A UNIFIED THEORY OF UNDERREACTION, MOMENTUM TRADING AND OVERREACTION IN ASSET MARKETS

Harrison Hong
Jeremy C. Stein

Working Paper **6324**

NBER WORKING PAPER SERIES

A UNIFIED THEORY OF UNDERREACTION, MOMENTUM TRADING AND OVERREACTION IN ASSET MARKETS

Harrison Hong
Jeremy C. Stein

Working Paper 6324
http://www.nber.org/papers/w6324

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1997

A Unified Theory of Underreaction, Momentum Trading and Overreaction in Asset Markets
Harrison Hong and Jeremy C. Stein
NBER Working Paper No. 6324
December 1997

ABSTRACT

We model an asset market populated by two groups of boundedly rational agents--"newswatchers" and "momentum traders"--each of whom can only process a subset of all available information. The bounded rationality of the newswatchers creates a tendency for prices to underreact to private information in the short run. This underreaction in turn means that the momentum traders can make money by trend-chasing. However, if they are restricted to following simple (i.e., univariate) strategies, their best attempts at arbitrage, though profitable, must inevitably lead to overreaction at long horizons. In addition to providing a unified account of asset-market under- and overrreactions, the model generates a number of other distinctive testable implications.

Harrison Hong
Graduate School of Business
Department of Finance
Stanford University
Stanford, CA 94305-5015
harrison_hong@gsb.stanford.edu

Jeremy C. Stein
Sloan School of Management, E52-434
Massachusetts Institute of Technology
50 Memorial Drive
Cambridge, MA 02142-1347
and NBER
jcstein@mit.edu

1. Introduction

Over the last several years, a large volume of empirical work has documented a variety of ways in which asset returns can be predicted based on publicly available information. Although different studies have used a host of different predictive variables, many of the results can be thought of as belonging to one of two broad categories of phenomena. On the one hand, there is a tendency for asset prices to underreact to news in the short run. On the other hand, there is also a tendency for prices to overreact to news in the long run.[1]

It is becoming increasingly accepted that the accumulated facts on underreaction and overreaction are hard to reconcile with standard renditions of the efficient markets hypothesis (EMH) in which asset prices reflect all public information. In the context of the EMH, all of the predictable patterns--both at short and long horizons--must be explained by variations in asset risk. And there is essentially no affirmative evidence at this point that any of the patterns discussed above can be rationalized based on measured variations in risk.

The shortcomings of the EMH suggest that a new theory is needed to explain the empirical facts about asset prices. Almost by definition, this new theory will have to be "behavioral", meaning that it will have to involve some departure from the EMH assumptions of strict rationality and unlimited computational capacity on the part of investors. But there are a potentially huge number of such departures that one might entertain, so it is hard to know where to start. In order to impose some discipline on the process, it is useful to be clear about the criteria that a new theory should be expected to satisfy. There seems to be widespread agreement that to be successful, any candidate theory will, at a minimum: 1) rest on assumptions

---

[1]We discuss this empirical work in detail and provide references in Section 2 below.

about investor behavior that are either a priori plausible or consistent with casual observation; 2) explain the existing evidence in a parsimonious and unified way; and 3) make a number of further predictions which can be subject to testing and which are ultimately validated.

In our view, the unified-model criterion is especially important. There already exist a number of behavioral models or informal stories which can each explain some pieces of the empirical evidence.[2] There has been less success to date in building models which offer an integrated explanation of both the underreaction and overreaction types of phenomena. Notably, however, a couple of recent papers do take up this unified-behavioral-model challenge. Both Barberis, Shleifer and Vishny (1997) and Daniel, Hirshleifer and Subrahmanyam (1997) take an approach that might be described as modelling the psychology of the representative investor. In other words, both papers assume that stock prices are determined by a single representative agent, and then posit a small number of cognitive biases that this representative agent might have. They then investigate the extent to which these biases are sufficient to simultaneously deliver both short-horizon underreaction and long-horizon overreaction.[3]

In this paper, we pursue the same goal as BSV and DHS, that of building a unified behavioral model. However, we adopt a somewhat different approach to the problem. Rather than trying to say much about the psychology of the representative investor, our premise is that the observed asset-pricing phenomena are inherently all about interactions between heterogeneous

[2]For example, pure noise shocks as in DeLong et al (1990a) can yield the negative autocorrelations in returns that are characteristic of overreaction. And any story wherein agents are slow to digest new information can generate positive short-run autocorrelations.

[3]We will have much more to say about these papers, as well as a number of other related works, in Section 6 below.

agents. To put it loosely, less of the action in our model comes from particular cognitive biases that we ascribe to individual investors, and more of it comes from the way these investors interact with one another.

More specifically, our model features two types of agents, whom we term "newswatchers" and "momentum traders". Neither type is fully rational in the usual sense. Rather, each is boundedly rational, with the bounded rationality being of an especially simple form: each type of agent is only able to "process" some subset of the available public information. Thus newswatchers make forecasts that are conditioned only on news that they privately observe about future fundamentals; they do not condition on current or past prices. Momentum traders, in contrast, condition only on some simple measures of past price changes, and do not observe news about fundamentals. Importantly, each type of agent makes the best possible use of the information that he does process. For example, momentum traders' forecasts will be the optimal forecasts given the limited conditioning information that they use.

We begin by showing that when only newswatchers are active, prices adjust gradually to new information--i.e., there is underreaction (positively correlated returns at short horizons) but never any overreaction (returns are never negatively correlated). There is not much to this result. Indeed, as will become clear, this baseline level of underreaction is essentially built into the model by virtue of the assumption that newswatchers do not extract information from prices.

Things get more interesting when we add the momentum traders. It is tempting to conjecture that because the momentum traders <u>can</u> condition on past prices, they will arbitrage away any underreaction left behind by the newswatchers. In other words, if returns are positively autocorrelated, momentum traders should rationally adopt a "trend-chasing" strategy of buying after price increases, and selling after price decreases. If the momentum traders are sufficiently risk-tolerant, one might expect that this would eliminate the underreaction, and lead the market to be approximately efficient.

It turns out that this intuition is incomplete, if momentum traders are restricted to following relatively simple strategies. For example, suppose that a momentum trader at time t must base his trade only on the price change over some prior interval, say that from t-2 to t-1. We show that in this case, momentum traders' attempts to profit from the underreaction caused by newswatchers lead to a somewhat perverse outcome: the initial reaction of prices in the direction of fundamentals is indeed accelerated, but this comes at the expense of creating an eventual overreaction to any news. This is true even when momentum traders are risk neutral. Figure 1 illustrates the impulse response of prices to new information that arises in our model, for the cases with and without momentum traders.[4]

Thus our model ties together the underreaction and overreaction phenomena in the following way. We begin by more or less assuming some tendency toward underreaction on the part of some traders. We then ask what happens when we allow a second group of traders to try to take advantage of this underreaction. When the second group uses a simple (but optimal, given its simplicity) arbitrage strategy, they only partially mitigate the underreaction, and in so doing, create an excessive momentum in stock prices that must inevitably culminate in overreaction. In other words, the very existence of underreaction sows the seeds for overreaction, by making it profitable for momentum traders to enter the market.

The key to our results is the assumption that momentum traders use simple strategies--

[4]We will discuss how this example is calibrated below.

i.e., do not condition on all publicly available information. Continuing with the above example, if a momentum trader's order strategy at time t is restricted to being a function only of the price change from t-2 to t-1, it is easy to see that it must be an increasing function of this price change. On average, this simple trend-chasing strategy will make money in equilibrium. But if one could condition on more information, it would become apparent that the strategy does better in some circumstances than in others. In particular, a simple momentum strategy earns the bulk of its profits early in the "momentum cycle"--by which we mean shortly after substantial news has arrived to the newswatchers--and loses money late in the momentum cycle, by which time prices have already overshot long-run equilibrium values.

For concreteness, suppose that there is a single dose of good news at time t and no change in fundamentals after that. The newswatchers cause prices to jump at time t, but not far enough, so that they are still below their long-run values. At time t+1 there will be a round of momentum purchases, and those momentum buyers who get in at this time will make money. But this round of momentum trading creates a further price increase, which sets off more momentum buying, and so on. Later momentum buyers--i.e., those buying at t+i for some i--will lose money, because they will get in at a price above the long-run equilibrium.

Thus a crucial insight is that "early" momentum buyers impose a negative externality on "late" momentum buyers.[5] Ideally, one uses a momentum strategy because a price increase signals that there is good news about fundamentals out there that is not yet fully incorporated into prices. But sometimes, a price increase is the result not of news but just of previous rounds

[5]As we will explain in more detail below, this negative "momentum externality" is reminiscent of the herding models of Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992) and Scharfstein and Stein (1990).

of momentum trade. Because momentum traders cannot directly condition on whether or not news has recently arrived, they do not know whether they are early or late in the cycle. Hence they must live with this externality, and accept the fact that sometimes they will be buying when earlier rounds of momentum trading have pushed prices past long-run equilibrium values.

In what follows, we develop a simple infinite-horizon model that allows us to capture these ideas and to explore their empirical content. We begin in Section 2 by giving an overview of the empirical evidence that motivates our work, and briefly discussing why the EMH is unlikely to be able to explain it. In Section 3, we present and solve the basic model, and do a number of comparative statics experiments. Section 4 contains several extensions. In Section 5, we draw out the model's empirical implications, with particular emphasis on those that have not yet been tested. Section 6 discusses related work, and Section 7 concludes.

2. Evidence of Underreaction and Overreaction

2.A Underreaction

The underreaction evidence can be decomposed along the following lines. First, returns tend to exhibit unconditional positive serial correlation at short horizons on the order of six to twelve months. This is true both for aggregate indices (Cutler, Poterba and Summers 1991) and in cross-sections of individual stocks (Jegadeesh and Titman 1993). One possible interpretation of this unconditional evidence--which fits most closely with the spirit of the model we develop below--is that information which is initially private is incorporated into prices only gradually.

Second, conditional on observable public events, stocks tend to experience post-event drift in the same direction as the initial event impact. The types of events that have been examined in detail and that fit this pattern include: earnings announcements (perhaps the most-studied type of event in this genre; see e.g. Bernard 1992 for an overview); stock issues and repurchases; dividend initiations and omissions; and analyst recommendations.[6]

Recent work by Chan, Jegadeesh and Lakonishok (1996) shows that these two types of underreaction are distinct: in a multiple regression, both past returns and public earnings surprises help to predict subsequent returns at horizons of six months and one year. This seems to suggest that the market underreacts to both information which is initially private, as well as information which is made publicly available to all investors simultaneously.

2.B Overreaction

One of the first and most influential papers in the overreaction category is DeBondt and Thaler (1985), who find that stock returns are negatively correlated at long horizons. Specifically, stocks that have had the lowest returns over any given five-year period tend to have high returns over the subsequent five years, and vice-versa.[7] A common interpretation of this result is that when there is a sustained streak of good news about an asset, its price overshoots its "fundamental value", and ultimately must experience a correction. More recent work in the

---

[6]References include: Bernard and Thomas (1989, 1990) on earnings announcements; Loughran and Ritter (1995) and Spiess and Affleck-Graves (1995) on stock issues; Ikenberry, Lakonishok and Vermaelen (1995) on repurchases; Michaely, Thaler and Womack (1996) on dividend initiations and omissions; and Womack (1996) on analyst recommendations.

[7]DeBondt and Thaler's (1985) results generated a good deal of controversy, but they seem to have stood up well to scrutiny (see, e.g. Chopra, Lakonishok and Ritter 1992). There are also direct analogs to these results in the time series of aggregate market returns, although the statistical power is lower. See Fama and French (1988), Poterba and Summers (1988) and Cutler, Poterba and Summers (1991).

same spirit has continued to focus on long-horizon predictability, but has used what are arguably more refined indicators of fundamental value, such as book-to-market, and cashflow-to-price ratios. (See, e.g., Fama and French 1992, Lakonishok, Shleifer and Vishny 1994.)[8]

2.C Is It Risk?

In principle, the predictable patterns in asset prices discussed above could be consistent with the EMH, to the extent that they reflect variations in risk, either over time or across assets. Fama and French (1993, 1996) argue that many of the long-horizon overreaction types of results --such as return reversals, the book-to-market effect, and the cashflow-to-price effect--can be largely subsumed within a three-factor model that they interpret as a variant of the arbitrage pricing theory (APT) or intertemporal capital asset pricing model (ICAPM). However, this pro-EMH position has been controversial, since there is little affirmative evidence that the Fama-French factors correspond to any economically meaningful risks. Indeed, several recent papers demonstrate that the contrarian strategies that exploit long-horizon overreaction are not significantly riskier than average.[9]

There seems to be more of a consensus in the literature that the short-horizon underreaction evidence cannot be explained in terms of risk. Bernard and Thomas (1989) reject risk as an explanation for post-earnings-announcement drift. And even Fama and French (1996)

---

[8]Again, there are analogous "fundamental reversion" results in the time-series literature on aggregate market predictability. (See Campbell and Shiller 1988.)

