center of mass energies  $E \leq 1 \text{ GeV}$  tend to produce weaker limits on  $\theta$  (the effect is quadratically suppressed as  $E^2\theta$ ) unless the sensitivity of the experiment is extraordinary. We begin with atomic and dipole moment constraints, as these are among the most precisely measured quantities in physics.

### 3.3.1 Atomic Transitions

The authors in [67, 68] apply noncommutative quantum mechanics (NCQM) to the hydrogen atom to calculate the Lamb shift. In NCQM one replaces the position operator  $x_i$  with the  $\theta$ -deformed  $x'_i = x_i + \frac{1}{2\hbar} \theta_{ij} p_j$  and then applies the usual rules of QM. In particular, the Coulomb potential is modified to  $V(r) = -\frac{e^2}{r} - e^2 \frac{(r \times p) \cdot \overrightarrow{\theta}}{4\hbar r^3} + \mathcal{O}(\theta^2)$ . They find the constraint on  $\theta$  is:  $1/\sqrt{\theta} \ge 10 \ TeV$ . The Stark and Zeeman effects add nothing to this constraint. The physics of positronium being very similiar to hydrogen, in [69, 70] the positronium  $2^3S_1 \to 2^3P_2$  splitting is also considered; the strongest constraint is  $\theta \le 10^{-5}\lambda_e^2$ , or  $1/\sqrt{\theta} \ge GeV$ . Transitions in the Helium atom give a stronger constraint:  $1/\sqrt{\theta} \ge 30 \ GeV$  [71].

### 3.3.2 New CP-violation

Beyond the standard source of CP violation from the mismatch between the mass and weak quark (and possibly lepton) eigenstates, in the NCSM there is an additional source of CP violation: the parameter  $\theta$  itself is the CP violating object. To see this, consider (as in [72]) the transformation of the action (16) under the discrete symmetries P, C. Under the parity transformation,

leaves the action invariant. However under charge conjugation

$$\begin{array}{lll}
\partial_{\mu} & \rightarrow & \partial_{\mu} \\
A_{\mu} & \rightarrow & -A_{\mu} \\
\theta_{\mu\nu} & \rightarrow & -\theta_{\mu\nu}
\end{array} \tag{27}$$

is required to keep the field strength tensor in (17) unchanged. Therefore, under C and P combined  $\theta \rightarrow -\theta$ . More detailed work reveals that  $\theta$  is in fact proportional to the size of an effective particle dipole moment [73]. Therefore noncommutative geometry can actually *explain* the origin of CP violation.

#### 3.3.3 Electron Electric Dipole Moment

Since the SM predictions of the CP violating electric dipole moments (edms) are extremely small, we might expect that new sources of CP violating physics from noncommutative geometry would be observable. The noncommutative geometry provides in addition a simple explanation for this type of CP violation: the directional sense of the  $edm \ D$  derives from the different amounts of noncommutivity in different directions (*i.e.*  $D_i \propto \epsilon_{ijk} \theta^{jk}$ ) and the size of the edm, classically proportional to the spatial extent of a charge distribution, is likewise in noncommutative geometry proportional to  $\sqrt{\theta}$ , the inherent "uncertainty" of space. The effects of noncommutative geometry will be proportional to the typical momentum involved, which for an electron edm observation is  $\sim keV$ . A detailed analysis of the size of the edm appears in [46, 47]. The magnetic dipole moment, incidentally, receives a very small and (to leading order in  $\theta$ ) spin-independent contribution from noncommutative geometry which makes it extremely difficult to observe. A simple estimate of the expected electron electric dipole moment [48] yields a fairly strong bound:  $1/\sqrt{\theta} \ge 100 \ TeV$ . However, we cannot exclude the possibility that the actual edm is much smaller than the above naive estimate, a situation which can arise in supersymmetric models [74, 75].

#### 3.3.4 CP Violation in the Electroweak Sector

At the field theory level, it is the momentum-dependent phase factor appearing in the noncommutative theory which gives CP violation. For example, the NCSM W-quark-quark SU(2) vertex in the flavor basis is

$$\mathcal{L}_{Wqq} = \overline{u(p)} \gamma^{\mu} (1 - \gamma_5) e^{i p \cdot \theta \cdot p'} \ d(p') \ W_{\mu}$$
<sup>(28)</sup>



Figure 6: CP-violating effects of noncommutative geometry: if the Wolfenstein parameter  $\eta = 0$ , noncommutative geometry alone could account for  $\epsilon_K \approx 2.28 \cdot 10^{-3}$  if  $\xi \approx 0.04$  ( $\xi \equiv M_W \sqrt{\theta}$ , so this corresponds to  $1/\sqrt{\theta} \approx 2 \ TeV$ ).

