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Entropy of the Randall-Sundrum brane world with the generalized uncertainty principle

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ABSTRACT

By introducing the generalized uncertainty principle, we calculate the entropy of the bulk scalar field on the Randall-Sundrum brane background without any cutoff. We obtain the entropy of the massive scalar field proportional to the horizon area. Here, we observe that the mass contribution to the entropy exists in contrast to all previous results of the usual black hole cases with the generalized uncertainty principle.

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1 Introduction

Three decades ago, Bekenstein had suggested that the entropy of a black hole is proportional to the area of the horizon through the thermodynamic analogy [1]. Subsequently, Hawking showed that the entropy of the Schwarzschild black hole satisfies exactly the area law by means of Hawking radiation based on the quantum field theory [2]. After their works, 't Hooft investigated the statistical properties of a scalar field outside the horizon of a Schwarzschild black hole by using the brick wall method with the Heisenberg uncertainty principle [3]. However, although he obtained the entropy proportional to the horizon area, an unnatural brick wall cutoff was introduced to remove the ultraviolet divergence near horizon [4, 5, 6, 7, 8, 9]. A similar idea was also considered by the entanglement entropy interpretation with a momentum cutoff, which is almostly equivalent to the brick wall model [10]. Recently, many efforts [11] have been devoted to the generalized uncertainty relations, and its consequences, especially the effect on the density of states. Very recently, in Refs. [12, 13, 14], the authors calculated the entropy of black holes by using the novel equation of state of density motivated by the generalized uncertainty principle [11], which drastically solves the ultraviolet divergences of the just vicinity near the horizon without any cutoff.

On the other hand, much interests have been paid to the Randall and Sundrum model to resolve the gauge hierarchy problem [15, 16], which is based upon the fact that our universe may be embedded in higher-dimensional spacetimes [17]. Furthermore, various aspects [18] of this model have been studied including the cosmological applications [19]. To study quantum mechanical aspect of this black brane world, we may first consider its entropy, which is expected to satisfy the area law [2]. However, up to now, the statistical entropy of the black brane world was only studied by using the brick wall method [6, 7, 8, 9] with the Heisenberg uncertainty principle, which still has several difficulties including artificial ultraviolet and infrared cutoffs.

In this paper, we would like to study the entropy of a bulk scalar field on the black brane background avoiding the difficulty in solving the 5-dimensional Klein-Gordon wave equation by using the quantum statistical method. By using the novel equation of state of density motivated by the generalized uncertainty principle in the quantum gravity [12, 13, 14], we derive the free energy of a massive scalar field through the complete decomposition of the extra mode and the radial mode, which has been impossible in the brick

wall method. We then calculate the quantum entropy of the black hole via thermodynamic relation between the free energy and the entropy. As a result, we obtain the desired entropy proportional to the horizon area without any artificial cutoff and little mass approximation. Here, we newly observe that in contrast to all previous results [12, 13, 14] of the usual black hole cases with the generalized uncertainty principle, which is independent of the ordinary scalar field mass, the contribution of the bulk scalar mass to the entropy for the Randall and Sundrum brane case exists.

2 Scalar field on Randall and Sundrum brane Background

Let us start with the following action [15, 16] of the Randall and Sundrum model in $(4 + 1)$ dimensions,

$$S_{(5)} = \frac{1}{16\pi G_N^{(5)}} \int d^4x \int dy \sqrt{-g_{(5)}} \left[R^{(5)} + 12k^2 \right] - \int d^4x \left[\sqrt{-g_{(+)}} \lambda_{(+)} + \sqrt{-g_{(-)}} \lambda_{(-)} \right], \quad (1)$$

where $\lambda_{(+)}$ and $\lambda_{(-)}$ are tensions of the branes at $y = 0$ and $y = \pi r_c$, respectively, while $12k^2$ is a cosmological constant. We assume that orbifold S^1/Z_2 possesses a periodicity in the extra coordinate y , and identify $-y$ with y . Two singular points on the orbifold are located at $y = 0$ and $y = \pi r_c$. Two 3-branes are placed at these points. Note that the metric on each brane is defined as $g_{\mu\nu}^{(+)} \equiv g_{\mu\nu}^{(5)}(x^\mu, y = 0)$ and $g_{\mu\nu}^{(-)} \equiv g_{\mu\nu}^{(5)}(x^\mu, y = \pi r_c)$.