[9]See Lakonishok, Shleifer and Vishny (1994), and MacKinlay (1995). Daniel and Titman (1997) provide direct evidence against the Fama-French notion that the book-to-market effect can be given a multifactor risk interpretation.

acknowledge that the continuation results of Jegadeesh and Titman (1993) constitute the "main embarrassment" for their three-factor model. (p. 81)

3. The Model

3.A Price Formation With Newswatchers Only

As mentioned above, our model features two classes of traders, newswatchers and momentum traders. We begin by describing how the model works when only the newswatchers are present. At every time t, the newswatchers trade claims on a risky asset. This asset will pay a single liquidating dividend at some later time T. The ultimate realized value of this liquidating dividend can be written as: $D_T = D_0 + \sum^{T}_{j=0} \epsilon_j$, where all the ϵ's are independently distributed, mean-zero normal random variables. Throughout the paper, we will consider the limiting case where T goes to infinity. This simplifies matters by allowing us to focus on steady-state trading strategies--i.e., strategies that do not depend on how close we are to the terminal date.[10]

The newswatchers are divided into z equal-sized groups. At any time t, every one of these groups observes $D_t = D_0 + \sum^{t}_{j=0} \epsilon_j$. In addition, at time t, news about ϵ_{t+z-1} begins to come out. This news is broken into z smaller "packets", $\epsilon^1_{t+z-1} \ldots\ldots \epsilon^z_{t+z-1}$, which collectively sum to ϵ_{t+z-1}. Each of the z groups observes one of the packets (a different one) over each of the next

---

[10]A somewhat more natural way to generate an infinite-horizon formulation might be to allow the asset to pay dividends every period. The only reason we push all the dividends out into the infinite future is for notational simplicity. In particular, when we consider the strategies of short-lived momentum traders below, our approach allows us to have these strategies depend only on momentum traders' forecasts of price changes, and we can ignore their forecasts of interim dividend payments.

z periods. This implies that at time t, each of the packets is known by 1 group. At time t+1, each of the packets is known by 2 groups, and so forth until time t+z-1, at which point every one of the z groups has seen all of the information that comprises ϵ_{t+z-1}.

The most straightforward interpretation is that ϵ_{t+z-1} is information that is unavailable before time t, and that gradually diffuses to the investing public--in the sense of being directly observed by more and more people--over the period from t to t+z-1. By time t+z-1, everybody knows all the new information. As will become clear shortly, this interpretation is sufficient if our goal is to capture the sort of underreaction that shows up empirically as unconditional positive correlation in returns at short horizons (e.g., that documented by Cutler, Poterba and Summers 1991, and Jegadeesh and Titman 1993) However, if we are also interested in capturing phenomena like post-earnings-announcement drift--where there is apparently underreaction even to data that is made available to everyone simultaneously--we will need to embellish the story. We will discuss this embellishment later on; for now it is easiest to think of the model as only speaking to the unconditional evidence on underreaction.

All the newswatchers have CARA utility with the same risk-aversion parameter, and all live until the terminal date T. The riskless interest rate is normalized to zero, and the supply of the asset at all times is fixed at Q. So far, all these assumptions are completely standard. We now make two that are less conventional. First, at every time t, newswatchers formulate their demands for the asset based on the static-optimization notion that they will buy and hold until the liquidating dividend at time T.[11] Second, and more critically, while newswatchers can

[11]There is an element of time-inconsistency here, since in fact newswatchers may adjust their positions over time. Ignoring the dynamic nature of newswatcher strategies is more significant when we add momentum traders to the model, so we discuss this issue further in Section 3.B.

condition on the information sets described above, they do not condition on current or past prices. In other words, our equilibrium concept is a Walrasian equilibrium with private valuations, as opposed to a fully revealing rational expectations equilibrium.

As suggested in the Introduction, these two unconventional assumptions can be motivated based on a simple form of bounded rationality. One can think of the newswatchers as having their hands full just figuring out the implications of the ϵ's for the terminal dividend D_T . This leaves them unable to also use current and past market prices to form more sophisticated forecasts of D_T (our second assumption); it also leaves them unable to make any forecasts of future price changes, and hence unable to implement dynamic strategies (our first assumption).

Given these assumptions, and the symmetry of our set-up, the conditional variance of fundamentals is the same for all newswatchers, and it is easy to see that the price at time t is given by:

$$P_t = D_t + \{(z-1)\epsilon_{t+1} + (z-2)\epsilon_{t+2} + \ldots\ldots + \epsilon_{t+z-1}\}/z - \theta Q \tag{1}$$

where θ is a function of newswatchers' risk aversion and the variance of the ϵ's. For simplicity, we will normalize the risk aversion so that $\theta = 1$ hereafter. In words, equation (1) says that the new information works its way gradually into the price over z periods. This implies that there is positive serial correlation of returns over short horizons (of length less than z). Note also that prices never overshoot their long-run values, or equivalently, that there is never any negative serial correlation in returns at any horizon.

Even given the eminently plausible assumption that private information diffuses gradually across the population of newswatchers, the gradual-price-adjustment result in (1) hinges critically on the further assumption that newswatchers do not condition on prices. For if they did--and as long as Q was common knowledge--the logic of Grossman (1976) would imply a fully revealing equilibrium, with a price P^*_t, satisfying (for $\theta = 1$):

$$P^*_t = D_{t+z-1} - Q \tag{2}$$

Clearly, in the rational expectations setting, gradual information diffusion does not by itself generate gradual adjustment of prices--i.e., there is no serial correlation of returns at any horizon as long as Q is nonstochastic.[12]

We should therefore stress that we view the underreaction result embodied in equation (1) to be nothing more than a point of departure--it is neither terribly interesting in and of itself, nor satisfactory as a complete account of asset pricing. It is not so interesting because we have to some extent hardwired in the baseline level of underreaction with our combination of assumptions about gradual information diffusion and lack of conditioning on prices. It is not satisfactory as an account of asset prices, because it begs an obvious question: even if newswatchers are too busy processing fundamental data to incorporate prices into their forecasts, can't some other group of traders focus exclusively on price-based forecasting, and in so doing generate an outcome close to the rational expectations equilibrium of equation (2)? It is to this central question that we turn next, by adding the momentum traders into the mix.

---

[12]This is true even if we maintain the assumption that newswatchers rely on static strategies that only involve predicting the terminal dividend D_T.

3.B Adding Momentum Traders to the Model

Momentum traders also have CARA utility. Unlike the newswatchers, however, they have finite horizons. In particular, at every time t, a new generation of momentum traders enters the market. Every trader in this generation takes a position, and then holds this position for j periods--that is, until time t+j. For modelling purposes, we will treat the momentum traders' horizon j as an exogenous parameter; later we will discuss how it might be endogenized.

The momentum traders transact with the newswatchers by means of market orders. In other words, the momentum traders submit quantity orders, not knowing the price at which these orders will be executed. The price is then determined by the competition among the newswatchers, who double as market-makers in this set-up. Thus in deciding the size of their orders, the momentum traders at time t must try to predict $(P_{t+j} - P_t)$. To do so, they make forecasts based on past price changes. We assume that these forecasts take an especially simple form: the only conditioning variable is the cumulative price change over the past k periods, i.e., $(P_{t-1} - P_{t-k-1})$.

As it turns out, the exact value of k is not that important, so in much of what follows we simplify things by setting $k = 1$, and using as the time-t forecasting variable $(P_{t-1} - P_{t-2}) \equiv \Delta P_{t-1}$. What is much more significant is that we are restricting the momentum traders to making univariate forecasts based on past price changes. If, in contrast, we allowed them to make forecasts using n lags of price changes, giving different weights to each of the n lags, we suspect that for sufficiently large n, many of the results that we present below would go away. Again, the motivation for restricting the momentum traders to univariate strategies is a crude notion of bounded rationality: traders simply do not have the computational horsepower to run complicated multivariate regressions.

With $k = 1$, the order flow from generation-t momentum traders, F_t, is of the form:

$$F_t = A + \phi\Delta P_{t-1} \tag{3}$$

where the constant A and the elasticity parameter ϕ have to be determined from optimization on the part of the momentum traders. This order flow must be absorbed by the newswatchers. We assume that the newswatchers treat the order flow simply as an uninformative supply shock. This is the only way to be consistent with our previous assumption that the newswatchers do not condition on current or past prices. Given that the order flow is simply a linear function of past price changes, if we allowed the newswatchers to extract information from it, we would be indirectly allowing them to learn from prices.

To streamline things, the order flow from the newswatchers is the <u>only</u> source of supply variation in the model. Given that there are j generations of momentum traders in the market at any point in time, the aggregate supply S_t absorbed by the newswatchers is given by:

$$S_t = Q - \Sigma^j_{i=1} F_{t+1-i} = Q - jA - \Sigma^j_{i=1} \phi\Delta P_{t-i} \tag{4}$$

We continue to assume that, at any time t, the newswatchers act as if they will buy and hold until the liquidating dividend at time T. This implies that prices are given exactly as in (1), except that the fixed supply Q is replaced by the variable S_t, yielding:

$$P_t = D_t + \{(z-1)\epsilon_{t+1} + (z-2)\epsilon_{t+2} + \ldots\ldots + \epsilon_{t+z-1}\}/z - Q + jA + \sum^{j}_{i=1} \phi\Delta P_{t-i} \qquad (5)$$

In most of the analysis, the constants Q and A play no role, so we will disregard them when it is convenient to do so.

As noted previously, newswatchers' behavior is, in general, time-inconsistent. Although at time t they base their demands on the premise that they will never trade again, they violate this premise to the extent that they are active in the market in later periods. We adopt this time-inconsistent shortcut because it dramatically simplifies the analysis. Otherwise, we would face a complex dynamic programming problem, with newswatcher demands at time t depending not only on their forecasts of the liquidating dividend D_T, but also on their predictions for the entire future path of prices.

Two points can be offered in defense of our time-inconsistent simplification. First, as suggested above, it fits with the basic spirit of our approach, which is essentially to have newswatchers act as a simple, boundedly rational "underreaction machine". Second, we have no reason to believe that this shortcut colors any of our important qualitative conclusions. Loosely speaking, what we are doing is shutting down a "frontrunning" effect, whereby newswatchers buy more aggressively at time t in response to good news, since they know that the good news will kick off a series of momentum trades and thereby drive prices up further over the next several periods.[13] As we discuss in detail in Section 4 below, frontrunning by newswatchers may speed the response of prices to information, thereby mitigating the underreaction phenomenon, but in our set-up it can never wholly eliminate either underreaction

[13]This sort of frontrunning effect is at the center of DeLong et al (1990b).

or overreaction.

3.C The Nature of Equilibrium

With all of the assumptions in place, we are now ready to solve the model. The only task is to calculate the equilibrium value of ϕ. Disregarding constants, optimization on the part of the momentum traders implies:

$$\phi\Delta P_{t-1} = \gamma E_M(P_{t+j} - P_t)/\mathrm{var}_M(P_{t+j} - P_t) \qquad (6)$$

where γ is the aggregate risk tolerance of the momentum traders, and where E_M and var_M denote the conditional mean and variance given their information, which is just ΔP_{t-1}. Equation (6) can be rewritten as:

$$\phi = \gamma \mathrm{cov}(P_{t+j} - P_t, \Delta P_{t-1})/\{\mathrm{var}(\Delta P)\mathrm{var}_M(P_{t+j} - P_t)\} \qquad (7)$$

The definition of equilibrium is a fixed point such that ϕ is given by (7), while at the same time price dynamics satisfy (5). We will restrict ourselves to studying covariance-stationary equilibria. In the appendix, we prove that a necessary condition for a conjectured equilibrium process to be covariance stationary is that $|\phi| < 1$. Such an equilibrium may not exist for arbitrary parameter values, and we are also unable to generically rule out the possibility of multiple equilibria. However, we prove in the appendix that existence is guaranteed so long as the risk tolerance γ of the momentum traders is sufficiently small, since this in turn ensures that $|\phi|$ will be sufficiently small. Moreover, detailed experimentation suggests that a unique covariance-stationary equilibrium does in fact exist for a large range of the parameter space.[14]

In general, it is difficult to solve the model in closed form, and we will have to resort to a computational algorithm to find the fixed point. For an arbitrary set of parameter values, we always begin our numerical search for the fixed point at $j = 1$. Given this restriction, we can show that the condition $|\phi| < 1$ is both necessary and sufficient for covariance-stationarity. We also start with a small value of risk tolerance and an initial guess for ϕ of zero. The solutions in this region of the parameter space are well-behaved. Using these solutions, we then move to other regions of the parameter space. This procedure ensures that if there were to be multiple covariance-stationary equilibria, we would always pick the one with the smallest value of ϕ. We also have a number of sensible checks for when we have moved outside the covariance-stationary region of the parameter space. These are described in the appendix.

Even without doing any computations, we can make several observations about the nature of equilibrium. First, we have:

Lemma 1: In any covariance-stationary equilibrium, $\phi > 0$. That is, momentum traders must rationally behave as trend-chasers.

---

[14]Our experiments suggest that we only run into existence problems when both the risk tolerance γ and the information-diffusion parameter z simultaneously become very large--even an infinite value of γ poses no problem so long as z is not too big. The intuition will become clearer when we do the comparative statics, but loosely speaking, the problem is this: as z gets large, momentum trading becomes more profitable. Combined with high risk tolerance, this can make momentum traders behave so aggressively that our $|\phi| < 1$ condition is violated.

The lemma is proved in the appendix, but it is trivially easy to see why $\phi = 0$ cannot be an equilibrium. Suppose to the contrary it were. Then prices would be given as in the all-newswatcher case in equation (1). And in this case, $\text{cov}(P_{t+j} - P_t, \Delta P_{t-1}) > 0$, so that equation (7) tells us that $\phi > 0$, establishing a contradiction.

We are now in a position to make some qualitative statements about the dynamics of prices. First, let us consider the impulse response of prices to news shocks. The thought experiment here is as follows. At time t, there is a one-unit positive innovation ϵ_{t+z-1} that begins to diffuse among newswatchers. There are no further news shocks from that point on. What does the price path look like?

