

Coulomb sum rule in quasielastic region

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Within a relativistic single particle model, we calculate the Coulomb sum rule of inclusive electron scattering from ^{40}Ca and ^{208}Pb in quasielastic region. Theoretical longitudinal and transverse structure functions are extracted for three momentum transfers from 300 to 500 MeV/c and compared with the experimental data measured at Bates and Saclay. We find that there is no drastic suppression of the longitudinal structure function and that the Coulomb sum rule depends on nucleus in our theoretical model.

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Medium energy electron scattering in quasielastic region has been acknowledged one of most useful tools to investigate not only the nuclear structure and properties but also the nucleon properties in nuclear medium. It is of considerable interest in experimental and theoretical aspects to extract longitudinal and transverse structure functions as a function of energy transfer at fixed three momentum transfer, because both structure functions stand for the electric and magnetic responses of the target nucleus, respectively.

The Fermi gas model in the impulse approximation describes roughly the inclusive (e, e') cross sections but fails to reproduce the structure functions. In particular, there appeared to be a large suppression (about 50%) of the longitudinal structure function, referred to the missing strength in the Coulomb sum rule (CSR) [1], which states that the integration of the (charge response) longitudinal function over the full range of energy loss should be equal to the charge of the nucleus. Many results of the CSR for several nuclei such as ^{12}C [2], ^{40}Ca [1, 3, 4], and ^{208}Pb [5] have been reaped during the last two

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decades.

However the CSR could be affected by the nucleons' correlations, such as Pauli, long-range and short-range correlations, which are inescapable drawbacks in the mean field approach to the many nucleon systems[4]. For example, Pauli correlations simply due to the fermion nature of the nucleon cause some deviations of the CSR from the proton number Z at the low momentum transfer region [6]. Fabrocini and Fantoni [7], for instance, found that the CSR within the framework of a non-relativistic nuclear matter calculation was saturated much slower than the calculation by the Fermi gas model. The short-range correlations between nucleons may also lead to the deviations of the CSR from the proton number Z . But such correlations due to the bounded nucleons turned out to account for at most 10 % reduction from the Z number [8].

Therefore any further suppression would indicate some modifications of the nucleon by the nuclear medium. This causes the compelling and revivable motivation to investigate the Coulomb sum rule in both theoretical and experimental sides [9]. Of course, the prerequisite before meaningful debates about the medium effect is whether the suppression exist on the CSR, but the missing strength is still an unsolved problem in spite of lots of theoretical efforts.

The CSR on ^{40}Ca measured from Bates [3, 4] showed a suppression of about 30% in the q range from 300 MeV/c to 450 MeV/c. But the CSR on ^{208}Pb at Saclay [5] showed a reduction of about 50% in the effective momentum transfer from 350 MeV/c to 550 MeV/c. What these measurements depend on the nucleus might not be clearly understood. While Traini [10] insisted on the suppression of the CSR which depends on the nucleus, Jourdan [6] concluded that the longitudinal structure function is not suppressed and there is no A -dependent quenching for the Coulomb sum in the analysis of the experimental data for ^{12}C , ^{40}Ca , and ^{56}Fe .

Morgenstern and Meziani [11], on the contrary, reanalyzed the experimental data of ^{40}Ca , ^{48}Ca , ^{56}Fe , ^{197}Au , ^{208}Pb , and ^{238}U at the effective momentum transfer 500 MeV/c. They claimed that the suppression of the longitudinal structure function exists about 40 % at the effective momentum transfer 500 MeV/c, and tried to explain the suppression for a change of the nucleon properties inside the nuclear medium.

Under these unsettled and controversial situations, future progress on the CSR would be made only by the expected JLAB results at the high momentum transfer region, in

which the relevant correlations become small. But in that region, the Coulomb corrections between the electron and the nucleus have to be pinned down to extract the information of the structure functions to understand the CSR. As detailed in Ref. [8, 12], there are two different methods, effective momentum approximation (EMA) and full calculations just like Ohio group's calculations. Although at high momentum transfer region nearly same results are expected from both approaches [12, 13], there still remained some controversy below the momentum region, so that one keenly needs to compare both results at the region.

