

Four-quark structure of the excited states of heavy mesons

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We propose a four-quark structure for some of the excited states of heavy mesons containing a single charm or bottom quark. The four-quark wave functions are constructed based on a diquark-antidiquark form under the constraint that they form an antitriplet $\bar{\mathbf{3}}_f$ in $SU(3)_f$, which seems to be realized in some of the excited states listed in Particle Data Group. Depending on the structure of antidiquark, we construct two possible models for its wave functions: Model I) the antidiquark is symmetric in flavor ($\mathbf{6}_f$) and antisymmetric in color ($\mathbf{3}_c$) and Model II) the antidiquark is antisymmetric in flavor ($\mathbf{3}_f$) and symmetric in color ($\bar{\mathbf{6}}_c$). To test phenomenological relevance of these wave functions, we calculate the mass differences among the excited states of spin $J = 0, 1, 2$ using color-spin interactions. The four-quark wave functions based on Model I is found to reproduce the observed mass of the excited states of heavy mesons. Also, our four-quark model provides an interesting phenomenology relating to the decay widths of the excited states. To further pursue the possibility of the four-quark structure, we make a few predictions for open charm and open bottom states that may be discovered in future experiments. Most of them are expected to have broad widths, which would make them difficult to be identified experimentally. However, one resonance with $J = 1$ containing bottom and strange quarks is expected to appear as a sharp peak with its mass around $B_{1N}^{\bar{s}} \sim 5753$ MeV. Confirmation of the existence of such states in future experiments will shed light on our understanding of the structure of heavy meson excited states.

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I. INTRODUCTION

Multiquark states, which refer to hadrons composed of four or higher number of quarks, are very interesting subjects in hadron physics. Although the ground states of hadrons can be well described by the conventional picture of quark-antiquark systems for mesons and three-quark systems for baryons, there has been a controversy over the existence of exotic states including multiquarks and/or glueballs in hadron spectroscopy. This is because the conventional quark models taking into account color and flavor degrees of freedom do not rule out the possible existence of multiquark states. Indeed, there have been various experiments reporting the candidates of exotic states, which include $X(3872)$ [1], $Y(4260)$ [2], and $Z(4430)$ [3]. For those mesons, among various interpretations, the four-quark scenarios containing two heavy and two light quarks are quite promising [4–6]. Also pentaquark states triggered by the experiments of the LEPS Collaboration at SPring-8 [7] are still under debate both theoretically and experimentally. Existence of hybrid mesons with gluonic excitations will also be investigated by the Hall-D experiments at Thomas Jefferson National Accelerator Facility [8].

The pure exotic states can be distinguished by their

unique quantum numbers but the existence of crypto-exotic states is hard to identify as their quantum numbers can also be produced by the conventional pictures of hadrons. Therefore, some crypto-exotic multiquark states, other than the newly discovered exotic state candidates, may already be observed and listed in the current edition of Particle Data Group (PDG) [9], especially in hadron excited states. The pioneering work along this direction may be the diquark-antidiquark model advocated by Jaffe in the 1970s [10, 11], who proposed the four-quark structure for the scalar meson nonet, $a_0(980)$, $f_0(980)$, $\sigma(600)$, and $\kappa(800)$. (For a review, see Ref. [12].) In this model, diquarks, belonging to a color antitriplet and flavor antitriplet having spin 0, are claimed to be tightly bound and they combine with antidiquarks to form four-quark states. Thus, the four-quark states constructed in this way are, if there is no orbital excitations, restricted to have spin zero. Though this model was confronted with different suggestions based on two-quark picture such as the P -wave $\bar{q}q$ [13] or the mixture of various configurations [14], there are other calculations favoring the four-quark picture as well [15, 16].

The lesson from the light quark system certainly provides theoretical motivations for the possibility of the four-quark structure in the excited states of heavy mesons containing c or b quark. Experimentally, the excited states of heavy mesons, which were scarcely explored in the past, become much richer thanks to recent experimental investigations and during the last decade or so, the excited states in the open-charm and open-bottom sectors listed in PDG keep accumulating with various de-

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caying properties. This can provide a nice environment in investigating the structure of heavy meson excited states.

Indeed, there have been various theoretical investigations for the four-quark structure in the excited states of open charm mesons. These include the phenomenological model studies based on the relativistic quark model [17], Glozman-Riska hyperfine interaction [18], 't Hooft interaction [19], QCD sum rules [20, 21], etc. Even though there are other suggestions based on the two-quark picture [22] or mixing configurations between two-quark and four-quark states [23], it is still worthwhile to pursue additional signatures for four-quark structure in the excited states of heavy meson systems, and this is the main motivation of the present investigation.

Our approach for four-quark states is quite phenomenological rather than dynamical. By closely examining the current data of heavy meson spectroscopy, we will postulate a plausible flavor structure for the excited states of heavy mesons. Then possible four-quark wave functions will be constructed accordingly based on a diquark-antidiquark picture. Here the diquark is composed of one heavy and one light quark, and the antidiquark is a system of two light antiquarks.

In the present study, we do not restrict our consideration for the antidiquark state to the scalar type which belongs to the color triplet and flavor triplet having spin zero. Instead, we extend our consideration to a more general case by allowing various possible antidiquark states to see their role in heavy meson excited states. Based on the observation that the excited heavy meson states listed in PDG have spin 0, 1, 2, we allow other antidiquark structures other than the scalar state and look for plausible scenarios which can accommodate all those spin states within one framework. To test the phenomenological relevance of various four-quark models generated from this approach, the mass differences among heavy mesons will be calculated using color-spin interactions and compared with the experimental data.

The paper is organized as follows. In Sec. II, we examine the excited states of heavy mesons in PDG and motivate the four-quark picture. The four-quark wave functions constructed accordingly will be presented in Sec. III. After a brief introduction of color-spin interactions in Sec. IV, we present our calculations of the hyperfine masses from the four-quark wave functions in Sec. V. Results and discussions are given in Sec. VI and we summarize in Sec. VII.

II. HEAVY MESON SPECTROSCOPY

We start with examining D and B meson spectroscopy compiled by the Particle Data Group, which motivates the possible four-quark structure for the excited states of heavy mesons. Listed in Tables I and II are open charm and open bottom mesons that can be found in the compilation of PDG [9]. The lowest-lying states listed in Table I are found to have negative parity. Their isospins are

Family	Lowest-lying states			
	Meson	$I(J^P)$	Mass (MeV)	Γ (MeV)
D	D^0	$\frac{1}{2}(0^-)$	1864.86	-
	D^\pm	$\frac{1}{2}(0^-)$	1869.62	-
	D^{*0}	$\frac{1}{2}(1^-)$	2006.99	< 2.1
	$D^{*\pm}$	$\frac{1}{2}(1^-)$	2010.29	0.096
D_s	D_s^\pm	$0(0^-)$	1968.50	-
	$D_s^{*\pm}$	$0(1^-)$	2112.3	< 1.9
B	B^\pm	$\frac{1}{2}(0^-)$	5279.25	-
	B^0	$\frac{1}{2}(0^-)$	5279.58	-
	B^*	$\frac{1}{2}(1^-)$	5325.2	-
B_s	B_s^0	$0(0^-)$	5366.77	-
	B_s^*	$0(1^-)$	5415.4	-

TABLE I. The lowest-lying resonances with $J^P = 0^-, 1^-$ in D, D_s, B, B_s families listed in PDG [9].

Family	Excited states			
	Meson	$I(J^P)$	Mass (MeV)	Γ (MeV)
D	D_0^{*0}	$\frac{1}{2}(0^+)$	2318.29	267
	$D_0^{*\pm}$	$\frac{1}{2}(0^+)$	2403	283
	D_1^0	$\frac{1}{2}(1^+)?$	2421.4	27.4
	D_1^\pm	$\frac{1}{2}(1^+)$	2423.2	25
	D_1^0	$\frac{1}{2}(1^+)$	2427	384
	D_2^{*0}	$\frac{1}{2}(2^+)$	2462.6	49
	$D_2^{*\pm}$	$\frac{1}{2}(2^+)$	2464.3	37
	D_s	$D_{s0}^{*\pm}$	$0(0^+)$	2317.8
D_{s1}^\pm		$0(1^+)$	2459.6	< 3.5
D_{s1}^\pm		$0(1^+)$	2535.12	0.92
$D_{s2}^{*\pm}$		$0(2^+)$	2571.9	17
B	B_1^0	$\frac{1}{2}(1^+)$	5723.5	-
	B_2^0	$\frac{1}{2}(2^+)$	5743	23
	B_J^*	$?(?^?)$	5698	128
B_s	B_{s1}^0	$0(1^+)$	5828.7	-
	B_{s2}^0	$0(2^+)$	5839.96	1.56

TABLE II. The low-lying excited states with $J^P = 0^+, 1^+, 2^+$ in D, D_s, B, B_s families collected from PDG. According to PDG, the quantum numbers (I, J, P) of most excited mesons are yet to be confirmed. $D_1^\pm(2423)$, whose J^P is unknown, is assigned to have $J^P = 1^+$ in our analysis because of its similar mass with D_1^0 .

either $I = 1/2$ or $I = 0$, and their spins are 0 or 1. There are 4 (2) mesons in D (D_s) family, 3 (2) mesons in B (B_s) family. The excited states, which refer to the resonances with higher masses, are listed in Table II. There are 7 (4) mesons in D (D_s) family, 3 (2) in B (B_s) family.¹ The excited states listed in Table II have interesting features to be noted. Their parity is positive, which is opposite to the lowest-lying case, isospins of all the resonances are either $I = 1/2$ or $I = 0$ as in the lowest-lying states, and their spins are $J = 0, 1, 2$. Within each family, there is a

¹ Some mesons are not included in this list because their quantum numbers are unknown and their masses are higher than the states that we are considering in this work.

hierarchy in the mass spectrum, i.e., the mass increases with spin J , namely, $m_{J=0} < m_{J=1} < m_{J=2}$.

As anticipated, the spectrum of the lowest-lying states is consistent with the conventional $Q\bar{q}$ picture. They form an antitriplet in $SU(3)_f$ as one can see from Table III, where the mesons are regrouped according to their spin and parity J^P . In most cases, there are three mesons for each J^P , composed by two members in isodoublet ($I = 1/2$) and one member in isosinglet ($I = 0$). The mass splitting Δm between $I = 1/2$ and $I = 0$ members is about 90-100 MeV, which, though somewhat smaller than the quark mass difference $m_s - m_u$, still supports the formation of $\bar{\mathbf{3}}_f$. The only exception is the B -mesons in the $J^P = 1^-$ channel where one member in isodoublet ($I = 1/2$) is missing. But the mass splitting between $B^*(5325)$ and $B_s^*(5415)$ is again 90 MeV, which is similar in magnitude to those of other $\bar{\mathbf{3}}_f$ multiplets. Even though one more member is anticipated in this channel, we expect that it would be discovered soon at current experimental facilities and one can safely claim that the B -mesons of $J^P = 1^-$ also form $\bar{\mathbf{3}}_f$. This antitriplet structure is consistent with the two-quark systems having a charm (or a bottom) and a light antiquark, namely $c\bar{q}$ ($q = u, d, s$) (or $b\bar{q}$) being in relative S -wave state. The negative parity comes out naturally with this quark composition.