The answer can be seen by decomposing the price at any time into two components: that attributable to newswatchers, and that attributable to momentum traders. Newswatchers' aggregate estimate of D_T rises from time t to time t+z-1, by which time they have completely incorporated the news shock into their forecasts. Thus by time t+z-1, the price would be just right in the absence of any order flow from momentum traders. But with $\phi > 0$, any positive news shock must generate an initially positive impulse to momentum-trader order flow. Moreover, the cumulative order flow must be increasing until at least time t+j, since none of the momentum trades stimulated by the shock begin to be unwound until t+j+1. This sort of reasoning leads to the following conclusions:

<u>Proposition 1</u>: In any covariance-stationary equilibrium, given a positive one-unit shock ϵ_{t+z-1} that first begins to diffuse among newswatchers at time t:

i) there is always overreaction, in the sense that the cumulative impulse response of prices peaks at a value that is strictly greater than one;

ii) if $j \geq z-1$, the cumulative impulse response peaks at $t+j$, and then begins to decline, eventually converging to one;

iii) if $j < z-1$, the cumulative impulse response peaks no earlier than $t+j$, and eventually converges to one.

In addition to the impulse response function, it is also interesting to consider the autocovariances of prices at various horizons. We can develop some rough intuition about these autocovariances by considering the limiting case where the risk tolerance of the momentum traders γ goes to infinity. In this case, equation (7) implies that the equilibrium must have the property that $\text{cov}(P_{t+j} - P_t, \Delta P_{t-1}) = 0$. Expanding this expression, we can write:

$$\text{cov}(\Delta P_{t+1}, \Delta P_{t-1}) + \text{cov}(\Delta P_{t+2}, \Delta P_{t-1}) + \ldots\ldots \text{cov}(\Delta P_{t+j}, \Delta P_{t-1}) = 0 \tag{8}$$

Equation (8) allows us to state the following:

<u>Proposition 2</u>: In any covariance-stationary equilibrium, if prices changes are positively correlated at short horizons (e.g., if $\text{cov}(\Delta P_{t+1}, \Delta P_{t-1}) > 0$), then with risk-neutral momentum traders they are negatively correlated at a horizon no longer than $j+1$--i.e., it must be that $\text{cov}(\Delta P_{t+i}, \Delta P_{t-1}) < 0$ for some $i \leq j$.

It is useful to explore the differences between Propositions 1 and 2 in some detail, since at first glance, it might appear that they are somewhat contradictory. On the one hand, Proposition 1 says that in response to good news, there is continued upward momentum in prices for at least j periods, and possibly more (if $j < z-1$). On the other hand, Proposition 2 suggests that price changes begin to be reversed within $j+1$ periods, and quite possibly sooner than that.

The two propositions can be reconciled by noting that the former is a conditional statement--i.e., it talks about the path of prices from time t onward, conditional on there having been a news shock at time t. Thus Proposition 1 implies that if a trader somehow knew for sure that there had been a news shock at time t, he could make a strictly positive expected profit by buying at this time and holding until time $t+j$. One might term such a strategy "buying early in the momentum cycle"--i.e., buying immediately on the heels of news arrival. But of course, such a strategy is not available to the momentum traders in our model, since they cannot condition directly on the ϵ's.

In contrast, Proposition 2 is an unconditional statement about the autocovariance of prices. It flows from the requirement that if a trader buys at time t in response to an unconditional price increase at time $t-1$, and then holds until $t+j$, he makes zero profits on average. This zero-profit requirement in turn must hold when momentum traders are risk-neutral, because the unconditional strategy is available to them.

There is a simple reason why an unconditional strategy of buying following any price increase does not work as well as the conditional strategy of buying only following directly observed good news: not all price increases are news-driven. In particular, a trader who buys based on a price increase observed at time t runs the following risk. It may be "late" in the momentum cycle, in the sense that there has not been any good news for the last several periods.

Say the last good news hit at t-i. If this is the case, the price increase at time t is just due to a late round of momentum buying. And those earlier momentum purchases kicked off by the news at t-i will begin to be unwound in the very near future (specifically, at t-i+j+1) causing the trader to experience losses well before the end of his trading horizon.

This discussion highlights the key spillover effect that drives our results. A momentum trader who is fortunate enough to buy shortly after the arrival of good news imposes a negative externality on those that follow him. He does so by creating a further price increase that the next generation partially misinterprets as more good news. This causes the next generation to buy, and so on. At some point, the buying has gone too far, and the price overshoots the level warranted by the original news. Given the inability of momentum traders to condition directly on the ϵ's, everybody in the chain is behaving as rationally as possible, but the externality creates an apparently irrational outcome in the market as a whole.

3.D Winners and Losers

A natural question in this context is whether the bounded rationality of either the newswatchers or the momentum traders causes them to systematically lose money. In general, both groups can earn positive expected returns--and hence plausibly survive in the long run--as long as the net supply Q of the asset is positive.

Consider first the case where Q = 0. In this case, it is straightforward to show that the momentum traders earn positive returns, as long as their risk aversion is non-zero. Because with Q = 0, this is a zero-sum game, it must therefore be that the newswatchers generally lose money. The one exception is when momentum traders are risk-neutral, and both groups break even.[15]

When Q > 0, the game becomes positive-sum, as there is a return to risk-sharing that can be divided between the two groups. Thus even though the newswatchers may effectively lose some money on a trading basis to the momentum traders, this can be more than offset by their returns from risk-sharing, and they can make a net profit. Again, in the limit where the momentum traders become risk-neutral, both groups break even. The logic is similar to that with Q = 0, because risk-neutrality on the part of momentum traders dissipates all the risk-sharing profits, restoring the zero-sum nature of the game.

3.E Numerical Comparative Statics

In order to develop a more detailed feeling for the properties of the model, we now perform a variety of numerical comparative statics exercises.[16] We begin in Table 1 by investigating the effects of changing the momentum traders' horizon j. We hold the information-diffusion parameter z fixed at 12 months, and set the standard deviation of the fundamental ϵ shocks equal to .5 per month. Finally, we assume that the aggregate risk tolerance of the momentum traders, γ, equals 1/3.[17] We then experiment with values of j ranging from 3 to

---

[15]This result is related to the fact that newswatchers have time-inconsistent strategies, so that in formulating their demands they ignore the fact that they will be transacting with momentum traders who will be trying to take advantage of them. Thus in some sense, the newswatchers are more irrational than the momentum traders in this model.

[16]The appendix briefly discusses our computational methods.

[17]Campbell, Grossman and Wang (1993) suggest that this value of risk tolerance is about right for the market as a whole. Of course, for individual stocks, arbitrageurs may be more risk-tolerant, since they may not have to bear systematic risk. As we demonstrate below, our results on overreaction tend to become more pronounced when risk tolerance is increased.

18 months. For each set of parameter values, we display the following five types of numbers: i) the equilibrium value of ϕ; ii) the unconditional standard deviation of monthly returns ΔP; iii) the standard deviation of the pricing error relative to a rational expectations benchmark, $(P_t - P^*_t)$; iv) the cumulative impulse response (up to 35 months) of prices to a one-unit ϵ shock; and v) the autocorrelations of returns (up to a horizon of 25 months).

As a baseline, let us focus first on the case where j = 12 months. Consistent with Proposition 1, the impulse response function peaks 12 months after an ϵ shock, reaching a value of 1.342. In other words, at the peak, prices overshoot the change in long-run fundamentals by 34.2%. After the peak, prices eventually converge back to 1.00, although not in a monotonic fashion--rather, there are a series of damped oscillations as the momentum-trading effects gradually wring themselves out. Consistent with the intuition from Proposition 2, the autocorrelations begin to turn negative somewhat before the 12-month mark; in this case, the first negative autocorrelation occurs at a lag of 10 months.

Now lets us ask what happens as j is varied. As can be seen from the table, (see also Figure 2 for a graphical illustration) the effects on the impulse response function are non-monotonic. For example, with j = 6, the impulse response peaks at 1.265, while with j = 18, the peak reaches 1.252, neither as high as in the case where j = 12. This non-monotonicity arises because of two competing effects. On the one hand, an increase in j means that there will be more generations of momentum traders active in the market at any one time; hence their cumulative effect should be stronger, all else equal. On the other hand, the momentum traders rationally recognize the dangers of having a longer horizon--there is a greater risk that they will be caught trading late in the momentum cycle. As a result, they trade less aggressively, so that

ϕ is decreasing in j.

A more clear-cut comparative statics result appears to emerge when we consider the effect of j on the pattern of autocorrelations. The smaller is j, the faster the autocorrelations begin to turn negative. For example, with $j = 6$, the first negative autocorrelation occurs at a lag of 6 months, while with $j = 18$, the first negative autocorrelation occurs at a lag of 12 months. Again, this suggests that the rough intuition from Proposition 2 carries over to the case of non-zero risk aversion.

In Table 2, we examine the effect of changing momentum traders' risk tolerance. We set $j = z = 12$ months, and allow γ to vary from 1/13 to 1/3. The effects here are for the most part intuitive. As risk tolerance increases, momentum traders respond more aggressively to past price changes--i.e., ϕ increases. This causes the impulse response function to reach higher peak values. (See Figure 2.) Also, the unconditional volatility of monthly returns rises monotonically. Note, however, that the effect of risk tolerance on the pricing error $(P_t - P^*_t)$ is U-shaped: the pricing error first falls, and then rises, as risk tolerance is increased. On the one hand, more aggressive momentum trading accelerates the reaction of prices to information, which reduces underreaction and thereby decreases the average pricing error. On the other hand, as we have just seen, more aggressive momentum trading also exacerbates the overreaction problem, which tends to increase the average pricing error. Evidently, these two effects interact in such a way as to produce a non-monotonic pattern.[18]

[18]This fact--that more aggressive momentum trading can increase both the volatility of returns and the magnitude of pricing errors--serves as another counterexample to Friedman's (1953) famous contention that an increase in profitable speculation must necessarily stabilize prices. See also Hart and Kreps (1986), Stein (1987), and DeLong et al (1990b).

Finally, in Table 3, we allow the information-diffusion parameter z to vary. Increasing z has a monotonic effect on the intensity ϕ of momentum trade. This makes sense: the slower are newswatchers to figure things out, the greater are the profit opportunities for momentum traders, because returns are more strongly autocorrelated at short horizons. In the range of the parameter space where $j \geq z-1$, the induced increase in ϕ in turn has a monotonic effect on the peak impulse response--more aggressive momentum trade leads to more pronounced overshooting, and correspondingly, to negative autocorrelations that are generally larger in absolute value during the reversal phase.[19]

4. Extensions of the Basic Model

4.A Momentum Forecasts Based on k-Period Past Price Changes

As noted above, our basic results do not hinge on the specific assumption that momentum traders use a one-period lagged price change to make their forecasts. What matters more is that these forecasts be based on univariate regressions. To illustrate this point, in Table 4 we experiment with some variations on the basic model where the momentum traders make forecasts at any time t based on the cumulative price change over the past k periods, $(P_{t-1} - P_{t-k-1})$.

It turns out that increases in k work very much like reductions in the risk tolerance parameter γ: as k goes up, momentum traders behave less aggressively (ϕ falls), the impulse response peaks at a lower value, the standard deviation of returns falls, and the pricing error follows a U-shaped pattern. Simply put, longer lags of past price changes have less predictive

[19]When $j < z-1$, there is no longer a monotonic link between ϕ and the degree of overshooting. This is because the biggest momentum trades are already being unwound before newswatchers have fully incorporated a news shock into their forecasts.

content for future returns, and so are less helpful to momentum traders.[20] By the time k reaches about 9 months (given that z = 12 months) the momentum traders are essentially completely on the sidelines, and the model is reduced to one with just newswatchers.

4.B Other Trading Styles: Contrarianism

We have emphasized repeatedly that our results are ultimately attributable to the bounded-rationality notion that momentum traders can only make "simple" forecasts--i.e., they are effectively only able to run univariate regressions. But even if one accepts this univariate-regression-restriction at face value, it begs the following question: why do all traders have to use the same single forecasting variable? Why not allow for some heterogeneity in trading styles, with different groups focusing on different predictive variables?

Given the existence of the newswatchers and the underreaction that they create, it is certainly natural to begin an examination of simple arbitrage strategies with the sort of momentum-trading style that we have considered thus far. However, once it is understood that the momentum traders must--if they are the only arbitrageurs active in the market--ultimately cause prices to overreact, we then ought to think about the effects of second-round "contrarian" strategies that might be designed to exploit this overreaction.

To incorporate such contrarian strategies into our model in a tractable fashion, we proceed as follows. As before, we assume that there is a total risk tolerance of γ available to engage in arbitrage activity. We also continue to assume that all arbitrageurs have horizons of

[20]This suggests that if we expanded the model to let momentum traders choose a value of k, they would likely opt for k = 1, which is the case we have been emphasizing throughout.

j periods. But there are now two arbitrage styles. A fraction w of the arbitrageurs are momentum traders, who use ΔP_{t-1} to forecast $(P_{t+j} - P_t)$. The remaining (1-w) are contrarians, who use ΔP_{t-1-c} to forecast $(P_{t+j} - P_t)$. If we choose the lag length c properly, the contrarians will in equilibrium put negative weight on ΔP_{t-1-c} in making these forecasts.

