# Theory of light-induced current in molecular-tunneling junctions excited with intense shaped pulses

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A theory for light-induced current by strong optical pulses in molecular-tunneling junctions is described. We consider a molecular bridge represented by its highest occupied and lowest unoccupied levels, HOMO and LUMO, respectively. We take into account two types of couplings between the molecule and the metal leads: electron transfer that gives rise to net current in the biased junction and energy transfer between the molecule and electron-hole excitations in the leads. Using a Markovian approximation, we derive a closed system of equations for the expectation values of the relevant variables: populations and molecular polarization that are binary, and exciton populations that are tetradic in the annihilation and creation operators for electrons in the molecular states. We have proposed an optical control method using chirped pulses for enhancing charge transfer in unbiased junctions where the bridging molecule is characterized by a strong charge-transfer transition. An approximate analytical solution of the resulting dynamical equation is supported by a full numerical solution. When energy transfer between the molecule and electron-hole excitations in the leads is absent, the optical control problem for inducing charge transfer with linearly chirped pulse can be reduced to the Landau-Zener transition to a decaying level. When chirp is fast with respect to the rate of the electron transfer, the Landau theory is recovered. The proposed control mechanism is potentially useful for developing novel opto-electronic single-electron devices with optical gating based on molecular nanojunctions.

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## I. INTRODUCTION

Molecular electronics is one of the most promising substitutes for today's semiconductor electronics. In this relation molecular conduction nanojunctions have been under intense study in the last few years [1, 2, 3, 4]. Recently, light induced switching behavior in the conduction properties of molecular nanojunctions has been demonstrated [5, 6, 7, 8, 9, 10].

However, the use of an external electromagnetic field as a controlling tool in the small nanogap between two metal leads is difficult to implement. Currently techniques available to achieve high spatial resolution with laser illumination are limited by diffraction to about half of the optical wavelength. The introduction of near-field scanning optical microscopes (NSOM) and tip-enhanced NSOM [11] has extended the spatial resolution beyond the diffraction limit. The latter technique uses the strongly confined electromagnetic field generated by optically exciting surface plasmons localized at the apex of a sharp metallic tip, increasing spatial resolution to better than 10 nm [11]. Recently, spatial resolution at the atomic scale has been also achieved in the coupling of light to single molecules adsorbed on a surface, using scanning tunneling microscopy (STM) [8].

If experimental setups that can couple biased molecular wires to the radiation field could be achieved, general questions concerning current through the molecular nanojunctions in nonequilibrium situations come to mind. Recently Galperin and Nitzan investigated a class of molecules characterized by strong charge-transfer transitions into their first excited state [12]. The dipole moment of such molecules changes considerably upon excitation, expressing a strong shift of the electronic charge distribution. For example, the dipole moment of 4dimethylamino-4'-nitrostilbene (DMEANS) is 7 D in the ground state and  $\sim 31$  D in the first excited singlet state [13]. For all-trans retinal in polymethyl methacrylate films the dipole increases from  $\sim 6.6$  to 19.8 D upon excitation to the  ${}^{1}B_{u}$  electronic state [14] and 40 Å CdSe nanocrystals change their dipole from  $\sim 0$  to  $\sim 32$  D upon excitation to their first excited state [15]. In the independent electron picture this implies that either the highest occupied, or the lowest unoccupied, molecular orbitals (HOMO,  $|1\rangle$ , or LUMO,  $|2\rangle$ , see Fig.1) is dominated by atomic orbitals of larger amplitude (and better overlap with metal orbitals) on one side of the molecule than on the other and therefore, when used as molecular wires connecting two metal leads, stronger coupling to one of the leads. They have shown that when such molecular wire connects between two metal leads, weak steady-state optical pumping can create an internal driving force for charge flow between the leads.

A theory of light-induced effects by weak CW radiation in molecular conduction was developed in Ref.[16]. However, there are reasons to consider also molecular junctions subjected to strong electromagnetic fields. First, the structure of such junctions is compatible with configurations considered for large electromagnetic field as in tip enhanced NSOM [11]. Secondly, it was demonstrated in Ref.[11] that the combination of near-field optics and ultrafast spectroscopy is readily achieved, and the observation of photo-induced processes, such as charge transfer, energy transfer or isomerization reactions on the nanoscale is feasible [17]. Third, consideration of junction stability and integrity suggests that strong radiation fields should be applied as sequences of well separated pulses to allow for sufficient relaxation and heat dissipation. Finally, consideration of strong time dependent pulses makes it possible to study ways to optimize the desired effect, here the light induced electron tunneling, i.e. to explore possibilities of coherent control of charge flow between the leads. Our objective in the present work is to extend the theory of Refs. [12, 16] to strong fields and to apply the theory to studies of coherent control of nanojunction transport.