We then speculate the structure of excited states listed in Table II. Since these states have a positive parity, one can think about two possible ways to construct such states. The first way is based on the two-quark picture. Here, the states with positive parity can be constructed by orbitally exciting the lowest-lying states ($\ell = 1$). By combining with the spin of the two-quark $j = (0, 1)$, one can generate the total spin $J = 0, 1, 2$ for positive parity states. Then the mass splitting among the excited states can be generated by spin-orbit forces. In particular, one can expect that the mass splitting between $J^P = 1^+$ and $J^P = 0^+$ members is expected to be about half of the one between $J^P = 2^+$ and $J^P = 1^+$ members [24]. We see from Table II that this expectation works well for $D_0^{*\pm}(2403)$, $D_1^{*\pm}(2423)$, and $D_2^\pm(2464)$ but fails for $D_0^*(2318)$, $D_1^0(2421)$, and $D_2(2463)$.

Another way to construct the positive-parity excited states, which we want to pursue in the present work, is to make the product of the $SU(3)_f$ singlet of $q\bar{q}$ of negative-parity and the ground states of $c\bar{q}$ (or $b\bar{q}$). The resulting states contain four quarks and they obviously form a $\bar{\mathbf{3}}_f$ in $SU(3)_f$. Of course, the states constructed in this way are close to the two-meson molecular states. Motivated by this observation, however, what we want to investigate in this work is the general features of four-quark resonance states in heavy quark sector. Thus, a similar approach like the diaquonia model [10, 11, 25] will be adopted for quantitative estimates.

The present investigation is also motivated by the $\bar{\mathbf{3}}_f$ structure observed explicitly in the excited states of Table III. In the $J^P = 2^+$ channel of the ‘ D or D_s ’ family, there are three members, namely, $D_2^\pm(2464)$, $D_2^{*0}(2463)$,

and $D_{s2}^{*\pm}(2572)$ with the isospins expected from the $\bar{\mathbf{3}}_f$ multiplet. The mass splitting between $I = 1/2$ and $I = 0$ members is about 108 MeV, which is similar to the splitting in the lowest-lying mesons. Thus, the three resonances in $J^P = 2^+$ seem to form a $\bar{\mathbf{3}}_f$.

In the $J^P = 1^+$ channel of the ‘ D or D_s ’ family, $D_1^\pm(2423)$, $D_1^0(2421)$, and $D_{s1}^{*\pm}(2535)$ seem to form a $\bar{\mathbf{3}}_f$ with the mass splitting Δm of 113 MeV. However, there is another state, $D_{s1}^\pm(2460)$ of $I = 0$, which is hard to be classified as a member of $\bar{\mathbf{3}}_f$. Later we will discuss the importance implied by the existence of this state. We will find that, in the four-quark picture with $\bar{\mathbf{3}}_f$, there are two possible ways to make the spin-1 states, and, after taking care of the mixing between the two, $D_{s1}^\pm(2460)$ fits nicely with the member in the spin-1 channel.

In the $J^P = 1^+$ channel from the ‘ B or B_s ’ family, there are three resonances. Here, the $B_j^*(5698)$ may not be a member of an isodoublet with $B_1^0(5724)$ because of their large mass difference of 26 MeV. But its existence as well as its quantum number is not well-established yet. The other two, $B_1^0(5724)$ and $B_{s1}^0(5829)$, have mass splitting around 106 MeV, similar to the mass splitting expected from the structure of $\bar{\mathbf{3}}_f$. Also in the $J^P = 2^+$ channel from the ‘ B or B_s ’ family, there are only two resonances with the mass splitting 97 MeV, again similar magnitude expected from $\bar{\mathbf{3}}_f$. So even though one member in the isodoublet is missing, the two resonances seem to be members of $\bar{\mathbf{3}}_f$.

A somewhat puzzling situation can be seen in the $J^P = 0^+$ channel. In the charm sector, even though we have three resonances, the mass of $D_{s0}^\pm(2318)$ is almost similar to that of $D_0^{*0}(2318)$. This shows that the D_{s0}^\pm can not be a member of $\bar{\mathbf{3}}_f$ and it may not be described by our four-quark model with $\bar{\mathbf{3}}_f$. Also $D_0^{*\pm}(2403)$, because of its large mass, may not form an isodoublet with $D_0^{*0}(2318)$. This observation shows that we may need mixing of various configurations for fully describing the excited heavy meson states, which is, however, beyond the scope of this work. In the bottom sector, there are no resonances reported from ‘ B or B_s ’ family in the $J^P = 0^+$ channel. As we will see later, the resonances belonging to $J^P = 0^+$, if they are constructed with our four-quark picture, are found to have strong components in the pseudoscalar-pseudoscalar decay channels with low-invariant masses. Because of this, they can have large decay widths, which make them difficult to be discovered experimentally. Indeed, we note that $D_0^{*0}(2318)$ has a broad width of 267 MeV and was listed in PDG only recently.²

In this Section, we have examined the excited states of positive parity listed in PDG, which shows that there are several reasons to believe that most excited states form $\bar{\mathbf{3}}_f$ in flavor space. Though some resonances are still missing in PDG, this examination motivates us to pursue

² This resonance was not listed in PDG before 2010.

	Family	J^P	I	Meson	Δm (MeV)
Lowest-lying states	D or D_s	0^-	$\frac{1}{2}$	$D^\pm(1870), D^0(1865)$	101
			0	$D_s^\pm(1968)$	
		1^-	$\frac{1}{2}$	$D^{*\pm}(2010), D^{*0}(2007)$	104
			0	$D_s^{*\pm}(2112)$	
	B or B_s	0^-	$\frac{1}{2}$	$B^\pm(5279), B^0(5280)$	87
			0	$B_s^0(5367)$	
		1^-	$\frac{1}{2}$	$B^*(5325), ?$	90
			0	$B_s^*(5415)$	
Excited states	D or D_s	0^+	$\frac{1}{2}$	$D_0^{*\pm}(2403), D_0^{*0}(2318)$	-0.2
			0	$D_{s0}^{*\pm}(2318)$	
		1^+	$\frac{1}{2}$	$D_1^\pm(2423), D_1^0(2427), D_1^0(2421)$	37.3
			0	$D_{s1}^\pm(2460)$	
		0	$D_{s1}^\pm(2535)$	112.8	
		2^+	$\frac{1}{2}$	$D_2^{*\pm}(2464), D_2^{*0}(2463)$	108.4
		0	$D_{s2}^{*\pm}(2572)$		
	B or B_s	0^+	$\frac{1}{2}$?, ?	?
		0	?		
1^+		$\frac{1}{2}$	$B_1^0(5724), B_J^*(5698, ?)$	105.9	
	0	$B_{s1}^0(5829)$			
	2^+	$\frac{1}{2}$	$B_2^{*0}(5743), ?$	97	
		0	$B_{s2}^{*0}(5840)$		

TABLE III. D, D_s and in B, B_s families compiled by the quantum numbers J^P . Δm is the mass difference between the $I = 1/2$ and $I = 0$ members, which shows that most low-lying resonances in each spin channel form $\mathbf{3}_f$ with mass splitting around 100 MeV. For the excited states, since the mass difference between the $I = 1/2$ states is not small, the mass splitting Δm is calculated using the underlined members in $I = 1/2$ as the reference point. The B_J^* meson in Table II is placed with the question mark in the $J^P = 1^+$ channel as its quantum numbers are unknown.

a possible four-quark structure based on $\bar{\mathbf{3}}_f$ for the study of the excited states of heavy mesons containing a charm or a bottom quark.

III. FOUR-QUARK WAVE FUNCTIONS

In this Section, we construct four-quark wave functions for the excited mesons in D and D_s families. As we have discussed in the previous Section, most excited states of heavy mesons listed in PDG have positive parity with $I = (0, 1/2)$ and $J = (0, 1, 2)$. In addition, they seem to have the flavor structure of $\mathbf{3}_f$. Purely from the phenomenological point of view, these properties can be generated by multiplying an SU(3) singlet $\bar{q}^i q_i$ to the two-quark systems, $Q\bar{q}^i$ ($q_i = u, d, s$), where Q stands for a heavy quark. Therefore, $Q = c$ for D and D_s families and

$Q = b$ for B and B_s families. To construct four-quark resonance states instead of molecular states, we follow the diquark-antidiquark approach [10, 11] and impose the phenomenological aspect of the $\mathbf{3}_f$ structure mentioned above. Such four-quark states can be schematically expressed as $Qq_i\bar{q}^j\bar{q}^i$. To construct the tetra-quark structure, therefore, the possible flavor, color, and spin configurations of each diquark should be determined.

As far as flavor is concerned, one can separate the antidiquark into two terms: namely, symmetric ($\bar{\mathbf{6}}_f$) and antisymmetric ($\mathbf{3}_f$) combinations as

$$\begin{aligned} \bar{q}^j\bar{q}^i &= \frac{1}{2}(\bar{q}^j\bar{q}^i + \bar{q}^i\bar{q}^j) + \frac{1}{2}(\bar{q}^j\bar{q}^i - \bar{q}^i\bar{q}^j) \\ &\equiv (\bar{q}^j\bar{q}^i)_+ + (\bar{q}^j\bar{q}^i)_-. \end{aligned} \quad (1)$$

Since these two combinations are orthogonal to each other, we have two possible flavor wave functions for four-quark states:

$$\text{Case 1: } D\bar{q}^j \Big|_{\text{flavor}} = \frac{1}{\sqrt{2}} \sum_{q_i=u,d,s} Qq_i(\bar{q}^j\bar{q}^i)_+ = \frac{1}{\sqrt{2}}[Qu(\bar{q}^j\bar{u})_+ + Qd(\bar{q}^j\bar{d})_+ + Qs(\bar{q}^j\bar{s})_+], \quad (2)$$

$$\text{Case 2: } D\bar{q}^j \Big|_{\text{flavor}} = \sum_{q_i=u,d,s} Qq_i(\bar{q}^j\bar{q}^i)_- = Qu(\bar{q}^j\bar{u})_- + Qd(\bar{q}^j\bar{d})_- + Qs(\bar{q}^j\bar{s})_-. \quad (3)$$

Here $Q = c$ so that these wave functions denote the ex-

cited states of D -mesons. When $\bar{q}^j = \bar{u}$ or \bar{d} , these four-

quark wave functions may represent the excited states in D family, and when $\bar{q}^j = \bar{s}$, they may be the excited states in D_s family. Clearly from this equation, we see that $D^{\bar{q}^j}$ in either case form $\bar{\mathbf{3}}_f$ separately in flavor space.

