The Mathematics of Computerized Tomography

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Preface

By computerized tomography (CT) we mean the reconstruction of a function from its line or plane integrals, irrespective of the field where this technique is applied. In the early 1970s CT was introduced in diagnostic radiology and since then, many other applications of CT have become known, some of them preceding the application in radiology by many years.

In this book I have made an attempt to collect some mathematics which is of possible interest both to the research mathematician who wants to understand the theory and algorithms of CT and to the practitioner who wants to apply CT in his special field of interest. I also want to present the state of the art of the mathematical theory of CT as it has developed from 1970 on. It seems that essential parts of the theory are now well understood.

In the selection of the material I restricted myself—with very few exceptions—to the original problem of CT, even though extensions to other problems of integral geometry, such as reconstruction from integrals over arbitrary manifolds are possible in some cases. This is because the field is presently developing rapidly and its final shape is not yet visible. Another glaring omission is the statistical side of CT which is very important in practice and which we touch on only occasionally.

The book is intended to be self-contained and the necessary mathematical background is briefly reviewed in an appendix (Chapter VII). A familiarity with the material of that chapter is required throughout the book. In the main text I have tried to be mathematically rigorous in the statement and proof of the theorems, but I do not hesitate in giving a loose interpretation of mathematical facts when this helps to understand its practical relevance.

The book arose from courses on the mathematics of CT I taught at the Universities of Saarbrücken and Münster. I owe much to the enthusiasm and diligence of my students, many of whom did their diploma thesis with me. Thanks are due to D. C. Solmon and E. T. Quinto, who, during their stay in Münster which has been made possible by the Humboldt–Stiftung, not only read critically parts of the manuscript and suggested major improvements but also gave their advice in the preparation of the book. I gratefully acknowledge the help of A. Faridani, U. Heike and H. Kruse without whose support the book would never have been finished. Last but not least I want to thank Mrs I. Berg for her excellent typing.

Münster, July 1985 Frank Natterer

Glossary of Symbols

Symbol	*Explanation*	*References*
$\mathbb{R}^n$	n-dimensional euclidean space	
Ω^n	unit ball of $\mathbb{R}^n$	
S^{n-1}	unit sphere in $\mathbb{R}^n$	
Z	unit cylinder in $\mathbb{R}^{n+1}$	II.1
T	tangent bundle to S^{n-1}	II.1
$\theta^\perp$	subspace or unit vector perpendicular to θ	
D^l	derivative of order $l = (l_1, \ldots, l_n)$	
$x \cdot \theta$	inner product	
$\lvert x \rvert$	euclidean norm	
$\mathbb{C}^n$	complex n-dimensional space	
$\hat{f}, \check{f}$	Fourier transform and its inverse	VII.1
C_l^λ	Gegenbauer polynomials, normed by $C_l^\lambda(1) = 1$	VII.3
Y_l	spherical harmonics of degree l	VII.3
$N(n, l)$	number of linearly independent spherical harmonics of degree l	VII.3
J_k	Bessel function of the first kind	VII.3
U_k, T_k	Chebyshev polynomials	VII.3
δ, δ_x	Dirac's δ-function	VII.1
sinc_b	sinc function	III.1
$\eta(\vartheta, b)$	exponentially decaying function	III.2
Γ	Gamma function	
$0(M)$	Quantity of order M	
$\mathscr{S}$	Schwartz space on $\mathbb{R}^n$	VII.1
	Schwartz space on Z, T	II.1
$\mathscr{S}'$	tempered distributions	VII.1
C^m	m times continuously differentiable functions	
C^∞	infinitely differentiable functions	
C_0^∞	functions in C^∞ with compact support	
H^α, H_0^α	Sobolev spaces of order α on $\Omega \subseteq \mathbb{R}^n$	VII.4
	Sobolev spaces of order α on Z, T	II.5

$L_p(\Omega)$	space with norm $\left(\int\limits_{\Omega} \lvert f \rvert^p \, dx\right)^{1/2}$	
$L_p(\Omega, w)$	same as $L_p(\Omega)$ but with weight w	
$\langle u_1, \ldots, u_m \rangle$	span of $u_1, \ldots, u_m$	
$\mathbf{R}$, $\mathbf{R}_\theta$	Radon transform	II.1
$\mathbf{P}$, $\mathbf{P}_\theta$	X-ray transform	II.1
$\mathbf{D}_a$	divergent beam transform	II.1
$\mathbf{R}^\#$ etc.	dual of **R**, etc.	II.1
$\mathbf{I}^\alpha$	Riesz potential	II.2
$\mathbf{R}_\mu$	attenuated Radon transform	II.6
$\mathbf{T}_\mu$	exponential Radon transform	II.6
A^*	adjoint of operator A	
A^T	transpose of matrix A	
$\mathbb{Z}^n$	n-tupels of integers	
$\mathbb{Z}^n_+$	n-tupels of non-negative integers	
$f \perp g$	f perpendicular to g	
$\mathbf{H}$	Hilbert transform	VII.1
$\mathbf{M}$	Mellin transform	VII.3
$\square$	end of proof	