

# The $\eta_c(3654)$ and hyperfine splitting in charmonium

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## Abstract

The hyperfine splitting for the 2S charmonium state is calculated and the predicted number is  $\Delta_{\text{HF}}(2S) = 57 \pm 8$  MeV, being by derivation the lower bound of this splitting. It results in  $M(\eta_c(2S)) = 3630 \pm 8$  MeV, which is smaller by two standard deviations than found in the Belle experiment [1], but close to the  $\eta_c(2S)$  mass observed by the same group in the experiment  $e^+e^- \rightarrow J/\psi \eta_c$  [6] where  $M(\eta_c(2S)) = 3622 \pm 12$  MeV was found.

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## I. INTRODUCTION

Recently the Belle Collaboration has observed a new charmonium state, the  $\eta_c(2S)$ , in exclusive  $B \rightarrow KK_S K^- \pi^+$  decays [1]. The measured mass of the  $\eta_c(2S)$ ,  $M(2^1S_0) = 3654 \pm 14$  MeV, has appeared to be rather close to the  $\psi(2S)$  mass,  $M_2 = 3686$  MeV, giving rise to a small hyperfine splitting for the  $2S$  state,

$$\Delta_{\text{HF}}(2S, \text{exp}) = M_2 - M(2^1S_0) = 32 \pm 14 \text{ MeV}. \quad (1.1)$$

Compared to the hyperfine splitting for the  $1S$  state,  $\Delta_{\text{HF}}(1S, \text{exp}) = 117.2 \pm 1.5$  MeV [2], the number (1.1) is 3.5 times smaller,

$$\frac{\Delta_{\text{HF}}(2S, \text{exp})}{\Delta_{\text{HF}}(1S, \text{exp})} = 0.273 \pm 0.123 \quad (1.2)$$

and has a large experimental error coming from the error in the  $\eta_c(2S)$  mass. However, in the ratio (1.2) even the upper limit is rather small and it is of interest to compare this small ratio with the predictions in the conventional theory of the spin-spin interaction in QCD.

In this rapid communication we shall discuss the problem posed in a model-independent way and show that the experimental value (1.2) puts a strong restriction on the coupling  $\alpha_{\text{HF}}(2S)$ , determining the spin-spin interaction. In particular, the central value of  $\Delta_{\text{HF}}(2S)$  corresponds to  $\alpha_{\text{HF}}(2S) \cong 0.18 \div 0.19$  which implies the very large value for the renormalization scale  $\mu_2 = 2M_2 \cong 7.4$  GeV. This scale appears to be drastically different from that for the  $1S$  state ( $J/\psi$ ) where the scale in the strong coupling determining the HF interaction is  $\mu_1 \cong \frac{1}{2}M_1 = 1.55$  GeV ( $M_1 = M(J/\psi)$ ) and the coupling constant  $\alpha_{\text{HF}}(\mu_1) \cong 0.335$  is rather large. We shall show that if one uses for the  $2S$  state the same prescription as for  $J/\psi$ , then our theoretical prediction is  $\Delta_{\text{HF}}(2S, \text{theory}) = 57 \pm 8$  MeV, which by derivation is the lower bound of  $\Delta_{\text{HF}}(2S)$  and gives rise to the mass value  $M(\eta_c(2S)) = 3630 \pm 8$  MeV, being two standard deviations off the value obtained from the Belle experiment [1].

## II. MODEL-INDEPENDENT CALCULATION OF $\Delta_{\text{HF}}(2S)$

In one-loop approximation in the  $\bar{M}S$  renormalization scheme, the hyperfine splitting,  $\Delta_{\text{HF}}(nS)$ , is given by the well-known expression:

$$\Delta_{\text{HF}}(nS) = \frac{8}{9} \frac{\alpha_{\text{HF}}(\mu_n)}{m_c^2} |R_n(0)|^2 \left( 1 + \frac{\alpha_{\text{HF}}(\mu_n)}{\pi} \xi_{\text{HF}} \right), \quad (2.1)$$

where the factor  $\xi_{\text{HF}}$  comes from the one-loop corrections,  $\xi_{\text{HF}} = \frac{5}{12}\beta_0 - \frac{8}{3} - \frac{3}{4}\ln 2$  [3], and for  $n_f = 4$ ,  $\xi_{\text{HF}}(n_f = 4) = 0.2857$  is small, so that the  $\alpha_{\text{HF}}$  correction in the brackets turns out to be less than 3% ( $\alpha_{\text{HF}}(\mu_n) \lesssim 0.35$ ) and can be neglected in the ratio (1.2).