In color space, the diquark belongs to either $\bar{\mathbf{3}}_c$ or $\mathbf{6}_c$ and the antiquark to $\mathbf{3}_c$ or $\bar{\mathbf{6}}_c$. Thus, to make colorless four-quark states, the diquark and antiquark should be in either $(\bar{\mathbf{3}}_c, \mathbf{3}_c)$ or $(\mathbf{6}_c, \bar{\mathbf{6}}_c)$. Possible spins of the diquark and antiquark, represented by J_{12} and J_{34} , respectively, are 0, 1. By combining these spins, one can generate the total spin states for the four-quark states as $J = 0, 1, 2$ since $\mathbf{J} = \mathbf{J}_{12} + \mathbf{J}_{34}$. Depending on specific flavor combination we choose, we can determine the possible color and spin configurations.

A. Antiquark: flavor symmetric case $(\bar{q}^j \bar{q}^i)_+$

We first discuss the case when the antiquark is symmetric in flavor, i.e., $(\bar{q}^j \bar{q}^i)_+$. Since the antiquark should be totally antisymmetric when spin, flavor, and color are considered all together, it can be either $\mathbf{3}_c$ or $\bar{\mathbf{6}}_c$ in color space. When it is in $\mathbf{3}_c$, since this is antisymmetric in color indices, the antiquark spin is re-

stricted to $J_{34} = 1$ in order to make totally antisymmetric $(\bar{q}^j \bar{q}^i)_+$ systems. On the other hand, the Qq diquark that contains a heavy quark is not constrained by the Pauli principle. Thus, if the four-quark state (namely diquark-antiquark system) has spin zero, possible spin configuration for the Qq diquark and the $\bar{q}\bar{q}$ antiquark is $J_{12} = 1$, and $J_{34} = 1$, respectively, which we denote as $|J, J_{12}, J_{34}\rangle = |011\rangle$. For spin-1 case, we have two spin configurations: i) $|J, J_{12}, J_{34}\rangle = |101\rangle$ and ii) $|J, J_{12}, J_{34}\rangle = |111\rangle$. If this situation is realized in the meson spectroscopy, the physical states should be mixing states of these two states in the $J = 1$ channel. For $J = 2$, the only possible spin configuration is $|J, J_{12}, J_{34}\rangle = |211\rangle$. Thus, if the four-quark states are constructed under the assumption that the antiquark is in flavor symmetric and color antisymmetric state ($\mathbf{3}_c$), there are one state with $J = 0$, two states with $J = 1$, and one state with $J = 2$. These numbers of states are seemingly consistent with the experimental spectra observed for the D and D_s family as one can see from Table III, suggesting that this model is promising for the excite states of open charm mesons.

Given the flavor part of the four-quark wave function in Eq. (2), it is straightforward to incorporate the color part. Since the diquark (antiquark) belongs to $\bar{\mathbf{3}}_c$ ($\mathbf{3}_c$) in color, we obtain the four-quark wave function as

$$D_J^{\bar{q}^j} [(\bar{q}^j \bar{q}^i)_+ \in \mathbf{3}_c] = \frac{1}{\sqrt{24}} \sum_{q_i=u,d,s} \left\{ \sum_{a,b,d,e,f} \varepsilon_{abd} \varepsilon^{aef} [(Q)^b (q_i)^d]_{J_{12}=0,1} [(\bar{q}^j)_e (\bar{q}^i)_f]_{J_{34}=1} \right\}, \quad (4)$$

where a, b, d, e, f are color indices. The numerical factor $1/\sqrt{24}$ in Eq. (4) includes the color normalization $1/\sqrt{12}$ as well as the flavor normalization $1/\sqrt{2}$ from Eq. (2). We have also indicated that the Qq diquark can have spin 0 or 1, but the $\bar{q}\bar{q}$ antiquark in the present configuration can have spin 1 only.

When the antiquark is in a color symmetric state of $\bar{\mathbf{6}}_c$, its spin is restricted to an antisymmetric state, i.e., $J_{34} = 0$. Then the possible spin configurations are $|J, J_{12}, J_{34}\rangle = |000\rangle$ for $J = 0$, and $|J, J_{12}, J_{34}\rangle = |110\rangle$ for $J = 1$. This model with $\bar{\mathbf{6}}_c$ cannot generate $J = 2$ state and thus this scenario alone cannot explain the observed excited states whose spins range from 0 to 2. If one wants to describe all the states with spin 0,1,2 within the same framework, one should construct a model allowing both color configurations, $\mathbf{3}_c$ and $\bar{\mathbf{6}}_c$, for the antiquark, since the two configurations can mix each other. This is the only way that the $\bar{\mathbf{6}}_c$ configuration can enter into the framework. With this mixing scheme, however, though we can generate all the spin states, number of states generated from this scenario seems too many. There should be two states in spin 0, three states in spin 1, and one state in spin 2, which is not consistent with the observed

excited states. For example, in Table III, if one counts the number of mesons in D family with charge zero, there are one meson in spin 0, two mesons in spin 1, and one meson in spin 2. For charged mesons in D family, there are one meson in spin 0, one in spin 1, and one in spin 2. Therefore, the mixing scheme requires additional two or three mesons of similar masses to be discovered in D family, which seems to be inconsistent with the present observations. Thus, the mixing scheme, allowing both color states $\mathbf{3}_c$ and $\bar{\mathbf{6}}_c$ for the antiquark, may be implausible for the excited states. In the present work, when the antiquark is flavor symmetric $(\bar{q}^j \bar{q}^i)_+$, we consider the antiquark with the color state $\mathbf{3}_c$ only. This model will be referred to as Model I. Our discussion on colors and possible spin configurations for diquark and antiquark, when the antiquark is in flavor symmetric state, is summarized in Table IV.

B. Antiquark: flavor antisymmetric case $(\bar{q}^j \bar{q}^i)_-$

The other flavor configuration of the $\bar{q}\bar{q}$ antiquark is antisymmetric combination, $(\bar{q}^j \bar{q}^i)_-$. Again the Pauli

Qq_i		$(\bar{q}^j \bar{q}^i)_+$		$Qq_i(\bar{q}^j \bar{q}^i)_+$
Spin ($= J_{12}$)	Color	Spin ($= J_{34}$)	Color	$ J, J_{12}, J_{34}\rangle$
0	$\mathbf{3}_c$	1	$\mathbf{3}_c$	$ 101\rangle$
1	$\mathbf{\bar{3}}_c$	1	$\mathbf{3}_c$	$ 011\rangle, 111\rangle, 211\rangle$
0	$\mathbf{6}_c$	0	$\mathbf{6}_c$	$ 000\rangle$
1	$\mathbf{6}_c$	0	$\mathbf{\bar{6}}_c$	$ 110\rangle$

TABLE IV. Possible spins and colors of the Qq diquark, the $\bar{q}\bar{q}$ antidiquark, and four-quark states when the $\bar{q}\bar{q}$ antidiquark is symmetric in flavor, $(\bar{q}^j \bar{q}^i)_+$. The case with the antidiquark in the color state of $\mathbf{3}_c$ is referred to as Model I.

principle requires that the antidiquark is antisymmetric when spin, flavor, and color degrees of freedom are considered all together. We begin with the color symmetric state $\mathbf{6}_c$. Since the antidiquark is flavor antisymmetric, its spin state is restricted to the symmetric state

$$D_J^{\bar{q}^j} [(\bar{q}^j \bar{q}^i)_- \in \mathbf{6}_c] = \frac{1}{\sqrt{24}} \sum_{q_i=u,d,s} \sum_{a,b} \left\{ [(Q)_a(q_i)_b]_{J_{12}=0,1} [((\bar{q}^j)^a(\bar{q}^i)^b)_-]_{J_{34}=1} + [(Q)_a(q_i)_b]_{J_{12}=0,1} [((\bar{q}^j)^b(\bar{q}^i)^a)_-]_{J_{34}=1} \right\}, \quad (5)$$

where the possible spins for the diquark J_{12} and antidiquark J_{34} are indicated explicitly. Here, the factor $1/\sqrt{24}$ comes from the color part.

When the antidiquark is in a color antisymmetric state of $\mathbf{3}_c$, since we are considering flavor antisymmetric wave function for the antidiquark, its spin is restricted to an antisymmetric state, i.e., $J_{34} = 0$. With this constraint, the possible spin configurations are $|J, J_{12}, J_{34}\rangle = |000\rangle$ for $J = 0$, and $|J, J_{12}, J_{34}\rangle = |110\rangle$ for $J = 1$. We can not generate the spin 2 state in this configuration. Here we have similar situation discussed in the last part of the previous subsection. With a similar argument, this scheme with $\mathbf{3}_c$, even if we allow the mixing among the $\mathbf{3}_c$ and $\mathbf{6}_c$ cases, may not be relevant for the excited states. In this work, when the antidiquark is flavor antisymmetric $(\bar{q}^j \bar{q}^i)_-$, we consider the color state with $\mathbf{6}_c$ only. This model is referred to as Model II from now on. Our discussion on colors and possible spin configurations for Qq diquark and $\bar{q}\bar{q}$ antidiquark, when the antidiquark is in flavor antisymmetric state, is summarized in Table V.

IV. COLOR-SPIN INTERACTIONS

To test the four-quark wave functions constructed in the previous Section, we now use the color-spin interaction to estimate the mass splittings among heavy mesons of our concern. The color-spin interaction takes the following simple form [26–30]

$$V = \sum_{i < j} v_0 \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i m_j}, \quad (6)$$

of $J_{34} = 1$ in order to make a totally antisymmetric $(\bar{q}^j \bar{q}^i)_-$ system. Since the spin of the Qq diquark can be $J_{12} = 0, 1$, the spin of the four-quark states can be $J = 0, 1, 2$. If the four-quark state (namely diquark-antidiquark system) has spin zero, the possible spins for the diquark and the antidiquark are $J_{12} = 1, J_{34} = 1$ so that the spin configuration of the four-quark system is $|J, J_{12}, J_{34}\rangle = |011\rangle$. When $J = 1$, however, we again have two spin configurations: i) $|J, J_{12}, J_{34}\rangle = |101\rangle$ and ii) $|J, J_{12}, J_{34}\rangle = |111\rangle$. When $J = 2$, the only possible spin configuration is $|J, J_{12}, J_{34}\rangle = |211\rangle$. Thus, in this scenario, one can construct one state in spin-0, two states in spin-1, and one state in spin-2, again seemingly agreeing with the excited meson spectra in charm sector.

The four-quark wave function can be constructed straightforwardly. Incorporating the color part into Eq. (3), we obtain the four-quark wave functions as

Qq_i		$(\bar{q}^j \bar{q}^i)_-$		$Qq_i(\bar{q}^j \bar{q}^i)_-$
Spin ($= J_{12}$)	Color	Spin ($= J_{34}$)	Color	$ J, J_{12}, J_{34}\rangle$
0	$\mathbf{3}_c$	0	$\mathbf{3}_c$	$ 000\rangle$
1	$\mathbf{\bar{3}}_c$	0	$\mathbf{3}_c$	$ 110\rangle$
0	$\mathbf{6}_c$	1	$\mathbf{6}_c$	$ 101\rangle$
1	$\mathbf{6}_c$	1	$\mathbf{\bar{6}}_c$	$ 011\rangle, 111\rangle, 211\rangle$

TABLE V. Possible spins (and colors) of the Qq diquark, the $\bar{q}\bar{q}$ antidiquark, and four-quark states when the antidiquark is antisymmetric in flavor, $(\bar{q}^j \bar{q}^i)_-$. The case with the antidiquark in the color state of $\mathbf{6}_c$ is referred to as Model II.

when the spatial dependence is integrated out. Here λ_i denotes the Gell-Mann matrix, J_i the spin, and m_i the constituent mass of the i -th quark. The overall strength of the color-spin interaction is controlled by the parameter v_0 , which needs to be determined from the experimental data. This interaction is basically a generalization of the dipole-dipole electromagnetic interaction to take into account effectively the gluon exchange among constituent quarks.

