Modeling Price Transmission between Farm and Retail Prices: A Soft Switches Approach

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Abstract

Vector error correction models (VECM) are used to model price transmission when farm and retail prices are cointegrated. To allow for non-linearity in the cointegration process, researchers may specify thresholds to break the error correction process into regimes according to whether the retail price is above, below, or close to its equilibrium value given farm prices. However, because the coefficients in a VECM can change when there is movement from one regime to another, the model can be discontinuous. This implies sudden, “hard” regime changes. In this study, we extend the threshold VECM to include features of STAR models. Our approach allows for gradual, soft regime changes. An empirical application to retail cheese and farm milk price is presented.

JEL classifications: C3, C4, Q1

*keywords: S*mooth transition autoregressive; Price transmission; Asymmetry; Cheddar cheese; Mozzarella cheese

1. Introduction

A perennial issue in agricultural economics is the relationship between retail food prices and the prices farmers receive for their products. Economic statistics on these relationships date back to at least 1913 (USDA, 1945). For years, a question of particular interest to researchers, policymakers, and farm groups alike is whether farm product price shocks are transmitted "symmetrically" or "asymmetrically" to retail food prices. The American Farm Bureau Federation (AFBF) asserts that, when the farm price of milk increases, marketers quickly pass the increases on to consumers. By contrast, when the farm milk price declines retail prices are adjusted downward slowly in order to increase marketers’ profits (AFBF, 2003). In general, there is a suspicion that asymmetry of price transmission contributes to lower farm prices. As recent as 2010, special hearings were held by the U.S. Department of Justice (USDOJ) and the

U.S. Department of Agriculture (USDA) where price transmission and other related concerns were addressed (USDOJ and USDA, 2010).

In a 2004 survey article, Meyer and von Cramon-Taubadel note that applied literature on asymmetric price transmission has been dominated by vector error correction models (VECM) when retail and farm-level prices are cointegrated. VECMs capture the inherent tendency of cointegrated variables to revert to their long-run relationship aftershocks to one or more of them. To allow for non-linearity in the cointegration process, the speed of adjustment may be allowed to differ according to whether the retail price is substantially above, below, or close to its equilibrium value given the farm price. This is accomplished by specifying thresholds that break the error correction process into distinct regimes, each of which may have different speeds of adjustment. Threshold VECMs (TVECM) are generally estimated in two steps and require cointegrated data. In the applied literature, they have typically been used to model the relationship between a single retail price and a single farm price and allow for 2 or 3 different price-transmission regimes. However, researchers seek still more flexible and general models.

Hassouneh et al. (2012) proposes extending the TVECM to include features of smooth-transition autoregressive (STAR) models. The basic approaches underlying the STAR approach have been applied to vector systems as well (Djik et al., 2002). In this study, we extend the TVECM to include features of STAR models as suggested by Hassouneh et al. Based on the present literature, this study is perhaps the first to take advantage of STAR models in this manner.

The model is estimated in a single step and does not require cointegration. The empirical application presented includes asymmetric and threshold interactions among retail prices for two types of cheese and the farm value of milk used to make those retail products. There are three cheese price models, Cheddar, Mozzarella, and farm. This additional contribution is considered to be significant since marketers commonly transform individual farm products such as milk, cattle, hogs, and poultry, among others, into multiple retail food products. Moreover, unlike a traditional TVECM model, a total of 9 regimes are analyzed, which include 3 regimes for each of our 3 prices.

2. Why Study Dairy Prices? Data and Descriptive Statistics

Production of natural cheese is the major use of milk produced in the United States. The quantity of milk used in natural cheeses, defined by two categories, American and Other-than-American, has grown steadily from about 68 billion pounds in 2000 to just over 86 billion pounds in 2011. According to the USDA Economic Research Service (ERS), annual sales of fluid milk and cream products have been relatively steady during the same period, hovering between 59 and 62 billion pounds of milk. Thus, in 2011, fluid milk and cream processing and cheese manufacturing together absorbed three quarters (75.2 percent) of total U.S. milk production. American (Cheddar) styles dominated cheese production until the mid-1980s. Now the “Other than American” styles are more important, especially Italian types like Mozzarella.

Data on the farm price of milk used in making different types of dairy products is available from USDA’s Agricultural Marketing Service (AMS). The AMS administers the Federal Milk Market Order (FMMO) system, a set of pricing regulations that establishes minimum classified prices. Through its administration of this program, the AMS generates data on milk supplies, utilization, sales, and plant prices, among other things. Current FMMO regulations include four class prices. The “Class III” milk price represents the minimum price paid for milk used to make cheese.

How much Class III Milk is needed to make a pound of Cheddar and Mozzarella? The challenge here is that milk leaving the farm has two economically significant components—fat solids and skim solids. Each of the milk solids furthermore has its own farm price. One hundred pounds of farm milk produced in the U.S. typically contains, on average, approximately 3.7 pounds of fat solids and 8.6 pounds of skim solids. Cheeses, including Cheddar and Mozzarella, contain relatively more fat and less skim solids than fluid milk. Using the Van Slyke formula, the amount of Class III milk needed to make a pound of each type of cheese is calculated, as well as the amount of skim solids left over.[1] Two farm prices were created originally, one for Cheddar and one for Mozzarella, based on the Class III price and the butterfat differential. However, the two were nearly perfectly correlated, >99.9%. The cost of farm milk to make a pound of Mozzarella is approximately 9.8% higher than the cost of farm milk to make Cheddar, which is the same relationship used in the model.

Movements in farm-level prices must be compared with movements in retail prices. Past studies for cheese have used retail price data from the US Department of Commerce's Bureau of Labor Statistics (BLS). The BLS collects and reports retail price data for a wide selection of food items as a part of its Consumer Price Index (CPI) program. These retail prices include all-city average prices charged by supermarkets and other retail food outlets for one pound of American-style Cheddar cheese.