While these problems are of general and fundamental interest, we note that this study is related to efforts to develop novel optoelectronic single-electron devices, such as a photon–electron conversion device, optical memory, and single-electron transistors with optical gating [6]. In addition, the potential significance of molecular nanojunctions for device applications lies in the possibility of creating all-optical switches [18] that could be incorporated in future generations of optical communications systems. It is conceivable that these devices will employ coherent optical manipulations, because the speed of coherent manipulations greatly exceeds that of currently available electronic devices.

The outline of the paper is as follows. In Sec.II we introduce our model. In Sec.III we derive a closed set of equations for the expectation values of binary and tetradic variables of the annihilation and creation operators for electrons in molecular states  $|1\rangle$  and  $|2\rangle$ , and get formulas for the current and charge transferred during the electromagnetic pulse action. In Sec.IV we calculate a current induced by quasistationary intense light pulse. Optical control of current and transferred charge with chirped pulses is considered in Sec.V. We summarize our results in Sec.VI. In the Appendices we show that in the absence of the radiative and nonradiative energy transfer couplings, the equations of motion derived in the paper lead to the well known Landauer formula for the current and present auxiliary calculations.

### **II. THE MODEL HAMILTONIAN**

We consider a system that comprises a molecule represented by its highest occupied molecular orbital (HOMO),  $|1\rangle$ , and lowest unoccupied molecular orbital (LUMO),  $|2\rangle$ , positioned between two leads represented by free electron reservoirs L and R and interacting with

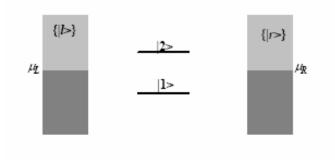


FIG. 1: A model for light induced effects in molecular conduction. The right  $(R = |\{r\}\rangle)$  and left  $(L = |\{l\}\rangle)$  manifolds represent the two metal leads characterized by electrochemical potentials  $\mu_R$  and  $\mu_L$  respectively. The molecule is represented by its highest occupied molecular orbital (HOMO),  $|1\rangle$ , and lowest unoccupied molecular orbital (LUMO),  $|2\rangle$ .

the radiation field (Fig.1). In the independent electron picture a transition between the ground and excited molecular states corresponds to transfer of an electron between levels  $|1\rangle$  and  $|2\rangle$ . The electron reservoirs (leads) are characterized by their electronic chemical potentials  $\mu_L$  and  $\mu_R$ , where the difference  $\mu_L - \mu_R = e\Phi$  is the imposed voltage bias. The Hamiltonian is

$$\hat{H} = \hat{H}_0 + \hat{V} \tag{1}$$

where

$$\hat{H}_0 = \sum_{m=1,2} \varepsilon_m \hat{n}_m + \sum_{k \in \{L,R\}} \varepsilon_k \hat{n}_k \tag{2}$$

contains additively terms that correspond to the isolated molecule (m) and the free leads (k). Here  $\hat{n}_i = \hat{c}_i^+ \hat{c}_i$  is the population operator in state i, the operators  $\hat{c}$  and  $\hat{c}^+$  are annihilation and creation operators of an electron in the various states.

The interaction term  $\hat{V}$  can be written as

$$\hat{V} = \hat{V}_M + \hat{V}_P + \hat{V}_N \tag{3}$$

where  $\hat{V}_P$  accounts for the effect of the external radiation field. The latter is represented by the (classical) function

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}^{(+)}(t) + \mathbf{E}^{(-)}(t) = \frac{1}{2}\mathbf{e}\mathcal{E}(t)\exp[-i\omega t + i\varphi(t)] + c.c.$$
(4)

characterized by the pulse envelope  $\mathcal{E}(t)$ , carrier frequency  $\omega$  and (possibly) time dependent phase  $\varphi(t)$ . The time dependent phase corresponds to time evolution of the pulse frequency (chirp)  $\omega(t) = \omega - d\varphi(t)/dt$ . Introducing bilinear operators of the excitonic type

$$b_M^+ = \hat{c}_2^+ \hat{c}_1, \ b_M = \hat{c}_1^+ \hat{c}_2, \ b_{ij}^+ = \hat{c}_i^+ \hat{c}_j = b_{ji}, \ (i \neq j), \ (5)$$

the molecule-radiation field coupling,  $\hat{V}_P$ , can be written as follows in the resonance or rotating wave approximation (RWA)

$$\hat{V}_P = -\frac{1}{2} (\mathbf{d} \cdot \mathbf{e}) \{ b_M^+ \mathcal{E}(t) \exp[-i\omega t + i\varphi(t)] + h.c. \}$$
(6)

where  $\mathbf{d}$  is the transition dipole moment.

