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Multichannel Auction Strategies in Online Advertising With a Profit Model

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ABSTRACT In the online advertising market that has increasingly emerged with the Internet's rapid development, supply side platforms (SSP) use multichannel auctions to sell demand side platform (DSP) advertisements. To explore which channel is dominant, we analyze three channels including direct bidding, bidding with costly information and proxy bidding. We develop an analytical model for studying the SSP's channel decisions and expected profit that will be generated in the context of such multichannel auctions. First, we construct the SSP profit models for these different channels. Then, comparing the profits, we analyze the formulation of strategies for these different situations. Importantly, for the three channels, we study how the probability that bidders will attach a higher value to advertisements influences the multichannel strategy of the SSP. The results show the following: (1) each channel has a market share; (2) it is better for SSPs to use the costly bidding channel if the probability that the bidders are higher value bidders is low or very high; and (3) the SSPs should consider the use of proxy bidding when the reserve price is high and the probability of having higher value bidders is not high.

INDEX TERMS Online advertising, multichannel auction, strategy analysis, expect profit.

I. INTRODUCTION

With the development of the Internet, increasingly more deals are made through the Internet [1]. As an emerging product, online advertising [2]–[4] has also increasingly attracted attention from advertising demanders. In the market for online advertising, an advertisement publisher on a supply side platform (SSP) has many channels to auction off these impressions or slots. To achieve the maximum profit, the development of a channel selection strategy is important for the SSP.

Multichannel auction strategies have been discussed in many papers and in different fields. Wu and Zhou [5] proposed a multi-channel double auction algorithm based on the heterogeneity of the spectrum; their strategy included multi-channel auctions and heterogeneous spectrum auctions. Liu *et al.* [6] examined how bidders choose their bidding strategies in multichannel, sequential business-to-business (B2B) auctions, although many empirical works have focused on business-to-consumer (B2C) auctions. Etzion and Moore [7] studied a case in which goods were sold in two channels, namely, a posted-price channel and an open ascending-bid uniform-price auction channel, and developed a model of consumer behavior when the consumers were faced with making a choice. Moreover, for online selling, multichannel selling is more successful than single-channel online selling [8].

As a modern marketing technology, online advertising has many positive effects, which have improved the consumers' evaluation of advertised products or services [9]–[11]. Much literature has focused on the optimization of the online advertising auction. Qin *et al.* [12], [13] proposed a new mechanism to improve a real-time bidding (RTB) auction in order to enhance the total revenue of both SSPs and DSPs and validated this new mechanism by a computational experimental approach. Moreover, online advertising generally requires good predictions of user behavior. Many researchers found that fast and robust methods were necessary for evaluating performance [14] and that auctions needed to be carefully optimized [15] to adapt to the changing market place. Vasile *et al.* [16] analyzed the relationship between optimizing the utility metric and the log loss, presenting a new cost weighting scheme to optimize online advertising auctions. Hummel and Mcafee [17] characterized the optimal loss

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functions for predicted click-through rates in auctions for online advertising, which could improve economic efficiency. However, they focused on optimization and ignored the profits of different channels. According to the empirical research on optimal reserve prices for online advertising, the interaction of marketplaces with each other and with different channels impacts revenue [18]. From a profit viewpoint, how to choose the correct channel is a significant issue for SSPs and is the focus of our study.

Moreover, many researchers have studied multichannel management and multichannel marketing. The former studies include research on the customers' motivations [19] and on multichannel coordination strategies [20], [21]. The latter studies provide guidance for firms in developing a multichannel mind-set and marketing programs [22]. In addition, many studies on multichannel auctions have generally focused on the retail channel. For example, Zafari and Soyer [23] proposed a Bayesian approach to study the retail secondary market of online B2B auctions. Kuruzovich and Etzion provided a framework for studying how the characteristics of the demand in an offline retail sales channel impacted the seller's optimal reserve price in the auction channel [24]. Prospect theory was adopted by Brunner *et al.*, who found that the average auctioneer revenues were above current retail prices and not subject to the sunk cost fallacy [25]. However, as research focusing on decision making in multichannel auctions of online advertising is scarce, studying the multichannel auctions of online advertising has become important.

Typically, SSPs sell their inventory via a data management system (DMS). Some impressions are sold through a third party, some are sold through directly made deals, and others are successfully traded with costly information. To depict the differences among auction channels, our paper introduces the following three channels: exogenous bidding, bidding with costly information acquisition and proxy bidding. The details of these three channels are discussed in the following paragraphs.

A. EXOGENOUS BIDDING

Exogenous bidding is direct bidding without any other costs or fees and is referred to in our paper as the traditional second-sealed prices auction method. Most research studies also use the same terminology to refer to this type of bidding [26]-[28]. This channel is more time-consuming than is proxy bidding, and it is difficult to guarantee the delivery effect. However, it still maintains a share of the online advertising market and is used in some situations. Some portal sites and social media platforms, such as Weibo, Sohu and Today Headlines, have thousands of news platforms that can be launched with one click. The price of each advertisement is different, and advertisements are generally purchased in a package. For example, to deliver advertisements, the web site ''SHENG-SE-DIAO'' has a channel named ''MICROBLOGGING FANS THROUGH''. To advertise on this channel, the advertisers just need to pay money and maintain a contract. The bidder who pays the highest money will have the most publicity. The advertising process is accomplished through the exogenous bidding (direct bidding) channel and provides a convenient way for both auctioneer and bidders to trade online advertising. However, as this method is time consuming, it is on the decline. In the rapid development of computers and the Internet, even though this traditional way is convenient, it cannot satisfy the demand for a large number of deals.

B. BIDDING WITH COSTLY INFORMATION ACQUISITION

To make an accurate evaluation, paying costs for the acquisition of additional information is one kind of bidding behavior [29]. This situation always happens, as some bidders' make imperious demand to obtain information regarding the selling of goods. This channel is similar to the situation of an auction with entry fees. For this channel, we assume that each bidder's cost is the same and cannot be returned. For example, the company ''SunTeng technology'' provides whole process marketing technology solutions and is one of the largest DSP platforms in China. It has developed a global business in which bidders can consult with the company to acquire accurate and valuable information about online advertising before bidding to enhance the efficiency of purchase. Some researchers have also focused on costly information acquisition. Szech [30] discussed how the seller optimally delivers costly information to bring more profits. Furthermore, Pancs [31] explained that if the seller cannot charge bidders for the information about the other bidders' bid, then a sequential second-price auction with a reserved price is optimal. Comparing the static and dynamic formats, Compte and Jehiel [32] found that dynamic formats made the option to acquire information valuable because these formats allow bidders to observe the number of competitors left throughout the selling process. Representing another difference among channels, the second sealed price auction is also used in the costly information auction. In this paper, we do not explain the cost function in detail, although we assume that cost has a cumulative distribution function defined as Fc. The cost information only comes from the SSP; moreover, the money stream just flows into the SSP.