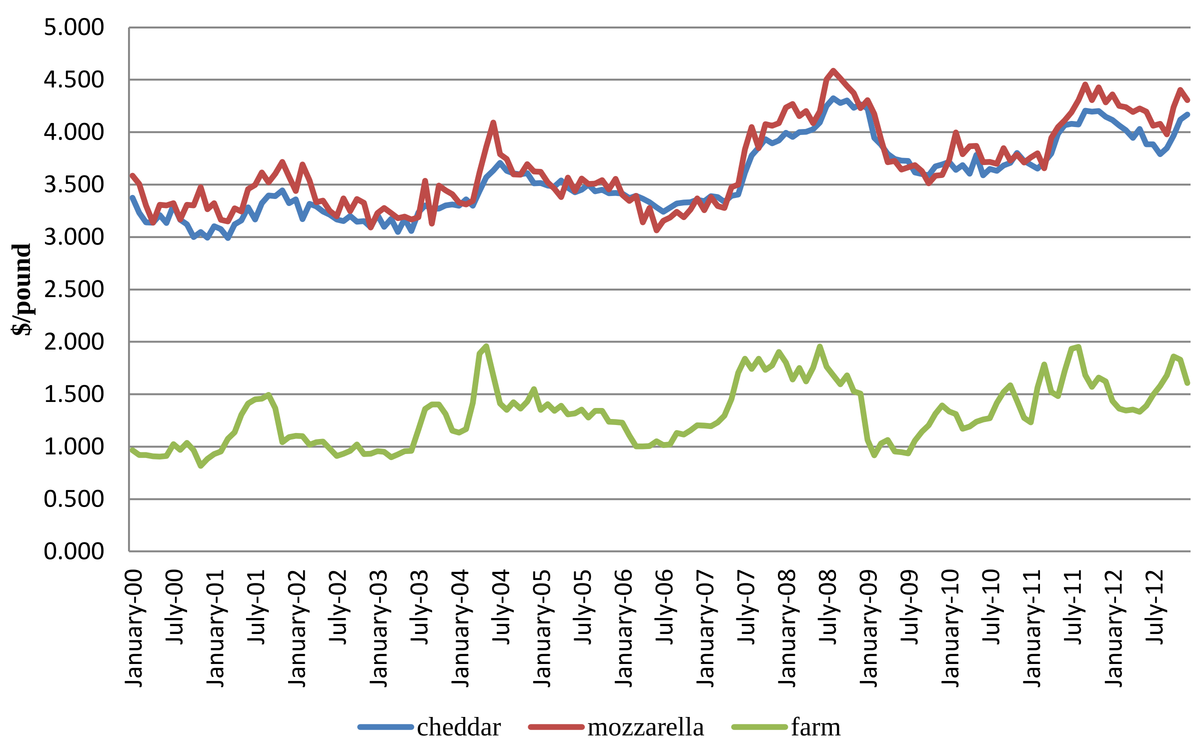
The National Consumer Panel (NCP) is another source of retail food prices. Information Resources, Inc. and Nielsen jointly maintain a panel of households that is demographically and geographically representative of the continental United States. Participating households are given a scanner to keep in their home. After a shopping occasion, panelists use these scanners to record their purchases including the quantities bought and the amount of money paid. Though

[1] Formulae and worksheets deriving farm value of each product are available upon request.

households may make mistakes when reporting information (e.g., some may fail to report all purchases), Einav et al. (2008) find that errors in these data are of the same order of magnitude as reporting errors in government-collected data sets commonly used to measure earnings and employment status. For this study, we used NCP data from 2000 to 2012. The advantage of using NCP data is that we can examine almost any dairy product, including Mozzarella cheese, whereas BLS releases average retail price data only for a relatively small number of dairy products.

Annual average retail prices and the farm values for a pound of Cheddar and a pound of Mozzarella cheese for the 2000 to 2012 time period can be seen in Figure 1. Over the long-run, the retail and farm values appear to move together, though not necessarily so in the short run. It is also clear that retail values for the two types of cheese always move essentially in tandem.

Figure 1-- Prices for retail cheeses and the farm cost of milk to make 1 pound of cheddar



3. Previous Studies

Price transmission is *asymmetric* when the speed and/or completeness of price adjustments to changes in economic conditions depend on the direction of the adjustment. For example, the retail price of a food may adjust more quickly to farm product price increases than to farm price decreases. Meyer and von Cramon-Taubadel (2004) identified two major approaches to modeling price transmission. The first is based on Wolffram (1971) and empirical analyses based on his method were popular in the 1970s and 1980s. Kinnucan and Forker (1987) used a Wolffram-based approach in their seminal study of price transmission in dairy markets that found price transmission is asymmetric for milk and manufactured dairy products like cheese.

von Cramon-Taubadel (1998) criticized Wolffram-based approaches as being inconsistent with cointegration. The second approach, which has been dominant in the applied literature in recent years, is based on Engel and Granger's (1987) error correction model—or in the terminology of this paper, the vector error-correction model (VECM). The VECM is designed to deal with cointegrated data and prices in related markets often are cointegrated. Milk-price transmission studies using ECM frameworks include Capps and Sherwell (2007), Awokose and Wang (2009), and Stewart and Blayney (2011).

Engel and Granger developed a 2-step procedure for estimating a simple VECM that has since been expanded to accommodate threshold model specifications. Here we outline the basic idea behind the VECM[2] using a simple example with two endogenous variables. One implication of cointegration is that while the two endogenous variables each have a unit root, a linear combination of them exists that does not have a unit root. For example, consider the following equation:

[2] Unless otherwise noted, the 1987 work by Engel and Granger is the source for statements in the next several paragraphs.

$$y_{2,t} = \theta_2 y_{1,t} + u_{2,t} \tag{1}$$

where θ_2 is a coefficient that determines the long-term relationship between $y_{2,t}$ and $y_{1,t}$, and $u_{2,t}$ is the first-stage error term. (Numerical subscripts have been attached to θ and u_t because the implication also applies to more than 2 endogenous variables, provided that all the terms are cointegrated.) The error term in equation (1) has a mean of 0 but will generally have some autocorrelation. Error-correction comes into play when an error that is large in period t is followed by smaller errors (in absolute value) in the following time periods.

If the data are cointegrated, one can estimate the first-order regression as in equation (1) and obtain a consistent estimate of θ_2. The method of testing for cointegration involves first testing the individual endogenous variables to see if they have unit roots and then estimating regressions such as equation (1) to test the resulting error terms for unit roots. If the endogenous variables have unit roots but the estimated errors do not, the data are cointegrated.

Engle and Granger wrote the VECM as:

$$\Delta y_{j,t} = \sum_i \sum_l \Delta y_{i,t-l}\beta_{j,i,l} + \pi_j u_{2,t-1} + \varepsilon_{j,t}, \text{ where } j=1,2,\ i=1,2,\ l=1,\ldots \mathrm{L}. \tag{2}$$

The model has "*L*" lags of endogenous variables and the $\beta_{i,j,l}$ are the coefficients of the ECM. The term π_j multiplies the lagged, estimated error term and $\varepsilon_{j,t}$ is a random error term with mean 0. When the lagged error is negative, $y_{1,t}$ is large relative to $y_{2,t}$. To get the prices closer to their long-run relationship, π_1 has to be positive and π_2 has to be negative.