The other terms in Eq.(3) describe coupling between the molecule and the metal electronic subsystems. In terms of the excitonic operators defined in Eq.(5), they are given by

$$\hat{V}_M = \sum_{K=L,R} \sum_{m=1,2;k \in K} (V_{km}^{(MK)} b_{mk} + h.c.), \quad (7)$$

$$\hat{V}_N = \sum_{K=L,R} \sum_{k \neq k' \in K} (V_{kk'}^{(NK)} b_{k'k} b_M^+ + V_{k'k}^{(NK)} b_M b_{k'k}^+), \quad (8)$$

L and R denote the left and right leads, respectively, and h.c. denotes Hermitian conjugate.  $\hat{V}_M$  and  $\hat{V}_N$ , Eqs. (7) and (8), respectively, denote two types of couplings between the molecule and the metal leads:  $\hat{V}_M$  describes electron transfer that gives rise to net current in the biased junction, while  $\hat{V}_N$  describes energy transfer between the molecule and electron-hole excitations in the leads. The latter interaction strongly affects the lifetime of excited molecules near metal surfaces.  $\hat{V}_N$  is written in the near field approximation, disregarding retardation effects that will be important at large molecule-lead distances.

## **III. EQUATIONS OF MOTION**

The physics of the system can be described within different approaches. One is the method of nonequi-

librium Green's functions [12, 16, 19]. It has advantages of a formal treatment due to the possibility of a diagrammatic representation, and it is particularly well suited for stationary processes where the Dyson equation can be cast in the energy representation. For timedependent processes, such as are the subject of this work, a method based on the equations of motion for the expectation values of the operators provides a more transparent approach, since the quantities are more directly related to physical observables. Such a method is adopted here. Using a Markovian approximation for the relaxation induced by the molecule-metal leads coupling, we derive a closed set of equations for the expectation values of binary  $\langle \hat{n}_m \rangle = n_m$  and  $\langle b_M \rangle = p_M$ , and tetradic  $\langle b_M^+ b_M \rangle = N_M$  variables of the annihilation and creation operators for electrons in molecular states  $|1\rangle$  and  $|2\rangle$ . The first expression is simply the population of electrons in molecular state m, the second gives the molecular polarization and the third represents the molecular excitation, referred to below as the molecular exciton population.

Using the Heisenberg equations of motion one obtains the equation for the expectation value of any operator  $\hat{F}$ 

$$\frac{d}{dt}\langle \hat{F}\rangle = \frac{i}{\hbar}\langle [\hat{H}_0 + \hat{V}, \hat{F}]\rangle \equiv \frac{i}{\hbar}Tr([\hat{H}_0 + \hat{V}, \hat{F}]\rho) \quad (9)$$

where  $\rho$  is the density matrix. Straightforward operator algebra manipulations yield for  $n_m$  and  $p_M$  in RWA

$$\frac{dn_m}{dt} = (-1)^m \operatorname{Im} \{\Omega^*(t) p_M \exp[i\omega t - i\varphi(t)]\} - -\frac{2}{\hbar} \operatorname{Im} \sum_{K=L,R} \sum_{k \in K} V_{km}^{(MK)} \langle b_{mk} \rangle - -\frac{2}{\hbar} \operatorname{Im} \sum_{K=L,R} \sum_{k \neq k' \in K} [\delta_{2m} V_{k'k}^{(NK)} \langle b_M b_{k'k}^+ \rangle + \delta_{1m} V_{kk'}^{(NK)} \langle b_M b_{k'k}^+ \rangle^*]$$
(10)

$$\frac{dp_M}{dt} = \frac{i}{\hbar} (\varepsilon_1 - \varepsilon_2) p_M + \frac{i}{2} \Omega(t) \exp[-i\omega t + i\varphi(t)] (n_1 - n_2) + \frac{i}{\hbar} \sum_{K=L,R} \sum_{k \in K} (V_{k1}^{(MK)} \langle b_{k2}^+ \rangle - V_{2k}^{(MK)} \langle b_{k1} \rangle) + \frac{i}{\hbar} \sum_{K=L,R} \sum_{k \neq k' \in K} V_{kk'}^{(NK)} \langle b_{k'k} (\hat{n}_2 - \hat{n}_1) \rangle$$
(11)

where  $\Omega(t) = (\mathbf{d} \cdot \mathbf{e}) \mathcal{E}(t)/\hbar$  is the Rabi frequency. The equations of motion include couplings to additional correlations of the second order  $\langle b_{mk} \rangle$  due to the electron-transfer interaction  $\hat{V}_M$ , and to higher-order correlations  $\langle b_M b_{k'k}^+ \rangle$  etc. due to the energy transfer  $\hat{V}_N$ . To obtain

expressions for these correlations, we now compute their equations of motion, using the Markovian approximation for the relaxations induced by the molecule-metal leads couplings,  $\hat{V}_M$  and  $\hat{V}_N$ . In this work we assume that the relaxation processes due to  $\hat{V}_M$  and  $\hat{V}_N$  are not interdependent and also do not depend on the external electromagnetic field. We shall discuss the last approach in Sec.VI