Suppose provisionally that one takes the fraction w as fixed. Then the equilibrium is a natural generalization of that seen above. In particular, prices will be given by:

$$P_t = D_t + \{(z-1)\epsilon_{t+1} + (z-2)\epsilon_{t+2} + \ldots.. + \epsilon_{t+z-1}\}/z + \Sigma^j_{i=1} (\phi^M \Delta P_{t-i} + \phi^C \Delta P_{t-c-i}) \quad (9)$$

where ϕ^M and ϕ^C now denote the trading elasticities of the momentum traders and the contrarians respectively. These elasticities in turn satisfy:

$$\phi^M = w\gamma \mathrm{cov}(P_{t+j} - P_t, \Delta P_{t-1})/\{\mathrm{var}(\Delta P)\mathrm{var}_M(P_{t+j} - P_t)\} \quad (10)$$

$$\phi^C = (1-w)\gamma \mathrm{cov}(P_{t+j} - P_t, \Delta P_{t-1-c})/\{\mathrm{var}(\Delta P)\mathrm{var}_C(P_{t+j} - P_t)\} \quad (11)$$

Equilibrium now involves finding a two-dimensional fixed point in (ϕ^M, ϕ^C) such that price dynamics are given by (9), while at the same time (10) and (11) are satisfied. Although this is a more complicated problem than before, it is still straightforward to implement numerically. Of course, this is no longer the end of the story, since we still need to endogenize w. This can be done by imposing an indifference condition: in an interior solution where $0 < w < 1$, it must be that the utilities of the momentum traders and contrarians are equalized, so nobody wants to switch styles.

It turns out that the equal-utility condition can be simply rewritten in terms of either conditional variances or covariances of prices. (See the appendix for a proof.) This gives us:

Proposition 3: In an interior solution with $0 < w < 1$, it must be that:

i) $\text{var}(P_{t+j} - P_t \mid \Delta P_{t-1}) = \text{var}(P_{t+j} - P_t \mid \Delta P_{t-1-c})$; or equivalently

ii) $|\text{cov}((P_{t+j} - P_t), \Delta P_{t-1})| = |\text{cov}((P_{t+j} - P_t), \Delta P_{t-1-c})|$; or equivalently

iii) $\text{cov}(\Delta P_{t+1}, \Delta P_{t-1}) + \text{cov}(\Delta P_{t+2}, \Delta P_{t-1}) + \ldots\ldots \text{cov}(\Delta P_{t+j}, \Delta P_{t-1}) =$

$-\text{cov}(\Delta P_{t+1}, \Delta P_{t-1-c}) - \text{cov}(\Delta P_{t+2}, \Delta P_{t-1-c}) - \ldots\ldots \text{cov}(\Delta P_{t+j}, \Delta P_{t-1-c})$.

The essence of the proposition is that in order for contrarians to be active in equilibrium (i.e., to have $w < 1$) there must be as much profit opportunity in the contrarian strategy as in the momentum strategy. Loosely speaking, this amounts to saying that the negative autocorrelations in the reversal phase must cumulatively be as large in absolute magnitude as the positive autocorrelations in the initial underreaction phase. Thus adding the option of a contrarian strategy to the model cannot overturn the basic result that if there is underreaction in the short run, there must eventually be overreaction at some later point.

As it turns out, for a large range of parameter values, we can make a much stronger statement: the contrarian strategy is not used at all, for any choice of c. Rather, we get a corner solution of $w = 1$, in which all arbitrageurs endogenously opt to use a momentum strategy. This is in fact the outcome for every set of parameters that appears in Tables 1-4. Thus our previous numerical solutions are wholly unaffected by adding contrarians to the mix.

In order to get contrarian strategies to be adopted in equilibrium, we have to crank up the aggregate risk tolerance γ to a very high value. Beginning from a situation where everybody is a momentum trader, this does two things: first, it drives down the expected profits to the momentum strategy; and second, it causes the degree of overreaction to increase. Both of these effects raise the relative appeal of being a contrarian to the point that some arbitrageurs will eventually switch over from the momentum strategy. Table 5 illustrates. The table considers a situation where $z = 3$, $j = 1$, and where the contrarians trade at a lag that is $c = 2$ periods greater than the momentum traders. We experiment with three relatively extreme values of the risk tolerance parameter γ: 1/.5; 1/.3; and 1/.1. In each case, there are two columns. One corresponds to an all-momentum-trader equilibrium (i.e., we have forced $w = 1$ as in our previous analysis); the other corresponds to the interior solution concept where we allow w to be such that the conditions of Proposition 3 are satisfied.

As can be seen, in the least risk-tolerant of the three cases, very few traders--only about 9% of the total--opt to become contrarians. The remaining 91% remain as momentum traders. Consequently, the contrarians have little impact on the nature of prices. For example, the impulse response function peaks at 1.174 when there are only momentum traders, and this is figure declines only slightly, to 1.156, when we allow for contrarian strategies. Indeed, if we reduce the risk tolerance just a bit below that in this example, we hit a corner where, as in all the previous tables, the contrarian strategy endogenously disappears.

By pushing the risk tolerance still higher, we can make contrarianism more attractive. In the most risk-tolerant case considered in the table, roughly one-third of all traders become contrarians. Nevertheless, in equilibrium, price dynamics are still remarkably similar to what we have seen throughout. Even in this case, the impulse response function peaks at 1.134, not all that much different than before. The return autocorrelations also look quite simila cross the three cases. This underscores our key point: across a wide range of parameter lues, allowing for contrarian strategies does not alter the important qualitative features of o nodel.

4.C Other trading styles: frontrunning

In our set-up, newswatchers are restricted to using the information about fundamentals that they observe to predict the terminal dividend D_T. An alternative trading strategy, which we call "frontrunning", would be to use these privately observed ϵ's to forecast short-run price changes. However, as long as frontrunners are somewhat risk-averse, and (like newswatchers) do not extract information from prices, our basic results are not altered.

Qualitatively, adding frontrunning to the model has an impact similar to reducing the information-diffusion parameter z. Frontrunners speed the incorporation of ϵ-shocks into prices, but as long as they are risk-averse, they never completely eliminate underreaction. Thus the existence of frontrunning reduces--but does not eliminate--the appeal of momentum trading, leading to lower equilibrium values of ϕ. This is the same basic effect analyzed in Table 3 above, where we considered the comparative statics of the model with respect to changes in z.[21]

5. Empirical Implications

We will not belabor the fact that our model delivers the right first-order predictions for

[21] The one noteworthy distinction is that with frontrunning, we should no longer have the unnatural feature that prices adjust linearly to ϵ-shocks in the absence of momentum trade.

asset returns: positive correlations at short horizons, and negative correlations at longer horizons. After all, it was designed to do just that. More interesting are the auxiliary implications of the model, which in principle should allow it to be tested against other candidate theories of underreaction and overreaction.

5.A Trading Horizons and the Pattern of Return Autocorrelations

One interesting and novel feature of our model is that we draw an explicit link between momentum traders' horizons and the time pattern of return autocorrelations. This link is loosely suggested by Proposition 2, and it emerges clearly in the comparative statics results of Table 1: the longer is the horizon j, the longer it should take for the autocorrelations to switch from positive to negative.

The first thing to note in this regard is that our model seems to get the average magnitudes about right. For example, Jegadeesh and Titman (1993) find that autocorrelations for stock portfolios are positive for roughly 12 months, and then turn systematically negative. According to Table 1, this is what one should expect if j is on the order of 12-18 months, which sounds like a very plausible value for the horizon of a trading strategy.[22]

A second observation is that we can make cross-sectional predictions, to the extent that we can identify exogenous factors that influence the trading horizon j. One natural candidate for such a factor is trading costs. It seems plausible to conjecture that as trading costs increase, momentum traders will choose to hold their positions for longer. If so, we would expect stocks

[22]As a benchmark, turnover on the NYSE has been in the range of 50-60 percent in recent years, implying an average holding period of 20-24 months. Of course, momentum traders may have shorter horizons than the average investor.

with relatively high bid-ask spreads to have autocorrelations that stay positive for longer periods of time before turning negative. Or going across assets classes, we would expect the same thing for assets such as houses, and collectibles, where trading costs are no doubt significantly higher. Some evidence on this latter point is provided by Cutler, Poterba and Summers (1991). They find that, in contrast to equities, the autocorrelations for house and farm prices are positive at lags of up to 3 years, and for collectibles, at lags of up to 2 years.

5.B The Speed of Information Diffusion and the Magnitude of Overreaction

In Table 3 we saw that more gradual information diffusion makes stocks more attractive to momentum traders, and thus for a wide range of parameter values leads to more pronounced overshooting and stronger reversals. One possible crude proxy for the speed of information diffusion would be something like analyst coverage. If one accepts this proxy, then our cross-sectional prediction is that stocks with scantier coverage--presumably smaller-cap stocks--should be expected to manifest greater long-run overreaction-type predictability. This is consistent with recent work which finds that much of the long-horizon predictability that has been documented in the stock market is attributable to smaller-cap companies.[23]

5.C Differential Dynamics in Response to Public vs. Private News Shocks?

As stressed above, the most natural interpretation of the ϵ's in our model is that they represent information that is initially private, and that gradually diffuses across the population

[23]See Fama (1997). Fama argues--and we agree--that this sort of evidence is problematic for other existing behavioral models, as they do not clearly predict that overreaction should be concentrated in smaller stocks.

of investors. Thus our primary contribution has been to shown that the equilibrium impulse response to such private information must be hump-shaped, with underreaction in the short run giving way to eventual overreaction. But what about news that is simultaneously observed by all investors, such as earnings announcements? What does the impulse response look like in this case?

It is easy to embellish our model so that it generates short-run underreaction to public news, much the same as it generates short-run underreaction to private information. Essentially, all one has to do is to add an additional behavioral assumption about the newswatchers, to the effect that they only gradually update their priors based on public news.[24] On the one hand, this sort of patch adds an element of descriptive realism, given the large body of empirical evidence on post-event drift. But of course, such realism cannot be thought of as much of a victory for the model, since we will have just built it in by assumption.

The much more interesting and subtle question is this: if we just brute-force assume in some underreaction to public news, what then does the model have to say about whether or not there will be <u>overreaction</u> in the longer run to this same news? In other words, will the impulse response function look hump-shaped as before, or will prices just drift gradually to the correct level without going too far?

Unlike with private news, the answer now is, it depends. The key point is that the inference problem for momentum traders is potentially simplified. Recall from above that with private news, a momentum trader never knows whether he is buying early or late in the

[24]This can be justified based on the "conservatism" bias identified by Edwards (1968). See Barberis, Shleifer and Vishny (1997) for a recent discussion.

momentum cycle--i.e. he cannot tell if a price increase is the result of recent news or of past rounds of momentum trade. This uncertainty is central to our overreaction results. But if momentum traders are able to condition on the fact that there was a public news announcement at some given date t, they can refine their strategies. In particular, they can make their strategies time-dependent, so that they only trend-chase aggressively in the periods right after public news, and lay low at other times. If they do this, there need be no overreaction to public news in equilibrium; rather, the impulse response function may be everywhere increasing.

Of course, it is conceivable that momentum traders will not be so sophisticated, and will continue to use strategies that do not depend on how recently public news was released. If so, our previous results will apply exactly, and the impulse response to public news will also be hump-shaped. But the important point is this: the logic of our model admits (even strongly suggests) the possibility that the impulse response to public news will look different than that to private information. This is clearly a testable proposition.

5.D Anecdotal Evidence on Momentum Strategies

In our model, momentum traders have two key characteristics: 1) aside from their inability to run multiple regressions, they are rational maximizers who make money on average; and 2) they impose a negative externality on others. The latter feature arises because someone entering the market at any time t does not know how heavily invested momentum traders are in the aggregate at this time, and hence cannot predict whether or not there will be large-scale unwinding of momentum positions in the near future.

Anecdotal evidence supports both of these premises. With regard to the almost-rationality of momentum strategies, it should be noted that a number of large and presumably sophisticated money managers use what are commonly described as momentum approaches, that "emphasize accelerating sales, earnings, or even stock prices...and focus less on traditional valuation measures such as price-to-earnings ratios..."[25] This contrasts with the more pejorative view of positive-feedback trading that prevails in the previous academic literature.[26]

With regard to the negative externalities, it seems that other professional investors do in fact worry a lot about the dangers of momentum traders unwinding their positions. The following quotes from money managers illustrate these concerns: "Before I look at a stock, I take a look at the (SEC) filings to see who the major shareholders are. If you see a large amount of momentum money in there, you have to accept that there's a high risk..."; "If you're in with managers who are very momentum oriented....you have to be aware that's a risk going in. They come barreling out of those stocks, and they're not patient about it."[27]

In addition to these two premises, anecdotal evidence is also consistent with one of the key predictions of our model: that momentum traders will tend to be more active in small stocks, where analyst coverage is thinner and information diffuses more slowly. According to a leading pension fund consultant: "Most of the momentum players play in the small and mid-cap stocks." And a well-known momentum investor says that he typically focuses on small companies because

---

[25]The quote is from Ip (1997). Among the large investors who are commonly labeled momentum players are Nicholas-Applegate Capital Management, Pilgrim Baxter & Associates, and Friess Associates. Also noteworthy is Richard Driehaus, who manages more than $1 billion as president of Driehaus Capital Management, and who was ranked first among 1,200 managers of all styles for the five years ended December 1995 by Performance Analytics, a pension advisory firm. See Rehfeld (1996).

[26]For example, DeLong et al (1990b), which we discuss below.

[27]See Ip (1997).

"the market is inefficient for smaller companies."[28]

6. Comparison to Related Work

As noted in the Introduction, this paper shares the same goal as recent work by Barberis, Shleifer and Vishny (1997) and Daniel, Hirshleifer and Subrahmanyam (1997)--i.e., to construct a plausible model that delivers a unified account of asset-price underreaction and overreaction. However, the approach taken here is quite different. Both BSV and DHS use representative agent models, while our results are driven by the externalities that arise when heterogeneous traders interact with one another.[29] Consequently, many of the auxiliary empirical implications of our model are distinct.