C. PROXY BIDDING

The advertisers who comprise the demand side platform (DSP) desire to acquire the impressions or advertisements and often rely on a third party to implement the proxy bidding [2]. This channel, proxy bidding, does not include any additional charges after fees and bidding prices have been paid. In addition, many auction web sites provide support for the use of automated proxies or agents [33]. The DoubleClick products of Google provide a variety of advertising management and advertising solutions to help enterprises purchase, produce or sell online advertising. They provide dynamic advertising reports, a target positioning advertising service platform and deep expertise. Moreover, DoubleClick products can help customers implement their digital media strategies more efficiently on media planning,

search advertising management, rich media, video and mobile advertising. Many marketers, publishers, ad networks and agencies are using DoubleClick products as the cornerstone of their online advertising business. Moreover, this channel of bidding has attracted much research attention. Hsu *et al.* [34] proposed a system model of a pairing-based proxy signcryption, which includes two cryptographic schemes, and applied the proposed schemes to the online proxy auction system for comparing two authorized online proxy auction policies with different applications of short message and long message. Bose and Daripa [35] used a proxy-bidding format, characterized the time at which such late bidding occurred, and showed the existence of a late-bidding equilibrium. Additionally, last minute bidding can also be found in a myopic bidding strategy. However, when a hard-close rule is in place, this strategy fails to support an equilibrium of simultaneous ascending proxy auctions for heterogeneous items. The reasons that this method is prevalent are as follows: (1) The method saves time. The use of DMS has made up for this disadvantage. Moreover, trading is not limited in form and the troubling process of matching is totally solved by the computer and the Internet. (2) The method is efficient. Through performing a constant matching in a short timeframe, Cloud computing for big data produces the trading results and presents them easily for both sides. In addition, the third party is needed in the process. We assume that this channel also uses second sealed price auctions.

The above three channels can be found in the online advertising market; how to select the channel is important for auctioneers (SSP). Therefore, it is necessary to discuss differences among these channels. Our research contributes to the understanding of how SSPs develop strategies for these three channels. We not only build the profit models for the different channels but by comparing the expected profit from each channel, we also determine which specific situation is appropriate for the use of a certain channel. In addition, we discuss the effect of some parameters that determine the optimal channel strategy. We make a contribution in three aspects. First, we create the profit model for three channels and compare their profits in each potential situation to find the complete scenarios that can be analyzed after discarding paradoxes. Second, to obtain the optimal strategy, we assess different feasible situations that include a Poisson distribution parameters and the sensitivity of a ''high type'' bidder probability. However, there are different choices associated with various parameters. Last, each channel has its own market share. We explain why the Ad Exchange has become mainstream in auctions.

To better describe the relationships of different channels, the conceptual framework is shown in Figure 1.

This paper is organized as follows. Section 2 depicts the problem and lays out the basic model for the three different channels. In Section 3, three channels are compared by using profits attained by SSP, which shows which channel is more dominant in different situations. Section 4 analyses the SSP strategies that are developed based on different parameters. Some concluding remarks and research extensions are shown in Section 5.

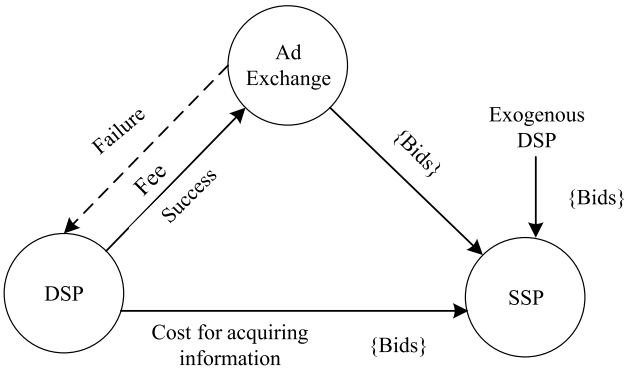

FIGURE 1. Conceptual framework.

II. MATERIALS AND METHODS

A. PROBLEM DESCRIPTION

In the auction of online advertising at each period t, there are three participants taking part in the auction: the DSP, the SSP and Ad Exchange. In this online advertising process, the online publisher releases the identity of the user to only a subset of the advertisers. The unique identities and behaviors that advertisers use to place corresponding impressions are possessed by the SSP. In detail, the DSP comprises many bidders who want to purchase advertisement slots to show their goods, i.e., advertisers that want to bid for advertisement slots. The SSP who collects the online publishers' slots provides impressions to potential bidders. Ad Exchange, as the third participant, is indispensable for both sides in the proxy bidding. We establish the profit models of the three SSP channels, through which an SSP sells the impressions to a set of advertisers. The channels are the following: exogenous (direct) bidding, costly bidding, and proxy bidding. These different channels are described in the following paragraphs.

1) DEMAND SIDE PLATFORM (DSP)

The quantity of potential bidders can be drawn from a Poisson distribution with a mean $m > 0$, and the probability of having j bidders in an auction is $g_{(i)}^{(t)}$ in time t, which is shown in the following.

$$g_{(i)}^{(t)} = \frac{m_j}{j!} e^{-m}, \quad j = 0, 1, 2 \ldots n \tag{1}$$

Moreover, we suppose that each channel has the same distribution and that the auction is a sealed second price auction. We assume that the probability that the bidder whose price is higher than the reserved price R_t has a high value, where $v = H$, for the advertisement is $\lambda \in (0,1)$, which we call a ''high type'' probability. The probability of a ''low type'' can be described as $1 - \lambda$, and its value is denoted by $v = L$. Moreover, we suppose that the set $P = \{p_1, p_2 \ldots p_i \ldots\}$ denotes all the bidding prices in the auction. The bidder J submits the first highest price, $J = \arg\max_j p_{(i)}^{(t)}, j = 1, 2 \ldots n$; bidder K submits the second highest price as follows: $K = \arg\max_{k, k \neq j} p_{(k)}^{(t)}, k = 1, 2 \ldots n$, and $K < J$.

If the bidder wins the auction, he needs to pay the first highest price; therefore, his expected bidding price from three channels can be represented as follows:

$$EP = \lambda\{\theta^{(t)}b_J^{(t)} + \tau^{(t)}b_J^{'(t)} + \sigma^{(t)}b_J^{''(t)}\}, \quad (2)$$

where $b_J^{(t)}, b_J^{'(t)}, b_J^{''(t)}$ are the first highest prices in the three channels and $\theta^{(t)}, \tau^{(t)}, \sigma^{(t)}$ are the probabilities of entering the three channels. Moreover,

$$\theta^{(t)} + \tau^{(t)} + \sigma^{(t)} = 1. \quad (3)$$

As this is the second sealed price auction, if a bidder enters and wins the auction, his expected payment for the advertisements is EU:

$$EU = \lambda\theta^{(t)}b_K^{(t)} + \lambda\tau^{(t)}b_K^{'(t)} + \tau^{(t)}c_J^{(t)} + \lambda\sigma^{(t)}(b_K^{''(t)} + f_J^{(t)}) \quad (4)$$

where $c_J^{(t)}$ is the price bidder J pays for obtaining more information when choosing to use costly bidding and $f_J^{(t)}$ is the fee for proxy bidding. In addition, $b_K^{(t)}, b_K^{'(t)}, b_K^{''(t)}$ are the second highest prices in the three channels.

2) SUPPLY SIDE PLATFORM (SSP)

The supply side platform (SSP) has many advertisement slots or impressions to sell; hence, the SSP is the source in the auction chain. Its expected profit $E\pi_t$ is the focus of our paper. More channel analysis details are provided in the following paragraphs. Further, in our study, the reserved price R_t is also a strategic decision made by the SSP.

The SSP profit in three different channels is developed based on the following assumptions: (1) If the bidder chooses the direct bidding channel, the SSP receives only the bidding price as profit. (2) If the bidder desires information about the true value or the reserved price, the SSP can obtain money by providing this valuable information. (3) If a bidder employs a proxy to bid, the SSP will exercise minor, additional care in the bidding auction, as Ad Exchange maintains a cooperative relationship with the SSP. This relationship is also discussed in the following description of Ad Exchange.