For light mesons the relativistic version of the expression (2.1) also exists, it was derived in Refs. [4] and can be useful for charmonium, since the current  $c$ -quark mass is not large,  $m_c = 1.3 \pm 0.2$  GeV [2],

$$\Delta_{\text{HF}}^R(nS) = \frac{8}{9} \frac{\alpha_{\text{HF}}(\mu_n)}{\omega_n^2} |R_n(0)|^2. \quad (2.2)$$

Here  $\omega_n$  is the kinetic energy matrix element:

$$\omega_n = \langle \sqrt{p^2 + m_c^2} \rangle_{nS}, \quad (2.3)$$

which plays the role of the constituent quark mass. The HF splitting (2.1) or (2.2) strongly depends on the  $c$ -quark mass chosen and  $\omega_n^2$  can change by a factor of 2 for different choices of  $m_c$ . To escape the problem of a correct choice of  $m_c$  it is convenient to consider the ratio of hyperfine splittings with relativistic corrections:

$$\frac{\Delta_{\text{HF}}^R(2S)}{\Delta_{\text{HF}}^R(1S)} = \frac{\alpha_{\text{HF}}(\mu_2)}{\alpha_{\text{HF}}(\mu_1)} \left( \frac{\omega_1}{\omega_2} \right)^2 \left| \frac{R_2(0)}{R_1(0)} \right|^2. \quad (2.4)$$

Note that the difference between  $\omega_1$  and  $\omega_2$  in charmonium which takes into account relativistic corrections, is not large, e.g., for the spinless Salpeter equation  $(\omega_1/\omega_2)^2 \cong 0.94$ , however, it will be useful to keep this factor in our analysis, since due to this factor the ratio (2.4) and therefore the hyperfine splitting is 6% smaller. However, the difference between  $\omega(nS)$  and the current (pole) mass  $m_c$  coming from relativistic corrections, is not small, being about 200 MeV and 250 MeV for the 1S and 2S states, respectively (see Section IV).

The ratio of the wave functions at the origin occurring in Eq. (2.4) can be extracted in a model-independent way from the leptonic widths:

$$\Gamma_{e^+e^-}(nS) = \frac{4\alpha^2 e_q^2}{M_n^2} |R_n(0)|^2 \gamma_n, \quad (2.5)$$

where the factor  $\gamma_n$  is

$$\gamma_n = 1 - \frac{16}{3\pi} \alpha_s(M_n), \quad (2.6)$$

and in  $\gamma_n$  the strong coupling  $\alpha_s$  is taken at the scale  $\mu_n = M_n$  ( $M_1 = M(J/\psi)$ ,  $M_2 = M(\psi(2S))$ ). Note that just this prescription for  $\alpha_s(\mu)$  (in the QCD factor  $\gamma$ ) provides the minimal value of the extracted wave function at the origin. For  $\Lambda^{(4)}(3\text{-loop}) = 280$

MeV ( $n_f = 4$ ), which corresponds to  $\alpha_s(M_z) = 0.117$ , the values of  $\alpha_s(M_n)$  in 3-loop approximation are the following,

$$\begin{aligned}\alpha_s(M_1) &= 0.247, & \alpha_s(M_2) &= 0.232, \\ \gamma_1 &= 0.581, & \gamma_2 &= 0.606.\end{aligned}\tag{2.7}$$

Note that for another choice:  $\gamma_1 = \gamma_2$  which is often used, the value of  $\Delta_{\text{HF}}(2S)$  would be 4% larger than in our consideration.