Since we are interested in black holes, let us assume the bulk metric as

$$ds_{(5)}^2 = e^{-2ky\Theta(x)} g_{\mu\nu}(x) dx^\mu dx^\nu + T^2(x) dy^2, \quad (2)$$

where $T(x)$ is the moduli field. Note that we use $\mu, \nu = 0, 1, \dots, 3$ for brane indices. Among possible solutions satisfying the equations of motion [7], let us consider the 4-dimensional Schwarzschild black hole solution as a slice of AdS spacetime as a brane solution,

$$ds^2 = e^{-2ky} \left[- \left(1 - \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 d\Omega_{(2)}^2 \right] + dy^2, \quad (3)$$

where $d\Omega_{(2)}^2$ is a metric of unit 2-sphere and we set $G_{(4)} = 1$ for convenience. In fact, it is a black string solution intersecting the brane world, which describes a black hole placed on the hypersurface at the fixed extra coordinate.

In this brane background, let us first consider a bulk scalar field with mass m , which satisfies the Klein-Gordon equation,

$$(\nabla_{(5)}^2 - m^2)\Phi = 0, \quad (4)$$

which is explicitly given as

$$e^{2ky} \left[-\frac{1}{f} \partial_t^2 \Phi + \frac{1}{r^2} \partial_r (r^2 f \partial_r \Phi) + \frac{1}{r^2 \sin \theta} \partial_\theta (\sin \theta \partial_\theta \Phi) + \frac{1}{r^2 \sin^2 \theta} \partial_\phi^2 \Phi \right] + e^{4ky} \partial_y (e^{-4ky} \partial_y \Phi) - m^2 \Phi = 0, \quad (5)$$

where $f = 1 - \frac{2M}{r}$. If we set

$$e^{4ky} \partial_y (e^{-4ky} \partial_y \chi) - m^2 \chi + \mu^2 e^{2ky} \chi = 0 \quad (6)$$

with $\Phi(t, r, \theta, \phi, y) \equiv \Psi(t, r, \theta, \phi) \chi(y)$, then the separation of variables is easily done, and the reduced form of the effective field equation becomes

$$-\frac{1}{f} \partial_t^2 \Psi + \frac{1}{r^2} \partial_r (r^2 f \partial_r \Psi) + \frac{1}{r^2 \sin \theta} \partial_\theta (\sin \theta \partial_\theta \Psi) + \frac{1}{r^2 \sin^2 \theta} \partial_\phi^2 \Psi - \mu^2 \Psi = 0. \quad (7)$$

Note that the above eigenvalue μ^2 plays a role of the effective mass on the brane. Substituting the 4-dimensional wave function $\Psi(t, r, \theta, \phi) = e^{-i\omega t} \psi(r, \theta, \phi)$, we find that the 4-dimensional Klein-Gordon equation becomes

$$\partial_r^2 \psi + \left(\frac{1}{f} \partial_r f + \frac{2}{r} \right) \partial_r \psi + \frac{1}{f} \left(\frac{1}{r^2} \left[\partial_\theta^2 + \cot \theta \partial_\theta + \frac{1}{\sin^2 \theta} \partial_\phi^2 \right] + \frac{\omega^2}{f} - \mu^2 \right) \psi = 0. \quad (8)$$