Using the color-spin interaction, the hadron mass can be calculated by

$$M_H \sim \sum_i m_i + \langle V \rangle, \quad (7)$$

where the hyperfine mass $\langle V \rangle$ is obtained by using an appropriate hadron wave function. A nice aspect of this approach is that, even though Eq. (7) is not precise enough to reproduce the experimental masses, the mass differences among hadrons are successfully explained by the

	Δm from data	Δm from $\langle V \rangle$
$\Delta - N$	292	292 (fit)
$\Sigma - \Lambda$	77	66.2
$\Sigma^* - \Sigma$	192	192.7
$\Xi^* - \Xi$	211	192.7
$\Sigma_c - \Lambda_c$	167	151.8
$\Sigma_c^* - \Sigma_c$	65	64.5
$\Sigma_b - \Lambda_b$	194	181
$\Sigma_b^* - \Sigma_b$	19	20.5

TABLE VI. The hyperfine mass splittings, given in MeV, are compared with the experimental mass differences of baryons. The coupling strength in color-spin interaction, v_0 , is fitted from the $\Delta - N$ mass difference and is used to determine the mass splittings of other resonances.

	Δm from data	Δm from $\langle V \rangle$
$\rho - \pi$	635	635 (fit)
$K^* - K$	396	419.1
$D^* - D$	140	139.7
$D_s^* - D_s$	144	92.2
$B^* - B$	45.8	44.6
$B_s^* - B_s$	48.6	29.4

TABLE VII. The hyperfine mass splittings, given in MeV, are compared with the experimental mass differences of mesons. The coupling strength v_0 fixed from the $\rho - \pi$ mass difference is used to determine the mass splittings of other resonances.

differences in the hyperfine masses,

$$\Delta M_H \sim \Delta \langle V \rangle. \quad (8)$$

To illustrate this feature, the computed mass differences among several baryons are presented in Table VI with the experimental mass splittings. In the baryon sector, the overall strength v_0 of the color-spin interaction is fitted from the measured $\Delta - N$ mass splitting, which leads to $v_0 \sim (-199.6)^3 \text{ MeV}^3$. We use this value to calculate the hyperfine masses of other baryons. For the constituent quark masses, we take the conventional values, $m_u = m_d = 330 \text{ MeV}$, $m_s = 500 \text{ MeV}$, $m_c = 1500 \text{ MeV}$, and $m_b = 4700 \text{ MeV}$. As one can see from Table VI, the splittings from hyperfine masses are consistent with the experimental mass splittings quite well. The largest error is found in the mass difference of $\Sigma_b - \Lambda_b$. But, even in this case, the experimental mass gap is only 13 MeV higher than the calculated hyperfine

mass gap. Therefore, Table VI shows that the hyperfine mass splittings are useful to calculate the mass splittings between baryons with different spins and different spin configurations but with the same flavor.³

Similar calculations can be performed for the meson sector and the results are given in Table VII. In this case, we fit v_0 from the observed $\rho - \pi$ mass splitting which leads to $v_0 \sim (-235)^3 \text{ MeV}^3$. This strength is somewhat different from the one fixed in the baryon sector. There could be various reasons for this difference. In particular, it is often believed that the pseudoscalar mesons involved in the analysis acquire contributions from the instanton-induced interactions. Moreover, the pion mass calculated from Eq. (7) involves the hyperfine mass about 480 MeV, which is comparable in magnitude with the leading quark mass contribution. This situation is rather different from the baryon case where the hyperfine masses are much smaller than the quark mass contribution. Nevertheless, if we use this value to calculate the hyperfine masses of the other mesons, then the mass differences among them seem to be comparable to the experimental ones. As one can see from Table VII, the hyperfine masses generate the experimental mass splittings of $K^* - K$, $D^* - D$, $B^* - B$ very well, although the agreement is not as good for $D_s^* - D_s$ and $B_s^* - B_s$.

V. HYPERFINE MASSES FROM FOUR-QUARK SYSTEMS

In Sec. III, we have constructed the four-quark wave functions which are relevant for our study on heavy meson excited states. Depending on the symmetric aspect of the antiquark, we come up with the following two plausible models for the four-quark wave functions:

Model I: The antiquark is symmetric in flavor ($\bar{\mathbf{6}}_f$) and belongs to color state $\mathbf{3}_c$. In this model, the four-quark wave functions are given by Eq. (4).

Model II: The antiquark is antisymmetric in flavor ($\mathbf{3}_c$) and belongs to color state $\bar{\mathbf{6}}_c$. In this model, the four-quark wave functions are given by Eq. (5).

The hyperfine masses of the four-quark systems are matrix elements of the hyperfine potential V between these four-quark wave functions. To explain our calculation in detail, we write the color-spin interaction for the four-quark systems as

$$V = v_0 \left[\lambda_1 \cdot \lambda_2 \frac{J_1 \cdot J_2}{m_1 m_2} + \lambda_3 \cdot \lambda_4 \frac{J_3 \cdot J_4}{m_3 m_4} + \lambda_1 \cdot \lambda_3 \frac{J_1 \cdot J_3}{m_1 m_3} + \lambda_1 \cdot \lambda_4 \frac{J_1 \cdot J_4}{m_1 m_4} + \lambda_2 \cdot \lambda_3 \frac{J_2 \cdot J_3}{m_2 m_3} + \lambda_2 \cdot \lambda_4 \frac{J_2 \cdot J_4}{m_2 m_4} \right], \quad (9)$$

³ Note that the Λ baryon contains a spin-0 diquark while Σ has a spin-1 diquark. Thus Λ and Σ have different spin configurations

although the both have spin-1/2 [24].

where the indices 1,2,3,4 refer to Q , q_i , \bar{q}^j , and \bar{q}^i in Eqs. (4) and (5). Thus, 1,2 quarks form the diquark (Q, q_i) and 3,4 quarks form the antidiquark (\bar{q}^j, \bar{q}^i). The corresponding quark masses are denoted by m_1, m_2, m_3 , and m_4 , respectively. Given one specific flavor combination, one can calculate the color part and spin part separately.

A. Color part

Here we calculate the color part $\lambda_i \cdot \lambda_j$ in the potential V . In the case of Model I, where the wave function is given by Eq. (4), the antidiquark (namely [3,4] quarks) is in color triplet state $\mathbf{3}_c$, which restricts the diquark (namely [1,2] quarks) to be in $\mathbf{3}_c$ in order to make colorless four-quark states. Thus, the expectation values of $\lambda_1 \cdot \lambda_2$ and $\lambda_3 \cdot \lambda_4$ can be calculated as

$$\langle \lambda_1 \cdot \lambda_2 \rangle_{\bar{\mathbf{3}}_c, \mathbf{3}_c} = \langle \lambda_3 \cdot \lambda_4 \rangle_{\bar{\mathbf{3}}_c, \mathbf{3}_c} = -\frac{8}{3}. \quad (10)$$

In the case of Model II, where the wave function is given by Eq. (5), the antidiquark is in $\bar{\mathbf{6}}_c$, which restricts the diquark to be in the color state $\mathbf{6}_c$. The expectation

values of $\lambda_1 \cdot \lambda_2$ and $\lambda_3 \cdot \lambda_4$ can be calculated in the [1,2][3,4] basis as

$$\langle \lambda_1 \cdot \lambda_2 \rangle_{\mathbf{6}_c, \bar{\mathbf{6}}_c} = \langle \lambda_3 \cdot \lambda_4 \rangle_{\mathbf{6}_c, \bar{\mathbf{6}}_c} = \frac{4}{3}. \quad (11)$$

To calculate the expectation values of other operators like $\lambda_1 \cdot \lambda_3$ and $\lambda_2 \cdot \lambda_3$, etc, it is necessary to rearrange the wave function of definite color states in the diquark-antidiquark ([1,2][3,4]) basis into the [1,3][2,4] basis or the [1,4][2,3] basis. This can be done by using the following decomposition

$$q_a \bar{q}^b = \underbrace{q_a \bar{q}^b - \frac{1}{3} \delta_a^b q_d \bar{q}^d}_{\mathbf{8}_a^b} + \underbrace{\frac{1}{3} \delta_a^b q_d \bar{q}^d}_{\mathbf{1}} = \mathbf{8}_a^b + \delta_a^b \mathbf{1}, \quad (12)$$

which expresses a quark-antiquark pair in terms of an octet and a singlet in color space.

When the diquark and the antidiquark are in $(\bar{\mathbf{3}}_c, \mathbf{3}_c)$ as in Eq. (4), we find

$$\begin{aligned} \langle \lambda_1 \cdot \lambda_3 \rangle_{\bar{\mathbf{3}}_c, \mathbf{3}_c} &= \langle \lambda_2 \cdot \lambda_4 \rangle_{\bar{\mathbf{3}}_c, \mathbf{3}_c} \\ &= \langle \lambda_2 \cdot \lambda_3 \rangle_{\bar{\mathbf{3}}_c, \mathbf{3}_c} = \langle \lambda_1 \cdot \lambda_4 \rangle_{\bar{\mathbf{3}}_c, \mathbf{3}_c} = -\frac{4}{3}. \end{aligned} \quad (13)$$

Inserting all the factors into Eq. (9) leads to

$$\langle V \rangle_{\bar{\mathbf{3}}_c, \mathbf{3}_c} = -\frac{8}{3} v_0 \left[\frac{J_1 \cdot J_2}{m_1 m_2} + \frac{J_3 \cdot J_4}{m_3 m_4} + \frac{J_1 \cdot J_3}{2m_1 m_3} + \frac{J_1 \cdot J_4}{2m_1 m_4} + \frac{J_2 \cdot J_3}{2m_2 m_3} + \frac{J_2 \cdot J_4}{2m_2 m_4} \right]. \quad (14)$$

When the diquark and the antidiquark are in $(\mathbf{6}_c, \bar{\mathbf{6}}_c)$, the expectation values are obtained as

$$\langle \lambda_1 \cdot \lambda_3 \rangle_{\mathbf{6}_c, \bar{\mathbf{6}}_c} = \langle \lambda_2 \cdot \lambda_4 \rangle_{\mathbf{6}_c, \bar{\mathbf{6}}_c} = \langle \lambda_2 \cdot \lambda_3 \rangle_{\mathbf{6}_c, \bar{\mathbf{6}}_c} = \langle \lambda_1 \cdot \lambda_4 \rangle_{\mathbf{6}_c, \bar{\mathbf{6}}_c} = -\frac{10}{3}, \quad (15)$$

which leads to

$$\langle V \rangle_{\mathbf{6}_c, \bar{\mathbf{6}}_c} = \frac{4}{3} v_0 \left[\frac{J_1 \cdot J_2}{m_1 m_2} + \frac{J_3 \cdot J_4}{m_3 m_4} - \frac{5}{2} \left(\frac{J_1 \cdot J_3}{m_1 m_3} + \frac{J_1 \cdot J_4}{m_1 m_4} + \frac{J_2 \cdot J_3}{m_2 m_3} + \frac{J_2 \cdot J_4}{m_2 m_4} \right) \right]. \quad (16)$$

B. Spin part

The spin parts can be calculated in a similar way. For an illustration, we take the four-quark wave function of spin 0, which has the spin configuration $|J, J_{12}, J_{34}\rangle = |011\rangle$ in the [1,2][3,4] basis. The calculation for the other spin configurations can be done similarly. The spin interactions, $J_1 \cdot J_2$ and $J_3 \cdot J_4$, can be calculated directly on $|011\rangle$. For instance, since the diquark [1,2] is in spin-1 state, $J_1 \cdot J_2$ acting on $|011\rangle$ is

$$J_1 \cdot J_2 |011\rangle = \frac{1}{2} (J_{12}^2 - J_1^2 - J_2^2) |011\rangle = \frac{1}{4} |011\rangle. \quad (17)$$

Similarly, $J_3 \cdot J_4 |011\rangle = \frac{1}{4} |011\rangle$ since the antidiquark [3,4] is also in the spin-1 state.