First and most obviously, it is impossible for any representative agent model to make predictions linking trading horizons to the temporal pattern of autocorrelations, as we do in Section 5.A above. Second, neither the BSV nor the DHS model would seem to be able to easily generate our prediction that overreaction is more likely to occur in stocks with thinner analyst coverage (Section 5.B). A further difference with BSV is that our model allows for a differential impulse response to public and private shocks (Section 5.C), while theirs only considers public news. Finally, in the regime-switching model of BSV, it appears that

---

[28]The consultant is Robert Moseson of Performance Analytics, quoted in Jereski and Lohse (1996). The momentum investor is Richard Driehaus, quoted in Rehfeld (1996).

[29]BSV develop a regime-switching learning model, where investors wind up oscillating between two states: one where they think that earnings shocks are excessively transitory; and one where they think that earnings shocks are excessively persistent. DHS emphasize the idea that investors are likely to be overconfident in the precision of their private information, and that this overconfidence will vary over time as they learn about the accuracy of their past predictions.

overreaction can only occur if there is a series of shocks all of the same sign--i.e., an extended run of either good or bad news. In contrast, in our model, the impulse response of prices to any single private news shock involves both underreaction and overreaction.

In terms of its focus on the interaction of different types of traders--including those who behave in a trend-chasing fashion--this paper is closer in spirit to earlier models of positive-feedback trading by DeLong et al (1990b) and Cutler, Poterba and Summers (1990). However, there are significant differences with this work as well. For example, in the framework of DeLong et al, positive-feedback trading is extremely irrational, and the positive-feedback traders get badly exploited by a group of rational frontrunners.[30] In contrast, in our model, the momentum traders are very nearly rational, and actually manage to take advantage of the other group of traders, the newswatchers. This distinction is closely related to the fact that in the model of DeLong et al, there is never any underreaction. There is short-run positive correlation of returns, but this reflects an initial overreaction, followed by even more overreaction.[31]

At a more general level, this paper revisits several themes that have been prominent in previous theoretical work. The notion that one group of optimizing traders might create a negative informational externality, and thereby destabilize prices even while they are making

---

[30]In Cutler, Poterba and Summers, positive-feedback traders can in some cases make a profit, since there is an underlying gradual adjustment of prices to fundamentals, much as in our model. However, because they--like DeLong et al--do not derive positive-feedback trading as arising from the optimization of any sort of objective function, one cannot use their model to generate the sort of distinctive empirical implications that we discuss in Section 5.

[31]Also, it should be noted that the model of DeLong et al does not really endogenously deliver long-run reversals. Rather, prices are just forced back to fundamentals on a terminal date when all information is revealed. In contrast, in our model, the reversal phase is more endogenous, corresponding to the unwinding of momentum traders' positions. It also involves more complex dynamics, with the sort of damped oscillations seen in the figures.

profits, also shows up in Stein (1987). Stretching a bit further, there is an interesting analogy here with the ideas of Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992) on informational cascades. In these models, agents move sequentially. In equilib..um, each rationally bases his decision on the actions of the agent before him, even though t' s inflicts a negative informational externality on those that follow. Very much the same thir could be said of the generations of momentum traders in this model.

7. Conclusions

At the outset, we argued that any new "behavioral" theory of asset pricing should be judged according to three criteria: 1) it should rest on assumptions about investor behavior that are either a priori plausible or consistent with casual observation; 2) it should explain the existing evidence in a parsimonious and unified way; and 3) it should make a number of further predictions which can be subject to testing and which are ultimately validated.

How well have we done on these three scores? With respect to the first, we believe that our particular rendition of bounded rationality--as the ability to process a small subset of the available information in an unbiased way--is both plausible and intuitively appealing. Moreover, in our framework, this sort of bounded rationality implies a widespread reliance by arbitrageurs on simple momentum strategies. As we have discussed, this implication appears to be strongly consistent with what is observed in the real world.

In terms of the parsimony/unity criterion, it should be emphasized that everything in our model is driven by just one primitive type of shock: news about future fundamentals. There are no other exogenous sources of investor sentiment, and no liquidity disturbances. Our main conceptual contribution has been to show that if there is ever any short-run underreaction to these news shocks on the part of one set of traders, then (given the simple nature of arbitrage strategies) there must eventually be overreaction in the longer run as well.

Finally, our model does deliver a number of testable auxiliary implications. Among the most noteworthy of these predictions are the following: 1) there should be a relationship between momentum traders' horizons and the time pattern of return autocorrelations; 2) long-run reversals should be more pronounced in those (smaller-cap?) stocks where there is thinner analyst coverage and where, as a consequence, information diffuses more gradually in the short run; and 3) there may (though there need not be) more long-run overreaction to information which is initially private than to public news announcements. We hope to explore some of these hypotheses in future work.

A. Appendix

A.1 ARMA Representation of the Return Process

Let us begin by recalling Equation (5) from the text (suppressing constants):

$$P_t = D_t + \frac{(z-1)}{z}\epsilon_{t+1} + \ldots + \frac{1}{z}\epsilon_{t+z-1} + \sum_{i=1}^{j} \phi \Delta P_{t-i}. \tag{A.1}$$

It follows that

$$\Delta P_t = \frac{\sum_{i=0}^{z-1} \epsilon_{t+i}}{z} + \phi \Delta P_{t-1} - \phi \Delta P_{t-(j+1)}. \tag{A.2}$$

Assuming that ϕ satisfies proper conditions to be specified, ΔP is a covariance stationary process. Let

$$\alpha_k \equiv E[\Delta P_t \Delta P_{t-k}]$$

(i.e. the unconditional autocovariance lagged k periods). When $k=0$, we have the unconditional variance. The autocovariances of this process satisfy the following Yule-Walker equations:

$$\alpha_0 = E[\sum_{i=0}^{z-1} \frac{\epsilon_{t-i}}{z} \Delta P_t] + \phi \alpha_1 - \phi \alpha_{j+1}. \tag{A.3}$$

And for $k > 0$, we have

$$\alpha_k = E\left[\sum_{i=0}^{z-1} \frac{\epsilon_{t+i}}{z} \Delta P_{t-k}\right] + \phi\alpha_{k-1} - \phi\alpha_{k-(j+1)}. \qquad (A.4)$$

It is not hard to verify that for k > z-1,

$$E\left[\sum_{i=0}^{z-1} \frac{\epsilon_{t+i}}{z} \Delta P_{t-k}\right] = 0. \qquad (A.5)$$

And for k ≤ z-1, we have

$$E\left[\sum_{i=0}^{z-1} \frac{\epsilon_{t+i}}{z} \Delta P_{t-k}\right] = \frac{(z-k)}{z^2} + \phi E\left[\sum_{i=0}^{z-1} \frac{\epsilon_{t+i}}{z} \Delta P_{t-(k+1)}\right] - \phi E\left[\sum_{i=0}^{z-1} \frac{\epsilon_{t+i}}{z} \Delta P_{t-(k+j+1)}\right]. \qquad (A.6)$$

Solving the Yule-Walker equations reduces to solving a system of j+2 linear equations. Next, the optimal strategies of the momentum traders are given by

$$\theta_t = \frac{\gamma E[P_{t+j} - P_t | \Delta P_{t-1}]}{Var[P_{t+j} - P_t | \Delta P_{t-1}]}, \qquad (A.7)$$

where

$$P_{t+j} - P_t = \Delta P_{t+j} + ... + \Delta P_{t+1}.$$

In equilibrium,

$$\theta_t = \phi \Delta P_{t-1}. \qquad (A.8)$$

Finally, it follows that

$$Cov(\Delta P_{t-1}, P_{t+j} - P_t) = \alpha_{j+1} + ... + \alpha_2,$$

and

$$Var(P_{t+j} - P_t) = j\alpha_0 + 2(j-1)\alpha_1 + ... + 2(j-(j-1))\alpha_{j-1}.$$

Using these formulas, the problem is reduced to finding a fixed point in ϕ that satisfies the equilibrium condition (A.8). Given the equilibrium ϕ, we then need to verify that the resulting equilibrium ARMA process is in fact covariance stationary (since all of our formulas depended crucially on this assumption).

A.2 Stationarity

We next provide a characterization for the covariance stationarity of a conjectured return process. This condition is just that the roots of

$$1 - \phi x + \phi x^{j-1} = 0 \qquad (A.9)$$

lie outside the unit circle (see, e.g., Hamilton (1994)).

Lemma A.1 The return process specified in (A.2) is a covariance stationary process only if $|\phi| < 1$.

Proof. Proof is by induction on j. For j=1, the return process follows an ARMA(2,z). So, the conditions for covariance stationarity is (1) -2 ϕ < 1 and (2) -1 < ϕ < 1, (see, e.g. Hamilton (1994)). The stated result follows for j=1. Apply the inductive hypothesis and assume the result holds for j=k. From (A.9), it follows that the roots x of

$$1 - \phi x + \phi x^{k-1} = 0$$

must lie outside the unit circle (e.g. $|x| > 1$). It follows that

$$|1 - \phi x| = |\phi||x|^{k+1}.$$

Hence, as k increases, it follows that $|\phi|$ decreases. The stated result follows for arbitrary j. QED

We use this result to characterize a number of properties of a conjectured covariance stationary equilibrium.

Proof of Lemma 1. We prove that $\phi > 0$ in a covariance stationary equilibrium by contradiction. Suppose not, so that $\phi \leq 0$. It is easy to verify from (A.4) and from Lemma A.1 that

$$\alpha_k \geq 0 \quad \forall k \quad \rightarrow \quad \alpha_2 + \alpha_3 + ... + \alpha_{j+1} > 0,$$

implying that $\phi > 0$, leading to a contradiction. QED

A.3 Existence and Numerical Computation

An equilibrium ϕ satisfying the covariance stationary condition in Lemma A.1 does not exist for arbitrary parameter values. It is easy to verify however that a covariance stationary equilibrium does exist for sufficiently small γ.

Lemma A.2 For γ sufficiently small, there exists a covariance stationary equilibrium.

Proof. It is easy to show that for γ sufficiently small, we can apply Brouwer's fixed point theorem. QED

In general, the equilibrium needs to be solved numerically. For the case of $j=1$, we can always verify that the resulting ϕ leads to covariance stationarity. For arbitrary j, we only have a necessary condition although the calculations for the autocovariances would likely explode for a ϕ which does not lead to a covariance stationary process. So, we always begin our calculations for $j=1$ and γ small and use the resulting solutions to bootstrap our way to other regions in the parameter space. The solutions are gotten easily. When we move outside the covariance stationary region of the parameter space, autocovariances take on non-sensible values such as negative values for the unconditional variance or autocovariances that do not satisfy the standard property that

$$|\alpha_k| < |\alpha_0|, \qquad k>0.$$

We have not had much problems finding fixed points for wide parameter regions around those exhibited in the text.

A.4 Proof of Proposition 4

The equilibrium condition to determine w is for the utilities from the two strategies to be equal. Given our assumptions on the preferences of the momentum and contrarian investors and the distributions of the ϵ's, it follows from Grossman and Stiglitz (1980) that this is equivalent to the conditional variance of the j-period returns being equal across the two strategies. Given all investors have j-period horizons, it follows that this is equivalent to the conditional covariance of the j-period returns being equal across the two strategies. QED

References

Banerjee, Abhijit 1992. "A Simple Model of Herd Behavior," Quarterly Journal of Economics 107: 797-817.

Barberis, Nicholas, Andrei Shleifer and Robert Vishny 1997. "A Model of Investor Sentiment," University of Chicago working paper.

Bernard, Victor L. 1992. "Stock Price Reactions to Earnings Announcements," in R. Thaler ed., Advances in Behavioral Finance, Russell Sage Foundation, New York.

Bernard, Victor L. and J. Thomas 1989. "Post-Earnings Announcement Drift: Delayed Price Response or Risk Premium?" Journal of Accounting Research, Supplement 27: 1-48.

Bernard, Victor L. and J. Thomas 1990. "Evidence that Stock Prices do not Fully Reflect the Implications of Current Earnings for Future Earnings," Journal of Accounting and Economics 13: 305-340.

Bikhchandani, Sushil, David Hirshleifer and Ivo Welch 1992. "A Theory of Fads, Fashions, Custom and Cultural Change as Informational Cascades," Journal of Political Economy 100: 992-1026.

Campbell, John Y., Sanford J. Grossman and Jiang Wang 1993. "Trading Volume and Serial Correlation in Stock Returns," Quarterly Journal of Economics 108: 905-939.

Campbell, John Y. and Robert Shiller 1988. "Stock Prices, Earnings, and Expected Dividends," Journal of Finance 43: 661-676.

Chan, Louis K.C., Narasimhan Jegadeesh and Josef Lakonishok 1996. "Momentum Strategies," Journal of Finance 51: 1681-1713.

Chopra, Navin, Josef Lakonishok and Jay R. Ritter 1992. "Measuring Abnormal Performance: Do Stocks Overreact?" Journal of Financial Economics 31: 235-268.

Cutler, David M., James M. Poterba and Lawrence H. Summers 1990. "Speculative Dynamics and the Role of Feedback Traders," American Economic Review Papers and Proceedings 80: 63-68.

Cutler, David M., James M. Poterba and Lawrence H. Summers 1990. "Speculative Dynamics," Review of Economic Studies 58: 529-546.

Daniel, Kent D., David Hirshleifer and Avanidhar Subrahmanyam 1997. "A Theory of Overconfidence, Self-Attribution, and Security Market Under- and Over-reactions," Northwestern University working paper.

Daniel, Kent D. and Sheridan Titman 1997. "Evidence on the Characteristics of Cross-Sectional Variation in Stock Returns," Journal of Finance 52: 1-33.

DeBondt, Werner F.M., and Richard H. Thaler 1985. "Does the Stock Market Overreact?" Journal of Finance 40: 793-808.