The 2-step procedure of Engel and Granger involves estimating equation (1) or its equivalent in step 1 and using the lagged error estimated in that step to estimate equation (2) in step 2. Shortly after the Engel and Granger article was published, Johansen (1988) demonstrated that the ECM can be estimated in one step. He wrote the ECM as:

$$\Delta y_{j,t} = \sum_i \sum_l \Delta y_{i,t-l}\beta_{j,i,l} + \sum_i y_{i,t-1}\beta_{j,i,0} + \varepsilon_{j,t}. \tag{3}$$

In a two-variable, 1-unit root case, $\beta_{j,i,0}$ is a function of θ_2 and π_j. Johansen also showed that one could test this set of equations for unit roots by comparing the likelihood of the model with cointegrating restrictions imposed on $\beta_{j,i,0}$ to the likelihood of a model with an unrestricted $\beta_{i,j,0}$. Johansen also derived the distribution of these test statistics[3].

While the VECM can be estimated in a single step, the "classic" TVECM continues to be estimated in two steps (e.g., Balke and Fomby, 1997; Stewart and Blayney, 2011). In TVECM specifications, the coefficients of the VECM shift depending on the value of the lagged error terms. A typical, 3-regime TVECM can be written:

$$\Delta y_{j,t} = \sum_i \sum_l \Delta y_{i,t-l}\beta_{j,i,l,B} + \pi_{j,B}u_{2,t-1} + \varepsilon_{j,t}, \text{ when } u_{2,t-1} < \alpha_B,$$

$$\Delta y_{j,t} = \sum_i \sum_l \Delta y_{i,t-l}\beta_{j,i,l,W} + \pi_{j,W}u_{2,t-1} + \varepsilon_{j,t}, \text{ when } \alpha_B \le u_{2,t-1} \le \alpha_A,$$

$$\Delta y_{j,t} = \sum_i \sum_l \Delta y_{i,t-l}\beta_{j,i,l,A} + \pi_{j,A}u_{2,t-1} + \varepsilon_{j,t}, \text{ when } \alpha_A < u_{2,t-1}. \tag{4}$$

In this 3-regime model, the number of coefficients has basically been tripled and another set of subscripts: *B*, for **B**elow, *W* for bet**W**een, and *A* for **A**bove have been added. We also identified two threshold terms α_B and α_A.

Greb et al. (2012) go into more detail on how the TVECM is estimated and discuss some of the problems in its estimation. While their basic model has intercepts and exogenous variables, this study uses a more parsimonious specification for the purpose of simplifying the equations.

What is of particular interest is the "between" case or the "zone of inaction". Balke and Fomby (1997) originally proposed that the error correction process may contain a middle zone they called the zone of inaction in which the cointegrating relationship is inactive, possibly because fixed costs prevent economic agents from altering their behavior. In this zone, the two

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[3] Johansen derived his results for models with an arbitrary number of endogenous variables and for more than one unit root. Higher-order systems can share more than 1 unit root.

endogenous variables are close enough to their equilibrium relationship that we would expect minimal (or even zero) amounts of error correction; π_{jW} will be 0 for both j if this is a true zone of inaction. The cointegrating relationship again becomes effective when the system is far enough from equilibrium (i.e., in the Above and Below zones).

Because all the coefficients in (3) change when there is movement from one regime to another, the general TVECM can be discontinuous at its threshold levels. Some analysts find these discontinuities implausible. Mainardi (2001), who investigated international wheat prices over the period 1973 to 1999, argued that prices in spatially separated wheat markets like the United States and Australia tend to follow very similar movements and long-run trends. Small price gaps for wheat from different countries are not unusual because transportation and other transactions costs may limit arbitrage opportunities. That is, when price differences do not exceed transaction costs, arbitrage is not profitable and a zone of inaction may exist.

However, if traders and investors respond heterogeneously to changes in transaction costs, especially to changes occurring in the proximity of the thresholds, then the threshold points would become blurred.“ Mainardi (2001) concludes that gradual regime changes would make more sense than sudden regime switches. The estimation of smooth transition, non-linear error-correction models would also be preferred to a TVECM. One approach he suggested is the use of polynomial adjustments:

$$\Delta y_{j,t} = \sum_i \sum_l \Delta y_{i,t-l}\beta_{j,i,l} + \pi_{1,j}u_{2,t-1} + \pi_{2,j}u_{2,t-1}^2 + \pi_{3,j}u_{2,t-1}^3 + \varepsilon_{j,t} \qquad (5)$$

developed by Von Cramon-Taubadel (1996) and Escribano (2004).

In their study of fluid milk and cheese prices, Stewart and Blayney (2011) compared results for various ECMs. They found that both a three-regime TVECM and a cubic polynomial VECM better explained movements in cheese prices than either a two-regime threshold or a single-regime (linear) model. However, they preferred the cubic polynomial VECM on theoretical grounds.

Milk used to make cheese may pass from the farm gate to a first manufacturer who may deliver barrels or blocks to a second manufacturer for further processing. One of these firms may then negotiate prices with a firm still further downstream. Prices downstream firms negotiate may bear no relationship to either the current farm price of milk or the price paid for the milk now in the cheese. Those costs are sunk. Milk was bought, possibly, more than one month ago, made into cheese, and aged. Instead, if farm prices were to increase, manufacturers may reduce production. Total supply of cheese would then start to decrease, and firms would subsequently be able to negotiate higher prices from their customers.

However, if some firms along this supply chain face costs for adjusting their production levels then, following Balke and Fomby's (1997) theory, this process would not be a continuous one. Firms would instead wait until input prices had changed enough to outweigh adjustment costs. Moreover, it is plausible that different firms at different stages of the marketing chain may respond differently to input price changes because, for example, they have different costs for adjusting their production levels.

4. Our "Soft-Switched" Approach

As noted previously, this study follows the approach suggested in Hassouneh et al. (2012) to use a STAR-type modeling approach for analyzing asymmetric price transmission and the effects of threshold behavior. STAR models can be written as explicit switching models, as can the TVECM outlined in equation (4). In equation (4) the switches will be 0-1 variables that are referred to as "hard switches". The STAR-based switches are continuous variables that lie on the closed interval [0,1], which we refer to as "soft switches." Technically, STAR models are continuous by nature and thus this model is specified so that it would be continuous even if it were hard-switched like a TVECM as in equation (4).

The basic model will have exogenous variables, generalizing the discussion to date, but a limited lag structure and rests on specification of a vector, partial adjustment model. Both TVECM and STAR are pure time series models, models that generally have a limited number of exogenous variables such as intercepts and seasonal variables. Our model incorporates a more complete set of explanatory variables. The vector of exogenous variables is denoted as X_t, where "t" is a numbered index for a month. As before, the endogenous variables are written in scalar form as $y_{t,i}$. Here the index "i" is defined over three prices: farm, Cheddar, and Mozzarella.