For the other spin interactions, $J_1 \cdot J_3$ and $J_2 \cdot J_4$ etc, it is necessary to write the spin state $|011\rangle$ in the [1,3][2,4]

basis using Racah coefficients. To do this, we first write $|011\rangle$ in terms of the diquark spin and its projection $|J_{12} M_{12}\rangle$, and the antidiquark part $|J_{34} M_{34}\rangle$, with appropriate Clebsch-Gordan coefficients, namely,

$$\begin{aligned} |011\rangle_{[12][34]} &= \frac{1}{\sqrt{3}} \left[|11\rangle_{12} |1-1\rangle_{34} - |10\rangle_{12} |10\rangle_{34} \right. \\ &\quad \left. + |1-1\rangle_{12} |11\rangle_{34} \right]. \end{aligned} \quad (18)$$

Here the subscripts in the kets indicate the participating quarks or antiquarks in making the designated spin state. Then, after writing down each spin state in terms of spinors of participating quarks, we reorganize the $|011\rangle$ state with respect to $|J_{13}, M_{13}\rangle$ and $|J_{24}, M_{24}\rangle$. This procedure applied to Eq. (18) yields the spin wave functions,

$$|011\rangle_{[13][24]} = \frac{\sqrt{3}}{6} \left[|10\rangle_{13} |10\rangle_{24} + 3 |00\rangle_{13} |00\rangle_{24} \right]$$

$$- |11\rangle_{13} |1-1\rangle_{24} - |1-1\rangle_{13} |11\rangle_{24} \Big] \quad (19)$$

in the $[1,3][2,4]$ basis. Of course, this state is not an eigenstate of J_{13} as it should be. Similarly, one can write Eq. (18) in terms of $[1,4][2,3]$ spin basis, $|J_{14}, M_{14}\rangle$ and $|J_{23}, M_{23}\rangle$, which gives

$$|011\rangle_{[14][23]} = \frac{\sqrt{3}}{6} \left[|10\rangle_{14} |10\rangle_{23} + 3|00\rangle_{14} |00\rangle_{23} - |11\rangle_{14} |1-1\rangle_{23} - |1-1\rangle_{14} |11\rangle_{23} \right] \quad (20)$$

in the $[1,4][2,3]$ basis. Using these expressions, it is now straightforward to calculate the expectation values of the spin operators of concern in this particular four-quark state, $\langle 011|J_2 \cdot J_4|011\rangle$, etc. They are obtained as

$$\begin{aligned} \langle 011|J_1 \cdot J_4|011\rangle &= \langle 011|J_2 \cdot J_4|011\rangle \\ = \langle 011|J_2 \cdot J_3|011\rangle &= \langle 011|J_2 \cdot J_4|011\rangle = -\frac{1}{2}. \end{aligned} \quad (21)$$

One interesting remark is that, under the change of basis, one can identify the decay channels of the four-quark state of concern. For example, in Eq. (20), the $[1,4]$ indices correspond to $Q\bar{q}(q = u, d, s)$ and the $[2,3]$ correspond to $q\bar{q}$. The spin state $|00\rangle_{14}|00\rangle_{23}$ in Eq. (20) contains a Fock space of pseudoscalar-pseudoscalar particles, which can decay, for instance, to πD for $Q = c$ if the decay occurs through a ‘fall-apart’ mechanism. The colors of course should be combined into a singlet separately in $[1,4]$ and $[2,3]$ for such a decay to happen. The other spin states in Eq. (20) correspond to vector-vector channel like the ρD^* channel. From this change of spin basis, we see that the state $|011\rangle$ consists of pseudoscalar-pseudoscalar and vector-vector components with the probability ratio of 3:1. Thus, this four-quark state in spin-0 channel has

large component in the pseudoscalar-pseudoscalar channel like πD . Usually the invariant mass of this decay channel is expected to be quite lower than the possible four-quark mass. This means that the four-quark state with $|011\rangle$ may have a large decay width, which would make them difficult to be observed experimentally. Indeed, as we mentioned in Sec. II, $D_0^{*0}(2318)$, which is one candidate of four-quark states, has the broad width of 267 MeV. Also by applying the same argument to the bottom sector, we expect that B -meson excited states with spin-0 are expected to be broad. Currently, B -mesons with spin-0 are missing in PDG (see Table II), which might be due to experimental difficulties coming from their broad widths.

Our prescription for evaluating the spin part can be similarly applied to the other spin states, which include the spin-1 state with two possible configurations, $|101\rangle$ and $|111\rangle$, and the spin-2 state with the configuration $|211\rangle$. The two configurations in $J = 1$, $|101\rangle$ and $|111\rangle$ can mix because of nonzero mixing term $\langle 101|V|111\rangle$. Therefore, one needs to diagonalize the 2×2 matrix in order to calculate physical hyperfine masses in the spin-1 channel.

C. Flavor part

The hyperfine masses for a general flavor combination, $q_1 q_2 \bar{q}_3 \bar{q}_4$, are presented in Table VIII where the corresponding spin configurations as well as color structure of the antiquark are given. Using these formulas, one can calculate $\langle V_D \rangle_J^{\bar{u}}$, $\langle V_D \rangle_J^{\bar{d}}$, and $\langle V_D \rangle_J^{\bar{s}}$. The final hyperfine masses corresponding to the states $D_J^{\bar{u}}$, $D_J^{\bar{d}}$, and $D_J^{\bar{s}}$ can be obtained by summing over all the flavor combinations according to Eq. (2) for Model I and Eq. (3) for Model II. To be specific, in the case of Model I, the hyperfine masses $\langle V_D \rangle_J^{\bar{u}}$, $\langle V_D \rangle_J^{\bar{d}}$, and $\langle V_D \rangle_J^{\bar{s}}$ are calculated schematically as

Model I:

$$\begin{aligned} \langle V_D \rangle_J^{\bar{u}} &= \frac{1}{8} [4\langle V_D \rangle_{Qu\bar{u}\bar{u}} + \langle V_D \rangle_{Qd\bar{u}\bar{d}} + \langle V_D \rangle_{Qd\bar{d}\bar{u}} + \langle V_D \rangle_{Qs\bar{u}\bar{s}} + \langle V_D \rangle_{Qs\bar{s}\bar{u}}], \\ \langle V_D \rangle_J^{\bar{d}} &= \frac{1}{8} [\langle V_D \rangle_{Qu\bar{d}\bar{u}} + \langle V_D \rangle_{Qu\bar{u}\bar{d}} + 4\langle V_D \rangle_{Qd\bar{d}\bar{d}} + \langle V_D \rangle_{Qs\bar{d}\bar{s}} + \langle V_D \rangle_{Qs\bar{s}\bar{d}}], \\ \langle V_D \rangle_J^{\bar{s}} &= \frac{1}{8} [\langle V_D \rangle_{Qu\bar{s}\bar{u}} + \langle V_D \rangle_{Qu\bar{u}\bar{s}} + \langle V_D \rangle_{Qd\bar{s}\bar{d}} + \langle V_D \rangle_{Qd\bar{d}\bar{s}} + 4\langle V_D \rangle_{Qs\bar{s}\bar{s}}]. \end{aligned} \quad (22)$$

The specified flavor combination and the associated numerical factors follow from Eq. (2). Here each term with specified flavors, for example, the term like $\langle V_D \rangle_{Qu\bar{u}\bar{u}}$, can be obtained from the general formulas given in Ta-

ble VIII with Model I. The isospin symmetry requires $\langle V_D \rangle_J^{\bar{u}} = \langle V_D \rangle_J^{\bar{d}}$.

In the case of Model II, the hyperfine masses can be calculated schematically as

Model II:

$$\langle V_D \rangle_J^{\bar{u}} = \frac{1}{4} [\langle V_D \rangle_{Qd\bar{u}\bar{d}} + \langle V_D \rangle_{Qd\bar{d}\bar{u}} + \langle V_D \rangle_{Qs\bar{u}\bar{s}} + \langle V_D \rangle_{Qs\bar{s}\bar{u}}],$$

$ J, J_{12}, J_{34}\rangle$	Color of $\bar{q}_3\bar{q}_4$	Hyperfine mass $\langle V \rangle_{q_1q_2\bar{q}_3\bar{q}_4}$
011)	$\mathbf{3}_c$ (Model I)	$-\frac{2}{3}v_0 \left[\frac{1}{m_1m_2} + \frac{1}{m_3m_4} - \frac{1}{m_1m_3} - \frac{1}{m_1m_4} - \frac{1}{m_2m_3} - \frac{1}{m_2m_4} \right]$
101)		$-\frac{2}{3}v_0 \left[-\frac{3}{m_1m_2} + \frac{1}{m_3m_4} \right]$
111)		$-\frac{2}{3}v_0 \left[\frac{1}{m_1m_2} + \frac{1}{m_3m_4} - \frac{1}{2m_1m_3} - \frac{1}{2m_1m_4} - \frac{1}{2m_2m_3} - \frac{1}{2m_2m_4} \right]$
211)		$-\frac{2}{3}v_0 \left[\frac{1}{m_1m_2} + \frac{1}{m_3m_4} + \frac{1}{2m_1m_3} + \frac{1}{2m_1m_4} + \frac{1}{2m_2m_3} + \frac{1}{2m_2m_4} \right]$
Mixing (101>, 111>)		$-\frac{\sqrt{2}}{3}v_0 \left[-\frac{1}{m_1m_3} - \frac{1}{m_1m_4} + \frac{1}{m_2m_3} + \frac{1}{m_2m_4} \right]$
011)	$\bar{\mathbf{6}}_c$ (Model II)	$\frac{v_0}{3} \left[\frac{1}{m_1m_2} + \frac{1}{m_3m_4} + \frac{5}{m_1m_3} + \frac{5}{m_1m_4} + \frac{5}{m_2m_3} + \frac{5}{m_2m_4} \right]$
101)		$\frac{v_0}{3} \left[-\frac{3}{m_1m_2} + \frac{1}{m_3m_4} \right]$
111)		$\frac{v_0}{6} \left[\frac{2}{m_1m_2} + \frac{2}{m_3m_4} + \frac{5}{m_1m_3} + \frac{5}{m_1m_4} + \frac{5}{m_2m_3} + \frac{5}{m_2m_4} \right]$
211)		$\frac{v_0}{6} \left[\frac{2}{m_1m_2} + \frac{2}{m_3m_4} - \frac{5}{m_1m_3} - \frac{5}{m_1m_4} - \frac{5}{m_2m_3} - \frac{5}{m_2m_4} \right]$
Mixing (101>, 111>)		$\frac{5\sqrt{2}}{6}v_0 \left[\frac{1}{m_1m_3} + \frac{1}{m_1m_4} - \frac{1}{m_2m_3} - \frac{1}{m_2m_4} \right]$