Then from the ratio of leptonic widths (2.5) it follows that

$$\left| \frac{R_2(0)}{R_1(0)} \right|^2 = \left( \frac{M_2}{M_1} \right)^2 \frac{\Gamma_{e^+e^-}(2S)}{\Gamma_{e^+e^-}(1S)} \frac{\gamma_1}{\gamma_2}.\tag{2.8}$$

Taking the experimental values:  $\Gamma_{e^+e^-}(1S) = 5.26 \pm 0.37$  keV and  $\Gamma_{e^+e^-}(2S) = 2.19 \pm 0.15$  keV [2], this ratio with a good accuracy is

$$\left| \frac{R_2(0)}{R_1(0)} \right|^2 = (0.59 \pm 0.08) \frac{\gamma_1}{\gamma_2}.\tag{2.9}$$

The values of the wave functions at the origin extracted from the leptonic widths are the following:  $|R_1(0)|^2 = 0.917 \text{ GeV}^3$  and  $|R_2(0)|^2 = 0.518 \text{ GeV}^3$  and at this point it is important to stress that for the  $2S$  state the extracted value of the wave function at the origin already takes into account the influence of the close lying  $D\bar{D}$  channel on the wave function in an implicit way, and it is the same both for the hyperfine splitting and the leptonic width. Due to the nearby  $D\bar{D}$  threshold the experimental value of  $|R(0)|^2$  as well as the leptonic widths typically appear to be smaller by about 20% than in the theoretical calculations (one-channel approximation) and the influence of the  $D\bar{D}$  channel on the  $\psi(2S)$  mass was discussed in Ref. [5].

Now combining the expressions (2.9) and (2.4) one obtains

$$\frac{\Delta_{\text{HF}}^R(2S)}{\Delta_{\text{HF}}^R(1S)} = (0.59 \pm 0.08) \eta_{\text{HF}}\tag{2.10}$$

with the factor

$$\eta_{\text{HF}} = \left( \frac{\omega_1}{\omega_2} \right)^2 \frac{\alpha_{\text{HF}}(\mu_2) \gamma_1}{\alpha_{\text{HF}}(\mu_1) \gamma_2}.\tag{2.11}$$

From the analysis of the leptonic width and hyperfine splitting for the  $J/\psi$  it is known that the renormalization scale  $\mu_1$  in  $\alpha_{\text{HF}}(1S)$  is different from  $M_1$  and a good description of the  $\Delta_{\text{HF}}(1S)$  can be reached if in Eq. (2.1) or (2.2) the scale is taken to be  $\mu_1 \cong \frac{1}{2}M_1 = 1.55$

GeV, which with  $\Lambda^{(4)}(3\text{-loop}) = 280\text{ MeV}$  gives the value  $\alpha_{\text{HF}}(\frac{1}{2}M_1) = 0.335$ . Now with the same prescription  $\mu_2 = \frac{1}{2}M_2$ , the value  $\alpha_{\text{HF}}(\frac{1}{2}M_2) = 0.307$  is obtained, and then from Eqs. (2.7) and (2.11) it follows that

$$\eta_{\text{HF}} = 0.826, \quad \text{for} \quad \left(\frac{\omega_1}{\omega_2}\right)^2 = 0.94, \quad \frac{\gamma_1}{\gamma_2} = 0.96. \quad (2.12)$$

As a result from Eq. (2.10) the "theoretical" ratio of the hyperfine splittings is

$$\frac{\Delta_{\text{HF}}^R(2S)}{\Delta_{\text{HF}}^R(1S)} = (0.48 \pm 0.07) \quad (2.13)$$

