# Numerical Aperture of Single-Mode Photonic Crystal Fibers

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Abstract— We consider the problem of radiation into free space from the end-facet of a single-mode photonic crystal fiber (PCF). We calculate the numerical aperture NA =  $\sin \theta$  from the half-divergence angle  $\theta \sim \tan^{-1}(\lambda/\pi w)$  with  $\pi w^2$  being the effective area of the mode in the PCF. For the fiber first presented by Knight *et al.* we find a numerical aperture NA ~ 0.07 which compares to standard fiber technology. We also study the effect of different hole sizes and demonstrate that the PCF technology provides a large freedom for NA-engineering. Comparing to experiments we find good agreement.

Keywords— Photonic crystal fiber, numerical aperture, Gaussian approximation

## I. INTRODUCTION

**P**HOTONIC CRYSTAL FIBERS (PCF) constitute a completely new class of optical fibers consisting of pure silica with air-holes distributed in the cladding. Among many remarkable properties [1] PCFs are believed to have a potential for high-numerical aperture (NA) applications. Here we report a calculation of the NA for the class of PCFs first fabricated by Knight *et al.* [2], [3]. For this particular fiber we find a numerical aperture up to NA ~ 0.07. We also demonstrate how the NA may be controlled by the hole size for a given pitch and wavelength.

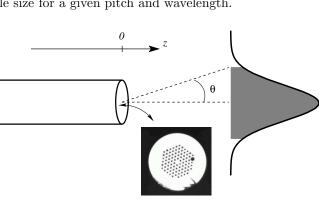


Fig. 1

Coupling of light from end-facet of fiber (z = 0) into free space. The insert shows a micrograph of the end-facet of a PCF.

The paper is organized as follows: First we consider the problem of radiation into free space from the end-facet of a single-mode optical fiber with a mode approximated by a Gaussian of width w. Solving the scattering problem at the end-facet of the fiber exactly we check the range of validity

of the text-book result  $\theta \simeq \tan^{-1}(\lambda/\pi w)$  (see e.g. Ref. [4]) where  $A_{\text{eff}} = \pi w^2$  is the effective area. For  $w \ll \lambda$  we find deviations whereas nice agreement is found for  $w > \lambda$ . We then turn to the application of the PCF of Knight *et al.* [2] which belongs to the latter regime with  $w > \lambda$ . Finally, we compare our calculations to experiments.

# II. NUMERICAL APERTURE IN THE GAUSSIAN APPROXIMATION

The numerical aperture NA =  $\sin \theta$  (see Fig. 1) may be defined in various ways, but often one defines it in the far-field limit  $(z \to \infty)$  from the half-divergence angle  $\theta_{\nu}$ between the z-axis and the  $\nu$ -intensity point  $r_{\nu}(z)$ , *i.e.* 

$$\tan \theta_{\nu} = \lim_{z \to \infty} \frac{r_{\nu}(z)}{z},\tag{1}$$

with  $r_{\nu}(z)$  determined from

$$\frac{\left|\Psi_{>}(z,r_{\perp}=r_{\nu})\right|^{2}}{\left|\Psi_{>}(z,r_{\perp}=0)\right|^{2}} = \nu.$$
(2)

For a Gaussian field  $\Psi$  of width w one has the standard approximate expression for  $\nu = 1/e^2 \simeq 13.5\%$  [4]

$$\tan \theta_{1/e^2} \simeq \frac{2}{kw} = \frac{\lambda}{\pi w}.$$
(3)

For the  $\nu = 5\%$  intensity point

$$\tan\theta_{5\%} = \sqrt{\frac{\ln 20}{2}} \times \tan\theta_{1/e^2} \tag{4}$$

which is often the one used experimentally. Eqs. (3,4) are valid for  $kw \gg 1$ , but in order to check the validity in the limit with kw of order unity we solve the scattering problem at the end-facet of the fiber exactly. In the fiber (z < 0) the field is of the form

$$\Psi_{<}(r) \propto \psi(r_{\perp}) \left( e^{i\beta(\omega)z} + \mathscr{R}e^{-i\beta(\omega)z} \right), \ z < 0 \tag{5}$$

where the transverse field is approximated by a Gaussian

$$\psi(r_{\perp}) \propto e^{-(r_{\perp}/w)^2},\tag{6}$$

which has an effective area  $A_{\text{eff}} = \pi w^2$  at frequency  $\omega$ . At the end-facet of the fiber (z = 0) the field couples to the free-space solution

$$\Psi_{>}(r) \propto \int d\boldsymbol{k}_{\perp} \mathscr{T}(\boldsymbol{k}_{\perp}) e^{i\boldsymbol{k}_{\perp} \cdot \boldsymbol{r}_{\perp}} e^{i\boldsymbol{k}_{\parallel} z}, \ z > 0 \qquad (7)$$

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which is a linear combination of plane waves with  $\omega = ck = c(2\pi/\lambda)$  and  $\mathbf{k} = \mathbf{k}_{\perp} + \mathbf{k}_{\parallel}$ .