The initial assumption is a set of exogenous variables determines the full-adjustment or "target" value of the prices. In our model, as in a TVECM, there may be only partial adjustment in the short run. However, the expected price changes tend move prices closer to their target values-relationships over time. Our basic, partial-adjustment relationship without switches is:

$\Delta y_{i,t} = \sum_j \beta_{i,j}(X_t C_j - y_{j,t-1}) + e_{i,t}$, or

$\Delta y_{i,t} = D_i X_t + \sum_j \beta_{i,j} y_{j,t-1} + e_{i,t}$, where $D_i = \sum_j \beta_{i,j} C_j$. (6)

Compared with the VECM in equation (3), the partial-adjustment equation in equation (6) has exogenous variables not included in equation (3) but has only a single lag. In equation (6), C_j is a vector of coefficients that determine the target value for price j.

One reason we begin with equation (6) is that the form resembles the VECM; it has changes in the endogenous variables on the left-hand-side and lagged endogenous variable levels on the right. Also, note equation (3) has lagged price changes but no exogenous variables. Some of our "experimental" versions of the model in equation (6) do have lagged price changes; however, we found much better fit by specifying the error term as a second-order VAR.

The basic equation in equation (6) is linear in its coefficients, albeit with non-linear coefficient restrictions. Note that the current price changes are negatively related to the β. It is expected that all the own-price terms, β_{ii}, are positive. Large lagged endogenous prices are likely to be larger than their target values—implying that the price needs to decrease. If the cross-price terms, β_{ij}, $i \neq j$ are positive, when the price j is above its target and needs to decrease, price i also decreases. A negative β_{ij} implies that high lagged prices for j tend to lead to higher current prices for i. One could impose and test unit roots on equation (6) using Johansen's approaches and restricting the matrix made of the β coefficients.

Although this type of model is called a "partial adjustment" model, it may exhibit "complete" or even "over adjustment" depending on the magnitude of β. If β_{ii} is 1 and the two β_{ij} are 0, complete adjustment is achieved. β_{ii} <1 implies partial adjustment, and β_{ii} >1 implies over adjustment. The cross-price effects can lead to complex[4] patterns.

4.1 Imposing Markup Relationships on the Target Prices

Absent any cross-equation restrictions on the coefficients of the C_j vectors, the D_i vectors exactly identify the C. A markup relationship is imposed on the target values of the three prices. Recall that the farm price is determined based on the amount of milk required to produce a pound of Cheddar cheese. It takes slightly over 9% more farm milk to produce a pound of Mozzarella. The Mozzarella multiplier is called m. The target value of Cheddar is the farm price plus a markup; the target price of Mozzarella is m times the farm price plus a (potentially) different markup. Our X vector is divided into two subsets; $X_{m,t}$. the subset that covers marketing costs, and $X_{g,t}$, the subset for the other variables. C_i can also be divided into $C_{m,i}$ and $C_{g,i}$. $C_{g,i}$ is then restricted using:

[4] By complex we mean both complicated and complex numbers, i.e. c+/-di. Complex roots imply a cyclical type of adjustment

$$C_{g,cheddar} = C_{g,farm}, \text{ and} \tag{7c}$$

$$C_{g,mozzarella} = mC_{g,farm}. \tag{7m}$$

Table 1, below, shows the exogenous variables included in the final version of our model and how these are divided into the *m* and *g* groups. A large set of exogenous variables is used while dropping all insignificant variables.

Table 1—Exogenous variables in the model

code	*explanation*	*group*[1,2]
X0	intercept	m
X1	trend	m
H1	long harmonic, cosine making 1 rotation in the sample period	m
H2	long harmonic, sine making 1 rotation in the sample period	m
H3	long harmonic, cosine making 2 rotations in the sample period	m
H4	long harmonic, cosine making 2 rotations in the sample period	m
sin1	seasonal variable, the sine of an angle making 1 turn per year	g
BeefWhl	USDA-ERS Choice beef wholesale value	g
BeefGFV	USDA-ERS Choice beef gross farm value	g
lagQ	lagged LN of milk production	g

[1] The "m" group drives the deflated price spreads for mozzarella and cheddar. The "g" group shifts the general price level of the three prices.
[2] The authors started out with a larger number of potential demand and supply shifters and kept only the 4 "g" terms listed in the table. Contact authors for a list of the insignificant shifters.

All prices are deflated using the Bureau of Economic Analysis (BEA) personal consumption deflator. Variables that measure marketing costs are simple functions of time: the intercept, trend and long harmonics. Deflating the prices and using simple functions of time as cost shifters basically makes the markup from farm to retail prices a function of the general price level-inflation rate.

4.2 Adding switches to the model

A general model is set up so that it will nest the partial adjustment model defined in equation (6). The switches in the model are driven by the predictable, non-error component of (6). To make the switched model specification simpler, the term $f_{i,t}$ is defined as:

$$f_{i,t} = X_t D_i - \sum_j \beta_{i,j} y_{j,t-1} \tag{8}$$

which is the right-hand-side of equation (6) without the error term. Rather than having 3 regimes for the system as a whole, we have 3 regimes for each equation. Each equation has its own set of thresholds.

In our original TVECM specification we had three sets of coefficients defined over the set {below, between, above}, $\{b,w,a\}$ for short. A 3-regime STAR is generally specified using 2 switches; our coefficient set is defined as {linear, below, above} or $\{l, b, a\}$. The subset {below, above} or $\{b,a\}$ will be called the "outside" subset—meaning outside the thresholds. Our switching model can now be written as:

$$\Delta y_{i,t} = \lambda_{i,l} f_{i,t} + s_{t,i,b}\lambda_{i,b}(f_{i,t} - \alpha_{i.b}) + s_{t,i,a}\lambda_{i,a}(f_{i,t} - \alpha_{i.a}) + e_{i,t}. \tag{9}$$

In equation (9) the λs are adjustment parameters and each *s* represents a different switch. The αs are double-subscripted now as they vary by their position and the equation. For each equation $\alpha_{i,b} \leq \alpha_{i,a}$.

Notably, λ, β, and α are not identified for either the hard-switched or our soft-switched model. For instance, it is possible double β and α in any equation, then halve its λ and get the exact same behavior for the $\Delta y_{i,t}$. The following arbitrary restriction is used to identify the λ for each equation:

$$\lambda_{i,l} + (\lambda_{i,b} + \lambda_{i,a})\tfrac{1}{2} = 1. \tag{10}$$

In addition to equation (10) all the λs are required to be positive. Requiring the λs to be positive assists us in 2 ways. First, if all λs in an equation are positive, then the change in the endogenous variable is monotonic in *f*. Second, it is still possible to have monotonic adjustment with a negative λ. However, a stronger adjustment is expected the further a price is from its target value. Requiring all λs to be positive insures that adjustment outside the thresholds is at least as strong as adjustment between the thresholds. The thresholds for each price have an inequality restriction that the lower threshold cannot be above the upper threshold for any of the products: $\alpha_{ib} \leq \alpha_{ia}$. In our estimation program, the "below" threshold is restricted to be non-positive and the "above" to be non-negative. Both could be 0 at the same time.