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers and Robert Waldmann 1990a. "Noise Trader Risk in Financial Markets," Journal of Political Economy 98: 703-738.

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers and Robert Waldmann 1990b. "Positive Feedback Investment Strategies and Destabilizing Rational Speculation," Journal of Finance 45: 379-395.

Edwards, W. 1968. "Conservatism in Human Information Processing," in B. Kleinmutz ed., Formal Representation of Human Judgment, New York, John Wiley and Sons.

Fama, Eugene F. 1997. "Market Efficiency, Long-Term Returns, and Behavioral Finance," CRSP working paper #448, University of Chicago.

Fama, Eugene F. and Kenneth R. French 1988. "Permanent and Temporary Components of Stock Prices," Journal of Political Economy 96: 246-273.

Fama, Eugene F. and Kenneth R. French 1992. "The Cross-Section of Expected Stock Returns," Journal of Finance 47: 427-465.

Fama, Eugene F. and Kenneth R. French 1993. "Common Risk Factors in the Returns on Stocks and Bonds," Journal of Financial Economics 33: 3-56.

Fama, Eugene F. and Kenneth R. French 1996. "Multifactor Explanations of Asset Pricing Anomalies," Journal of Finance 51: 55-84.

Friedman, Milton 1953. "The Case for Flexible Exchange Rates," in Essays in Positive Economics, Chicago, University Of Chicago Press.

Grossman, Sanford J. 1976. "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information," Journal of Finance 31: 573-85.

Grossman, Sanford J. and Joseph Stiglitz 1980. "On the Impossibility of Informationally Efficient Markets," American Economic Review 70: 393-408.

Hamilton, James D. 1994. Time Series Analysis, Princeton N.J., Princeton University Press.

Hart, Oliver D. and David M. Kreps 1986. "Price Destabilizing Speculation," Journal of Political Economy 94: 927-52.

Ikenberry, David, Josef Lakonishok and Theo Vermaelen 1995. "Market Underreaction to Open Market Share Repurchases," Journal of Financial Economics 39: 181-208.

Ip, Greg 1997. "Red Flag on Wall Street: Momentum Investors," Wall Street Journal February 24, p. C1.

Jegadeesh, Narasimhan and Sheridan Titman 1993. "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," Journal of Finance 48: 65-91.

Jereski, Laura and Deborah Lohse 1996. "Momentum-Investing Fever is Sending Some Mutual Funds to their Sickbeds," Wall Street Journal July 26, p. C1.

Lakonishok, Josef, Andrei Shleifer and Robert Vishny 1994. "Contrarian Investment, Extrapolation and Risk," Journal of Finance 49: 1541-1578.

Loughran, Tim and Jay R. Ritter 1995. "The New Issues Puzzle," Journal of Finance 50: 23-51.

MacKinlay, A. Craig 1995. "Multifactor Models Do Not Explain Deviations From the CAPM," Journal of Financial Economics 38: 3-28.

Michaely, Roni, Richard H. Thaler and Kent L. Womack 1995. "Price Reactions to Dividend Initiations and Omissions: Overreaction or Drift?" Journal of Finance 50: 573-608.

Poterba, James M. and Lawrence H. Summers 1988. "Mean Reversion in Stock Returns: Evidence and Implications," Journal of Financial Economics 22: 27-59.

Rehfeld, Barry 1996. "Mutual Funds: What Goes Up, Might Go Up More," New York Times, February 25, section 3, p. 6.

Scharfstein, David S. and Jeremy C. Stein 1990. "Herd Behavior and Investment," American Economic Review 80: 465-479.

Spiess, D. Katherine and John Affleck-Graves 1995. "Underperformance in Long-Run Stock Returns Following Seasoned Equity Offerings," Journal of Financial Economics 38: 241-67.

Stein, Jeremy C. 1987. "Informational Externalities and Welfare-Reducing Speculation," Journal of Political Economy 95: 1123-1145.

Womack, Kent L. 1996. "Do brokerage Analysts' Recommendations Have Investment Value?" Journal of Finance 51: 137-168.

Table 1: Comparative Static w.r.t. Momentum Trader's Horizon j*

Momentum Trader's Horizon		j=3	j=6	j=9	j=12	j=15	j=18
ϕ		0.5550	0.4455	0.3262	0.2605	0.2263	0.2015
Standard Deviation of ($P_t - P_{t-1}$)		0.2229	0.2322	0.2179	0.2028	0.1908	0.1833
Standard Deviation of ($P_t - P_t^*$) w/ ϕ=0		0.9373	0.9373	0.9373	0.9373	0.9373	0.9373
Standard Deviation of ($P_t - P_t^*$)		0.8011	0.8438	0.9103	0.9365	0.9524	0.9604
Cumulative Impulse Response at Lag	0	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833
	1	0.2129	0.2038	0.1939	0.1884	0.1855	0.1835
	2	0.3682	0.3408	0.3132	0.2991	0.2920	0.2870
	3	0.5377	0.4851	0.4355	0.4113	0.3994	0.3912
	4	0.6688	0.6328	0.5588	0.5238	0.5071	0.4955
	5	0.7530	0.7819	0.6823	0.6365	0.6147	0.5999
	6	0.7969	0.9317	0.8059	0.7492	0.7225	0.7042
	7	0.8105	1.0446	0.9296	0.8618	0.8302	0.8086
	8	0.8287	1.1246	1.0533	0.9745	0.9379	0.9129
	9	0.8753	1.1825	1.1769	1.0872	1.0456	1.0173
	10	0.9602	1.2273	1.2734	1.1999	1.1533	1.1217
	11	1.0830	1.2649	1.3522	1.3126	1.2610	1.2260
	12	1.1412	1.2151	1.3389	1.3420	1.2854	1.2471
	13	1.1475	1.1263	1.2947	1.3279	1.2909	1.2513
	14	1.1040	1.0364	1.2401	1.2969	1.2921	1.2522
	15	1.0116	0.9607	1.1820	1.2600	1.2924	1.2523
	16	0.9281	0.9012	1.1227	1.2211	1.2736	1.2524
	17	0.8782	0.8547	1.0630	1.1817	1.2462	1.2524
	18	0.8747	0.8173	1.0032	1.1420	1.2160	1.2524
	19	0.9240	0.8228	0.9433	1.1024	1.1848	1.2356
	20	0.9977	0.8648	0.8923	1.0627	1.1534	1.2120
	21	1.0663	0.9235	0.8500	1.0230	1.1219	1.1864
	22	1.1063	0.9834	0.8405	0.9833	1.0904	1.1603
	23	1.1012	1.0366	0.8518	0.9436	1.0589	1.1340
	24	1.0574	1.0810	0.8733	0.9038	1.0274	1.1076
	25	0.9951	1.1175	0.8993	0.8859	0.9959	1.0813
	26	0.9382	1.1313	0.9271	0.8848	0.9644	1.0550
	27	0.9096	1.1187	0.9557	0.8926	0.9329	1.0286
	28	0.9180	1.0870	0.9845	0.9043	0.9202	1.0023
	29	0.9572	1.0461	1.0134	0.9175	0.9161	0.9759
	30	1.0105	1.0042	1.0395	0.9312	0.9149	0.9496
	31	1.0560	0.9658	1.0618	0.9451	0.9146	0.9401
	32	1.0766	0.9324	1.0722	0.9590	0.9187	0.9373
	33	1.0663	0.9114	1.0719	0.9730	0.9259	0.9365
	34	1.0309	0.9076	1.0648	0.9870	0.9344	0.9364
	35	0.9861	0.9201	1.0540	1.0010	0.9433	0.9363
Return Autocorrelations at Lag	1	0.8630	0.9331	0.9458	0.9466	0.9465	0.9436
	2	0.5441	0.7888	0.8415	0.8504	0.8673	0.8634
	3	0.1706	0.6063	0.7048	0.7388	0.7813	0.7782
	4	(0.1203)	0.4039	0.5561	0.6220	0.6907	0.6920
	5	(0.2115)	0.1888	0.4031	0.5036	0.5870	0.6054
	6	(0.1035)	(0.0293)	0.2492	0.3846	0.4774	0.5182
	7	0.1284	(0.2138)	0.0963	0.2654	0.3658	0.4285
	8	0.3635	(0.3233)	(0.0522)	0.1464	0.2537	0.3270
	9	0.4735	(0.3637)	(0.1883)	0.0276	0.1415	0.2208
	10	0.4096	(0.3533)	(0.3027)	(0.0898)	0.0300	0.1137
	11	0.1910	(0.3051)	(0.3708)	(0.2027)	(0.0784)	0.0087
	12	(0.0958)	(0.2200)	(0.3955)	(0.2994)	(0.1741)	(0.0846)
	13	(0.3160)	(0.0850)	(0.3589)	(0.3385)	(0.2162)	(0.1215)
	14	(0.4026)	0.0574	(0.2985)	(0.3348)	(0.2452)	(0.1465)
	15	(0.3294)	0.1696	(0.2289)	(0.3088)	(0.2697)	(0.1690)
	16	(0.1297)	0.2376	(0.1560)	(0.2729)	(0.2873)	(0.1909)
	17	0.1034	0.2632	(0.0823)	(0.2332)	(0.2792)	(0.2125)
	18	0.2808	0.2532	(0.0098)	(0.1919)	(0.2595)	(0.2330)
	19	0.3387	0.2108	0.0582	(0.1502)	(0.2355)	(0.2485)
	20	0.2599	0.1318	0.1177	(0.1083)	(0.2096)	(0.2402)
	21	0.0869	0.0331	0.1594	(0.0663)	(0.1803)	(0.2224)
	22	(0.1076)	(0.0608)	0.1810	(0.0245)	(0.1488)	(0.2016)
	23	(0.2477)	(0.1329)	0.1762	0.0170	(0.1165)	(0.1801)
	24	(0.2817)	(0.1765)	0.1549	0.0573	(0.0838)	(0.1583)
	25	(0.2046)	(0.1914)	0.1252	0.0929	(0.0510)	(0.1363)

* All cases are for z=12, σ=.5, γ=1/3, k=1.

Table 2: Comparative Static w.r.t. Risk Tolerance*

Risk Tolerance		$\gamma=1/13$	$\gamma=1/11$	$\gamma=1/9$	$\gamma=1/7$	$\gamma=1/5$	$\gamma=1/3$
ϕ		0.1316	0.1453	0.1625	0.1848	0.2152	0.2605
Standard Deviation of ($P_t - P_{t-1}$)		0.1662	0.1691	0.1731	0.1786	0.1872	0.2028
Standard Deviation of ($P_t - P_t^*$) w/ ϕ=0		0.9373	0.9373	0.9373	0.9373	0.9373	0.9373
Standard Deviation of ($P_t - P_t^*$)		0.9070	0.8998	0.8999	0.9021	0.9102	0.9365
Cumulative Impulse Response at Lag	0	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833
	1	0.1776	0.1788	0.1802	0.1821	0.1846	0.1884
	2	0.2734	0.2760	0.2793	0.2836	0.2897	0.2991
	3	0.3693	0.3734	0.3787	0.3857	0.3957	0.4113
	4	0.4652	0.4709	0.4782	0.4879	0.5018	0.5238
	5	0.5612	0.5684	0.5777	0.5902	0.6080	0.6365
	6	0.6572	0.6659	0.6772	0.6924	0.7142	0.7492
	7	0.7531	0.7634	0.7767	0.7946	0.8203	0.8618
	8	0.8491	0.8609	0.8762	0.8968	0.9265	0.9745
	9	0.9450	0.9584	0.9757	0.9990	1.0327	1.0872
	10	1.0410	1.0559	1.0752	1.1013	1.1389	1.1999
	11	1.1369	1.1534	1.1747	1.2035	1.2451	1.3126
	12	1.1496	1.1676	1.1909	1.2224	1.2679	1.3420
	13	1.1403	1.1575	1.1800	1.2105	1.2549	1.3279
	14	1.1266	1.1422	1.1625	1.1900	1.2303	1.2969
	15	1.1122	1.1259	1.1435	1.1675	1.2024	1.2600
	16	1.0977	1.1093	1.1243	1.1444	1.1736	1.2211
	17	1.0832	1.0928	1.1050	1.1213	1.1446	1.1817
	18	1.0687	1.0762	1.0857	1.0981	1.1155	1.1420
	19	1.0541	1.0596	1.0664	1.0750	1.0864	1.1024
	20	1.0396	1.0430	1.0471	1.0518	1.0572	1.0627
	21	1.0251	1.0265	1.0278	1.0286	1.0281	1.0230
	22	1.0105	1.0099	1.0085	1.0055	0.9990	0.9833
	23	0.9960	0.9933	0.9892	0.9823	0.9699	0.9436
	24	0.9815	0.9767	0.9698	0.9591	0.9408	0.9038
	25	0.9779	0.9723	0.9641	0.9514	0.9296	0.8859
	26	0.9786	0.9731	0.9649	0.9521	0.9300	0.8848
	27	0.9805	0.9754	0.9679	0.9560	0.9354	0.8926
	28	0.9827	0.9781	0.9715	0.9609	0.9425	0.9043
	29	0.9849	0.9809	0.9752	0.9661	0.9503	0.9175
	30	0.9871	0.9838	0.9789	0.9713	0.9582	0.9312
	31	0.9893	0.9866	0.9827	0.9766	0.9662	0.9451
	32	0.9915	0.9894	0.9864	0.9818	0.9741	0.9590
	33	0.9937	0.9922	0.9901	0.9871	0.9821	0.9730
	34	0.9959	0.9950	0.9939	0.9923	0.9901	0.9870
	35	0.9981	0.9978	0.9976	0.9976	0.9981	1.0010
Return Autocorrelations at Lag	1	0.9337	0.9352	0.9370	0.9393	0.9423	0.9466
	2	0.8410	0.8419	0.8430	0.8445	0.8467	0.8504
	3	0.7437	0.7430	0.7422	0.7413	0.7401	0.7388
	4	0.6456	0.6432	0.6401	0.6360	0.6304	0.6220
	5	0.5474	0.5431	0.5376	0.5302	0.5198	0.5036
	6	0.4492	0.4430	0.4351	0.4244	0.4091	0.3846
	7	0.3510	0.3429	0.3326	0.3186	0.2983	0.2654
	8	0.2527	0.2429	0.2301	0.2127	0.1875	0.1464
	9	0.1545	0.1428	0.1276	0.1070	0.0769	0.0276
	10	0.0565	0.0430	0.0255	0.0017	(0.0330)	(0.0898)
	11	(0.0403)	(0.0554)	(0.0749)	(0.1013)	(0.1398)	(0.2027)
	12	(0.1281)	(0.1439)	(0.1644)	(0.1923)	(0.2329)	(0.2994)
	13	(0.1484)	(0.1662)	(0.1892)	(0.2203)	(0.2653)	(0.3385)
	14	(0.1424)	(0.1600)	(0.1830)	(0.2143)	(0.2599)	(0.3348)
	15	(0.1294)	(0.1456)	(0.1667)	(0.1956)	(0.2381)	(0.3088)
	16	(0.1149)	(0.1291)	(0.1477)	(0.1731)	(0.2105)	(0.2729)
	17	(0.1000)	(0.1122)	(0.1280)	(0.1495)	(0.1809)	(0.2332)
	18	(0.0852)	(0.0952)	(0.1082)	(0.1256)	(0.1508)	(0.1919)
	19	(0.0703)	(0.0782)	(0.0883)	(0.1016)	(0.1205)	(0.1502)
	20	(0.0554)	(0.0612)	(0.0684)	(0.0776)	(0.0901)	(0.1083)
	21	(0.0405)	(0.0442)	(0.0485)	(0.0537)	(0.0597)	(0.0663)
	22	(0.0257)	(0.0272)	(0.0286)	(0.0297)	(0.0294)	(0.0245)
	23	(0.0108)	(0.0102)	(0.0088)	(0.0058)	0.0008	0.0170
	24	0.0039	0.0066	0.0107	0.0177	0.0302	0.0573
	25	0.0174	0.0219	0.0285	0.0388	0.0566	0.0929