In order to solve the elastic scattering problem,  $\Delta \omega = \omega(\beta) - \omega(k) = 0$ , we apply appropriate boundary conditions at the end-facet of the fiber; continuity of  $\Psi$  and  $\partial \Psi/\partial z$ . At z = 0 we thus get two equations determining the reflection amplitude  $\mathscr{R}$  and the transmission amplitude  $\mathscr{T}$ . Eliminating  $\mathscr{R}$  and substituting the resulting  $\mathscr{T}$  into Eq. (7) we get

$$\Psi_{>}(r) \propto 2\pi k^2 \int_0^\infty d\chi \, \chi \frac{2n_{\text{eff}}}{\sqrt{1-\chi^2} + n_{\text{eff}}} \\ \times e^{-(\chi kw/2)^2} J_0(\chi kr_\perp) e^{i\sqrt{1-\chi^2}kz}.$$
(8)

Here,  $\chi = k_{\perp}/k$ ,  $J_0$  is the Bessel function of the first kind of order 0, and  $n_{\text{eff}} = \beta/k$  is the effective mode-index. Eq. (8) is the exact solution to the scattering problem and in contrast to many approximate text-book expressions (see *e.g.* Refs. [4]) we have here treated the scattering problem correctly including the small, but finite, backscattering in the fiber. Thus, we take into account the possible filtering in transmitted  $k_{\perp}$  at the fiber end-facet. The solution has similarities with the Hankel transform usually employed in the far-field inversion integral technique, see *e.g.* [5]. Numerically we have found that Eq. (8) gives a close-to-Gaussian field in the far-field limit.

In Fig. 2 we compare the two approximate solutions Eqs. (3,4) to a numerically exact calculation of  $\tan \theta_{\nu}$  from Eq. (8). The calculation is performed for the realistic situation with  $n_{\text{eff}} = \beta/k = 1.444$  corresponding to a silicabased fiber. For  $kw \sim 1$  the deviations increase because of the small, but finite, backscattering at the end-facet of the fiber. For kw somewhat larger than unity a very nice agreement is found. A typical all-silica fiber like the Corning SMF28 has kw > 10.

#### III. Application to photonic crystal fibers

We consider the class first studied in Ref. [2] which consists of pure silica with a cladding with air-holes of diameter d arranged in a triangular lattice with pitch  $\Lambda$ . For a review of the operation of this class of PCFs we refer to Ref. [6].

In applying Eq. (3) to PCFs we calculate w from the effective area  $A_{\text{eff}} = \pi w^2$  given by [7]

$$A_{\text{eff}} = \frac{\left[\int d\boldsymbol{r} \left|\boldsymbol{H}(\boldsymbol{r},z)\right|^2\right]^2}{\int d\boldsymbol{r} \left|\boldsymbol{H}(\boldsymbol{r},z)\right|^4}.$$
(9)

Indeed we find that the corresponding Gaussian of width w accounts well for the overall spatial dependence of the field. Of course we thereby neglect the satellite spots seen in the far-field [2], but because of their low intensity they only give a minor contribution to the NA [8].

For the field H of the PCF, fully-vectorial eigenmodes of Maxwell's equations with periodic boundary conditions are computed in a planewave basis [9].

Figure 3 illustrates the effective mode-index and effective area as a function of wavelength for a PCF with  $d/\Lambda$  =

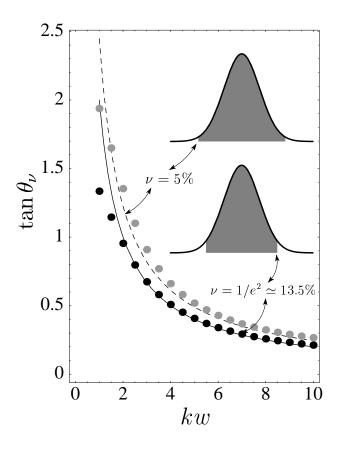


Fig. 2

PLOT OF  $\tan \theta$  as a function of the dimensionless parameter kw. The points are the results of a numerical exact calculation from Eq. (8) for a mode with effective index,  $n_{\rm eff} = \beta/k = 1.444$ . The full and dashed lines show the approximations Eqs. (3,4), respectively.