3) AD EXCHANGE (THE THIRD PARTY)

Ad Exchange, as a third party, needs to improve the profit or raise the enthusiasm of other parties in order to facilitate the movement of trade in the auction. Ad Exchange should increase the possibility of a deal and quickly match the counterparties in each auction. Only in this way is the SSP willing to accept Ad Exchange; DSP prefers to achieve the bidding process in this way. With the rapid development of the Internet, the intermediate is increasingly important in any online channel, especially due to the characteristics of big data.

If bidder i provides bidding prices by proxy, he or she will pay a service fee. To draw the bidders' attention to this channel, Ad Exchange will implement the auction that it return the full fee to bidders if the bidders fail to obtain the advertisement. Therefore, Ad Exchange not only attracts DSPs to enter the channel but also helps SSPs to obtain more profit in some scenarios. The latter scenario is described in the narrative on proxy bidding.

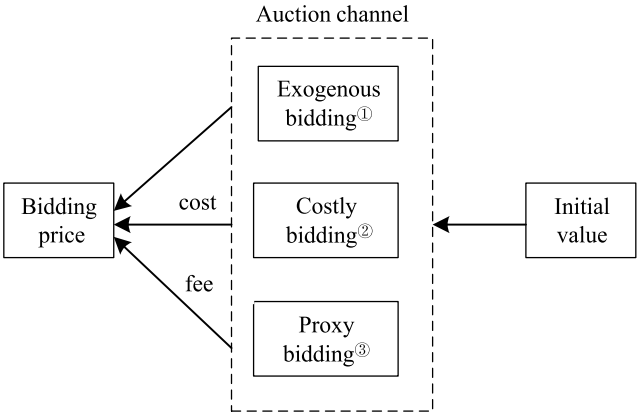

FIGURE 2. Descriptions of multichannel bidding.

B. METHODS AND MODELS

We use circled numbers to represent the three auction channels: channel ① is the exogenous (direct) bidding channel; channel ② is the costly bidding; and channel ③ is the proxy bidding channel. In all three channels, we adopt the Vickrey-Clarke-Groves (VCG) mechanism; therefore, bidders report their true value as their bidding prices. We introduce the profits of different channels in the following sections. In the exogenous bidding channel, the bidder only pays the bidding price. In the other channels, bidders pay additional charges. Bidders using the costly bidding channel need to pay for the cost of information, and the bidders in the proxy bidding channel will pay a fee if the deal is successful. Figure 2 depicts the differences among the three channels in multichannel bidding.

First, we depict the process and develop the model for channel ①, the exogenous bidding channel. In channel ①, the bidding price is just related to the private initial value estimated by the bidder in this channel. The SSP's expected auction profit, $E\pi_t$, can be written as:

$$\begin{aligned} E\pi_t = {} & \lambda g_{(1)}^{(t)} R_t + \sum_{j=2}^{N1} g_{(1)}^{(t)} \lambda (1-\lambda)^{j-1} R_t \\ & + \sum_{j=2}^{N1} \sum_{i=1}^{j} g_{(j)}^{(t)} \lambda^i (1-\lambda)^{j-i} b_K^{(t)} \\ & - \{g_{(0)}^{(t)} + (1-\lambda) g_{(1)}^{(t)} + \sum_{j=2}^{N1} g_{(1)}^{(t)} (1-\lambda)^{j-1}\} \\ & \times (1+\rho)^{-1} E\pi_{t+1} - \overline{C} \end{aligned} \quad (5)$$

The first term in Equation (5) is the profit in the case in which only one bidder who enters the auction has a bidding price higher than the reserved price, R_t, and the bidder just pays R_t to obtain the online advertising opportunity. The second term in Equation (5) is the expected profit in the case in which at least two bidders enter the auction but only one

of the bidders is a high type. This means that the bidder j just pays R_t. The third term in Equation (5) denotes the case in which at least two bidders have prices higher than R_t; due to the channel's adoption of the second price sealed auction mechanism, the bidder who gives the highest price just needs to pay the second highest price. The fourth term in Equation (5) denotes the case in which a loss is incurred as there is no sale in period t; the loss equals the discounted profit from period $t+1$. This case includes the situation in which no bidder appears and the situation in which only low-type bidders appear. When this phenomenon take place, the SSP advertises his advertising position to attract investment, which means he or she loses income that could have been obtained. Finally, the last term is the cost that SSP incurs to operate the auction.

Last, the time for allocating the advertising impression is 0.001 seconds. We assume that the seller treats the parameters as constants; i.e., the problem can be solved as a myopic problem [7]–[9]. We can substitute $E\pi_{t+1} = E\pi$ and $E\pi_t = E\pi$ into Equation (5) and solve $E\pi$ to obtain $\mathrm{E}\pi^1$:

$$\begin{aligned} E\pi^1 = \{&\sum_{j=1}^{N1} g_{(j)}^{(t)} \lambda (1-\lambda)^{j-1} R_t \\ &+ \sum_{j=2}^{N1} \sum_{i=1}^{j} g_{(j)}^{(t)} \lambda^i (1-\lambda)^{j-i} b_K^{(t)} - \overline{C}\} \\ &\div \{1 + \sum_{j=0}^{N1} g_{(1)}^{(t)} (1-\lambda)^j (1+\rho)^{-1}\} \end{aligned} \tag{6}$$

Second, in the following, we develop the model for channel ②, the bidding with costly information acquisition channel. If the advertiser wants to win the impression of the online advertising, he can also acquire some additional information that is only released by the SSP. The information may describe the preference of consumers through disclosing HTTP cookies [36].

Although all bidders are rational and do not pay too much to win this advertising in a period, the information is helpful for advertisers to make an appraisal of the advertising. Regarding the equality of the market price or the auction process, bidders may increase their rate of winning by purchasing more information from the SSP. Therefore, the situation in which some bidders make imperious demands for slots always occurs. The SSP welcomes more bidders who are willing to pay for the cost of information.

The ability to pay to acquire information that is directly submitted to the advertiser (DSP) will increase the probability of having high-type bidders λ_c, and we assume that if one bidder enters the channel ②, he or she will pay the cost $c_i^{(t)}$, $i = 1, 2 \ldots n$, where $c_i^{(t)}$ has a cumulative distribution F_c. On the one hand, the bidders' valuations v_i is increasing with the cost for information c_i; on the other hand, the SSP will establish a benevolent level [4] in period t, $\gamma_i^t \geq 0$. γ_i^t measures how benevolent the SSP is. When γ_i^t is low, the SSP cares only about his own expected profit; on the other extreme, when γ_i^t is high, the SSP has aligned with the advertiser. This benevolence may be rationalized by the phenomenon that the SSP is engaged in the long-time contractual relationship with the advertiser. Therefore, the parameter λ_c denoting "high types" in channel ② is increasing with c_i^t and with γ_i^t. Moreover, $\lambda_c(c_i^{(t)}, \gamma_i^t) \in [0, 1]$.