TABLE VIII. The hyperfine mass $\langle V \rangle$ for a given spin configurations of the four-quark states and the color states of the antiquark. Hyperfine masses presented here are for a general flavor combination, $q_1q_2\bar{q}_3\bar{q}_4$, without including the flavor normalization. Thus, to obtain the final hyperfine masses, one needs to combine all the flavor combination as well as the normalization according to Eqs. (2) and (3).

$$\begin{aligned}
\langle V_D \rangle_J^{\bar{d}} &= \frac{1}{4} [\langle V_D \rangle_{Qu\bar{d}\bar{u}} + \langle V_D \rangle_{Qu\bar{u}\bar{d}} + \langle V_D \rangle_{Qs\bar{d}\bar{s}} + \langle V_D \rangle_{Qs\bar{s}\bar{d}}], \\
\langle V_D \rangle_J^{\bar{s}} &= \frac{1}{4} [\langle V_D \rangle_{Qu\bar{s}\bar{u}} + \langle V_D \rangle_{Qu\bar{u}\bar{s}} + \langle V_D \rangle_{Qd\bar{s}\bar{d}} + \langle V_D \rangle_{Qd\bar{d}\bar{s}}],
\end{aligned} \tag{23}$$

where the specified flavor combination and the numerical factors follow from Eq. (3). Here each term with specified flavors is again obtained from the general formulas given in Table VIII with Model II.

VI. RESULTS AND DISCUSSION

We now present and discuss the results obtained from the two models using Eqs. (22) and (23). In our calculations, there are a few parameters to be fixed. For the constituent quark masses, we use $m_u = m_d = 330$ MeV, $m_s = 500$ MeV, $m_c = 1500$ MeV, and $m_b = 4700$ MeV as discussed in Sec. IV.

One additional parameter is the strength of the color-spin interaction v_0 . Our analyses in Sec. IV show that fitted parameter v_0 takes different values for the baryon sector and for the meson sector. With keeping this limitation in mind, we fix v_0 separately within our four-quark systems. Specifically, we fix this strength from the experimental mass difference between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$ by identifying $D_0^{\bar{u}}$ with $D_0^{*0}(2318)$ and $D_2^{\bar{u}}$ with $D_2^{*0}(2463)$. Of course, the extracted parameter v_0

depends on the two models presented above. Once v_0 is fixed, one can calculate the hyperfine masses of the other resonances such as spin-1 mesons without strangeness and spin-0,1,2 mesons with nonzero strangeness. The obtained mass difference will be compared with the measured data to test the idea of four-quark structure. To check the parameter dependence of our results, we will also show the results using the v_0 value fixed from the $\Delta - N$ mass difference.

The calculations are performed for the charm sector and for the bottom sector. For B -mesons, we will use a similar nomenclature, i.e., $B_J^{\bar{q}^j}$ represents a state and $\langle V_B \rangle_J^{\bar{q}^j}$ ($\bar{q}^j = \bar{u}, \bar{d}, \bar{s}$) is the corresponding hyperfine mass.

A. Results from Model I

In Model I, the antiquark is symmetric in flavor space and its color wave function belongs to $\mathbf{3}_c$. The hyperfine masses are calculated using Eq. (22). From the mass splitting between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$ we have $v_0 \sim (-193)^3$ MeV³ in Model I, which is somewhat close

to the one obtained from the $\Delta - N$ mass difference, $v_0 \sim (-199.6)^3 \text{ MeV}^3$. Using this parameter, we calculate the hyperfine masses from the four-quark states, $|J, J_{12}, J_{34}\rangle = |011\rangle, |101\rangle, |111\rangle, |211\rangle$ as well as the mixing terms between the two states in spin-1 channel. The resulting hyperfine masses are presented in Table IX.

We now discuss the results for the mesons with-

$$\begin{array}{c|cc} & |101\rangle & |111\rangle \\ \hline |101\rangle & 13.66 & 42.00 \\ |111\rangle & 42.00 & 0.82 \end{array} \xrightarrow{\text{diagonalization}} \begin{array}{c|cc} & |D_{1P}^{\bar{u}}\rangle & |D_{1N}^{\bar{u}}\rangle \\ \hline |D_{1P}^{\bar{u}}\rangle & 49.73 & 0.00 \\ |D_{1N}^{\bar{u}}\rangle & 0.00 & -35.24 \end{array} \quad (24)$$

$ J, J_{12}, J_{34}\rangle$	$\langle V_D \rangle_J^{\bar{u}}$	$\langle V_D \rangle_J^{\bar{s}}$	$\langle V_B \rangle_J^{\bar{u}}$	$\langle V_B \rangle_J^{\bar{s}}$
$ 011\rangle$	-47.37	-37.89	-40.80	-33.55
$ 211\rangle$	97.21	67.05	84.90	56.70
$ 101\rangle$	13.66	0.00	31.71	16.38
$ 111\rangle$	0.82	-2.91	1.10	-3.47
mixing ($ 101\rangle, 111\rangle$)	42.00	29.12	50.90	36.05
Charge	0	+1	-1	0

TABLE IX. The hyperfine masses obtained for open-charm ($D_J^{\bar{u}}, D_J^{\bar{s}}$) and open-bottom ($B_J^{\bar{u}}, B_J^{\bar{s}}$) excited mesons in Model I. Here the diquark and the antidiquark belong to the color state $\bar{\mathbf{3}}_c$ and $\mathbf{3}_c$, respectively. The strength of the color-spin interaction v_0 fixed by the mass difference $D_2^{*0}(2463) - D_0^{*0}(2318)$ is $v_0 \sim (-193)^3 \text{ MeV}^3$. We also indicate the charge of the four-quark states corresponding to the hyperfine masses.

Thus, in the spin-1 channel, the physical hyperfine masses are

$$\langle V_D \rangle_{1P}^{\bar{u}} = 49.73 \text{ MeV}, \quad \langle V_D \rangle_{1N}^{\bar{u}} = -35.24 \text{ MeV}. \quad (25)$$

Here we have denoted corresponding eigenstates as $D_{1P}^{\bar{u}}$ and $D_{1N}^{\bar{u}}$ where the subscript P (N) is introduced to indicate a positive (negative) hyperfine mass.

The hyperfine mass difference between $D_{1P}^{\bar{u}}$ and the spin-2 meson is $\langle V_D \rangle_{1P}^{\bar{u}} - \langle V_D \rangle_2^{\bar{u}} = -47.48 \text{ MeV}$, which means that the $D_{1P}^{\bar{u}}$ mass is lower than the spin-2 meson by about -48 MeV . If we use the experimental mass of the spin-2 meson, i.e., 2462.6 MeV , Model I predicts the mass of $D_{1P}^{\bar{u}}$ to be 2415 MeV , which is very close to the observed mass of $D_1^0(2421)$.

The other member in spin-1 channel, $D_{1N}^{\bar{u}}$, has a hyperfine mass of -35.24 MeV . The hyperfine mass difference from the spin-2 meson is then $\langle V_D \rangle_{1N}^{\bar{u}} - \langle V_D \rangle_2^{\bar{u}} = -132.45 \text{ MeV}$, which indicates that the $D_{1N}^{\bar{u}}$ mass should be around 2330 MeV . The current compilation of PDG does not list the resonance corresponding to $D_{1N}^{\bar{u}}$ of spin-1. The listed $D_1^0(2427)$ has a mass of 100 MeV larger than this estimation. We expect that this state, if it exists, has a large decay width coming from the kinematically favorable πD^* mode and, therefore, it may not be easy to be identified in experiments.

out strangeness, $D_J^{\bar{u}}$, using the corresponding hyperfine masses $\langle V_D \rangle_J^{\bar{u}}$. In spin-1 channel, because of the two spin configurations and the mixing between them, the hyperfine masses form a 2×2 matrix. The physical hyperfine masses can be obtained by diagonalization as shown below.

This can be explained by writing the two eigenstates in spin-1 with respect to the original spin configurations via

$$|D_{1P}^{\bar{u}}\rangle = \alpha|101\rangle + \beta|111\rangle, \quad (26)$$

$$|D_{1N}^{\bar{u}}\rangle = -\beta|101\rangle + \alpha|111\rangle. \quad (27)$$

The mixing parameters are calculated to be $\alpha = -0.76$ and $\beta = -0.65$. Because of the sign difference in Eqs. (26) and (27), the two spin configurations in $J = 1$ channel either add up or partially cancel in making the eigenstate $D_{1P}^{\bar{u}}$ or $D_{1N}^{\bar{u}}$. If the two spin configurations, $|101\rangle$ and $|111\rangle$, are rewritten in terms of the $[1,4][2,3]$ basis similarly as was done in Eq. (20), one can see that they contain the spin components, $(J_{14} = 1, J_{23} = 0)$, $(J_{14} = 0, J_{23} = 1)$, and $(J_{14} = 1, J_{23} = 1)$. The spin component $(J_{14} = 1, J_{23} = 0)$ contains the πD^* decay mode in addition to the kinematically forbidden mode $K D_s^*$. The πD^* decay mode is kinematically favorable because the threshold energy is about 150 MeV lower than the expected mass of $D_{1N}^{\bar{u}}$ which is around 2330 MeV . If we count only the spin part of the wave functions, the spin component containing the πD^* mode constitutes 25% in the configuration $|101\rangle$, while it is 50% in $|111\rangle$ before the mixing. After the mixing through Eqs. (26) and (27), this component is enhanced ($\sim 74\%$) in $D_{1N}^{\bar{u}}$ but strongly suppressed ($\sim 0.7\%$) in $D_{1P}^{\bar{u}}$. Because of strong enhancement of the component containing πD^* , $D_{1N}^{\bar{u}}$ is expected to have a large decay width. On the other hand, $D_{1P}^{\bar{u}}$ contains a small component containing the πD^* mode and is expected to be a sharp resonance. Indeed, the $D_1^0(2421)$, which we identify as $D_{1P}^{\bar{u}}$ in our model, has the decay width about only 27 MeV .