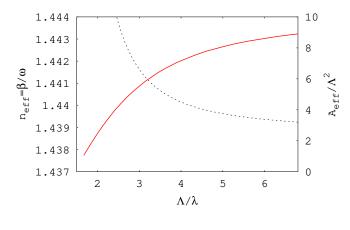


Fig. 3

Effective mode-index (solid line, left axis) and effective area (dashed line, right axis) of a PCF with  $d/\Lambda = 0.15$ .

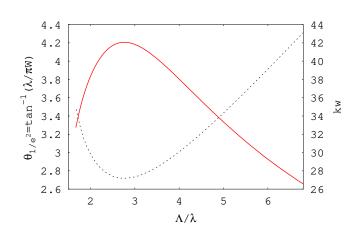


Fig. 4 Half-divergence angle (solid line, left axis) and kw (dashed line, right axis) of a PCF with  $d/\Lambda=0.15$ .

0.15. The first PCF fabricated by Knight *et al.* [2] of this kind had a pitch of  $\Lambda = 2.3 \,\mu\text{m}$  and was found to be single-mode in the range  $\Lambda/\lambda$  between 1.5 and 6.8. In Fig. 4 we show the corresponding half-divergence angle. We have also shown the value of the dimensionless parameter kw (dashed line, right axis); the magnitude justifies the application of the approximate result in Eq. (3) to PCFs. We note that for non-linear PCFs [10] the value of kw will approach the regime where deviations from Eq. (3) arise.

In Fig. 5 we show the half-divergence angle for different hole sizes where the fiber is endlessly single mode [11]. For small hole sizes  $d/\Lambda$  we note that in practice the operation is limited by a significant confinement loss for long wavelengths where the effective area increases [12]. In Fig. 5 this can be seen as a bending-down of  $\theta$  for small  $\Lambda/\lambda$ . In general the NA increases for increasing hole size and fixed pitch and wavelength. By adjusting the pitch  $\Lambda$  and the hole size d this demonstrates a high freedom in designing a fiber with a certain NA at a specified wavelength.

In order to verify our calculations experimentally a PCF with  $d/\Lambda \sim 0.53$  and  $\Lambda \simeq 7.2 \,\mu m$  has been fabricated. In Fig. 6 we compare our calculations to a measurement of the NA at the wavelength  $\lambda = 632$  nm. As seen the calculation agrees well with the measured value.

### IV. CONCLUSION

We have studied the numerical aperture (NA) of photonic crystal fibers (PCF). The calculations is based on the approximate "standard" result  $\theta \simeq \tan^{-1}(\lambda/\pi w)$  which we have found to be valid in the regime relevant to PCFs. As an example we have applied it to the fiber first fabricated by Knight *et al.* [2]. By studying the effect of different hole sizes we have demonstrated that the PCF technology have a strong potential for NA-engineering in the single-mode regime.

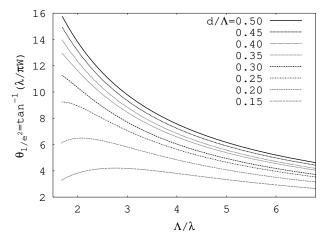
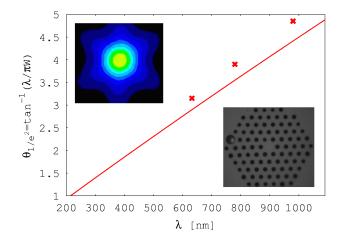


Fig. 5 Half-divergence angle of a PCF for different hole sizes.



#### Fig. 6

Half-divergence angle of a PCF with  $d/\Lambda \sim 0.53$  and  $\Lambda \simeq 7.2\,\mu m$ . The solid line is a calculation based on the ideal structure and the data points are measurements at

 $\lambda = 632 \text{ nm}, 780 \text{ nm}, \text{ AND } 980 \text{ nm}$  OF THE FIBER SHOWN IN THE

lower right insert. The upper left insert shows a near-field image at  $\lambda = 632 \, \mathrm{nm}.$ 

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