Once we perform rotations on the quark fields to diagonalize the Yukawa interactions, *i.e.*  $u_L \rightarrow U u_L$  and  $d_L \rightarrow V d_L$ , the above becomes

$$\mathcal{L}_{Wqq} = \overline{u(p)}\gamma^{\mu}(1-\gamma_5)e^{ip\cdot\theta\cdot p'}U^{\dagger}V \ d(p') \ W_{\mu}$$
<sup>(29)</sup>

Even if  $U^{\dagger}V$  is purely real, there will be some nonzero phases  $e^{ip\cdot\theta\cdot p'}$  in the Lagrangian whose magnitudes increase as the momentum flow in the process increases. Of course, the above phase factor has no effect at tree-level (suitably redefining all the fields) but will affect results at 1-loop and beyond. The overall effect is to introduce momentum-dependent phases into the CKM matrix. The authors in [48] consider such a matrix:

$$\overline{V}(p,p') \equiv \begin{pmatrix} 1 - \lambda^2/2 + ix_{ud} & \lambda + ix_{us} & A\lambda^3(\rho - i\eta) + ix_{ub} \\ -\lambda + ix_{cd} & 1 - \lambda^2/2 + ix_{cs} & A\lambda^2 + ix_{cb} \\ A\lambda^3(1 - \rho - i\eta) + ix_{td} & -A\lambda^2 + ix_{ts} & 1 + ix_{tb} \end{pmatrix}$$
(30)

where  $x_{ab} \equiv p_a{}^{\mu}\theta_{\mu\nu}p'_b{}^{\nu}$  for quarks a, b. This matrix is an approximation of the exact ncSM in the perturbative limit where we expand  $e^{ip\cdot\theta\cdot p'} \approx 1 + ip\cdot\theta\cdot p'$ . In the limit  $\theta \to 0$ , the  $x_{ab}$  all go to zero and  $\overline{V}$  becomes the CKM matrix V in the Wolfenstein parameterization [76] in terms of the small number  $\lambda \approx 0.22$ .

The authors in [48] calculate whether the  $\theta$ -dependent phases are observable in K- or B- physics: the conclusion is that if  $1/\sqrt{\theta} \approx 1 \ TeV$  then K- physics observables such as  $\epsilon_K$  and  $\epsilon'/\epsilon$  are sensitive to noncommutative geometry (see Figure 6), however B- physics observables, such as  $sin_2\beta$ , are not. However in D- physics, the asymmetry  $\mathcal{A}_s$  between the rates  $D_s^- \to K^- K_s$  and  $D_s^+ \to K^+ K_s$  could provide a measurable signal. The authors in [77] calculate

$$\mathcal{A}_{s} \equiv \frac{\left|A(D_{s}^{-} \to K^{-}K_{s})\right|^{2} - \left|A(D_{s}^{+} \to K^{+}K_{s})\right|^{2}}{\left|A(D_{s}^{-} \to K^{-}K_{s})\right|^{2} + \left|A(D_{s}^{+} \to K^{+}K_{s})\right|^{2}} \approx 2\Re\epsilon_{K} - 2\mathcal{J}R_{s}sin\delta_{s}$$
(31)

where  $\mathcal{J} \approx A^2 \lambda^6 \eta - \lambda^2 p^2 \theta$  (*p* being the typical momentum flow in the process),  $R_s \approx -1.2$ , and  $\delta_s$  is a phase from the strong dynamics. With  $p \approx M_{D^+} \approx 1.97 \ GeV$  [78], a measurement of this asymmetry would be sensitive to noncommutative effects for  $1/\sqrt{\theta} \leq 1 \ TeV$ .



Figure 7: A novel contribution to the muon's magnetic moment from noncommutative geometry considered in [82].



Figure 8: (a)Tree-level muon decay occurs through the exchange of a W-boson. When the electron is emitted with an energy close to its kinematical upper bound of  $m_{\mu}/2$  in the center-of-mass frame, the neutrinos are ejected from the decay vertex in the opposite direction and the spin of the electron becomes highly correlated with the spin of the muon. (b) The  $W - l - \nu$  vertex receives a  $\theta$ -dependent one-loop correction which can upset the aforementioned spin correlation [85].