As discussed in our previous papers [14, 15, 16], our approximate treatment of the electron Coulomb distortion from medium and heavy nuclei turned out to agree to a full distorted wave Born approximation (DWBA) within a few percent (see Ref. [14, 15] in detail). This approximation allows the separation of the cross section into a longitudinal term and a transverse term while the full DWBA calculation cannot yield the separation. In this work, this approximation is taken into account in all the calculations in order to include the electron Coulomb distortion.

In this work, therefore, along the Ohio model we investigate the CSR by comparing with Bates and Saclay data in the q range from 300 MeV/c to 500 MeV/c in quasielastic region, whose recent results and detailed discussions appeared Ref. [12]. In order to calculate the nuclear transition current, we use the relativistic single particle model for the bound state wave function in the presence of the strong scalar and vector potentials based on the $\sigma - \omega$ model generated by Horowitz and Serot [17].

In the plane wave Born approximation (PWBA) in which the electrons are described as Dirac plane waves, the cross section for the inclusive (e, e') scattering can be written as

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{q_\mu^4}{q^4} R_L(q, \omega) + \left(\tan^2 \frac{\theta}{2} - \frac{q_\mu^2}{2q^2} \right) R_T(q, \omega) \right], \quad (1)$$

where $q_\mu^2 = \omega^2 - \mathbf{q}^2 = -Q^2$ is the four momentum transfer, σ_M is the Mott cross section, and R_L and R_T are the longitudinal and transverse structure functions which depend only on the three momentum transfer q and the energy transfer ω . Note that the scalar and vector potentials for the outgoing nucleons are used as the energy-dependent form in Ref. [16]. These different potentials between the bound and continuum states result in non-conserved current. Hence instead of eliminating the z -component we directly calculate the z -component of the nucleon current. Notice that this energy-dependent potential still

includes the final state interaction for the outgoing nucleon.

From the measured cross section in Eq. (1), the total structure function is defined as

$$S_{tot}(q, \omega, \theta) = \left(\frac{\epsilon(\theta)}{\sigma_M} \right) \left(\frac{q^4}{Q^4} \right) \frac{d^2\sigma}{d\Omega d\omega}, \quad (2)$$

where the $\epsilon(\theta)$ is the virtual photon polarization.

Therefore, the total structure function in Eq. (2) becomes

$$S_{tot}(q, \omega, \theta) = \epsilon(\theta)R_L(q, \omega) + \left(\frac{q^2}{2Q^2} \right) R_T(q, \omega). \quad (3)$$

S_{tot} is described as a straight line in terms of the independent variable $\epsilon(\theta)$ with slope $R_L(q, \omega)$ and intercept proportional to $R_T(q, \omega)$ by keeping the momentum transfer q and the energy transfer ω fixed.

The CSR is defined as the integration of the total longitudinal structure function in Eq. (3) for inclusive (e, e') reaction by de Forest [18]

$$C(q) = \frac{1}{Z} \int_{\omega_{min}}^{\infty} \frac{R_L(q, \omega)}{\tilde{G}_E^2(Q^2)} d\omega, \quad (4)$$

with the electric form factor given by

$$\tilde{G}_E^2(Q^2) = \left[G_{Ep}^2(Q^2) + \frac{N}{Z} G_{En}^2(Q^2) \right] \frac{(1 + \tau)}{(1 + 2\tau)}, \quad (5)$$

where Z and N are number of protons and neutrons of the target, respectively. G_{Ep} and G_{En} are the Sachs electric form factors for the protons and neutrons, respectively. The last factor corresponds to the relativistic correction factor, in which τ is given by $\tau = Q^2/4M_N^2$ with the nucleon mass M_N . The lower limit ω_{min} in the integration includes all inelastic contributions but excludes the elastic peak.