and our prediction for the  $\Delta_{\text{HF}}(2S)$  is

$$\Delta_{\text{HF}}(2S, \text{theory}) = (57 \pm 8)\text{MeV}. \quad (2.14)$$

Owing to our derivation this number can be considered as the lower bound of  $\Delta_{\text{HF}}(2S)$  which was obtained in a model-independent way. The predicted mass of the  $\eta_c(2S)$ ,  $M(\eta_c(2S)) = 3630 \pm 8\text{ MeV}$ , differs by two standard deviations from the measured value in the Belle experiment [1], but is rather close to the  $\eta_c(2S)$  mass observed in another Belle experiment [6]  $e^+e^- \rightarrow J/\psi \eta_c$  where the measured  $\eta_c(2S)$  mass,  $M(\eta_c(2S)) = 3622 \pm 12\text{ MeV}$  is in good agreement with our prediction. Note that besides the dominant perturbative term in the HF splitting there exists also a nonperturbative (NP) contribution to  $\Delta_{\text{HF}}(nS)$ . However, the NP terms in charmonium can be calculated as in Ref. [7] and turn out to be small: we find  $\Delta_{\text{HF}}^{\text{NP}}(1S) = 3 - 5\text{ MeV}$  and  $\Delta_{\text{HF}}^{\text{NP}}(2S) = 1 - 2\text{ MeV}$ . It is of interest to note that the same model-independent estimate of the  $\Delta_{\text{HF}}(2S)$  only with  $\gamma_1 = \gamma_2$  and  $\alpha_{\text{HF}}(\mu_1) = \alpha_{\text{HF}}(\mu_2)$  was suggested many years ago at the time when the mass of the  $\eta_c(1S)$  was not correctly measured [8].

### III. THE RENORMALIZATION SCALE IN $\alpha_{\text{HF}}(\mu)$

To obtain the central value in Eq. (1.1) in the same procedure one needs to take the very small value  $\alpha_{\text{HF}}(\mu_2) = 0.187$ , which corresponds to the large renormalization scale:  $\mu_2 = 7.4\text{ GeV} \gtrsim 2M_2$ . It is difficult to point out any physical explanation for such different scales of  $\mu_2 = 2M_2$  for the  $2S$  state and  $\mu_1 \cong \frac{1}{2}M_1 = 1.55\text{ GeV}$  for the  $J/\psi$ .

Existing theoretical calculations of  $\Delta_{\text{HF}}$  mostly provide the  $2S$  splittings in the range 70-90 MeV [9, 10], while only in Ref. [11], where the modified (screened) color Coulomb

interaction was used, the calculated HF splitting is smaller:  $\Delta_{\text{HF}}(2S) = 38$  MeV. However, in Ref. [11] with the same modified Coulomb interaction the fine structure splittings for the  $\chi_c(1P)$  mesons turned out to be twice as small as the experimental data.

#### IV. CHARMONIUM SPECTRUM

Here we give as an illustration the spectrum and the constituent masses  $\omega_n$  in charmonium which were calculated solving the spinless Salpeter equation with a linear plus Coulomb potential,

$$\left(2\sqrt{\hat{p}^2 + m_c^2} + V_0(r)\right) \psi_{nL}(r) = M_{nL}^{(0)} \psi_{nL}(r). \quad (4.1)$$

In the static potential

$$V_0(r) = -\frac{4}{3} \frac{\alpha_{\text{st}}}{r} + \sigma r \quad (4.2)$$

the parameters  $\alpha_{\text{st}} = 0.42$  and  $\sigma = 0.18$  GeV<sup>2</sup> were taken. For the one-loop pole mass we used  $m_c = 1.42$  GeV. The remarkable feature of this set of parameters is that the meson mass,  $M(nL) = M^{(0)}(nL) - C_{SE}(nL)$ , contains a very small negative subtractive constant which can be strictly determined by the nonperturbative self-energy contribution:  $C_{SE} = -4\sigma \cdot 0.24/\pi\omega_{nL}$  [12] and its value  $C_{SE} = -33 \pm 3$  MeV is small and approximately equal for all  $nS$  states ( $n \leq 5$ ).

From Table I one can see that the energy  $\omega(nS)$  of the  $c$  quark, playing the role of the constituent mass of a given  $nS$  state, appears to be around 1.65-1.75 GeV, i.e., essentially larger than the current mass,  $m_c \cong 1.4$  GeV in the Salpeter equation and gives rise to a small suppression of the HF splittings. This fact can be considered as an explanation why in the nonrelativistic approach the constituent  $c$ -quark mass is usually taken to be larger, e.g., in Ref. [9]  $m_c = 1.84$  GeV. Note also that with  $\omega(1S) = 1.60$  GeV and neglecting the NP contribution of about 3-5 MeV, the value of  $\alpha_{\text{HF}}(1S) = 0.355$  is needed in order to obtain the experimental number for  $\Delta_{\text{HF}}(1S) = 117$  MeV.