4.3 Hard and Soft Switches

As of yet, the switching terms have not been defined in equation (9). Our STAR-based approach is used to define these switches as "soft" switching. Djik et al. (2002) note that it is possible to specify STAR-type models so that they closely approximate abrupt-hard switched cases. To make equation (9) into a hard-switched model, like a TVECM, the two switches are defined as:

$$s_{i,t,b} = \begin{array}{ll} 1 & if\ f_{i,t} < \alpha_{i,b}, \\ 0 & otherwise; \end{array} \text{ and}$$

$$s_{i,t,a} = \begin{array}{ll} 1 & if\ f_{i,t} > \alpha_{i,a}, \\ 0 & otherwise. \end{array} \tag{11}$$

In (11) the "below" switch turns on when the *f* is below its lower threshold, the "above" switch turns on when the *f* is above the higher threshold. Both the switches are off when the *f* is between the thresholds. The hard-switched equation can be written in the TVECM form as:

$$\Delta y_{i,t} = \begin{array}{ll} \lambda_{i,l} f_{i,t} + \lambda_{i,b}\left(f_{i,t} - \alpha_{i.b}\right) + e_{i,t}, & \text{for } f_{t,i} < \alpha_{i,b}, \\ \lambda_{i,l} f_{i,t} + e_{i,t}, & \text{for } \alpha_{i,b} \leq f_{t,i} \leq \alpha_{i,b}, \\ \lambda_{i,l} f_{i,t} + \lambda_{i,a}\left(f_{i,t} - \alpha_{i.a}\right) + e_{i,t},, & \text{for } \alpha_{i,a} < f_{t,i}. \end{array} \tag{12}$$

Equation (12) shows that $\Delta y_{i,t}$ is continuous and piecewise linear in $f_{t,i}$. The hard-switched model outlined in equation (12) cannot generally be estimated but it is possible to set up STAR switches so that they closely approximate hard-switched models.

Our model has 3 regimes per equation. For each equation, the "outside" cases are modeled using a linear-LOGIT type function and the between-the-thresholds case using a quadratic function. Our general approach to building these soft switches is to start with 3 functions of f_t whose values are always positive, $g_{b,i}(f_{t,i})$, $g_{w,i}(f_{t,i})$, and $g_a(f_{t,i})$. The "*w*" subscript here stands for function for when $f_{t,i}$ is bet**W**een the thresholds. The three functions we use for our estimates are:

$g_{b,i}(f_{i,t}) = e^{-\Gamma(f_{i,t}-\alpha_{i,b})}$,

$g_{w,i}(f_{i,t}) = e^{-\Gamma(f_{i,t}-\alpha_{i,b})*(f_{i,t}-\alpha_{i,b})}$,

$$g_{a,i}(f_{i,t}) = e^{\Gamma(f_{i,t}-\alpha_{i,a})}. \quad (13)$$

The first and third functions above are for outside cases, and are the same functions used in a typical logit analysis. The between function is inspired by the normal distribution in so far as it involves a squared term. It will achieve its maximum at the mid-point of the thresholds. At either threshold, the exponent of g_w is 0, so its value is 1. The outside functions are also 1 at their thresholds. The switches are defined as:

$$s_{i,t,k} = \frac{g_{k,i}(f_{i,t})}{g_{b,i}(f_{i,t})+g_{w,i}(f_{i,t})+g_{a,i}(f_{i,t})} \text{ for k=(b,a).} \quad (14)$$

The larger the value of Γ, the more the soft switch resembles the hard switch. In most STAR applications, Γ is an estimated coefficient, which is fixed at 100, so that the soft switches would closely resemble the hard switches[5].

4.4 Model Specification through Hypothesis Testing

The approach taken in developing our cheese price model (CPM) has been to develop the most general and flexible model possible. Thus, to begin, our model is defined as equation (9) with the switches as defined in equation (14). However, in order to achieve a more parsimonious specification, researchers can test hypotheses about the asymmetry of price adjustment by

[5] We experimented with different values of Γ to see how they performed. When Γ is too large, the software generates calculation errors. We found that making Γ =100 gave us relatively hard regime breaks with few other issues. (Poor starting values for the other coefficients could still cause problems.)

Generally, what constitutes a "large" Γ depends on how the data is scaled. For our data a change of 0.10 (10 cents) is large. The greater the dispersion in the data the smaller a "large" Γ is. Also, after the fact, we tried out a range of Γ values on our final model. Once Γ was larger than 9, the model likelihood changed little for different values between 9 and 120.

placing restrictions on the λs and αs. Notably, model estimates are maximum likelihood if the error terms are normally distributed. For our tests, the likelihood of the restricted and unrestricted models are compared.

Under a wide variety of circumstances, the coefficients of a regression are asymptotically normal and the likelihood ratio tests are chi-square. The coefficients can be normally-distributed even if the errors are not. However, the λ and α coefficients in our model have sign constraints that result in violation of the regularity conditions that insure asymptotic normality. This raises certain econometric issues as described below. The implications of specific restrictions and the associated econometric issues are demonstrated using the hard-switched model and we note if and how the soft-switched models differ. Complete details of the testing procedures are available in the on-line technical appendix. The authors will post computer code on the web as well.

4.4a Imposing linear adjustment and the nuisance-parameter problem

We specified the CPM as generally as possible so that it could nest the relatively simple partial model in equation (6). Equation (9) can be turned into equation (6) for any equation in the CPM by making both outside λs equal 0: $\lambda_{i,a} = \lambda_{i,b} = 0$. By equation (10), if we do set both outside λs equal to 0, then $\lambda_{i,l}$ is 1. However, eliminating either of the "outside" λs, leaves the matching α unidentified. This is the "nuisance parameter" problem first identified by Davies (1977). Most STAR specifications suffer from the above problem (Djik et al., 2002). Typically, analysts deal with this issue using numerical techniques. The most common approach is to use some sort of series expansion of the constraint. In this study, it is important to deal with the sign constraints on λ and α. To evaluate the "problem tests", a Monte-Carlo or parametric bootstrapping procedure is used.

4.4b Imposing a two-regime equation Model

A general equation in our CPM has 3 regimes. However, any or all of the equations in the CPM can be constrained to create a linear partial-adjustment equation. The constrained equation(s) will have only 1 regime. Similarly, if the two αs in an equation are both 0, then the hard-switched model has effectively two regimes, similar to a threshold autoregressive (TAR) model (e.g. Enders and Siklos, 2001). In this case, equation (10) is no longer sufficient to identify the equation's λ. Our solution for the hard-switched case would be to pick one of the 3 λs and force it to be 0. Even though a soft-switched equation's λs are *technically* identified when both αs are 0, we continue to use the "set-1-λ-to-0-constraint" when both αs in the equation are 0.