By using the Wenzel-Kramers-Brillouin (WKB) approximation [3] with $\psi \sim \exp[iS(r, \theta, \phi)]$, we have

$$p_r^2 = \frac{1}{f} \left(\frac{\omega^2}{f} - \mu^2 - \frac{p_\theta^2}{r^2} - \frac{p_\phi^2}{r^2 \sin^2 \theta} \right), \quad (9)$$

where

$$p_r = \frac{\partial S}{\partial r}, \quad p_\theta = \frac{\partial S}{\partial \theta}, \quad p_\phi = \frac{\partial S}{\partial \phi}. \quad (10)$$

Furthermore, we also obtain the square module momentum as follows

$$p^2 = p_i p^i = g^{rr} p_r^2 + g^{\theta\theta} p_\theta^2 + g^{\phi\phi} p_\phi^2 = \frac{\omega^2}{f} - \mu^2. \quad (11)$$

Then, the volume in the momentum phase space is given by

$$\begin{aligned}
V_p(r, \theta) &= \int dp_r dp_\theta dp_\phi \\
&= \frac{4\pi}{3} \sqrt{\frac{1}{f}(\frac{\omega^2}{f} - \mu^2)} \cdot \sqrt{r^2(\frac{\omega^2}{f} - \mu^2)} \cdot \sqrt{r^2 \sin^2 \theta (\frac{\omega^2}{f} - \mu^2)} \\
&= \frac{4\pi}{3} \frac{r^2 \sin \theta}{\sqrt{f}} \left(\frac{\omega^2}{f} - \mu^2 \right)^{\frac{3}{2}}
\end{aligned} \tag{12}$$

with the condition $\omega \geq \mu\sqrt{f}$.

3 Mode Spectrum

Recently, many efforts have been devoted to the generalized uncertainty relation [11] given by

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left(1 + \lambda \left(\frac{\Delta p}{\hbar} \right)^2 \right). \tag{13}$$

From now on, we take the units $\hbar = k_B = c \equiv 1$. Then, since one can easily get $\Delta x \geq \sqrt{\lambda}$, which gives the lowest bound, it can be defined to be the the minimal length near horizon, which effectively plays a role of the brick wall cutoff. Furthermore, based on the generalized uncertainty relation, the 3-dimensional volume of a phase cell is changed from $(2\pi)^3$ into

$$(2\pi)^3 (1 + \lambda p^2)^3, \tag{14}$$

where $p^2 = p^i p_i$ ($i = r, \theta, \phi$).

From the Eqs. (11) and (14), the number of quantum states related to the radial mode with energy less than ω is given by

$$\begin{aligned}
n_r(\omega) &= \frac{1}{(2\pi)^3} \int dr d\theta d\phi dp_r dp_\theta dp_\phi \frac{1}{\left(1 + \lambda \left(\frac{\omega^2}{f} - \mu^2 \right) \right)^3} \\
&= \frac{1}{(2\pi)^3} \int dr d\theta d\phi \frac{1}{\left(1 + \lambda \left(\frac{\omega^2}{f} - \mu^2 \right) \right)^3} V_p(r, \theta) \\
&= \frac{2}{3\pi} \int_{r_H} dr \frac{r^2}{\sqrt{f}} \frac{\left(\frac{\omega^2}{f} - \mu^2 \right)^{\frac{3}{2}}}{\left(1 + \lambda \left(\frac{\omega^2}{f} - \mu^2 \right) \right)^3}.
\end{aligned} \tag{15}$$

It is interesting to note that $n_r(\omega)$ is convergent at the horizon without any artificial cutoff due to the existence of the suppressing λ -term in the denominator induced from the generalized uncertainty principle.