The expected auction profit of SSP, $E\pi_t$, is as follows:

$$\begin{aligned} E\pi_t = &\lambda_c g_{(1)}^{(t)} R_t + \sum_{j=2}^{N2} g_{(1)}^{(t)} \lambda_c (1-\lambda_c)^{j-1} R_t \\ &+ \sum_{j=2}^{N2} \sum_{i=1}^{j} g_{(j)}^{(t)} \lambda_c^i (1-\lambda_c)^{j-i} b_K^{'(t)} + \sum_{j=2}^{N2} c_i^{(t)} \\ &- \{g_{(0)}^{(t)} + \sum_{j=1}^{N2} g_{(j)}^{(t)} (1-\lambda_c)^j\} (1+\rho)^{-1} E\pi_{t+1} - \overline{C} \end{aligned} \tag{7}$$

The components of Equation (7) are similar to those in Equation (5). We omit the depiction. The difference between Equation (7) and Equation (5) is that in Equation (7), the cost $(\sum_{j=2}^{N2} c_i^{(t)})$ from all the bidders can not be returned even though they failed in the process of obtaining the information.

We can also substitute $E\pi_{t+1} = E\pi$ and $E\pi_t = E\pi$ into Equation (7) and as in the former, we can solve $E\pi$ to obtain $\mathrm{E}\pi^2$:

$$\begin{aligned} \mathrm{E}\pi^2 = &\{\sum_{j=1}^{N2} g_{(j)}^{(t)} \lambda_c (1-\lambda_c)^{j-1} R_t + \sum_{j=2}^{N2} \sum_{i=1}^{j} g_{(j)}^{(t)} \lambda_c^i (1-\lambda_c)^{j-i} b_K^{'(t)} \\ &+ \sum_{j=2}^{N2} c_i^{(t)} - \overline{C}\} \div \{1 + \sum_{j=0}^{N2} g_{(j)}^{(t)} (1-\lambda_c)^j (1+\rho)^{-1}\} \end{aligned} \tag{8}$$

Last, we develop the profit model of proxy bidding, channel ③. Facing heterogeneous ad viewers, bidders in online auctions want to know who is more likely to buy the displayed online advertising. Ad Exchange adopts big data to predict the preferences and buying behaviors of viewers or potential buyers and finds the most likely buyer for the bidders (DSP). This process is only implemented on the DMP (data management platform).

The fees for employing a proxy to bid are paid to Ad Exchange. Unlike channel ①, in this channel, this will increase the winning rate. We assume the fees are fixed, $f_i^{(t)}, (i = 1, 2 \ldots n)$, where $f_i^{(t)}$ has a cumulative distribution F_f. In addition, we assume the parameter $\delta_i^{(t)}$ denotes the incentive level of Ad Exchange in period t, toward bidder i, and $\delta_i^{(t)} \geq 0$. The parameter measures how incented Ad Exchange is. When $\delta_i^{(t)}$ is low, Ad Exchange fails to help the bidders, and this situation must lead to bidder i failing in the deal; when $\delta_i^{(t)}$ is high, on the other extreme, this implies Ad Exchange has some methods to help bidder i as much as possible to obtain the goods. Therefore, λ_f is increasing with $f_i^{(t)}$ and with $\delta_i^{(t)}$. Moreover, $\lambda_f(f_i^{(t)}, \delta_i^t) \in [0, 1]$.

The incentive level is established from the long business relationship between Ad Exchange, which has a high brand

equity, and the SSP. They both have cooperated on many deals. Therefore, the SSP is willing to provide a billing discount for ads and Ad Exchange can provide advantageous prices that can attract more potential bidders. The fee flows into Ad Exchange's profit.

The expected auction profit of SSP, $E\pi_t$, is as follows:

$$\begin{aligned} E\pi_t = {} & \lambda_f g_{(1)}^{(t)} R_t + \sum_{j=2}^{N3} g_{(1)}^{(t)} \lambda_f (1-\lambda_f)^{j-1} R_t \\ & + \sum_{j=2}^{N3} \sum_{i=1}^{j} g_{(j)}^{(t)} \lambda_f^i (1-\lambda_c)^{j-i} b_K^{''(t)} \\ & - \{ g_{(0)}^{(t)} + (1-\lambda_f) g_{(1)}^{(t)} + \sum_{j=1}^{N2} g_{(j)}^{(t)} (1-\lambda_f)^j \} \\ & \times (1+\rho)^{-1} E\pi_{t+1} - \overline{C} \end{aligned} \tag{9}$$

The components of Equation (9) are similar to those of Equation (5). One point that differentiates Equation (9) from the Equation (5) is the rate of high-type bidders, λ_f. In addition, there is a proxy fee $f_i^{(t)}$ paid by each bidder. However, when bidder i fails in the auction, there is a full refund of the bidding price, and it does not belong to the SSP.

We can also substitute $E\pi_{t+1} = E\pi$ and $E\pi_t = E\pi$ into Equation (9), and similar to the former, we can solve $E\pi$ to obtain $E\pi^3$:

$$\begin{aligned} E\pi^3 = {} & \{ \sum_{j=1}^{N3} g_{(j)}^{(t)} \lambda_f (1-\lambda_f)^{j-1} R_t \\ & + \sum_{j=2}^{N3} \sum_{i=1}^{j} g_{(j)}^{(t)} \lambda_f^i (1-\lambda_f)^{j-i} b_K^{''(t)} \\ & + \sum_{j=2}^{N3} c_i^{(t)} - \overline{C} \} \div \{ 1 + \sum_{j=0}^{N3} g_{(j)}^{(t)} (1-\lambda_f)^j (1+\rho)^{-1} \} \end{aligned} \tag{10}$$

C. SIMPLIFICATION OF MODELS

The quantities of potential bidders from different channels are random variables; hence, we can regard $N_1, N_2, N_3 \to \infty$, whose technique is presented in literature [37]. In addition, for the goal of simplifying the calculation, we define the following:

$$C = \sum_{i=1}^{\infty} c_i^{(t)}; \ \xi_1^j = \sum_{i=1}^{j} \lambda^i (1-\lambda)^{j-i}; \tag{11}$$

$$\xi_2^j = \sum_{i=1}^{j} \lambda_c^i (1-\lambda_c)^{j-i}; \ \xi_3^j = \sum_{i=1}^{j} \lambda_f^i (1-\lambda_f)^{j-i}; \tag{12}$$

$$\beta_1 = \sum_{j=1}^{\infty} g_j^{(t)} \lambda^i (1-\lambda)^{j-1} = \frac{\lambda}{1-\lambda} e^{-m} (e^{m-m\lambda} - 1); \tag{13}$$

$$\begin{aligned} \beta_2 = {} & \sum_{j=2}^{\infty} \sum_{i=1}^{j} g_j^{(t)} \lambda^i (1-\lambda)^{j-i} \\ = {} & \sum_{j=2}^{\infty} g_j^{(t)} \xi_1^j = e^{-m} (e^{m\xi_1} - 1 - m\xi_1); \end{aligned} \tag{14}$$

$$\beta_3 = \sum_{j=1}^{\infty} g_j^{(t)} (1-\lambda)^j = e^{-m\lambda}; \tag{15}$$

$$\delta_1 = \sum_{j=1}^{\infty} g_j^{(t)} \lambda_c^i (1-\lambda_c)^{j-1} = \frac{\lambda_c}{1-\lambda_c} e^{-m} (e^{m-m\lambda_c} - 1); \tag{16}$$

$$\begin{aligned} \delta_2 = {} & \sum_{j=2}^{\infty} \sum_{i=1}^{j} g_j^{(t)} \lambda_c^i (1-\lambda_c)^{j-1} \\ = {} & \sum_{j=2}^{\infty} g_j^{(t)} \xi_2^j = e^{-m} (e^{m\xi_2} - 1 - m\xi_2); \end{aligned} \tag{17}$$

$$\delta_3 = \sum_{j=0}^{\infty} g_j^{(t)} (1-\lambda_c)^j = e^{-m\lambda_c}; \tag{18}$$

$$\tau_1 = \sum_{j=1}^{\infty} g_j^{(t)} \lambda_f^i (1-\lambda_f)^{j-1} = \frac{\lambda_f}{1-\lambda_f} e^{-m} (e^{m-m\lambda_f} - 1); \tag{19}$$

$$\begin{aligned} \tau_2 = {} & \sum_{j=2}^{\infty} \sum_{i=1}^{j} g_j^{(t)} \lambda_f^i (1-\lambda_f)^{j-1} \\ = {} & \sum_{j=2}^{\infty} g_j^{(t)} \xi_3^j = e^{-m} (e^{m\xi_3} - 1 - m\xi_3); \end{aligned} \tag{20}$$

and

$$\tau_3 = \sum_{j=0}^{\infty} g_j^{(t)} (1-\lambda_f)^j = e^{-m\lambda_f}; \tag{21}$$