We now discuss the results for the mesons with nonzero net strangeness, $D_J^{\bar{s}}$. From Table IX, we see that the hyperfine mass difference between $J = 0$ and $J = 2$ channels is $\langle V_D \rangle_2^{\bar{s}} - \langle V_D \rangle_0^{\bar{s}} = 104 \text{ MeV}$. If we identify $D_2^{\bar{s}}$ as $D_{s2}^{*\pm}(2572)$, the spin-0 resonance $D_0^{\bar{s}}$ must have a mass around 2470 MeV , i.e. about 105 MeV lower than $D_{s2}^{*\pm}(2572)$. As we have discussed in Sec. II, the current PDG listing does not have a corresponding spin-0 resonance in this nonzero strangeness channel. $D_{s0}^{*\pm}(2318)$ cannot be a candidate because this resonance does not

belong to $\bar{\mathbf{3}}_f$. Again, the absence of this resonance may be due to its large decay width, which makes $D_0^{\bar{s}}$ difficult to be identified experimentally. Careful inspection of Eq. (20) where the spin-0 wave function is written in the $[1,4][2,3]$ basis leads to that $D_0^{\bar{s}}$ contains a large component for the KD decay channel, namely $|00\rangle_{14}|00\rangle_{23}$ component. Since the KD threshold energy is 2364 MeV and

is less than the expected mass of $D_0^{\bar{s}}$, which is 2470 MeV, the KD decay channel is kinematically favorable, which again leads to a large decay width for $D_0^{\bar{s}}$.

On the other hand, very interesting phenomena can be foreseen in spin-1 resonance $D_1^{\bar{s}}$. The hyperfine mass matrix for $D_1^{\bar{s}}$ in the basis of spin configurations $|J, J_{12}, J_{34}\rangle = |101\rangle$ and $|111\rangle$ can be read off from Table IX, and its diagonalized form is as follows.

$$\begin{array}{c|cc} & |101\rangle & |111\rangle \\ \hline |101\rangle & 0.00 & 29.12 \\ |111\rangle & 29.12 & -2.91 \end{array} \xrightarrow{\text{diagonalization}} \begin{array}{c|cc} & |D_{1P}^{\bar{s}}\rangle & |D_{1N}^{\bar{s}}\rangle \\ \hline |D_{1P}^{\bar{s}}\rangle & 27.7 & 0.00 \\ |D_{1N}^{\bar{s}}\rangle & 0.00 & -30.61 \end{array} \quad (28)$$

Thus, the physical hyperfine masses are $\langle V_D \rangle_{1P}^{\bar{s}} = 27.7$ MeV and $\langle V_D \rangle_{1N}^{\bar{s}} = -30.61$ MeV, which correspond to two spin-1 mesons $D_{1P}^{\bar{s}}$ and $D_{1N}^{\bar{s}}$, respectively. The two eigenstates, $D_{1P}^{\bar{s}}$ and $D_{1N}^{\bar{s}}$, are related to the original spin configurations via

$$|D_{1P}^{\bar{s}}\rangle = \alpha|101\rangle + \beta|111\rangle, \quad (29)$$

$$|D_{1N}^{\bar{s}}\rangle = -\beta|101\rangle + \alpha|111\rangle, \quad (30)$$

where the mixing parameters are calculated as $\alpha = -0.725$ and $\beta = -0.689$.

These two states in the spin-1 channel, $D_{1P}^{\bar{s}}$ and $D_{1N}^{\bar{s}}$, seem to fit well with $D_{s1}^{\pm}(2535)$ and $D_{s1}^{\pm}(2460)$ of PDG. The predicted mass of $D_{1P}^{\bar{s}}$, determined from the hyperfine mass difference, $\langle V_D \rangle_{1P}^{\bar{s}} - \langle V_D \rangle_2^{\bar{s}} = -39$ MeV, is 2530 MeV. This is very close to the observed mass, 2535 MeV, of D_{s1}^{\pm} . For $D_{1N}^{\bar{s}}$, the predicted mass reads about 2475 MeV, which is only 15 MeV larger than the observed mass of $D_{s1}^{\pm}(2460)$.

One very interesting feature of this model is that $D_{1N}^{\bar{s}}$, which we identify as $D_{s1}^{\pm}(2460)$, has a narrow width (see Table II), while the corresponding state in the nonstrange

sector, $D_{1N}^{\bar{u}}$ discussed above, has a broad width. The reason of this feature is that the possible decay channel of $D_{1N}^{\bar{s}}$ with the lowest-invariant mass is kinematically forbidden. To illustrate this, we again reorganize the spin configurations $|101\rangle$ and $|111\rangle$ in terms of the $[1,4][2,3]$ basis. Because of the nonzero strangeness, one can see that, in the case of $D_{1N}^{\bar{s}}$, the decay channel with the lowest invariant mass is KD^* . This is in contrast to the case of $D_{1N}^{\bar{u}}$ where the lowest decay channel is πD^* . Since the KD^* threshold is ~ 2504 MeV and is larger than the predicted mass of $D_{1N}^{\bar{s}}$, that is ~ 2474 MeV, $D_{1N}^{\bar{s}}$ cannot decay into KD^* even if it acquires a large KD^* component from the mixing. For $D_{1P}^{\bar{s}}$, its predicted mass (2533 MeV) is larger than the KD^* threshold (2504 MeV). But in this case, the KD^* component is strongly suppressed through the mixing, which again leads to a narrow resonance. The agreement with the experimental masses as well as the possible explanation for their decay patterns provides strong support for the four-quark structure of excited heavy mesons.

This model can also be applied to B -meson systems and the results for the hyperfine masses read

$$J = 0: \quad \langle V_B \rangle_0^{\bar{u}} = -40.8 \text{ MeV}, \quad \langle V_B \rangle_0^{\bar{s}} = -33.55 \text{ MeV}, \quad (31)$$

$$J = 1: \quad \langle V_B \rangle_{1P}^{\bar{u}} = 69.56 \text{ MeV}, \quad \langle V_B \rangle_{1P}^{\bar{s}} = 43.84 \text{ MeV}, \quad (32)$$

$$J = 1: \quad \langle V_B \rangle_{1N}^{\bar{u}} = -36.75 \text{ MeV}, \quad \langle V_B \rangle_{1N}^{\bar{s}} = -30.94 \text{ MeV}, \quad (33)$$

$$J = 2: \quad \langle V_B \rangle_2^{\bar{u}} = 84.9 \text{ MeV}, \quad \langle V_B \rangle_2^{\bar{s}} = 56.7 \text{ MeV}. \quad (34)$$

We mention that the states with the superscript \bar{u} have charge -1 and the states with the superscript \bar{s} have charge 0 . Currently, PDG lists only 4 resonances in the excited states of B -mesons with relatively well-known spin, $B_1^0(5724)(J = 1)$, $B_2^{*0}(5743)(J = 2)$, $B_{s1}^0(5829)(J = 1)$, and $B_{s2}^{*0}(5840)(J = 2)$ as can be seen in Table II. Certainly, these are not enough to test the four-quark structure. But we can find that the four mesons listed in PDG seem to fit well with the four-

quark states $B_{1P}^{\bar{u}}$, $B_2^{\bar{u}}$, $B_{1P}^{\bar{s}}$, and $B_2^{\bar{s}}$. The experimental mass splitting between $B_2^{*0}(5743)$ and $B_1^0(5724)$ is about 20 MeV which is quite close to the corresponding value from the hyperfine mass difference $\langle V_B \rangle_2^{\bar{u}} - \langle V_B \rangle_{1P}^{\bar{u}} \simeq 15$ MeV. In B_s family, the mass difference between $B_{s2}^{*0}(5840)$ and $B_{s1}^0(5829)$ is about 10 MeV and this is again not so different from the hyperfine mass difference $\langle V_B \rangle_2^{\bar{s}} - \langle V_B \rangle_{1P}^{\bar{s}} \simeq 13$ MeV. Therefore, as far as the mass difference is concerned, the B -mesons in the current list

of PDG fit very well with our four-quark model. Since the information on the B -meson spectroscopy is accumulating year by year in PDG at these days, we expect that the predicted B -meson spectrum can be tested in near future.

The hyperfine mass differences obtained in this model are collected in Table X in the column of Model I. Two sets of the results are shown there depending on the value

$$\begin{aligned}
D_0^{\bar{u}} &= D_0^{*0}(2318), & D_{1P}^{\bar{u}} &= D_1^0(2421), & D_2^{\bar{u}} &= D_2^{*0}(2463), \\
D_{1P}^{\bar{s}} &= D_{s1}^{\pm}(2535), & D_{1N}^{\bar{s}} &= D_{s1}^{\pm}(2460), & D_2^{\bar{s}} &= D_{s2}^{*\pm}(2572), \\
B_{1P}^{\bar{u}} &= B_1^0(5724), & B_2^{\bar{u}} &= B_2^{*0}(5743), \\
B_{1P}^{\bar{s}} &= B_{s1}^0(5829), & B_2^{\bar{s}} &= B_{s2}^{*0}(5840).
\end{aligned} \tag{35}$$

Once the model parameter is fixed, we can make prediction on the masses of the unobserved mesons of spin-0 and spin-1, i.e., $B_0^{\bar{u}}$, $B_0^{\bar{s}}$, $B_{1N}^{\bar{u}}$, and $B_{1N}^{\bar{s}}$. For the $B_0^{\bar{u}}$ mass, using the fact that $\langle V_B \rangle_{1P}^{\bar{u}} - \langle V_B \rangle_0^{\bar{u}} \simeq 110$ MeV from Eqs. (31) and (32), the $B_0^{\bar{u}}$ mass should be 110 MeV smaller than the $B_{1P}^{\bar{u}}$ mass. Since $B_{1P}^{\bar{u}}$ is identified as $B_1^0(5724)$, the $B_0^{\bar{u}}$ mass is expected to be around 5613 MeV. One can also estimate the $B_0^{\bar{u}}$ mass from the spin-2 meson $B_2^{*0}(5743)$, which gives 5617 MeV. Thus, the two methods give a quite consistent prediction. We take the average value of the two values as our prediction. In a similar way, we have

$$\begin{aligned}
J = 0: & \quad B_0^{\bar{u}} \text{ mass} \sim 5615 \text{ MeV}, \\
& \quad B_0^{\bar{s}} \text{ mass} \sim 5751 \text{ MeV},
\end{aligned} \tag{36}$$

$$\begin{aligned}
J = 1: & \quad B_{1N}^{\bar{u}} \text{ mass} \sim 5619 \text{ MeV}, \\
& \quad B_{1N}^{\bar{s}} \text{ mass} \sim 5753 \text{ MeV}.
\end{aligned} \tag{37}$$