The sign constraints imposed on both of the αs raise an additional econometric issue when estimating a two-regime model. The "below" α has an upper bound of 0, while the "above" α has a lower bound of 0. The higher bound is allowed to lie above the lower bound. In practice, it is possible, perhaps even likely, that the optimal estimate of one or both αs will actually be 0. A freely-estimated value could even come out on the wrong side of 0. Even without the bounds at 0, it would be possible for both estimated terms to "collapse" on one another. In this case, the constrained and freely-estimated models could converge to the same estimates and the likelihood-ratio test would be 0. The likelihood ratio test for a two-regime model will have a mixed continuous-discrete distribution and if the estimated test statistic is 0, we will accept the null hypothesis of a two-regime model.

4.4c Imposing asymmetric price adjustment

Price adjustment is going to be *asymmetric* if the speed of adjustment depends on the direction of adjustment. That is, if the speeds of adjustments associated with the above and below λ are different, i.e., $\lambda_{i,a} \neq \lambda_{i,b}$, then the process is asymmetric. Symmetric adjustment is achieved if $\lambda_{i,a}$

$= \lambda_{i,b}$. The symmetric adjustment constraint is not generally one of our problem cases except when both outside λ*s* are equal to 0.

4.4d The zone of inaction

Like Balke and Fomby (1997) and Greb et al. (2013), a zone of inaction can be obtained in our hard-switched case for price *i*, if the linear adjustment coefficient, $\lambda_{i,i}$, is 0 and if either or both of the α are not 0. Technically, the soft-switched model will have no true zone of inaction. With our large-Γ case there is some minor amount of adjustment between the thresholds when $\lambda_{i,l}$, is 0.

The test statistics for $\lambda_{i,j}$=0 are mixed continuous-discrete random variables just like the test statistics for α=0. Making a $\lambda_{i,j}$=0 gives us another flavor of the nuisance-parameter problem. When $\lambda_{i,j}$=0 the equation's intercept and thresholds are not identified. The same small number can be added or subtracted to the intercept and both αs without changing the prediction for $\Delta y_{i,t}$ when $\lambda_{i,j}$=0. "Zone of inaction" cases are identified by making the equations' thresholds symmetric around 0, i.e. by requiring that $\alpha_{i,b}+\alpha_{i,a}=0$.

5. Empirical Analysis

Our CPM was estimated with the monthly data on retail and farm prices for 2000 through 2012 previously described. Below, the results of our hypothesis tests conducted to specify a parsimonious model are summarized followed by estimation results and model interpretation. As previously noted above, our model tests and procedures are outlined in the online appendix. The computer code will also be available online.

5.1 Model Specification

The process of identifying a parsimonious model involved estimating and testing our model in a series of steps. The first set of hypothesis tests were carried out to see whether our CPM could be written as a standard partial-adjustment model without soft switching. This hypothesis was rejected by our tests.

Next, the constraints on α and λ are examined. These coefficients all have sign or bound constraints, and there are three cases where parameter estimates consistently reached their bounds. Both retail prices consistently converged to estimates when their “linear” λ coefficients were 0. Making the “linear” λ equal to 0 for either or both retail prices gives a test statistic of 0—and we accept that the two retail prices have a zone of inaction. Each retail price equation’s thresholds are identified by making the two thresholds symmetric, i.e., $\alpha_{i,b}+\alpha_{i,a}=0$.

The farm price thresholds collapsed on one another. That is, the upper and lower thresholds were both 0. Additionally, the “above” λ was consistently equal to 0. For our subsequent analysis, we therefore forced $\alpha_{i,b}= \alpha_{i,a}= 0$ and further set $\lambda_{i,a}$ (the above case) to be 0 for the farm price. This set of farm-price restrictions allows us to specify the farm-price equation using only one switch, the “below” switch. This simplifies the program mathematically and speeds up convergence.

Our estimates establish unrestricted zones of inaction for the two retail prices while the farm-price model converges to a two-regime model. The tests for any or all of these restrictions are 0—we accepted these hypotheses. Notably, the α estimates for the retail prices are remarkably similar—identical to two or three significant digits. Thus, a hypothesis test is added to help determine whether they are the same. The test for making both retail prices have the same thresholds is 0 to 5 places; this is considered insignificant. Price adjustment for both retail prices will be characterized by the same zone of inaction.

It is also of interest to test for the asymmetry of price adjustment. As discussed above, we have chosen to allow for two-regimes in our farm price equation. If the speed of adjustment is the same in each of these two regimes, the error correction process would further simplify to a linear one. In our tests, we rejected the 1-regime model and likewise concluded that adjustment for farm price has to be asymmetric. The retail prices are also going to have symmetric adjustment if their "above" and "below" λ*s* are the same. It is accepted that the linear part of λ is 0; the two retail prices are going to be symmetric if their outside λ*s* are 1. Our Monte-Carlo evaluations of these tests show that each retail price's test is insignificant as is the joint test of retail price symmetry.

Table 3— Testing the cross-price Beta coefficients against zero

equation	lagged dependent	test	degrees of freedom	chi-square alpha	insignificant
retail cheddar	farm milk cheese	18.09	1	0.00%	
retail cheddar	retail mozzarella	0.43	1	51.20%	yes
farm milk cheese	retail cheddar cheese	13.73	1	0.02%	
farm milk cheese	retail mozzarella	22.18	1	0.00%	
retail mozzarella	retail cheddar	0.33	1	56.64%	yes
retail mozzarella	farm milk cheese	39.24	1	0.00%	
imposing two insignificant terms together		0.89	2	64.15%	yes

[1] The test statistic is the difference between the free model likelihood and constrained model likelihood.

In the last phase of model testing, we examined the off-diagonal elements of the β_{ij} coefficients. We tested whether $\beta_{ij} = 0$, for all the $i \neq j$. Analysts often look for lead-lag relationships in vector price-transmission models. Price *i* leads price j when the lagged value of *i* is in *j's* equation, $\beta_{ji} \neq 0$, but *j's* lag is not in *i's* equation, $\beta_{ij} = 0$. Table 3 shows the results of our tests for the individual β coefficients. Two of β coefficients are insignificant and the insignificant terms are the cross-prices between the two retail prices. Shown in the bottom row of table 3 are the results of our test forcing both of the insignificant retail cross-price terms to be

0 simultaneously. This joint restriction is also insignificant. Both lagged retail prices have a significant effect on the current farm price and lagged farm prices significantly influence current retail prices. The two retail prices can have only indirect influence on one another through their interactions with the farm price. Our estimates imply that there are no leaders or followers in these prices.