Therefore, the expected profit (Profit Model) from the three different channels can be rewritten as:

$$E\pi^1 = \{ \beta_1 R_t + \beta_2 b_K^{(t)} - \overline{C} \} \div \{ 1 + \beta_3 (1+\rho)^{-1} \} \tag{22}$$

$$E\pi^2 = \{ \delta_1 R_t + \delta_2 b_K^{'(t)} + C - \overline{C} \} \div \{ 1 + \delta_3 (1+\rho)^{-1} \} \tag{23}$$

$$E\pi^3 = \{ \tau_1 R_t + \tau_2 b_K^{''(t)} - \overline{C} \} \div \{ 1 + \tau_3 (1+\rho)^{-1} \} \tag{24}$$

III. RESULTS AND DISCUSSION

A. OPTIMAL STRATEGY FOR SSP IN DIFFERENT CHANNELS

For comparing the different channels' expected profit, without loss of generality, we define $y(\lambda, \lambda_c) = E\pi^1 - E\pi^2$, $y(\lambda, \lambda_f) = E\pi^1 - E\pi^3$, and $y(\lambda_c, \lambda_f) = E\pi^2 - E\pi^3$. In addition, some assumptions are given in the following:

$$A = \{ 1 + \beta_3 (1+\rho)^{-1} \}, \tag{25}$$

$$B = \{ 1 + \delta_3 (1+\rho)^{-1} \}, \tag{26}$$

$$D = \{ 1 + \tau_3 (1+\rho)^{-1} \}, \tag{27}$$

$$CB\beta_1 - A\delta_1 > 0, \tag{28}$$

$$DB\beta_1 - A\tau_1 > 0, \tag{29}$$

and

$$CB\tau_1 - D\tau_1 > 0. \tag{30}$$

If $y(\lambda, \lambda_c) = 0$, $y(\lambda, \lambda_f) = 0$, and $y(\lambda_c, \lambda_f) = 0$, we can obtain the following thresholds:

$$\hat{R}_t^1 = \frac{A\delta_2 b_K^{'(t)} + AC - B\beta_2 b_K^{(t)} + (B-A)\bar{C}}{B\beta_1 - A\delta_1} \tag{31}$$

$$\hat{R}_t^2 = \frac{A\tau_2 b_K^{''(t)} - D\beta_2 b_K^{(t)} + (D-A)\bar{C}}{B\beta_1 - A\tau_1} \tag{32}$$

$$\hat{R}_t^3 = \frac{D\tau_2 b_K^{'(t)} + DC - B\tau_2 b_K^{''(t)} + (B-D)\bar{C}}{B\tau_1 - D\delta_1} \tag{33}$$

However, it is too difficult to compare them authentically. Therefore, to show the effect caused by different scenarios, we describe all potential situations, except these paradoxes envisioned in Note 1.

Situation 1: $\hat{R}_t^1 \geq \max[\hat{R}_t^2, \hat{R}_t^3]$.

Proposition 1: The optimal strategy for SSP is given as follows.

(i) When $R_t \geq \hat{R}_t^1$, the strategy of choosing channel ① dominates over the strategies of channel ② and channel ③.

(ii) When $\hat{R}_t^1 > R_t \geq \max[\hat{R}_t^2, \hat{R}_t^3]$, the paradox is explained in Note 1.

(iii) When $\max[\hat{R}_t^2, \hat{R}_t^3] > R_t > \min[\hat{R}_t^2, \hat{R}_t^3]$, only when $\hat{R}_t^2 < \hat{R}_t^3$, the strategy of choosing channel ② dominates over the strategies of channel ① and channel ③; otherwise, the strategy of choosing channel ③ is dominant.

(iv) When $R_t \leq \min[\hat{R}_t^2, \hat{R}_t^3]$, the strategy of choosing channel ② dominates over the strategies of channel ① and channel ③.

The details of proof can be seen in the Appendix.

Situation 2: $\hat{R}_t^2 \geq \max[\hat{R}_t^1, \hat{R}_t^3]$:

Proposition 2: The optimal strategy for SSP is given as follows.

(i) When $R_t \geq \hat{R}_t^2$, the strategy of choosing channel ① dominates over the strategies of channel ② and channel ③.

(ii) When $\hat{R}_t^2 > R_t \geq \max[\hat{R}_t^1, \hat{R}_t^3]$, the strategy of choosing channel ③ dominates over the strategies of channel ① and channel ②.

(iii) When $\max[\hat{R}_t^1, \hat{R}_t^3] > R_t > \min[\hat{R}_t^1, \hat{R}_t^3]$, only when $\hat{R}_t^1 > \hat{R}_t^3$, the strategy of choosing channel ③ dominates over the strategies of channel ① and channel ②; if $\hat{R}_t^3 > \hat{R}_t^1$, the paradox is explained in Note 1.

(iv) When $R_t < \min[\hat{R}_t^1, \hat{R}_t^3]$, the strategy of choosing channel ② dominates over the strategies of channel ① and channel ③.

The details of proof can be seen in Appendix.

Situation 3: $\hat{R}_t^3 > \max[\hat{R}_t^1, \hat{R}_t^2]$:

Proposition 3: The optimal strategy for SSP is given as follows.

(i) When $R_t \geq \hat{R}_t^3$, the strategy of choosing channel ① dominates over the strategies of channel ② and channel ③.

(ii) When $\hat{R}_t^3 > R_t \geq \max[\hat{R}_t^1, \hat{R}_t^2]$, the strategy of choosing channel ① dominates over the strategies of channel ② and channel ③.

(iii) When $\max[\hat{R}_t^1, \hat{R}_t^2] > R_t > \min[\hat{R}_t^1, \hat{R}_t^2]$, only when $\hat{R}_t^1 > \hat{R}_t^2$, the strategy of choosing channel ② dominates over the strategies of channel ① and channel ③; if $\hat{R}_t^2 > \hat{R}_t^1$, the paradox is explained in Note 1.

(iv) When $R_t < \min[\hat{R}_t^1, \hat{R}_t^2]$, the strategy of choosing channel ② dominates over the strategies of channel ① and channel ③.

The details of the proof can be seen in the Appendix.

Note 1: For these paradoxes, we conclude the following different scenarios:

(1) In situation 1, the scenario $\hat{R}_t^1 > R_t \geq \max[\hat{R}_t^2, \hat{R}_t^3]$ demonstrates the relationship of the expected profit among the three channels: $\mathrm{E}\pi^2 < \mathrm{E}\pi^3 < \mathrm{E}\pi^1$, but $\mathrm{E}\pi^1 < \mathrm{E}\pi^2$. Hence, this is a paradox.