* All cases are for z=12, j=12, σ=.5, k=1.

Table 3: Comparative Static w.r.t. Information Diffusion Parameter z*

Information Diffusion Parameter		z=3	z=6	z=9	z=12	z=15	z=18
ϕ		0.0322	0.1293	0.2023	0.2605	0.3214	0.3785
Standard Deviation of ($P_t - P_{t-1}$)		0.2952	0.2317	0.2106	0.2028	0.1977	0.1907
Standard Deviation of ($P_t - P_t^*$) w/ ϕ=0		0.3727	0.6180	0.7935	0.9373	1.0620	1.1736
Standard Deviation of ($P_t - P_t^*$)		0.3744	0.6317	0.8087	0.9365	1.0331	1.0992
Cumulative Impulse Response at Lag	0	0.3333	0.1667	0.1111	0.0833	0.0667	0.0556
	1	0.6774	0.3549	0.2447	0.1884	0.1548	0.1321
	2	1.0218	0.5459	0.3828	0.2991	0.2497	0.2167
	3	1.0329	0.7373	0.5219	0.4113	0.3469	0.3042
	4	1.0333	0.9287	0.6611	0.5238	0.4448	0.3929
	5	1.0333	1.1201	0.8004	0.6365	0.5430	0.4821
	6	1.0333	1.1449	0.9397	0.7492	0.6412	0.5713
	7	1.0333	1.1481	1.0790	0.8618	0.7394	0.6607
	8	1.0333	1.1485	1.2183	0.9745	0.8376	0.7501
	9	1.0333	1.1486	1.2465	1.0872	0.9359	0.8395
	10	1.0333	1.1486	1.2522	1.1999	1.0341	0.9288
	11	1.0333	1.1486	1.2533	1.3126	1.1323	1.0182
	12	1.0333	1.1486	1.2536	1.3420	1.2306	1.1076
	13	1.0226	1.1270	1.2311	1.3279	1.3074	1.1760
	14	1.0111	1.0999	1.1996	1.2969	1.3704	1.2284
	15	0.9997	1.0717	1.1652	1.2600	1.3602	1.2718
	16	0.9989	1.0433	1.1302	1.2211	1.3256	1.3107
	17	0.9989	1.0148	1.0949	1.1817	1.2831	1.3474
	18	0.9989	0.9864	1.0596	1.1420	1.2378	1.3275
	19	0.9989	0.9795	1.0242	1.1024	1.1918	1.2862
	20	0.9989	0.9782	0.9889	1.0627	1.1454	1.2368
	21	0.9989	0.9780	0.9536	1.0230	1.0989	1.1842
	22	0.9989	0.9779	0.9407	0.9833	1.0524	1.1305
	23	0.9989	0.9779	0.9370	0.9436	1.0059	1.0763
	24	0.9989	0.9779	0.9360	0.9038	0.9594	1.0220
	25	0.9989	0.9779	0.9358	0.8859	0.9128	0.9676
	26	0.9992	0.9807	0.9402	0.8848	0.8732	0.9211
	27	0.9996	0.9846	0.9475	0.8926	0.8402	0.8837
	28	1.0000	0.9887	0.9560	0.9043	0.8329	0.8531
	29	1.0000	0.9929	0.9648	0.9175	0.8417	0.8268
	30	1.0000	0.9972	0.9737	0.9312	0.8581	0.8030
	31	1.0000	1.0014	0.9826	0.9451	0.8780	0.8015
	32	1.0000	1.0028	0.9916	0.9590	0.8992	0.8165
	33	1.0000	1.0032	1.0005	0.9730	0.9209	0.8409
	34	1.0000	1.0033	1.0095	0.9870	0.9428	0.8701
	35	1.0000	1.0033	1.0139	1.0010	0.9648	0.9014
Return Autocorrelations at Lag	1	0.6806	0.8661	0.9239	0.9466	0.9585	0.9666
	2	0.3410	0.6970	0.8155	0.8504	0.8794	0.9010
	3	0.0110	0.5233	0.6995	0.7388	0.7761	0.8178
	4	0.0004	0.3493	0.5787	0.6220	0.6640	0.7216
	5	0.0000	0.1775	0.4416	0.5036	0.5486	0.6116
	6	0.0000	0.0230	0.2981	0.3846	0.4321	0.4861
	7	0.0000	0.0000	0.1532	0.2654	0.3152	0.3547
	8	(0.0000)	(0.0230)	0.0112	0.1464	0.1983	0.2234
	9	(0.0000)	(0.0481)	(0.1148)	0.0276	0.0818	0.0979
	10	(0.0004)	(0.0739)	(0.1647)	(0.0898)	(0.0335)	(0.0202)
	11	(0.0110)	(0.0997)	(0.1983)	(0.2027)	(0.1455)	(0.1304)
	12	(0.0223)	(0.1249)	(0.2270)	(0.2994)	(0.2483)	(0.2311)
	13	(0.0330)	(0.1455)	(0.2482)	(0.3385)	(0.3352)	(0.3159)
	14	(0.0230)	(0.1309)	(0.2371)	(0.3348)	(0.3873)	(0.3692)
	15	(0.0117)	(0.1071)	(0.2130)	(0.3088)	(0.4071)	(0.3980)
	16	(0.0007)	(0.0815)	(0.1846)	(0.2729)	(0.3802)	(0.4097)
	17	(0.0000)	(0.0557)	(0.1544)	(0.2332)	(0.3356)	(0.4070)
	18	(0.0000)	(0.0302)	(0.1206)	(0.1919)	(0.2842)	(0.3855)
	19	(0.0000)	(0.0069)	(0.0847)	(0.1502)	(0.2302)	(0.3299)
	20	(0.0000)	(0.0009)	(0.0481)	(0.1083)	(0.1753)	(0.2591)
	21	0.0000	0.0029	(0.0120)	(0.0663)	(0.1201)	(0.1826)
	22	0.0000	0.0066	0.0208	(0.0245)	(0.0649)	(0.1062)
	23	0.0000	0.0104	0.0375	0.0170	(0.0101)	(0.0326)
	24	0.0004	0.0142	0.0477	0.0573	0.0435	0.0370
	25	0.0007	0.0180	0.0556	0.0929	0.0938	0.1015

* All cases are for j=12, σ=.5, γ=1/3, k=1.

Table 4: Comparative Static w.r.t. k*

		k=1	k=4	k=7	k=10	k=13	k=16
Window of Past Price Changes Used		k=1	k=4	k=7	k=10	k=13	k=16
ϕ		0.2605	0.1869	0.0969	0.0117	0.0000	0.0000
Standard Deviation of ($P_t - P_{t-1}$)		0.2028	0.1693	0.1515	0.1446	0.1443	0.1443
Standard Deviation of ($P_t - P_t^*$) w/ ϕ=0		0.9373	0.9373	0.9373	0.9373	0.9373	0.9373
Standard Deviation of ($P_t - P_t^*$)		0.9365	0.9103	0.9086	0.9326	0.9373	0.9373
Cumulative Impulse Response at Lag	0	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833
	1	0.1884	0.1667	0.1667	0.1667	0.1667	0.1667
	2	0.2991	0.2500	0.2500	0.2500	0.2500	0.2500
	3	0.4113	0.3333	0.3333	0.3333	0.3333	0.3333
	4	0.5238	0.4322	0.4167	0.4167	0.4167	0.4167
	5	0.6365	0.5312	0.5000	0.5000	0.5000	0.5000
	6	0.7492	0.6301	0.5833	0.5833	0.5833	0.5833
	7	0.8618	0.7290	0.6747	0.6667	0.6667	0.6667
	8	0.9745	0.8308	0.7661	0.7500	0.7500	0.7500
	9	1.0872	0.9326	0.8575	0.8333	0.8333	0.8333
	10	1.1999	1.0344	0.9490	0.9176	0.9167	0.9167
	11	1.3126	1.1363	1.0404	1.0020	1.0000	1.0000
	12	1.3420	1.1553	1.0484	1.0029	1.0000	1.0000
	13	1.3279	1.1743	1.0565	1.0039	1.0000	1.0000
	14	1.2969	1.1934	1.0653	1.0049	1.0000	1.0000
	15	1.2600	1.2124	1.0742	1.0059	1.0000	1.0000
	16	1.2211	1.2004	1.0831	1.0068	1.0000	1.0000
	17	1.1817	1.1884	1.0919	1.0078	1.0000	1.0000
	18	1.1420	1.1763	1.1008	1.0088	1.0000	1.0000
	19	1.1024	1.1643	1.0935	1.0098	1.0000	1.0000
	20	1.0627	1.1436	1.0862	1.0108	1.0000	1.0000
	21	1.0230	1.1228	1.0790	1.0118	1.0000	1.0000
	22	0.9833	1.1021	1.0718	1.0108	1.0000	1.0000
	23	0.9436	1.0814	1.0645	1.0098	1.0000	1.0000
	24	0.9038	1.0585	1.0573	1.0089	1.0000	1.0000
	25	0.8859	1.0356	1.0501	1.0079	1.0000	1.0000
	26	0.8848	1.0126	1.0406	1.0069	1.0000	1.0000
	27	0.8926	0.9897	1.0310	1.0060	1.0000	1.0000
	28	0.9043	0.9819	1.0214	1.0050	1.0000	1.0000
	29	0.9175	0.9741	1.0119	1.0040	1.0000	1.0000
	30	0.9312	0.9662	1.0023	1.0031	1.0000	1.0000
	31	0.9451	0.9584	1.0009	1.0021	1.0000	1.0000
	32	0.9590	0.9592	0.9994	1.0011	1.0000	1.0000
	33	0.9730	0.9599	0.9976	1.0001	1.0000	1.0000
	34	0.9870	0.9607	0.9958	1.0001	1.0000	1.0000
	35	1.0010	0.9615	0.9940	1.0000	1.0000	1.0000
Return Autocorrelations at Lag	1	0.9466	0.9328	0.9221	0.9170	0.9167	0.9167
	2	0.8504	0.8655	0.8442	0.8340	0.8333	0.8333
	3	0.7388	0.7983	0.7671	0.7519	0.7500	0.7500
	4	0.6220	0.7311	0.6900	0.6699	0.6667	0.6667
	5	0.5036	0.6281	0.6128	0.5878	0.5833	0.5833
	6	0.3846	0.5251	0.5434	0.5058	0.5000	0.5000
	7	0.2654	0.4222	0.4740	0.4237	0.4167	0.4167
	8	0.1464	0.3192	0.3818	0.3417	0.3333	0.3333
	9	0.0276	0.2202	0.2897	0.2597	0.2500	0.2500
	10	(0.0898)	0.1211	0.1976	0.1776	0.1667	0.1667
	11	(0.2027)	0.0221	0.1055	0.0927	0.0833	0.0833
	12	(0.2994)	(0.0770)	0.0135	0.0077	0.0000	0.0000
	13	(0.3385)	(0.1081)	(0.0000)	0.0058	0.0000	0.0000
	14	(0.3348)	(0.1391)	(0.0135)	0.0039	0.0000	0.0000
	15	(0.3088)	(0.1702)	(0.0298)	0.0019	0.0000	0.0000
	16	(0.2729)	(0.2013)	(0.0462)	0.0000	0.0000	0.0000
	17	(0.2332)	(0.1946)	(0.0626)	(0.0019)	0.0000	0.0000
	18	(0.1919)	(0.1878)	(0.0791)	(0.0039)	0.0000	0.0000
	19	(0.1502)	(0.1810)	(0.0955)	(0.0058)	0.0000	0.0000
	20	(0.1083)	(0.1743)	(0.0893)	(0.0077)	0.0000	0.0000
	21	(0.0663)	(0.1538)	(0.0831)	(0.0097)	0.0000	0.0000
	22	(0.0245)	(0.1333)	(0.0772)	(0.0116)	0.0000	0.0000
	23	0.0170	(0.1128)	(0.0713)	(0.0107)	0.0000	0.0000
	24	0.0573	(0.0922)	(0.0654)	(0.0097)	0.0000	0.0000
	25	0.0929	(0.0699)	(0.0603)	(0.0088)	0.0000	0.0000

* All cases are for z=12, j=12, σ=.5, γ=1/3.