5.2. Model Estimates

Table 4 shows the value of R-squared associated with each of our final-model estimates. The retail prices are better fit than the farm prices. Monte-Carlo techniques are used to evaluate the statistical properties of the final model estimates. The standard deviations or confidence intervals associated with our statistical estimates in the following tables are based on 5,000 Monte-Carlo iterations.

Table 4— R-squares in percent using two definitions of SST[1]

	level[2]	Naïve model[3]
retail cheddar	91.07	55.20
retail mozzarella	86.69	55.13
farm milk for cheese	81.79	23.82

[1] sum of squares total
[2] level calculates the SST as the variances of the prices
[3] Naïve model bases its SST on a non-change in price model. Its SST is the sum of squared price changes

The price adjustment implied by our model estimates is a function of its β, λ, and α parameters. Two of the retail prices have a zone of inaction but their adjustment is symmetric outside that zone. Their price adjustment outside the zone of inaction is driven by their β coefficients. The farm price has no zone of inaction, but does exhibit asymmetric adjustment. Farm price's adjustment will vary depending on whether it is likely to increase or decrease and is a function of its β and its λ.

Table 5 — Beta coefficient estimates

equation	lagged dependent	estimate	95% confidence interval[1] lower	upper
cheddar	cheddar	0.1929	0.1056	0.3915
	farm	0.1840	0.1128	0.3404
mozzarella	mozzarella	0.4642	0.3789	0.6463
	farm	-0.7654	-1.0887	-0.6278
	Cheddar	-1.2656	-1.7074	-0.8230
farm	mozzarella	0.7071	0.4508	1.0160
	farm	1.0888	0.9345	1.3285

[1]The confidence intervals are based on the 97.5 and 2.5 percentiles from 5,000 Monte-Carlo iterations.

Table 5 shows the β estimates. The Mozzarella effect on current Cheddar and the Cheddar effect on current Mozzarella are set to 0; these 0's are not shown in the table. As expected, each price's β_{ii}, the coefficient for the difference between its own target and lag, is positive. Cheddar's β_{ii} is less than 1, implying partial adjustment. Cheddar's lagged farm price coefficient is also positive implying that Cheddar prices will tend to increase along with the current farm price.

Like Cheddar, the price of Mozzarella exhibits partial adjustment to its target value. Mozzarella also has a large negative coefficient for its farm-price effect. This negative coefficient implies that Mozzarella is reacting to lagged farm prices: high farm prices last month mean high Mozzarella prices this month.

Price adjustment for the farm price is a product of its β and λ coefficients. The farm price has a β_{ii} that is close to 1. The Cheddar effect is large and negative, implying that the current farm price is reacting to the lagged Cheddar price; the Mozzarella effect is also positive and large implying that the current farm price tends to increase or decrease along with current Mozzarella prices. Note that the retail price βs in the farm price equation have the opposite signs of the farm-price βs in the retail price equations.

Table 6 — Lambda estimates for the farm price of milk[1]

	linear	below	"net" below effect
estimates	0.9184	0.1632	1.0816
lower bound, 95% confidence interval[2]	0.8336	0.0	1.0
upper bound	1.0	0.3328	1.1664
percentage of estimates at the bound	3.02%		

[1] The two retail prices have symmetric "outside" lambda whose values were fixed to 1. The upper bound for farm price's linear lambda is 1, the lower bound for the "below" lambda is 0.
[2] The confidence intervals are based on the 97.5 and 2.5 percentiles from 5,000 Monte-Carlo iterations.

Table 6 shows the λ estimates for the farm price. In those cases where farm prices would tend to increase, the β effects are multiplied by the linear λ. When the farm price would tend to decrease, the β are multiplied by the sum of the linear and below λ. Table 6 also shows the sum of the two farm-price λ. Farm prices tend to show partial adjustment to their own target when increasing and over-adjustment when decreasing.

No formal test was conducted to determine symmetrical adjustment for farm price. To make the farm-price adjustment symmetric, we would make its "below" λ=0, which would make its linear λ=1. Our Monte-Carlo simulations are based on an asymmetric farm-price adjustment. Table 6 shows that in slightly over 3% of our Monte-Carlo iterations, the linear λ estimate hit its upper bound of 1 simultaneously forcing the "below" λ to 0, its lower bound. This can be used as supporting evidence that the farm price is significantly asymmetric; however, the number of times we get symmetric estimates implies that the power of the asymmetry test is low.

Table 7 shows estimates and confidence intervals for the retail prices' common threshold parameters, the *α*. Because these two prices have no linear λ, the thresholds are identified by making them symmetric around 0. Table 7 shows only the upper threshold; the lower one is the negative of the upper.

Table 7 — The threshold-bound parameter α[1]

estimate		0.0537
lower bound	*95% confidence interval[2]*	0.0424
upper bound		0.0840

[1] This table shows the "above" threshold for the two retail prices. The below threshold is the negative of the above.
[2] The confidence intervals are based on 5,000 Monte-Carlo iterations. The lower bound is the 2.5% percentile value of those 5,000 iterated coefficients, the upper the 97.5% value.

5.3 The effects of asymmetry and thresholds on prices

Farm prices are subject to asymmetric price transmission that implies farm prices will tend to adjust more rapidly when decreasing than when increasing. Considering an observation made in the introduction, one may ask the question: Does this asymmetry make farm milk prices generally lower? Our model's estimates are used to simulate the effects of asymmetric price transmission on farm prices.

Our first set of simulations were made with the linear λ=1 and the below λ=0 for farm prices but kept the rest of the model parameters as estimated. The actual prices have error terms associated with them so we added in the estimated errors to our model simulations. One of the drawbacks of this simulation procedure is that it does not allow for real supply and demand responses. One of our exogenous variables is lagged milk production suggesting changes in the prices farmers receive will change the amount of milk they are willing to supply. Thus, it is expected that higher farm prices would lead to greater supply, which would in turn lead to lower prices.

For our second set of simulations, farm-price adjustment is made symmetric and the thresholds for the retail prices are eliminated. Basically, this means our estimates are used to run the linear, vector partial adjustment model. Error terms are also used in these simulations. Just as with the first set of simulations, this model simulation will not correct for supply or demand responses resulting from the price changes.

Table 8—summary statistics comparing the simulated with the actual prices

		cheddar	mozzarella	farm
	maximum change	0.33%	1.83%	4.66%
assuming symmetric	median change	-0.17%	0.14%	0.26%
farm adjustment	minimum change	-0.67%	-1.03%	-4.96%
symmetric farm and	maximum change	5.75%	7.33%	16.43%
the elimination of	median change	-2.53%	-1.35%	-1.62%
retail thresholds	minimum change	-5.85%	-7.40%	-13.47%

Calculated by the authors.