#### V. CONCLUSIONS

Thus we can conclude that the Belle experiment [1] on the  $\eta_c(2S)$  mass if it is confirmed, creates a number of interesting theoretical problems, in particular, about the correct choice of the renormalization scale of  $\alpha_s(\mu)$  for the excited states. In the conventional approach

TABLE I: Spin-averaged masses  $M(nL)$  for the spinless Salpeter equation with the static potential  $V_0(r)$  ( $m_c = 1.42$  GeV,  $\sigma = 0.18$  GeV,  $\alpha_{\text{st}} = 0.42$ ).

	$\omega_{nL}$ (in GeV)	$M(nL)^a$ in MeV	$M(nL)$ in MeV
		theory	experiment
1S	1.64	3069	$3067.6 \pm 0.05$
2S	1.69	3662	$3678.0 \pm 2.6$
3S	1.75	4083	$4040 \pm 10$
4S	1.80	4433	$4415 \pm 6$
1P	1.63	3526	$3525.3 \pm 0.2$
2P	1.70	3967	—
1D	1.66	3825	( $1^3D_1$ state)
			$3769.9 \pm 2.5$
2D	1.72	4200	$4159 \pm 20?$

<sup>a)</sup> The overall self-energy constant  $C_{SE} = -33$  MeV is used.

$\alpha_{\text{HF}}(\mu_2) \cong 0.30$  and  $\Delta_{\text{HF}}(2S) \cong (57 \pm 8)$  MeV and our prediction for the mass is  $M(\eta_c(2S)) = 3630 \pm 88$  MeV which appears to be in agreement with the result of the other Belle experiment [6]:  $e^+e^- \rightarrow J/\psi, \eta_c$ . To obtain decisive conclusions it would be important to measure the  $\eta_c(2S)$  mass with better accuracy. The confirmation of the value of  $\Delta_{\text{HF}}(2S)$  in the range close to 32 MeV would require a drastic reconsideration of our understanding of the scale in the HF interaction and possibly also of the QCD factor  $\gamma_n$  present in the leptonic width, since they are interconnected.

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[1] S.-K. Choi et al., (the Belle Collaboration), Phys. Rev. Lett., **89**, 142001 (2002)

[2] Particle Data Group, Phys. Rev., **D66**, 010001-1 (2002)

- [3] J. Pantaleone, S.-H.H Tye, and Y.Y. Ng, Phys. Rev., **D33**, 777 (1986)
- [4] Yu.A. Simonov, Proc. XVII Int. Sch. Phys., Lisbon, 29 Sept. – 4 Oct., 1999, p. 60, World Scientific (2000); hep-ph/9911239
- [5] A. Martin and J.M. Richard, Phys. Lett., **bf 115 B**, 323 (1982)
- [6] K. Abe et al., (the Belle Collaboration), hep-ex/0205104
- [7] A.M. Badalian and B.L.G. Bakker, Phys. Rev., **D 64**, 114010 (2001); hep-ph/0105156
- [8] L.B. Okun and A.Yu. Khodzhamiryan, JETP Lett., **23**, 46 (1976)
- [9] E.J. Eichten, K. Lane, and C. Quigg, Phys. Rev. Lett., **89** 162002 (2002)
- [10] G.S. Bali et al., Phys. Rev. **D56**, 2566 (1997)  
D. Ebert et al., Phys. Rev. **D62**, 034014 (2000)  
E.Y. Eichten and C. Quigg, Phys. Rev. **D49**, 5845 (1994)
- [11] T.A. Lahde and D.O. Riska, Nucl. Phys. **A707**, 425 (2002); hep-ph/0112131
- [12] Yu.A. Simonov, Phys. Lett. **B515**, 137 (2001); hep-ph/0105141