In reviewing the Bates and Saclay datasets we do not find experimental data at fixed ω and q values within given kinematics enough to obtain the longitudinal and transverse structure functions from the Rosenbluth separation in Eq. (3). Thus, expanding the range of the momentum transfer at fixed ω value, we choose that the q range at 350 MeV/c is between 320 MeV/c and 380 MeV/c and at $q=450$ MeV/c from 420 MeV/c to 480 MeV/c to obtain proper separation. Furthermore, Saclay group used the effective momentum approximation (EMA) to include the electron Coulomb distortion but this has been proved a poor approximation within the intermediate electron energy range from 300 to 600 MeV although it is good at high electron energy greater than 1.0 GeV [12, 14, 15].

Figures 1 and 2 show the longitudinal and transverse structure functions obtained from the slopes and the intercepts in Eq. (3) in terms of the energy transfer at the momentum transfer $q=350, 450$ MeV/c on ^{40}Ca . The solid curves (labelled DW) represent the results with inclusion of the electron Coulomb distortion in Ref. [16]. The dotted curves (labelled PW) are calculated without the electron Coulomb distortion which uses the plane wave for the electrons. The experimental data are taken from Bates [4]. Since the final electron energy decreases with higher energy transfer, the Coulomb effect in the transverse functions is larger than that in the longitudinal functions in this region. In particular, in this high energy transfer region, the short-range interactions lead to the correlations between nucleons in the mean field. Hence the effect of the Coulomb distortion affects the magnetization of nucleons.

Our theoretical results show good description of the experimental data in the longitudinal structure function but they do not in the transverse functions in the magnitude although they have similar shape. It should be noted that any other processes except the quasielastic scattering are not included in this calculation.

In Figs. 3 and 4, we compare our theoretical calculations with Saclay experimental data [5] of ^{208}Pb for the longitudinal and transverse structure functions in terms of ω at the momentum transfer $q=350, 450$ MeV/c. Like our previous works [14, 15], our results are not good descriptions of the longitudinal structure function with the experimental data even in shape even if we take into account that the Saclay data were given in terms of the effective momentum. However, they relatively well produce the transverse parts.

In order to calculate the CSR we need to know the lower and upper limits of the integration in Eq. (4). While the different values were used for the lower limit ω_{min} with each momentum transfer in Ref. [5], we choose $\omega_{min} = 10$ MeV which is enough to exclude the elastic process in all calculations. For the upper limit ω_{max} of the integration we use different values for each momentum transfer, $\omega_{max} = 200$ MeV for $q=300$ and 350 MeV/c, $\omega_{max} = 225$ MeV for $q=400$ MeV/c, and $\omega_{max} = 250$ MeV for $q=450$ and 500 MeV/c. In these upper limits we obtain the Coulomb sum rule $C(q)$ values within our theoretical model in Fig. 5. Our calculations of $C(q)$ for ^{40}Ca show tendency similar to the Bates data [4] but those for ^{208}Pb do not match with the Saclay data [5, 11]. From these results, there is no suppression of the longitudinal structure functions so much as 50 % obtained from Saclay group [5]. Our results exhibit the nucleus-dependence of the

CSR.

In this report, we examined the suppression of the longitudinal structure function in the CSR within the relativistic single particle model for the inclusive (e, e') reaction in the quasielastic region. Our theoretical calculations show a good description of the experimental data for the longitudinal structure function of ^{40}Ca but do not have good expressions for the transverse part on ^{40}Ca and for both functions on ^{208}Pb . The electron Coulomb distortion in the transverse part appears larger than that in the longitudinal part with higher energy transfer. Our model shows the nucleus-dependence of the CSR. We find that the suppression (about 50%) of the longitudinal structure function within our theoretical model is not so much as Saclay group insisted. Nevertheless our results show some deviation of the Coulomb sum rule from unity.