The resonances with the superscript \bar{u} have charge -1 and isospin $1/2$ (isodoublet) so its isospin partner should appear with the same mass. The others with the superscript \bar{s} have charge 0 and they have $I = 0$ (isosinglet). We note that the $J = 0$ resonances have masses quite close to their counterparts of $J = 1$, which may cause some difficulties in discovering these new resonances. Additionally, based on a similar discussion as in D -meson, we expect that the three resonances, $B_0^{\bar{u}}$, $B_0^{\bar{s}}$, and $B_{1N}^{\bar{u}}$

of the color-spin strength v_0 . The first set uses the v_0 value fixed from the mass splitting between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$ and the results are listed in the column of ‘ v_0 from 4-quark’. The other set uses the v_0 value fitted from the $\Delta - N$ mass splitting and the results are given in the column of ‘ v_0 from ΔN ’. In this calculation, we make use of the following identification of the four-quark states:

have broad widths, which hampers the discovery of these mesons. However, the resonance with $J = 1$ of nonzero strangeness, $B_{1N}^{\bar{s}}$ should appear as a sharp resonance, if exists. *Therefore, the discovery of $B_{1N}^{\bar{s}}$ at a mass of ~ 5750 MeV may be a good probe for understanding the structure of excited heavy mesons.*

B. Results from Model II

Another four-quark wave function that we have constructed in Sec. V is called Model II, where the antiquark is antisymmetric in flavor space and its color wave function belongs to $\bar{\mathbf{6}}_c$. Within this model, the formulas for the hyperfine masses of one specific flavor combination are given in Table VIII. Putting them into Eq. (23), we then calculate the hyperfine masses in Model II. Again, the strength of the color-spin interaction v_0 is determined by fitting the mass splitting between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$, which gives $v_0 \sim (-147.8)^3$ MeV³. Using this strength, we calculate the hyperfine masses of the four-quark states, $|J, J_{12}, J_{34}\rangle = |011\rangle, |101\rangle, |111\rangle, |211\rangle$ as well as the mixing term between the two spin-1 states. Again for spin-1 case, it is necessary to diagonalize the hyperfine masses in order to obtain the physical states.

Then we can make predictions on the excited heavy meson spectrum as we did for Model I. The hyperfine masses for D and D_s family are obtained as

$$J = 0: \quad \langle V_D \rangle_0^{\bar{u}} = -106.43 \text{ MeV}, \quad \langle V_D \rangle_0^{\bar{s}} = -108.82 \text{ MeV}, \tag{38}$$

$$J = 1: \quad \langle V_D \rangle_{1P}^{\bar{u}} = 18.18 \text{ MeV}, \quad \langle V_D \rangle_{1P}^{\bar{s}} = 24.58 \text{ MeV}, \tag{39}$$

$$J = 1: \quad \langle V_D \rangle_{1N}^{\bar{u}} = -79.2 \text{ MeV}, \quad \langle V_D \rangle_{1N}^{\bar{s}} = -83.34 \text{ MeV}, \tag{40}$$

$$J = 2: \quad \langle V_D \rangle_2^{\bar{u}} = 38.2 \text{ MeV}, \quad \langle V_D \rangle_2^{\bar{s}} = 41.36 \text{ MeV}. \tag{41}$$

The hyperfine masses for B and B_s family in Model II read

$$J = 0: \quad \langle V_B \rangle_0^{\bar{u}} = -91.65 \text{ MeV}, \quad \langle V_B \rangle_0^{\bar{s}} = -95.05 \text{ MeV}, \tag{42}$$

$$J = 1: \quad \langle V_B \rangle_{1P}^{\bar{u}} = 25.87 \text{ MeV}, \quad \langle V_B \rangle_{1P}^{\bar{s}} = 31.0 \text{ MeV}, \tag{43}$$

Mass difference	$\Delta m_{\text{expt.}} [9]$	Model I		Model II	
		v_0 from 4-quark	v_0 from $\Delta - N$	v_0 from 4-quark	v_0 from $\Delta - N$
$D_2^{*0}(2463) - D_0^{*0}(2318)$	144.6	144.6 (fit)	160.3	144.6 (fit)	356
$D_1^0(2421) - D_0^{*0}(2318)$	103.3	97.1	107.6	124.6	306.7
$D_2^{*0}(2463) - D_1^0(2421)$	41.3	47.5	52.6	20	49.3
$D_{s_2}^{*\pm}(2572) - D_{s_1}^{*\pm}(2535)$	36.8	39.4	43.6	16.78	41.3
$D_{s_2}^{*\pm}(2572) - D_{s_1}^\pm(2460)$	112.3	97.7	108.2	124.7	306.9
$D_{s_1}^{*\pm}(2535) - D_{s_1}^\pm(2460)$	75.5	58.3	64.6	107.9	265.6
$B_2^0(5743) - B_1^0(5724)$	19.5	15.3	17	6.98	16.7
$B_{s_2}^0(5840) - B_{s_1}^0(5829)$	10.3	12.9	14.3	5.7	14.0

TABLE X. The mass splittings among the excited heavy mesons in MeV. The results given under the column name ‘ v_0 from 4-quark’ are obtained with the v_0 value fixed from the mass difference of $D_2^{*0}(2463) - D_0^{*0}(2318)$, which gives $v_0 = (-192.9)^3 \text{ MeV}^3$ for Model I and $v_0 = (-147.8)^3 \text{ MeV}^3$ for Model II. The results given under the column name ‘ v_0 from $\Delta - N$ ’ are obtained with the v_0 value fixed from the $\Delta - N$ mass difference, which gives $(-199.6)^3 \text{ MeV}^3$ in both models. The experimental data are from Ref. [9].

$$J = 1: \quad \langle V_B \rangle_{1N}^{\bar{u}} = -82.57 \text{ MeV}, \quad \langle V_B \rangle_{1N}^{\bar{s}} = -86.58 \text{ MeV}, \quad (44)$$

$$J = 2: \quad \langle V_B \rangle_2^{\bar{u}} = 32.65 \text{ MeV}, \quad \langle V_B \rangle_2^{\bar{s}} = 36.69 \text{ MeV}. \quad (45)$$

Alternatively, within Model II, we can again calculate the mass differences by using the v_0 value determined by the $\Delta - N$ mass difference. Presented in Table X are the mass differences in Model II for these two values of v_0 . These results are compared with the experimental mass splittings as well as the predictions of Model I. As one can see in Table X, the results from Model I have a better agreement with the experimental data than those of Model II. Therefore, we conclude that the four-quark wave functions constructed in Model I are more reliable for the excited heavy meson states as far as the mass differences are concerned.

VII. SUMMARY

In this work, we have constructed four-quark wave functions, which might be relevant for excited states of open charm and open bottom mesons. The four-quark wave functions were constructed from a diquark-antidiquark picture under the assumption that they form the $\mathbf{3}_f$ multiplet in the SU(3) flavor space. Formation of $\mathbf{3}_f$ seems to be realized in some of the observed excited states. Within this approach, we propose two models for the four-quark wave functions, which we call Model I and Model II. In Model I, the antidiquark is symmetric in flavor ($\bar{\mathbf{6}}_f$) and antisymmetric in color ($\mathbf{3}_c$). On the contrary, in Model II, the antidiquark is antisymmetric in flavor ($\mathbf{3}_f$) and symmetric in color ($\bar{\mathbf{6}}_c$). In both models, the possible spin structures are found to be $|J, J_{12}, J_{34}\rangle = |011\rangle, |101\rangle, |111\rangle, \text{ and } |211\rangle$ where J is the spin of the four-quark system, J_{12} the diquark spin, J_{34} the antidiquark spin. There exists a mixing between the two spin-1 states, which is to be diagonalized for finding the physical states. To test these four-quark structure,

we calculated the hyperfine masses using the color-spin interactions and investigated whether they can reproduce the observed mass splittings among the excited states of D , D_s , B and B_s families listed in PDG.

By comparing with the experimental masses, we found that Model I gives a good description of the observed mass splittings as shown in Table X while Model II fails. It should be noted that all these results are obtained with only one model parameter v_0 which is fixed either by the mass splitting between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$ or by the $\Delta - N$ mass splitting. We found that Model I gives a nice description of the mass splittings with these two values of v_0 .

Another supporting result of four-quark structure is the appearance of two spin-1 states. This is indeed consistent with the two experimentally observed resonances, $D_{s_1}^\pm(2460)$ and $D_{s_1}^{*\pm}(2535)$, of which masses are well explained by our Model I. On the other hand, in the charm sector, one of the two spin-1 states fits nicely with the $D_1^0(2421)$ meson but there is a missing resonance. We have demonstrated that the missing spin-1 state may have a large component of πD^* decay mode which is substantially magnified through the mixing. Because of this decay channel, this resonance is expected to be a broad resonance and it may not be easily identified in experiments. However, the two states in D_s mesons have smaller decay widths. In this case, the decay mode with the lowest invariant mass is $K D^*$ which is kinematically forbidden in one state and, in the other state, this decay mode is strongly suppressed through the mixing.

Our Model I can predict some other resonances which are currently missing in PDG compilation. Motivated by this success to explain the observed spectroscopy, we

make predictions on some missing resonances as follows.

$$\begin{aligned}
 J = 0: & \quad D_0^{\bar{s}} \sim 2468 \text{ MeV}; \quad \text{broad resonance,} \\
 J = 1: & \quad D_{1N}^{\bar{u}}, D_{1N}^{\bar{d}} \sim 2330 \text{ MeV}; \quad \text{broad resonances,} \\
 J = 0: & \quad B_0^{\bar{u}}, B_0^{\bar{d}} \sim 5615 \text{ MeV}; \quad \text{broad resonances,} \\
 J = 0: & \quad B_0^{\bar{s}} \sim 5751 \text{ MeV}; \quad \text{broad resonance,} \\
 J = 1: & \quad B_{1N}^{\bar{u}}, B_{1N}^{\bar{d}} \sim 5619 \text{ MeV}; \quad \text{broad resonance,} \\
 J = 1: & \quad B_{1N}^{\bar{s}} \sim 5753 \text{ MeV}; \quad \text{narrow resonance.} \quad (46)
 \end{aligned}$$

This shows that most of these resonances are expected to have broad widths due to decay modes kinematically allowed. Therefore, those resonances may not be easily identified in experiments. However, there is one exception: $B_{1N}^{\bar{s}}$ of spin-1 is expected to be a narrow resonance because its possible decay mode KB^* is not kinematically allowed. So the discovery of $B_{1N}^{\bar{s}}(5753)$ in future experiments will shed light on our understanding of four-quark structure of excited heavy mesons.

Throughout the present work, our discussions are limited to the masses of resonances based on the group structure of four-quark systems. Then the next question would be the dynamical origin of such a structure, which may also provide a key to understand the reason why

Model I is better than Model II for explaining heavy meson excited states in four-quark picture. It is, therefore, highly desirable to test the four-quark picture based on dynamical model approaches to calculate full mass spectra and the couplings of meson resonances. Such studies should also address the question whether the real physical states would be mixtures of orbitally excited two-quark states and four-quark states. Testing the four-quark interpolating fields in QCD sum rules may also be interesting to compute the physical properties of excited heavy mesons and it will help us verify which structure has a strong overlap with the physical hadron states.

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