On the other hand, the exact quantization of Eq.(6) seems to be cumbersome. However, since in the WKB approximation [3], the wave number k_χ of the wave function $\chi(y)$ is easily written as

$$k_\chi^2(y, \mu) = \mu^2 e^{2ky} - m^2, \quad (16)$$

the number of extra mode for a given value μ is given by

$$\pi n_\chi(\mu) = \int_0^{\pi r_c} dy \sqrt{\mu^2 e^{2ky} - m^2}. \quad (17)$$

We then obtain the total number of extra mode with energy less than ω as follows

$$\begin{aligned} \pi n_\chi &= \int_m^{\frac{\omega}{\sqrt{f}}} d\mu \frac{dn_\chi(\mu)}{d\mu} \\ &= \frac{1}{k} \int_m^{\frac{\omega}{\sqrt{f}}} d\mu \frac{1}{\mu} \left(\sqrt{\mu^2 e^{2k\pi r_c} - m^2} - \sqrt{\mu^2 - m^2} \right). \end{aligned} \quad (18)$$

As a result, we could formally write down the proper total number of quantum states with energy less than ω as follows

$$N_T(\omega) = \int dN_T(\omega) = \int dn_r dn_\chi. \quad (19)$$

4 Free Energy and Entropy

For the bosonic case, the free energy at inverse temperature β is given by

$$e^{-\beta F} = \prod_K [1 - e^{-\beta \omega_K}]^{-1}, \quad (20)$$

where K represents the set of quantum numbers. Then, by using Eq. (15), we are able to obtain the equation of free energy as

$$\begin{aligned} F_T &= \frac{1}{\beta} \sum_K \ln [1 - e^{-\beta \omega_K}] \approx \frac{1}{\beta} \int dN_T(\omega) \ln [1 - e^{-\beta \omega}] \\ &= - \int_m^\infty \frac{d\omega}{\mu \sqrt{f}} \frac{N_T(\omega)}{e^{\beta \omega} - 1} \\ &= - \frac{2}{3\pi} \int_{r_H} dr \frac{r^2}{\sqrt{f}} \int_m^\infty \frac{d\omega}{\mu \sqrt{f}} \int_m^{\frac{\omega}{\sqrt{f}}} d\mu \frac{\left(\frac{\omega^2}{f} - \mu^2\right)^{\frac{3}{2}}}{(e^{\beta \omega} - 1) \left(1 + \lambda \left(\frac{\omega^2}{f} - \mu^2\right)\right)^3} \left(\frac{dn_\chi}{d\mu}\right) \\ &= - \frac{2}{3\pi} \int_{r_H} dr \frac{r^2}{\sqrt{f}} \Lambda_T \int_m^{\frac{\omega}{\sqrt{f}}} d\mu \left(\frac{dn_\chi}{d\mu}\right). \end{aligned} \quad (21)$$

Here, we have taken the continuum limit in the first line and integrated by parts in the second line in Eq. (21). Note that our free energy is exactly the same as Eq. (27) in Ref. [8] with the exception of the new suppressing λ -term that is introduced by the generalized uncertainty principle in the denominator, which drastically solves the ultraviolet divergence near the event horizon. In the last line, since $f \rightarrow 0$ near the event horizon, *i.e.*, in the range of $(r_H, r_H + \epsilon)$, $\frac{\omega^2}{f} - \mu^2$ becomes $\frac{\omega^2}{f}$. Although we do not require the little mass approximation, the integral equation of ω is naturally reduced to

$$\Lambda_T = \int_0^\infty d\omega \frac{f^{-\frac{3}{2}} \omega^3}{(e^{\beta\omega} - 1) \left(1 + \lambda \frac{\omega^2}{f}\right)^3}. \quad (22)$$

Therefore, the free energy can be rewritten as

$$F_T = -\frac{2}{3\pi} \int_{r_H} dr \frac{r^2}{f^2} \int_0^\infty d\omega \frac{\omega^3}{(e^{\beta\omega} - 1) \left(1 + \lambda \frac{\omega^2}{f}\right)^3} \int_m^{\frac{\omega}{\sqrt{f}}} d\mu \left(\frac{dn_\chi}{d\mu} \right). \quad (23)$$