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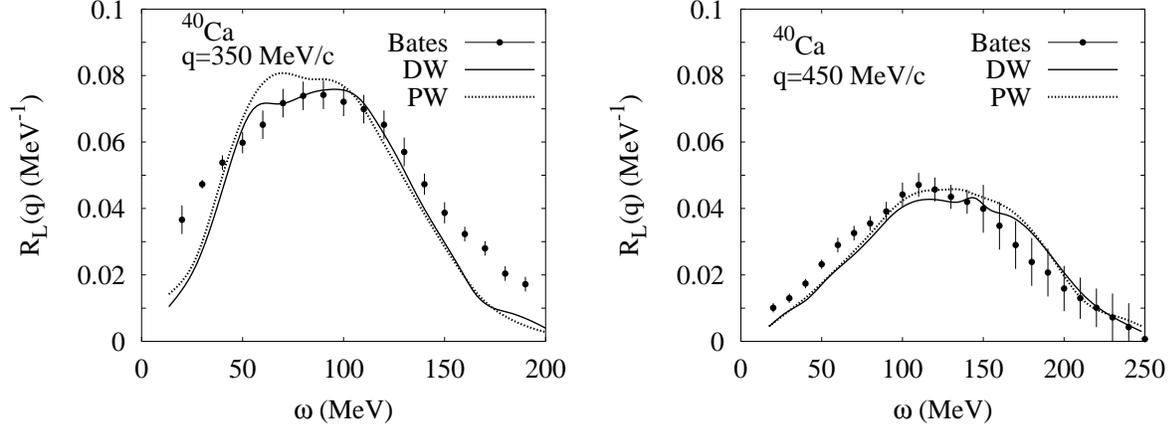


Fig. 1: The longitudinal structure functions of ^{40}Ca at $q=350, 450$ MeV/c. The solid lines are the results with the inclusion of the electron Coulomb distortion and the dotted curves are without the Coulomb distortion.

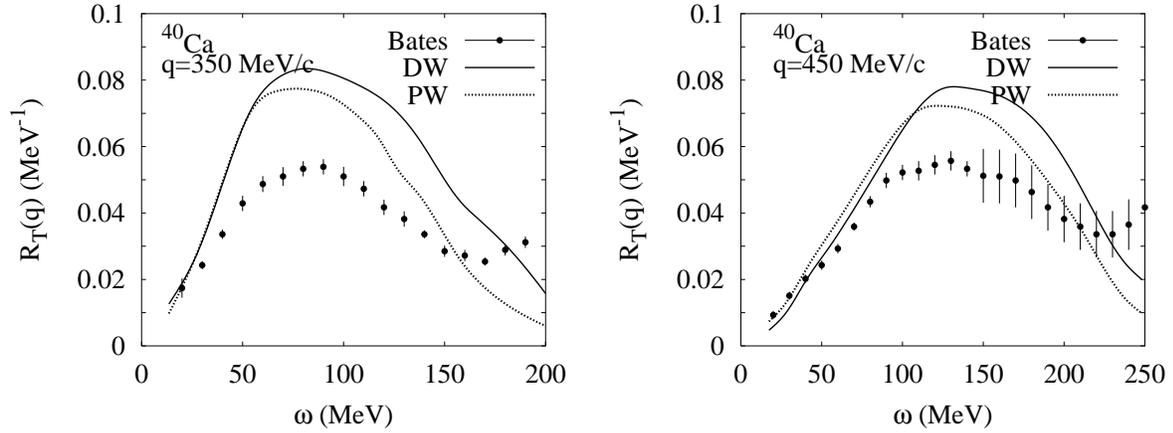


Fig. 2: The transverse structure functions of ^{40}Ca at $q=350, 450$ MeV/c. The solid lines are the results with the inclusion of the electron Coulomb distortion and the dotted curves are without the Coulomb distortion.