Other work in CP violation includes the enhancement of the decay  $\pi \to 3\gamma$  relative to the SM by many orders of magnitude [64]. Unfortunately, the branching ratio for this decay, assuming  $1/\sqrt{\theta} \approx 1 \ TeV$ , is  $\mathcal{O}(10^{-20})$ , whereas current experimental bounds are at  $\mathcal{O}(10^{-8})$ .

## **3.3.5** $(g-2)\mu$

Although the leading order contribution to a fermionic magnetic moment doesn't couple to the fermion's spin (and therefore not leading to precession in an external magnetic field, making it hard to observe), at second order in  $\theta$  this is no longer the case. There is a novel contribution to the magnetic moment of the muon coming from the diagram shown in Figure 7. In an experiment such as the one at BNL [83], where muons circulate in a ring at a very high momentum ( $\gamma \approx 30$ ), the second-order contribution to the anomalous magnetic moment of the muon has been calculated [82] in NCQED to be roughly of order  $\delta a_{\mu} \approx \alpha m_{\mu}^2 \theta^2 E_{\mu}^2 / 96\pi \gamma^4$  which, considering that the discrepancy between experiment and theory may be as large as  $\delta a_{\mu} \leq 10^{-9}$  [84], suggests a lower bound  $1/\sqrt{\theta} \geq m_{\mu}$ .

There is a much stronger bound in the literature which is derived from the  $\mathcal{O}(\theta)$  correction

to muon decay. The BNL experiment measures  $a_{\mu}$  by comparing the cyclotron frequency to the observed precession rate of the muon's spin; the difference between these frequencies is directly proportional to  $a_{\mu}$ . In this experiment the rate of precession of the muon's spin is measured indirectly from the decay of the muons into electrons. Electrons emerge from the decay vertex with a characteristic angular distribution which in the Standard Model (SM) has the following form in the rest frame of the muon:

$$dP(y,\phi) = n(y)(1 + A(y)\cos(\phi))dyd(\cos(\phi))$$
(32)

where  $\phi$  is the angle between the momentum of the electron e and the spin of the muon,  $y = 2p_e/m_\mu$ measures the fraction of the maximum available energy which the electron carries, and n(y), A(y)are particular functions which peak at y = 1. The detectors (positioned along the perimeter of the ring) accept the passage of only the highest energy electrons in order to maximize the angular asymmetry in (32). In this way, the electron count rate is modulated at the frequency  $a_{\mu}eB/(2\pi mc)$ , the difference between the cyclotron and precession frequencies of the muons. However each of the  $W - e - \nu$  vertices in the decay diagram receives a one-loop correction which is  $\theta - dependent$ , assuming the validity of a NCSM (see Sec 2.3 above). The appearance of the antisymmetric object  $\theta_{\mu\nu}$  in the decay amplitude leads to combinations of the muon and electron spins and momenta which alter the modulation frequency of the decay rate (32). Specifically, one anticipates factors of  $(\overrightarrow{pe} \cdot \overrightarrow{s}_{\mu})(\overrightarrow{pe} \cdot \theta \cdot \overrightarrow{s}_{\mu})$  which for electron momenta close to their kinetmatical limit (*i.e.* y = 1) behaves like  $\cos(\phi)\sin(\phi)$  and upsets the simple angular dependence in (32). In [85] the  $\theta$ -dependence of the decay is calculated and based on the degree to which this changes the apparent precession of muon spin the bound  $1/\sqrt{\theta} \geq 1 \ TeV$  is obtained. These bounds are subject to improvement in light of the ongoing analysis of the BNL data [86].