Now, since at this stage the n_χ mode part is completely decoupled with the n_r mode part for the μ coupling on the contrary to the previous results [7, 8], we are able to separately carry out the integral equation of μ in Eq. (18). As a result, we obtain

$$\begin{aligned} k\pi n_\chi &= \left(\sqrt{\frac{\omega^2 e^{2k\pi r_c}}{f} - m^2} - \sqrt{\frac{\omega^2}{f} - m^2} \right) - m\gamma \\ &- m \left(\tan^{-1} \sqrt{\frac{\omega^2 e^{2k\pi r_c}}{m^2 f} - 1} - \tan^{-1} \sqrt{\frac{\omega^2}{m^2 f} - 1} \right) \end{aligned} \quad (24)$$

with $\alpha_a = e^{ak\pi r_c} - 1$ ($a = 1, 2$) and $\gamma = \sqrt{\alpha_2} - \tan^{-1} \sqrt{\alpha_2} \geq 0$. Furthermore, near the event horizon as $f \rightarrow 0$, we get the integral from the Eq. (24) without little mass approximation as

$$\pi k n_\chi = \alpha_1 \frac{\omega}{\sqrt{f}} - m\gamma. \quad (25)$$

Hence, the value of free energy can be rewritten as

$$\begin{aligned} F_T &= -\frac{2\alpha_1}{3\pi^2 k} \int_{r_H} dr \frac{r^2}{f^2 \sqrt{f}} \int_0^\infty d\omega \frac{\omega^4}{(e^{\beta\omega} - 1) \left(1 + \lambda \frac{\omega^2}{f}\right)^3} \\ &+ \frac{2m\gamma}{3\pi^2 k} \int_{r_H} dr \frac{r^2}{f^2} \int_0^\infty d\omega \frac{\omega^3}{(e^{\beta\omega} - 1) \left(1 + \lambda \frac{\omega^2}{f}\right)^3}. \end{aligned} \quad (26)$$

From this free energy and Eq. (25), the entropy for the scalar field is given by

$$\begin{aligned}
S_T &= \beta^2 \frac{\partial F_T}{\partial \beta} \\
&= \beta^2 \frac{2\alpha_1}{3\pi^2 k} \int_{r_H} dr \frac{r^2}{f^2 \sqrt{f}} \int_0^\infty d\omega \frac{e^{\beta\omega} \omega^5}{(e^{\beta\omega} - 1)^2 (1 + \lambda \frac{\omega^2}{f})^3} \\
&\quad - \beta^2 \frac{2m\gamma}{3\pi^2 k} \int_{r_H} dr \frac{r^2}{f^2} \int_0^\infty d\omega \frac{e^{\beta\omega} \omega^4}{(e^{\beta\omega} - 1)^2 (1 + \lambda \frac{\omega^2}{f})^3} \\
&\equiv \frac{2\alpha_1}{3\pi^2 k} \int_{r_H} dr \frac{r^2}{\sqrt{f}} \Lambda_T^1 - \frac{2m\gamma}{3\pi^2 k} \int_{r_H} dr \frac{r^2}{\sqrt{f}} \Lambda_T^2,
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
\Lambda_T^1 &= \int_0^\infty dx \frac{f^{-2} \beta^{-4} x^5}{(1 - e^{-x})(e^x - 1) \left(1 + \frac{\lambda}{\beta^2 f} x^2\right)^3}, \\
\Lambda_T^2 &= \int_0^\infty dx \frac{f^{-\frac{3}{2}} \beta^{-3} x^4}{(1 - e^{-x})(e^x - 1) \left(1 + \frac{\lambda}{\beta^2 f} x^2\right)^3}
\end{aligned} \tag{28}$$

with $x = \beta\omega$.