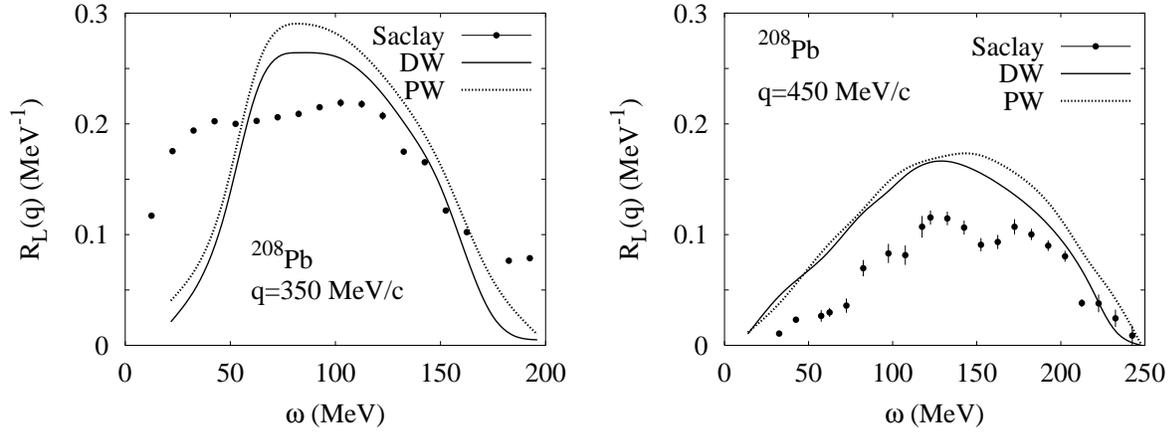


Fig. 3: The longitudinal structure functions of ^{208}Pb at $q=350, 450$ MeV/c. The solid lines are the results with the inclusion of the electron Coulomb distortion and the dotted curves are without the Coulomb distortion.

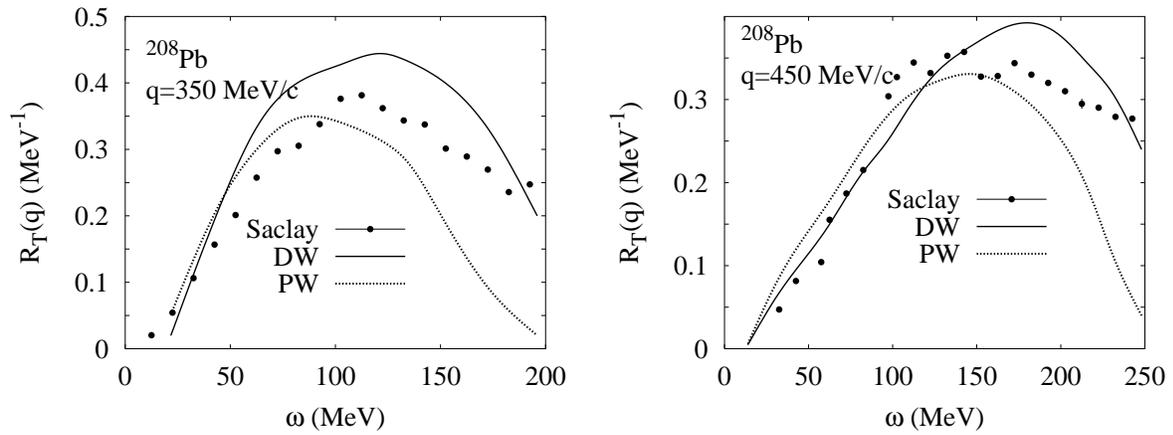


Fig. 4: The transverse structure functions of ^{208}Pb at $q=350, 450$ MeV/c. The solid lines are the results with the inclusion of the electron Coulomb distortion and the dotted curves are without the Coulomb distortion.

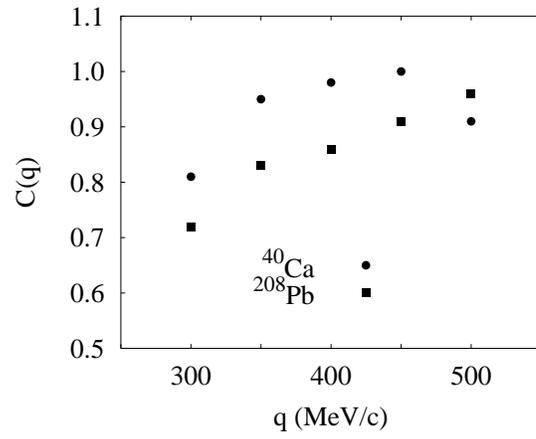


Fig. 5: The Coulomb sum rule for our model in terms of q values. The solid circles are for ^{40}Ca and the solid rectangles are for ^{208}Pb , respectively.