#### 3.3.6 Contributions from Two Loops

The authors in [87] derive a two loop contribution to the effective NCQED action of the form

$$\mathcal{L}_{eff} = \frac{3}{4} m \Lambda^2 \left(\frac{e^2}{16\pi^2}\right)^2 \theta^{\mu\nu} \overline{\psi} \sigma_{\mu\nu} \psi \tag{33}$$

where  $\Lambda$  is an effective cutoff. They conclude that if  $\Lambda \sim 1 \ TeV$  then experiments sensitive to background magnetic fields could constrain  $\theta < (10^{12-13} GeV)^{-2}$ . Such experiments may include the variation of spin precessions with time [66, 88]. Experiments typically look for sidereal variations of electronic or nuclear spin, for example in Cs or Hg, the latter's nuclear spin giving the stronger constraint. If  $\theta$  is constant in space over distances comparable to those the Earth moves through on a timescale of months, then its direction behaves as a background magnetic field, possibly giving rise to effective operators like  $\theta^{\mu\nu}\bar{q}\sigma_{\mu\nu}q$  which would cause noticable variations in the nuclear spin precession frequencies (observed on scales of months). To the extent that such variations are on the order of microhertz or hundreds of nanohertz [89, 90], bounds on the projection of  $\vec{\theta}$  on the axis of the Earth's rotation (see Section 2.1.3 above)such as  $1/\sqrt{\theta} \geq 10^{14} \ GeV$ ,  $10^{17} \ GeV$  are obtained [91]. Such bounds are on a weaker footing than the ones from NCQED as the complete formulation of NCQCD has not been thoroughly developed [66].

### 3.3.7 Other Precision Tests

Among the other ideas for testing noncommutivity are directly measuring the dispersion relation for photons, which is altered in a noncommutative space-time, in a Michelson-Morley-type interferometry experiment [92]. Those authors find that in principle one could be sensitive to  $1/\sqrt{\theta} \leq 10 \ TeV$  in an interference experiment with visible light in which the sum of the legs of the apparatus extended for one parsec in a background magnetic field of 1 Tesla; they note this distance, however, is impractically large to coherently maintain such a strong background field. Moreover there is the question of the uniformity of  $\theta$  over these distances. A similar bound is obtained from considering an Aharonov-Bohm effect with high energy electrons [93]. Reference [94] discusses noncommutative contributions to  $b \to s\gamma$  inclusive decays. In another paper [95] the effects of noncommutative geometry on the triple neutral gauge boson couplings is considered. They use an SU(5) unification scheme to motivate a particular choice of gauge transformation parameters left undetermined in the methods that [16] employ to construct consistent noncommutative non-Abelian gauge interactions in the NCSM. At first order in  $\theta$ , they find analytical contributions to couplings (e.g.  $\gamma\gamma\gamma$ ,  $\gamma\gamma Z$ ,  $\gamma gg$  ...) which are strictly forbidden in the SM. The work in [96] follows up on this and derives the partial width  $Z \to \gamma \gamma$  in the NCSM; this decay has a branching ratio of  $10^{-10}$  in the SM (current experimental bounds are  $B.R. < 10^{-4}$  [78]) so its observation could be a clear positive indicator for noncommutative geometry. High energy processes in stars may also provide some useful constraints: in [79] a limit of  $1/\sqrt{\theta} > 80 \ GeV$  is placed on the noncommutivity scale based on the criterion that any additional energy loss from the star due to noncommutivity not exceed the standard amount due to neutrinos. Some theoretical work [80] on the Hall effect indicates that one can test noncommutivity here, though as yet there is no direct comparison with exeriment. Finally, [81] presents a calculation of the effects of noncommutivity on synchrotron radiation and, although no numerical bounds are presented, asserts this could be a good way to search for noncommutative effects.

### 3.4 Cosmology and Extra Dimensions

Finally, we will say a word about developments in noncommutative cosmology and extra dimensions. Noncommutative effects in cosmology range from birefringence of light, a variable speed of light, an alterted cosmic microwave background, highly energetic photons and protons above the GZK [97, 98] cutoff (that is, the threshold on cosmic proton energies due to the reaction  $p + \gamma \rightarrow p + \pi$  with the cosmic microwave backround radiation (CMBR)), to the overall quantum structure of the cosmos [99, 100, 101, 102, 103]. In [102], for example, noncommutative geometry is a possible explanation of the detection of highly energetic photons ( $E > 20 \ TeV$  from sources  $> 100 \ Mpc$  distant [104]). It is a mystery how such energetic photons can reach detectors on Earth since the interaction with the infrared background via  $\gamma\gamma \rightarrow e^+e^-$  should impose a threshold energy of  $\approx 1 \ TeV$  on photons travelling over galactic distances. In considering a  $\theta$ -deformed spacetime, the authors of [102] derive modified relativistic kinematics introducing a breaking of Lorentz invariance. Their results indicate that the threshold can be pushed to much higher values if  $1/\sqrt{\theta} <= 1 \ TeV$ . The same analysis can account for the recent observation of highly energetic cosmic rays far above the GZK cutoff, a limit of about  $7 \cdot 10^{19} eV$  on the energy protons can have due to the interaction with the CMBR. A noncommutative scale as high as  $10^8 TeV$  can push the threshold above the observations.