Now, let us rewrite Eq. (27) as

$$S_T = \frac{2\alpha_1}{3\pi^2 k \lambda^{\frac{3}{2}}} \int_{r_H} dr \frac{r^2}{\sqrt{f}} \Lambda_T^1 - \frac{2m\gamma}{3\pi^2 k \lambda^{\frac{3}{2}}} \int_{r_H} dr \frac{r^2}{\sqrt{f}} \Lambda_T^2, \tag{29}$$

where

$$\begin{aligned}
\Lambda_T^1 &= \int_0^\infty dX \frac{a^2 X^5}{(e^{\frac{a}{2}X} - e^{-\frac{a}{2}X})^2 (1 + X^2)^3}, \\
\Lambda_T^2 &= \int_0^\infty dX \frac{a^2 X^4}{(e^{\frac{a}{2}X} - e^{-\frac{a}{2}X})^2 (1 + X^2)^3}
\end{aligned} \tag{30}$$

with $x = \beta\sqrt{\frac{f}{\lambda}}X \equiv aX$. Note that when $r \rightarrow r_H$, a goes to zero. Since we are interested in the contributions from just the vicinity of the horizon, the integrals in Eq. (28) are finally reduced as follows:

$$\begin{aligned}
\Lambda_T^1 &= \int_0^\infty dX \frac{X^3}{(1 + X^2)^3} = \frac{1}{4}, \\
\Lambda_T^2 &= \int_0^\infty dX \frac{X^2}{(1 + X^2)^3} = \frac{\pi}{16}.
\end{aligned} \tag{31}$$

On the other hand, we are also interested in the contribution from just the vicinity near the horizon in the range $(r_H, r_H + \epsilon)$, where ϵ is related to a proper distance of order of the minimal length, $\sqrt{\lambda}$ as follows

$$\sqrt{\lambda} = \int_{r_H}^{r_H + \epsilon} \frac{dr}{\sqrt{f(r)}} \approx \int_{r_H}^{r_H + \epsilon} \frac{dr}{\sqrt{2\kappa(r - r_H)}} = \sqrt{\frac{2\epsilon}{\kappa}}. \quad (32)$$

Here κ is the surface gravity at the horizon of the black hole, and it is identified as $\kappa = 2\pi\beta$.

Therefore, when $r \rightarrow r_H$, we finally get the desired entropy of the massive scalar field on the RS black brane background as follows

$$\begin{aligned} S_T &\approx \frac{2\alpha_1}{3\pi^2 k \lambda^2} \cdot r_H^2 \sqrt{\lambda} \cdot \frac{1}{4} - \frac{2m\gamma}{3\pi^2 k \lambda^{\frac{3}{2}}} \cdot r_H^2 \sqrt{\lambda} \cdot \frac{\pi}{16} \\ &= \left(\frac{\alpha_1}{6\pi^3 k} \frac{1}{(\sqrt{\lambda})^3} - m \frac{\gamma}{24\pi^2 k} \frac{1}{(\sqrt{\lambda})^2} \right) \frac{A}{4}, \end{aligned} \quad (33)$$

which is proportional to the area $A = 4\pi r_H^2$. Note that there is no divergence within the just vicinity near the horizon due to the effect of the generalized uncertainty relation on the quantum states.

It seems to be appropriate to comment on the entropy (33). First, by using the generalized uncertainty principle, this entropy is obtained from the contribution of the just vicinity near the horizon in the range $(r_H, r_H + \epsilon)$, which is neglected by the brick wall method. Second, since the entropy consists of the inverse power terms of the minimal length, it is non-perturbative. Moreover, the positive dominant leading term is proportional to $(\sqrt{\lambda})^{-3}$, while the negative sub-leading term, which gives the massive effect, is proportional to $(\sqrt{\lambda})^{-2}$.

In conclusion, we have investigated the massive bulk scalar field within the just vicinity near the horizon of a static black hole in the black brane world by using the generalized uncertainty principle. We have derived the free energy of a massive scalar field through the complete decomposition of the extra mode and the radial mode, which has been impossible in the brick wall method with the Heisenberg uncertainty principle. From this free energy, we have obtained the desired entropy proportional to the two-dimensional area of the black brane world without any artificial cutoff and little mass approximation. As a result, we have newly observed that the negative contribution of the bulk scalar mass to the entropy exists for this brane case in contrast to all previous results, which is independent of the mass of the ordinary scalar field, of the usual black hole cases with the generalized uncertainty principle.

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