CONFIDENCE LIMITS FOR A CROSS-PRODUCT RATIO[1]

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If the observations in a 2×2 table are distinctly out of proportion (and indeed in other cases also) we may wish to set limits to the true cross-product ratio, e.g. the observed table

10	3
2	15

gives a crude ratio of 25. How small could the true ratio be in reasonable consistency with the data?

If the expectations in the four classes were

$10-x$	$3+x$
$2+x$	$15-x$

the true ratio would be

$$(10-x)(15-x)/(3+x)(2+x),$$

and χ^2 for the observations would be

$$\chi^2 = x^2\left(\frac{1}{10-x}+\frac{1}{3+x}+\frac{1}{2+x}+\frac{1}{15-x}\right);$$

so, if x were $3 \cdot 0$

$$\chi^2 = 3^2(0\cdot59286) = 5\cdot3357$$

with one degree of freedom.

The exact probability of such a small sample of 30 giving 10 or more in the first quadrant is the partial sum of a hypergeometric series, and not easy to calculate, for if ξ stand for the theoretical product ratio, the frequencies of 0 to 12 in the quadrant will be proportional to the terms

$$1,\ \frac{13\times12}{1\times6}\xi,\ \frac{13\times12\times12\times11}{1\times2\times6\times7}\xi^2,\ \ldots\ \frac{13!12!5!}{(13-r)!(12-r)!(5+r)!r!}\xi^r\ \ldots$$

It would not be too difficult, as in the exact test for disproportionality, to calculate the last three terms for any chosen value of ξ, but for the ratio of these to the whole we would require the sum of the entire series, or

$$F(-13,\ -12,\ 6,\ \xi)$$

which would be best obtained by calculating all the terms and summing them, a process too lengthy to be recommended.

Using Yates' adjustment, however, we can at once find

$$\chi_c^2 = (2\cdot5)^2(0\cdot59286) = 3\cdot7054.$$

Further, taking $x = 3\cdot1$ we have

$$\chi_c^2 = (2\cdot6)^2(0.58897) = 3.9815.$$

Interpolating for the tabular entry $3\cdot841$, it appears that

$$x = 3.0491$$

and the cross-product ratio 2.720.

So that it may be inferred from the data that the true cross-product ratio exceeds 2.720, unless a coincidence of one in forty has occurred. Similar limits can be set in both directions and at all levels of probability.