(2) In scenario (iii) of situation 2, the condition $\hat{R}_t^2 > \hat{R}_t^3 > R_t > \hat{R}_t^1$ exhibits $\mathrm{E}\pi^1 < \mathrm{E}\pi^3 < \mathrm{E}\pi^2$ and $\mathrm{E}\pi^2 < \mathrm{E}\pi^1$. The scenario is also a paradox.

(3) In the scenario in situation 3, $\hat{R}_t^3 > \hat{R}_t^2 > R_t > \hat{R}_t^1$ immediately shows as (2): $\mathrm{E}\pi^1 < \mathrm{E}\pi^3 < \mathrm{E}\pi^2$, and $\mathrm{E}\pi^2 < \mathrm{E}\pi^1$.

To maintain its existence as a third party, Ad Exchange needs to ensure that its expected profit in channel ② is at least larger than that from one of the other channels. This means $\mathrm{E}\pi^2 \geq min[\mathrm{E}\pi^1, \mathrm{E}\pi^3]$. Only in this scenario can Ad Exchange maintain its operation as an intermediary in the auction market.

Corollary 1: As a third party, Ad Exchange operates in the auction market if at least one of the conditions $\hat{R}_t^2 \geq R_t$ and $R_t \geq \hat{R}_t^3$ is satisfied.

The details of proof can be seen in the Appendix.

Finally, we depict all situations by using the following figure (Figure 3). From Equations (7), (8) and (9), we evidently know that the expected profit monotonously increases with R_t. To perfectly show all situations mentioned above, we draw Figure 3. The abscissa axis is the reserved price, R_t, and the ordinate is the expected profit, $\mathrm{E}\pi$. If a paradox is present, the involved thresholds $\hat{R}_t^1$, $\hat{R}_t^2$ and $\hat{R}_t^3$ cannot satisfy the following equations: $\mathrm{E}\pi^1 = \mathrm{E}\pi^2$, $\mathrm{E}\pi^1 = \mathrm{E}\pi^3$, and $\mathrm{E}\pi^3 = \mathrm{E}\pi^2$.

It is obvious that when R_t is high enough, channel ① is superior to other channels. The phenomenon that appears in all scenarios reveals that direct bidding is preferred among three channels if only profit, not the actual quantity of deals, is considered. In addition, it is clear that this situation occurs when the deal price that is not lower than R_t is high. When R_t is low enough, the strategy of channel ②, i.e., bidding with costly information, is the best choice. This fact can also be seen in all scenarios. Therefore, if the reserved price of some slots is not high, costly bidding can generate more profit for the SSP. For Channel ③, its profit is higher than the others only in the situation in which $\hat{R}_t^2 > R_t > \hat{R}_t^3$, which can be found in (a1), (b1) and (b2). When R_t is in the interval $(\min[\hat{R}_t^1, \hat{R}_t^2, \hat{R}_t^3], \max[\hat{R}_t^1, \hat{R}_t^2, \hat{R}_t^3])$, there is no exclusive channel satisfying SSP. This easily causes the paradoxes.

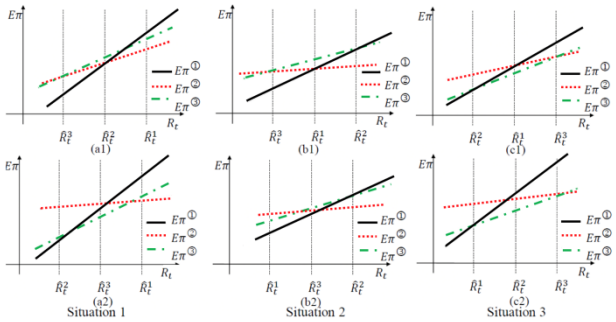

FIGURE 3. All situations description. (a1) and (a2) belong to situation 1. (b1) and (b2) belong to situation 2. (c1) and (c2) belong to situation 3.

B. DISCUSSION OF PARAMETERS FOR DETERMINING THE OPTIMAL STRATEGY

In this section, we analyze the relation between the relevant parameters and the profit. We first compare the scenarios under different Poisson distributions in order to examine the effect of the distribution in determining the optimal channel strategy. Then, to facilitate the subsequent discussion, we analyze situations based on the probability of high-type bids and of the parameters λ_c and λ_f, when both of them are denoted by different numbers. In this way, we examine the effect that various values of λ_c and λ_f have on the expected profits of channel ② and channel ③. Using channel ① as our baseline, we set $\lambda = 0.2$, as high-type bidders always exist in real life.

1) EFFECT OF DIFFERENT POISSON DISTRIBUTIONS (M)

First, we analyze the effect caused by different Poisson distribution parameters, which are denoted by $\lambda = 0.2$, $\lambda_c = 0.5$ and $\lambda_f = 0.25$. We set $\lambda < \lambda_f < \lambda_c$, as the DSPs who enter the costly bidding channel have paid for the cost of information and will remain in the auction to the end. This means that they are more willing to buy online advertising than are the bidders in the other two channels.

To compare the effect in more situations and to obtain distinguishing phenomena, we set the parameter m to 2, 5 and 9, which represent the SSP's low ebb, normal and peak flow of entering consumers [38]. In addition, R_t is in the interval [0,60].

Figure (a) reveals that exogenous bidding is preferred when R_t is not high, and channel ① dominates channel ② after a threshold of R_t. This result appears in Situation 3; i.e., $\hat{R}_t^3 > \hat{R}_t^1 > R_t > \hat{R}_t^2$, which is the situation also shown in the upper right of Figure (c) with the high R_t. The left and the right of Figure 4, (a) and (c), reveal a contrary scenario in which after a threshold of R_t, channel ② is superior to channel ①. The other channel, channel ③, is not. Furthermore, in the left Figure (a), although we do not draw the lines after $R_t = 60$, the trend of channel ③ exceeds that of channel ①, which is consistent with $R_t < \hat{R}_t^1, \hat{R}_t^2$ and $\hat{R}_t^3$. As the middle Figure (b) explains, it is likely that proxy bidding through Ad Exchange is conducted by bidders, as proxy bidding is more advantageous than channel ②, bidding with cost of information fees, which is described by $R_t > \hat{R}_t^1, \hat{R}_t^2$ and $\hat{R}_t^3$. As a whole, from a profit perspective, exogenous bidding captures market share. In the Figure (c), after a threshold of R_t, channel ① excels over channel ②. This is reflected in Situation 3: $\hat{R}_t^3 > \hat{R}_t^1 > R_t > \hat{R}_t^2$.

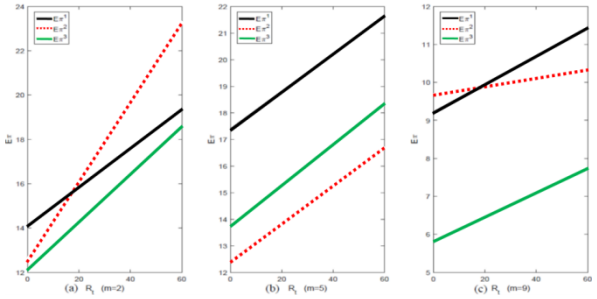

FIGURE 4. Different parameter (m) of Poisson distribution. (a) is $m = 2$. (b) is $m = 5$. (c) is $m = 9$.