The reason why we expect noncommutative geometry will have a large effect on a cosmic scale is two-fold. First, large distances are available between source and observer over which even a small change in a particle's dispersion relation can accumulate to a produce a large effect. Second, the evolution of the cosmos went through an epoch when temperatures were far above the noncommutative scale  $1/\sqrt{\theta}$ : noncommutative effects were non-negligable, perhaps even dominant, and may have left a lasting trace on the global structure of the cosmos after inflation. These are very interesting ideas to pursue, but also very difficult, as they involve nonperturbative effects of



Figure 9: The decay of a KK photon to a lighter KK photon plus  $f\overline{f}$  pair in the standard (non-noncommutative)  $T^2/Z_2$  extra dimensions scenario can receive corrections of over 100% from noncommutative effects in the non-standard noncommutative extra dimensional scenario according to the work in [105]. Partial width is in units of the compactification scale 1/R, and the parameter  $\xi$  denotes the ratio of the compactification radii of the torus.

noncommutative geometry.

Noncommutative extra dimensions may be a more attractive alternative to noncommutative 4space. The authors in [105] consider six-dimensional NCQED where the noncommutivity is confined to two bulk dimensions compactified on a toroidal orbifold  $T^2/Z_2$ . Such a scenario explicitly evades the tight four-dimensional Lorentz-violating bounds mentioned above in connection with nuclear spins, because ordinary matter doesn't exist in the bulk. They find that for heavier Kaluza-Klein (KK) modes (several TeV or more) the decay of a KK photon to a fermion-antifermion pair,  $\gamma^{n_a} \rightarrow \gamma^{n_b} f \overline{f}$ , may be observable at a Very Large Hadron Collider(VLHC) [106]. Decay rates may be up to 100% larger for noncommuting extra dimensions than for ordinary commuting ones (see Figure 9). We quote their claim that a 2 TeV initial state produced at a  $\sqrt{s} = 200 \ TeV$  VLHC with 100  $fb^{-1}$  integrated luminosity would enhance the production rate to allow the discrimination of commuting versus noncommuting extra dimensions at a level of 6.8  $\sigma$ . On the other hand, for lighter KK modes, a pair production process such as  $f\overline{f} \rightarrow \gamma^{n_a}\gamma^{n_b}$  is the more favorable one to observe at a collider such as the LHC.

## 4 Challenges and Conclusions

While the NCSM has some technical difficulties, particularly with regards to the IR/UV problem and in the gauge sector, it is possible to estimate the effects of such extensions in many cases as we have shown above. As discussed in Section 2.1, versions of noncommutative field theory involving supersymmetry are easier to develop theoretically.

Apart from the issue of the full theory there are other areas where work on noncommutative phenomenology is incomplete. In particular, there is as yet no compelling reason why  $\theta_{\mu\nu}$  should be constant; a theory with a space-time dependent  $\theta(x)_{\mu\nu}$  is feasible and is eventually necessary to investigate. In particular, if  $\theta_{\mu\nu}$  is space-dependent then there is no single direction it "points" in, and many of the stringent limits on  $\theta_{\mu\nu}$  which rely on this "direction" need reconsideration.

Constraints on the NCSM arise from two distint classes of experiments: very high precision measurments at relatively low energy that can see indirect effects and experiments whose energy scales approach that of the noncommutative physics that can see the direct effects. The latter involves experimentation at high energy colliders. In an  $e^+e^-$  collider effects show up as distortions in, for example the Bhabha, scattering rates. These experiments are expected to be sensitive to  $\sqrt{\theta} \sim 1/\sqrt{s}$ . Moreover there is as yet no phenomenological work on a noncommutative geometry at the LHC or other hadron colliders. Indirect constraints from measurements of magnetic moments can be very stringent but their interpretation is model dependent.

Finally, all the above ideas should be investigated more fully in the contex of noncommutative cosmology and extra dimensions to the extent where there are definite predictions. The work so far in these areas is encouraging.

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