Table 5: Comparative Static of Equilibrium (w) w.r.t. Risk Tolerance *

Risk Tolerance		$\gamma=1/.1$		$\gamma=1/.3$		$\gamma=1/.5$	
Equilibrium (w)/ All Momentum		0.6703	1	0.786	1	0.9079	1
ϕ^M		0.3078	0.4668	0.3291	0.4167	0.3466	0.3794
ϕC		(0.1514)	0.0000	(0.0896)	0.0000	(0.0351)	0.0000
Cumulative Impulse Response at Lag	0	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
	1	0.7693	0.8223	0.7764	0.8056	0.7822	0.7931
	2	1.1342	1.2282	1.1458	1.1968	1.1556	1.1744
	3	1.0619	1.1895	1.0917	1.1630	1.1177	1.1447
	4	0.9117	0.9819	0.9425	0.9859	0.9711	0.9887
	5	0.8985	0.9031	0.9178	0.9262	0.9361	0.9408
	6	1.0069	0.9632	0.9967	0.9751	0.9892	0.9818
	7	1.0561	1.0281	1.0393	1.0204	1.0236	1.0156
	8	1.0171	1.0303	1.0162	1.0189	1.0131	1.0128
	9	0.9716	1.0010	0.9853	0.9994	0.9945	0.9990
	10	0.9785	0.9864	0.9860	0.9919	0.9923	0.9947
	11	1.0080	0.9931	1.0023	0.9969	0.9996	0.9984
	12	1.0160	1.0032	1.0081	1.0021	1.0032	1.0014
	13	1.0014	1.0047	1.0019	1.0022	1.0013	1.0011
	14	0.9911	1.0007	0.9965	1.0000	0.9991	0.9999
	15	0.9956	0.9981	0.9977	0.9991	0.9991	0.9995
	16	1.0036	0.9988	1.0010	0.9996	1.0001	0.9999
	17	1.0040	1.0003	1.0016	1.0002	1.0004	1.0001
	18	0.9994	1.0007	1.0001	1.0002	1.0001	1.0001
	19	0.9974	1.0002	0.9992	1.0000	0.9999	1.0000
	20	0.9993	0.9998	0.9997	0.9999	0.9999	1.0000
	21	1.0013	0.9998	1.0003	1.0000	1.0000	1.0000
	22	1.0009	1.0000	1.0003	1.0000	1.0001	1.0000
	23	0.9996	1.0001	1.0000	1.0000	1.0000	1.0000
Return Autocorrelations at Lag	1	0.6123	0.6433	0.6346	0.6530	0.6524	0.6595
	2	0.0221	0.0268	0.0600	0.0657	0.0910	0.0937
	3	(0.2404)	(0.2878)	(0.2218)	(0.2447)	(0.2068)	(0.2147)
	4	(0.0221)	(0.1468)	(0.0600)	(0.1294)	(0.0910)	(0.1170)
	5	0.1565	0.0658	0.1047	0.0481	0.0599	0.0371
	6	0.0947	0.0992	0.0794	0.0740	0.0627	0.0584
	7	(0.0521)	0.0156	(0.0228)	0.0108	(0.0031)	0.0081
	8	(0.0722)	(0.0390)	(0.0484)	(0.0263)	(0.0281)	(0.0191)
	9	0.0032	(0.0255)	(0.0062)	(0.0155)	(0.0088)	(0.0103)
	10	0.0454	0.0063	0.0231	0.0045	0.0090	0.0033
	11	0.0161	0.0149	0.0119	0.0083	0.0070	0.0052
	12	(0.0204)	0.0040	(0.0075)	0.0016	(0.0014)	0.0007
	13	(0.0176)	(0.0051)	(0.0090)	(0.0028)	(0.0035)	(0.0017)
	14	0.0053	(0.0042)	0.0005	(0.0018)	(0.0007)	(0.0009)
	15	0.0126	0.0004	0.0049	0.0004	0.0013	0.0003
	16	0.0018	0.0022	0.0016	0.0009	0.0008	0.0005
	17	(0.0068)	0.0008	(0.0019)	0.0002	(0.0003)	0.0001
	18	(0.0038)	(0.0006)	(0.0015)	(0.0003)	(0.0004)	(0.0002)
	19	0.0026	(0.0007)	0.0004	(0.0002)	0.0000	(0.0001)
	20	0.0032	0.0000	0.0010	0.0000	0.0002	0.0000
	21	(0.0003)	0.0003	0.0001	0.0001	0.0001	0.0000
	22	(0.0020)	0.0002	(0.0004)	0.0000	0.0000	0.0000
	23	(0.0007)	(0.0001)	(0.0002)	0.0000	(0.0001)	0.0000
	24	0.0010	(0.0001)	0.0001	0.0000	0.0000	0.0000
	25	0.0008	0.0000	0.0002	0.0000	0.0000	0.0000

* All cases are for z=3, j=1, σ=1, k=1, c=2.

Figure 1: Generic Cumulative Impulse Response

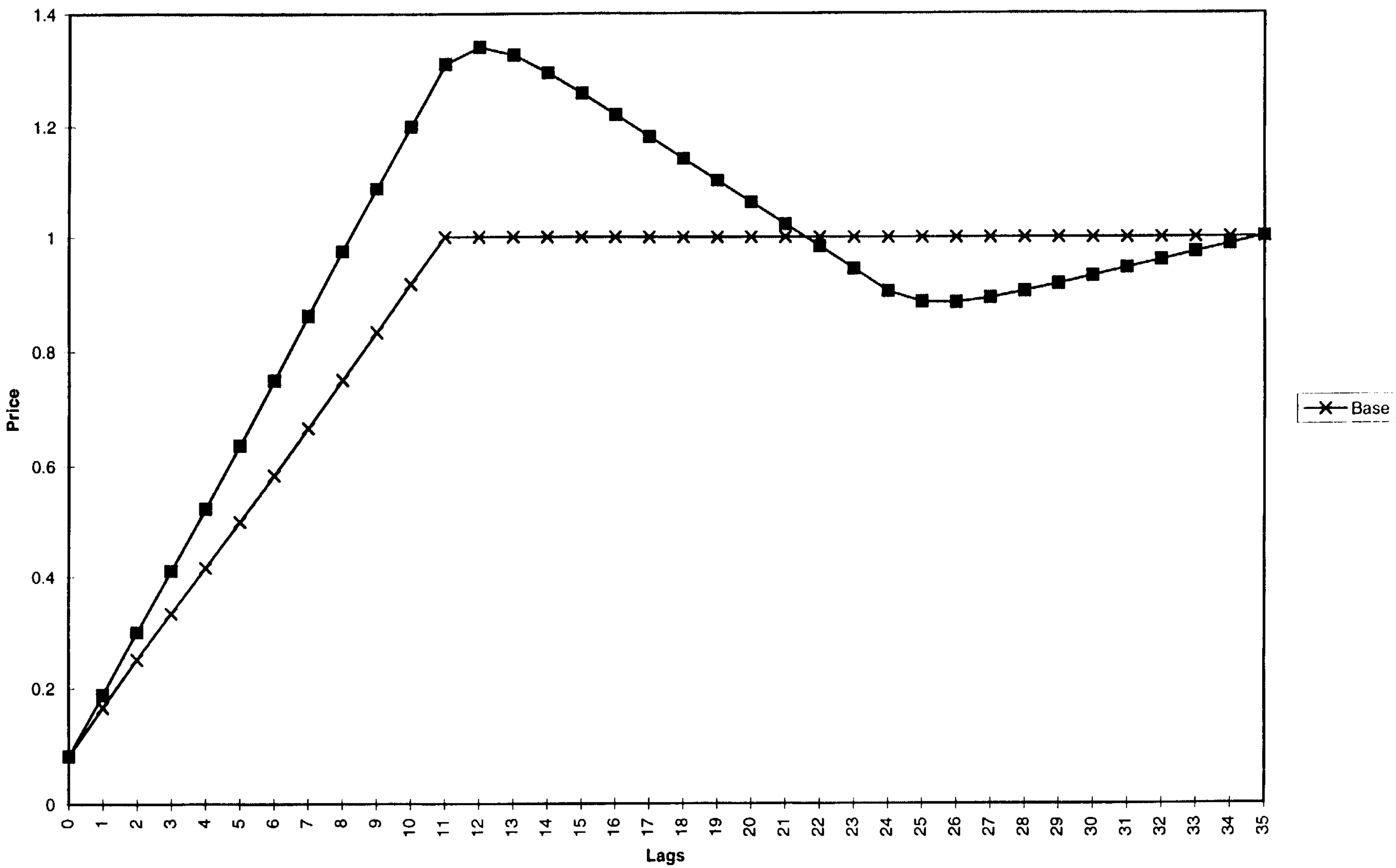

Figure 2: Cumulative Impulse Response for Momentum Trader's Horizon, j=6,12,18

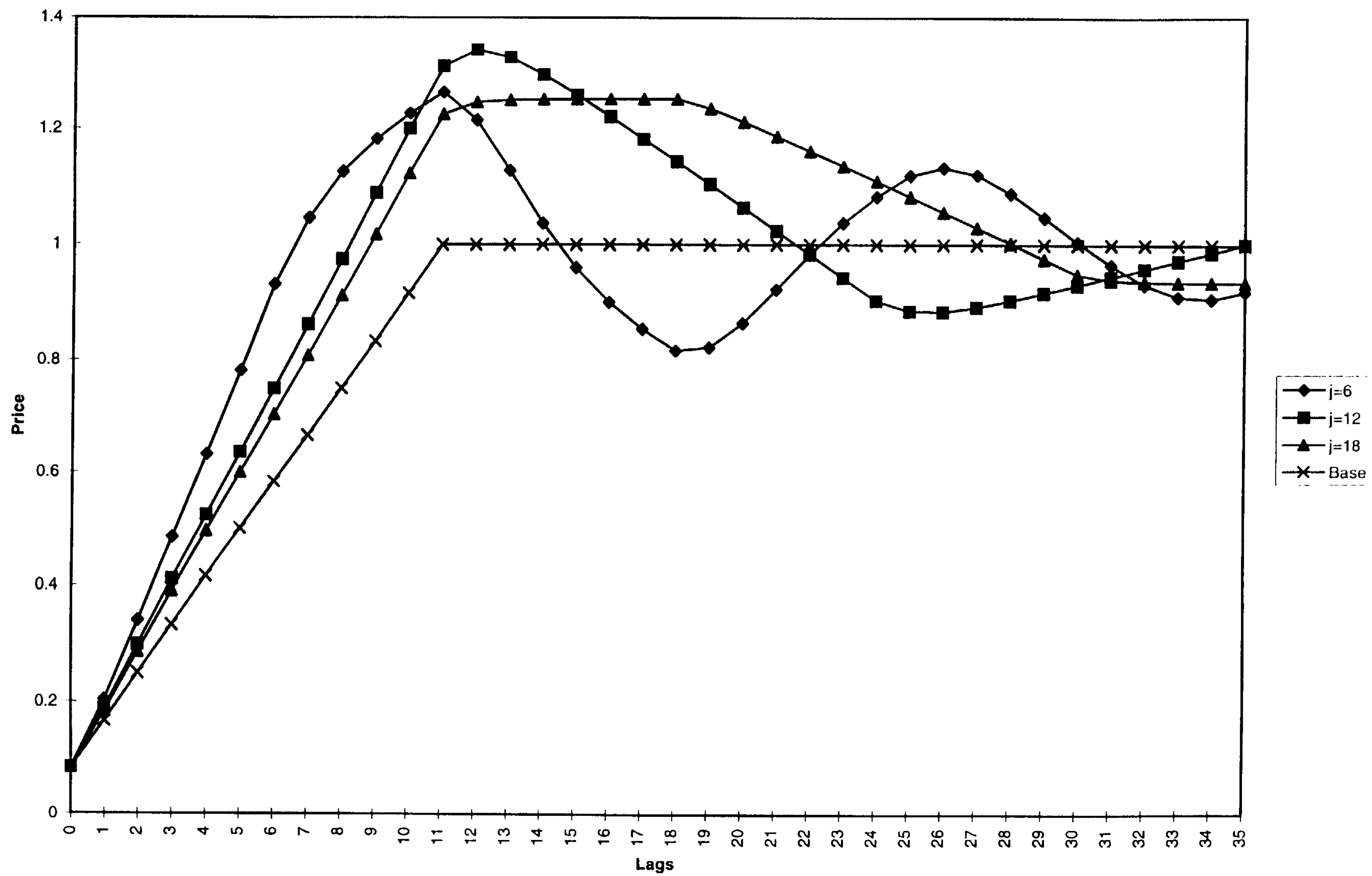

Figure 3: Cumulative Impulse Response for Risk Tolerance, gamma=1/11,1/7,1/3