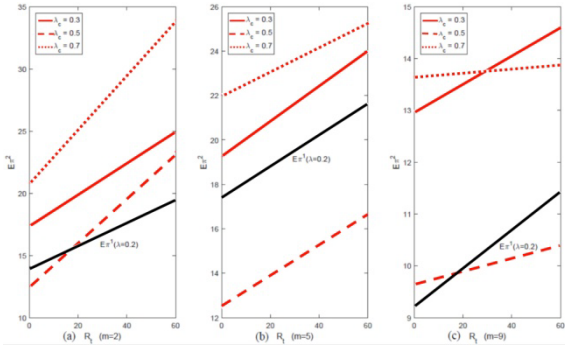

FIGURE 5. Sensitivity analysis of λ c. (a) is $m = 2$. (b) is $m = 5$. (c) is $m = 9$.

In conclusion, the online advertising market is unpredictable, especially the quantity of the potential bidders. Because of this fact, the advertising deal and profit reflect different situations at different times. Due to the complexity of the situations, each of three channels may capture some market share. Moreover, when only considering the profit, proxy bidding is not preferred. This is reflected in Figure (b), where the profit from proxy bidding is only higher than that from channel ②.

2) EFFECT OF HIGH TYPE PROBABILITY IN CHANNEL ② (λ_c)

To determine the effect of the probability of having high-type bidders, we set $\lambda_c = 0.3, 0.5$ and 0.7. $\lambda_c = 0.3$ means the channel is not satisfactory for some DSPs, but in our consideration, it still provides higher satisfaction than does the direct bidding channel ($\lambda = 0.2$). Moreover, $\lambda_c = 0.5$ is the situation in which bidders choose the costly bidding channel, and $\lambda_c = 0.7$ reflects a better situation; i.e., many DSPs are high-type bidders.

Moreover, the probability of a high-type bidder in channel ① is 0.2, which is our baseline, and the parameter in the costly bidding channel is set higher than 0.2 because the DSP who enters channel ② has an imperious demand to win in spite of the cost of information. As is shown in Figure 5, three scenarios of $m = 2, 5$ and 9, each of which has an expected profit of $\lambda_c = 0.3, 0.5$ and 0.7, respectively, explain

the complexity of indirectly comparing the profit of the three channels. For the base case, channel ①, we set $\lambda = 0.2$.

Figure 5, shows that when $\lambda_c = 0.3$ and 0.7, the expected profit of channel ① is smaller than that of channel ② in (a), (b) and (c). For $\lambda_c = 0.3$, 0.7 and 0.9, $\lambda_c = 0.7$ is the most dominant, except for the situation in which there is a lower value of R_t in (c). However, when $\lambda_c = 0.3$, channel ① is preferred to channel ②. Therefore, the probability of having a high-type bidder in channel ② is not very high. The phenomenon in which going too far is as bad as not going far enough is shown in (c) when $\lambda_c = 0.5$. Moreover, when λ_c increases from 0.3 to 0.5, the profit of channel ② decreases, although its profit is higher than that of channel ①. Therefore, it is totally unnecessary for the SSP to employ the costly bidding channel when there is a higher value of λ_c. The strategy based on the value of λ_c is accepted, but this does not mean that the higher the value is, the better. Once the condition $\lambda_c = 0.3$ can satisfy the use of channel ②, there is no need for the higher value of λ_c. Focusing on $\lambda_c = 0.5$, we found that there is a cross point of channel ① and ②, i.e., when $\lambda_c = 0.5$, $m = 2$ and $m = 9$. Contrary situations can be seen in (a) and (c). In the middle Figure, (b), although we do not draw the lines after $R_t = 60$, the trend of $\lambda_c = 0.5$ exceeds that of $\lambda_c = 0.3$.

Moreover, the potential bidders described by the Poisson distribution have a large impact on profit. When $m = 9$, if the SSP uses the costly bidding channel, he or she can attract more bidders to obtain a higher profit. When $m = 2$, the costly bidding channel should be used to stimulate bidders to bid higher prices and to create a competitive atmosphere.

3) EFFECT OF HIGH TYPE PROBABILITY IN CHANNEL ③ (λ_f)

For channel ①, we set $\lambda = 0.2$. There are three scenarios of $m = 2$, 5 and 9, and each has an expected profit of $\lambda_f = 0.25$, 0.4 and 0.7, respectively. Specifically, if $\lambda_f = 0.25$ is the base case situation, the situation in which $\lambda_f = 0.4$ and $\lambda_f = 0.7$ means that an enterprise has good word of mouth and that the probability of having a high-type bidder in the proxy channel may be higher. For excellent SSPs such as Baidu, we set $\lambda = 0.7$. Some other SSPs may not warrant this level, and they are assigned a value of 0.4. However, to generate a profit, third parties use many ways to help make deals for both sides; therefore, the number of high-type bidders in this channel is higher than those in the direct bidding channel, in which $\lambda = 0.2$.

On the left of Figure 6, in (a), when $\lambda_f = 0.7$, channel ③ has an intersection with channel ①. Therefore, there must be a market share for the Ad Exchange. The closer the business relationship the third party has with the SSP is, the higher the probability of having a high-type bidder. The Ad Exchange pays the money from the bidders' fees to the SSP. This makes the third party an assemblage where bidders use costly information to make bids.

For many situations in Figure 6, the profit of channel ③ is lower than that of channel ①. In addition, in the comparison with $\lambda_f = 0.4$ and 0.7, $\lambda_f = 0.25$ is dominant in (b) and (c),

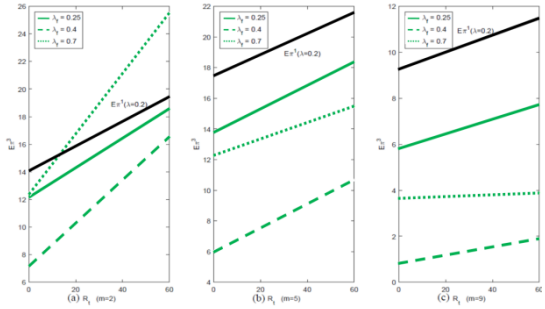

FIGURE 6. Sensitivity analysis of λ_f. (a) is $m = 2$. (b) is $m = 5$. (c) is $m = 9$.

because as shown in equation (8) and (9), some profit is transferred to the third party, the Ad Exchange, by fees. In channel ③, the fees are the cost of obtaining information and are received by the SSP. In addition, Figure (a) and (b) show that the profit of channel ③ will be higher than that of channel ① in the situation in which $\lambda_f = 0.25$, although we do not display the trends after $R_t = 60$.

When parameter λ in the direct bidding channel is 0.2, when R_t is high, the parameter λ_f that only increases to 0.25 is dominant for $m = 2$ and 5. Actually, currently, the price of online advertising is more than 60 yuan in some platforms, although the click on charge is low. Therefore, when the value of λ_f is not high, the SSP should consider employing the proxy channel for conducting an auction. Due to the cooperative relationship between the SSP and the Ad Exchange, the probability of a deal is considered higher by the DSPs, who when they enter this channel, are usually considered to be "high-types". In fact, an SSP can expect to earn a satisfactory profit if λ_f is not high. Moreover, occurring in just 0.001 seconds, online advertising is conducted quickly, but using the direct channel can take a few minutes or hours and even several days. Therefore, due to the time involved in generating a profit, SSPs prefer channel ③ over channel ①.