Table 8 summarizes the results of these two simulations. It shows the maximum, median and minimum percentage difference between the simulated and actual prices for all 3 products. Of the three prices, the farm price shows the largest percentage changes between the actual and simulated price results. Just making farm-price transmission symmetric would seem to have little effect on the price pattern over time. More extreme effects are generated when we also eliminate the thresholds. Eliminating thresholds would make all three prices generally, albeit inconsistently, lower.

A number of different simulations are done using the estimated coefficients from our models. It is usually the case that when one retail price is at its higher threshold, the other tends to be above its full-adjustment target as well. This in turn makes the farm price above its full-adjustment target. The opposite is true when a retail price is at its lower threshold. The thresholds tend to make prices stick at either high or low levels. Over the course of our sample it appears that the “stuck at low levels” scenarios are somewhat more common.

6. Conclusions

Recent applied literature on price transmission analyses has been dominated by error correction-type and related models. While these models allow for non-linearity in price transmission, researchers have sought still more flexible and general approaches. In this study, we extend the TVECM (threshold vector error correction model) to include features of STAR (smooth transition auto regressive) models. Our CPM (cheese price model) model is estimated in a single step and does not require data cointegration. Unlike most applications of TVECMs, we use our model to investigate asymmetric and threshold interactions among 3 prices: retail prices for two types of cheese and the Class III farm milk price.

Moreover, in order to increase flexibility, we allow for 9 regimes in total: 3 regimes for each of the 3 prices. Estimation results confirm that our model captures nuances of price transmission for milk and dairy products beyond what a TVECM could have captured. Finally, on the perennial question of whether price transmission is symmetric in the dairy industry, we find that it is asymmetric. But at the same time, our results suggest that this asymmetry is not economically significant to dairy farmers as there is no evidence that it reduces the price they receive for their milk.

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Appendix: Model Tests and Development

As noted above, we set up our cheese-price transmission model so that it tests a linear model. The likelihood-ratio test comparing the linear model to the most general one was 41.88. Testing the general model against its restricted linear form is complicated by identification issues and the sign constraints.

We used Monte-Carlo techniques to evaluate this test statistic. We used the β and D estimates from the linear model, the covariance matrix and VAR from the most general model, and the sample's exogenous variables to generate new sets of endogenous and lagged endogenous variables. We assumed normally distributed errors. We used this simulated data and re-estimated the linear and general models, saving the likelihood ratio tests. After every iteration, the program saved the test results to date in a file we could open—which we did occasionally.

We set up our program to do 5,000 iterations but stopped it after only 436. At that point only 4 other those iterated tests were larger than our actual test. Suppose that our actual test were the customary 5% significance level value. The odds of 4 or fewer 5% tests in a set of 436 random tries are under 3 in a million. We are confident that 41.88 is larger than the 5% value for this test.

We are unconcerned with discovering the 5% critical value of any of our tests; we are concerned with determining whether or not a test is significant at the 5% level. For our test analysis we use the binomial distribution to compare the number of Monte-Carlo tests exceeding our actual to what one would see if the actual **were** a 5% value. An unusually large number of Monte-Carlo tests over the actual value shows that the actual value is not significant; an unusually small number of tests over the actual, as above, is a sign that the actual test is over the 5% critical value. We use an under 1-in-10,000 or 0.00% (rounded) criteria to determine whether an actual test is significant or not. The worse-case scenario for our approach is an actual test statistic that has a 5% value.

Testing hypotheses about the λ and α

Four things consistently happened with our model estimates. First, both retail prices consistently converged to estimates where their "linear" λ coefficients were 0. As noted above, under these circumstances we need a constraint to identify the retail prices' thresholds—we required them to be symmetrical about 0. The model naturally converged to retail prices with a zone of inaction.

As noted above, we use likelihood ratio tests for our hypotheses. The test statistic for imposing a zone of inaction on either or both retail prices is 0. We consider this hypothesis accepted. For our subsequent analysis we forced the retail prices to have a zone of inaction.

The second thing was that the threshold estimates for the two retail prices were remarkably similar, the same to 2 or 3 decimal points. We decided to test whether or not they are the same.

The farm-price thresholds differed from the two retail price thresholds. The farm price thresholds collapsed on one another. That is upper and lower thresholds were the same and insignificantly different from 0. If this were a model with hard switches, the set of farm-price λ would be unidentified when the thresholds collapse on themselves. (One way to make the model identified would be to make any of the 3 farm-price λ equal to 0[6].) With the soft-switched case we get a kind of weak identification. The model consistency converged to a point where the "above" λ was 0. For our subsequent analysis, we forced farm's 2 α to 0 and its λ_{ia} (the above case) to be 0. One minor advantage of this set of farm-price restrictions is that it allows us to specify the farm-price equation using only one switch, the "below" switch. This gives the program less math with which to deal and speeds up convergence.

One may impose symmetry on the retail prices by requiring that their above and below λ be the same. Given our zone of inaction and our identifying restrictions, this means that a retail price is symmetric if its outside λ are 1. This is a 1-degree-of-freedom restriction on an equation. Requiring the retail prices to have the same thresholds-α is also a 1-degree-of-freedom restriction. The retail prices' symmetry and α restrictions[7] are consistent with the regularity conditions implying asymptotic normality. The farm price will be symmetric when its "below" λ is 0. Given the sign constraints this hypothesis test is going to have a mixed asymptotic distribution.

The test for making both retail prices have the same thresholds was 0 to 5 places; we consider that insignificant. Table 2 shows some of the tests for price adjustment symmetry. Mozzarella's test is insignificant at the 5% level; cheddar's test has a significance level of 2.72%. Jointly imposing symmetry on mozzarella and cheddar has an asymptotic significance of 2.61%.

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[6] We ran a model where we forced farm's linear λ to be 0; this restriction decreased the likelihood by 0.15, a small amount.
[7] They are consistent locally. It is possible that an α or λ estimate could hit a bound in a small sample.

In our experience, the small-sample distributions of hypothesis tests tend to have fatter tails than the asymptotic distributions. For this reason we used Monte-Carlo analysis to test the symmetry of the retail prices. We used a model with symmetric retail price adjustment to generate and test the hypotheses that both retail prices are symmetric by themselves and as a group. We saved the two-degree of freedom joint restriction tests and the largest of the individual equation tests. We then compared these Monte-Carlo tests to the actual joint and largest individual tests. Again, we were able to stop the program early after 1,928 iterations[8]. 213 of our joint tests exceeded the actual joint value and 358 of our largest individual tests exceeded the largest individual values; both are unusually large number numbers for a 5% value. We conclude that neither of the statistics is significant and accept that the retail prices have symmetric price adjustment.

[8] We don't obsessively check these things—apparently we were busier when running this analysis than we were with the first.