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(continued following index)

Stanley Osher Ronald Fedkiw

Level Set Methods and Dynamic Implicit Surfaces

With 99 Figures, Including 24 in Full Color



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Dedicated with love to Katy, Brittany, and Bobbie

Preface

Scope, Aims, and Audiences

This book, *Level Set Methods and Dynamic Implicit Surfaces* is designed to serve two purposes:

Parts I and II introduce the reader to implicit surfaces and level set methods. We have used these chapters to teach introductory courses on the material to students with little more than a fundamental math background. No prior knowledge of partial differential equations or numerical analysis is required. These first eight chapters include enough detailed information to allow students to create working level set codes from scratch.

Parts III and IV of this book are based on a series of papers published by us and our colleagues. For the sake of brevity, a few details have been occasionally omitted. These chapters do include thorough explanations and enough of the significant details along with the appropriate references to allow the reader to get a firm grasp on the material.

This book is an introduction to the subject. We have given examples of the utility of the method to a diverse (but by no means complete) collection of application areas. We have also tried to give complete numerical recipes and a self-contained course in the appropriate numerical analysis. We believe that this book will enable users to apply the techniques presented here to real problems.

The level set method has been used in a rapidly growing number of areas, far too many to be represented here. These include epitaxial growth, optimal design, CAD, MEMS, optimal control, and others where the simulation

of moving interfaces plays a key role in the problem to be solved. A search of “level set methods” on the Google website (which gave over 2,700 responses as of May 2002) will give an interested reader some idea of the scope and utility of the method. In addition, some exciting advances in the technology have been made since we began writing this book. We hope to cover many of these topics in a future edition. In the meantime you can find some exciting animations and moving images as well as links to more relevant research papers via our personal web sites: <http://graphics.stanford.edu/~fedkiw> and <http://www.math.ucla.edu/~sjo/>.

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Los Angeles, California
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