IV. CONCLUSION

In this paper, we modeled three online advertising auction channels through which a DSP could submit bid prices. These models should be helpful for SSPs in their development of strategies for different reserved prices and channels, which is beneficial to make strategies of SSPs, especially multi channels. The exogenous bidding channel is a traditional way to submit bids; its advantage is that there are no extra costs or fees, but its use may cost the SSP more time than would the other channels. With the increasing demands of the quick deals, the traditional way faces the risk of becoming obsolete. The channel of bidding involving costly information acquisition not only facilitates the presentation of bidding prices but also enables the receipt of fees for the cost of information. This channel is welcomed by SSPs. Moreover, a third party can play a proper role in accelerating the trading, because it is skilled at bid data analysis that is necessary for a large volume of quick deals. Some conclusions are as follows:

1) Each channel may be an excellent choice for different scenarios. For instance, when $m = 5$, direct bidding

is preferred among the basic situations shown in Figure 4(b); when $\lambda = 0.2$ and $\lambda_c = 0.3$, bidding with costly information is dominant, as shown in Figure 5(b); when $\lambda = 0.2$, $\lambda_f = 0.7$ and R_t is higher, proxy bidding may be the optimal strategy, as shown in Figure 6(b). The different probability of having a high-type bidder in a normal period ($m = 5$) can cause a channel strategy to dominate others. Therefore, when SSPs use the proxy channel, the improvement in synthetical strength can indirectly increase the probability of having a hype-type consumer in the DSP.

2) It is better for an SSP to use the channel of costly bidding if the probability of a high-type bidder is low or very high. From Figure 5, we can see that for most situations, the revenue is worse if $\lambda_f = 0.5$. The SSP needs to make a large effort to improve the probability of having a high-type bidder, which is unlikely unless the probability greatly increases, i.e., when $\lambda_f = 0.7$. However, it is obvious that many efforts have to be made, and these efforts may result in much unnecessary investment. For a SSP, $\lambda_f = 0.3$ may be better if he or she wants to use this channel. Moreover, Figure 6, reveals that in most situations, channel ③'s profit is lower than that from direct bidding, which again proves that it is better for the SSP to use the channel of costly bidding if the probability of having a high-type bidder is low or very high.

3) In the age of the Internet, if the probability of having a high-type bidder is not high, when R_t is high, an SSP should consider employing proxy bidding. This can be seen from Figure 5(b). To save time and cost, both the SSP and the DSP are willing to use a third party; the condition in which $\lambda_f = 0.25$ can be easily satisfied; therefore, the use of a third party is a mainstream practice in online advertising auctions. The detailed reasons are as follows. First, the trade of online advertising is so fast that a direct channel cannot accommodate it well. Second, long term profits are more vital than the profit from one deal. Most SSPs tend to use an Ad Exchange to obtain access to more DSPs. Last, the cooperative relationship between SSPs and Ad Exchanges can attract more DSPs to enter an auction.

However, due to the uncertainty of potential bidders, we don't provide concrete reserved price information. Though we regard the demand as a Poisson distribution, the parameter of m has immense variety during online advertising auctions that are completed within 0.001 seconds. For the potential bidders, the assumption of Poisson distribution has a limitation. In our research, there are still many aspects that can be improved. In the future, actual data will be adopted in our work to describe the different channels.

APPENDIX

Proof of Situation 1: One can immediately verify that the values $\hat{R}_t^1$, $\hat{R}_t^2$ and $\hat{R}_t^3$ are defined such that (a) $E\pi^1 > E\pi^2$, if and only if $R_t \geq \hat{R}_t^1$; (b)$E\pi^{\breve{n}} > E\pi^{\breve{o}}$, if and only if $R_t \geq \hat{R}_t^2$; and (c)$E\pi^3 > E\pi^2$, if and only if $R_t \geq \hat{R}_t^3$.

For part (i), if $R_t \geq \hat{R}_t^1$, it is obvious that $E\pi^1 > E\pi^2$, which according to the condition $\hat{R}_t^1 > \max[\hat{R}_t^2, \hat{R}_t^3]$, means that $E\pi^1 > E\pi^3$ and that $E\pi^3 > E\pi^2$. Moreover, $E\pi^1 > E\pi^3 > E\pi^2$. Therefore, $R_t \geq \hat{R}_t^1$ means the strategy ① is dominant and gives the proof of part (i).

For part (iii), $\hat{R}_t^1 > R_t > \min[\hat{R}_t^2, \hat{R}_t^3]$, there are two different results. One is that $E\pi^3 > E\pi^1 > E\pi^2$, if $\hat{R}_t^1 > \hat{R}_t^3 > R_t > \hat{R}_t^2$, which shows the SSP is more willing to choose channel ② than the other two channels. The other result is that $E\pi^1 < E\pi^2 < E\pi^3$, if $\hat{R}_t^1 > \hat{R}_t^2 > R_t > \hat{R}_t^3$, which reveals that channel ③ is preferred by the SSP.

For part (iv), if $R_t < \min[\hat{R}_t^2, \hat{R}_t^3]$, we can easily have $E\pi^1 < E\pi^2$, $E\pi^1 < E\pi^3$ and $E\pi^3 < E\pi^2$, which concludes that $E\pi^1 < E\pi^3 < E\pi^2$. There is no doubt that channel ② is more profitable.

Proof of Situation 2: Some verdicts in Proposition 1 can be applied to this proof. Moreover, the proofs in part (i) and part (iv) are the same as those in Proposition 1. Thus, we only depict part (ii) and part (iii).

For part (ii), when $R_t < \hat{R}_t^2, R_t \geq \hat{R}_t^1$ and $R_t \geq \hat{R}_t^3$, according to Proposition 1, which asserts $E\pi^1 < E\pi^3$, $E\pi^1 > E\pi^2$, and $E\pi^3 > E\pi^2$; we obtain $E\pi^2 < E\pi^1 < E\pi^3$. Therefore, the channel ③ is more profitable.

For part (iii), if $\hat{R}_t^1 > \hat{R}_t^3$, we know that $\hat{R}_t^2 > \hat{R}_t^1 > R_t > \hat{R}_t^3$, and $E\pi^1 < E\pi^2 < E\pi^3$, which reveals that channel ③ is preferred by SSP.

Proof of Situation 3: Based on the proof of Proposition 1, we omit the proof for part (i) and part (iv).

For part (ii), if $\hat{R}_t^3 > R_t \geq \max[\hat{R}_t^1, \hat{R}_t^2]$, we obtain $E\pi^3 < E\pi^2 < E\pi^1$. The channel ① is the dominant strategy.

For part (iii), if $\hat{R}_t^3 > \hat{R}_t^1 > R_t > \hat{R}_t^2$, we see that $E\pi^3 < E\pi^1 < E\pi^2$, and the preferred strategy is choosing channel ②.

Proof of Corollary 1: If channel ③, i.e., proxy bidding with a fee, wants to be utilized in the auction market, it must show its profit superiority.

When $\hat{R}_t^2 \geq R_t$ is satisfied, it means that an SSP obtains more profit from using a proxy auction than from using exogenous (direct) bidding. When $R_t \geq \hat{R}_t^3$, it implies that an SSP can obtain more profit from proxy auction than from costly bidding. Both channels even inspire the SSP to encourage Ad Exchange, the third party, to be present in the auction market. Furthermore, if $\hat{R}_t^2 > R_t > \hat{R}_t^3$, channel ③ dominates other channels, and the Ad Exchange is prosperous and develops rapidly.

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