# A. Calculation of terms associated with the electron transfer interaction $\hat{V}_M$ in the equations for $n_m$ and $p_M$

In evaluating the effect of the relaxation processes associated with the electron transfer and energy transfer terms in the Hamiltonian,  $\hat{V}_M$  and  $\hat{V}_N$  respectively, we make the approximation (known as the non-crossing approximation) that these processes do not affect each other. A similar assumption is made with respect to the effect of the external field. With this in mind we consider the expectation values  $\langle b_{km} \rangle$  and  $\langle b_{mk} \rangle$  that enter the terms containing  $\hat{V}_M$  on the right-hand side of Eqs.(10) and (11) and omit  $\hat{V}_P$  and  $\hat{V}_N$  terms in the equations of motion that describe their evolution. This leads to

$$\frac{d}{dt}\langle b_{mk}\rangle = \frac{i}{\hbar}(\varepsilon_k - \varepsilon_m)\langle b_{mk}\rangle + \frac{i}{\hbar}\sum_{m'=1,2} V_{m'k}^{(MK)}\langle \hat{c}_{m'}^+ \hat{c}_m\rangle - \frac{i}{\hbar}V_{mk}^{(MK)}f_K(\varepsilon_k)$$
(12)

where we assumed that the leads are in equilibrium with the expectation values

$$\langle \hat{c}_k^+ \hat{c}_{k'} \rangle = f_K(\varepsilon_k) \delta_{kk'}, \qquad (13)$$

$$f_K(\varepsilon_k) = [\exp((\varepsilon_k - \mu_K)/k_BT) + 1]^{-1}$$

is the Fermi function,  $\delta_{kk'}$  is the Kronecker delta. Formally integrating Eq.(12), we get

$$\langle b_{mk} \rangle = \frac{i}{\hbar} \int_{-\infty}^{t} dt' \exp[\frac{i}{\hbar} (\varepsilon_k - \varepsilon_m)(t - t')] \times \\ \times \left[ \sum_{m'=1,2} V_{m'k}^{(MK)} \langle \hat{c}_{m'}^{\dagger} \hat{c}_m \rangle(t') - V_{mk}^{(MK)} f_K(\varepsilon_k) \right]$$
(14)

In the absence of the  $\hat{V}_M$  coupling this results in a set of integro-differential equations for  $\langle \hat{c}_m^+ \hat{c}_m \rangle = n_m$ and  $\langle \hat{c}_1^+ \hat{c}_2 \rangle = p_M$ . The dynamics contains memory effects, and therefore is non-Markovian. Next we make a Markovian approximation by transforming  $\langle \hat{c}_{m'}^+ \hat{c}_m \rangle$  to the interaction representation:  $\langle \hat{c}_{m'}^+ \hat{c}_m \rangle(t') =$  $\langle \hat{c}_{m'}^+ \hat{c}_m \rangle^{int}(t') \exp[\frac{i}{\hbar} (\varepsilon_{m'} - \varepsilon_m)t']$  and assuming that slowly varying function  $\langle \hat{c}_{m'}^+ \hat{c}_m \rangle^{int}(t')$  can be moved as  $\langle \hat{c}_{m'}^+ \hat{c}_m \rangle^{int}(t)$  to outside the integral [44]. Eq.(14) then becomes

$$\langle b_{mk} \rangle = \langle b_{km} \rangle^* = \frac{i}{\hbar} \{ \sum_{m'=1,2} V_{m'k}^{(MK)} \langle \hat{c}_{m'}^+ \hat{c}_m \rangle (t) \times \\ \times [\frac{i\hbar P}{\varepsilon_k - \varepsilon_{m'}} + \hbar\pi \delta(\varepsilon_k - \varepsilon_{m'})] - \\ - V_{mk}^{(MK)} f_K(\varepsilon_k) [\frac{i\hbar P}{\varepsilon_k - \varepsilon_m} + \hbar\pi \delta(\varepsilon_k - \varepsilon_m)] \}$$
(15)

where P denotes the principal value. Substituting the last result into the corresponding terms containing  $\hat{V}_M$  on the right-hand side of Eqs.(10) and (11) and keeping only resonant terms, we have

$$\frac{2}{\hbar} \operatorname{Im} \sum_{K=L,R} \sum_{k \in K} V_{km}^{(MK)} \langle b_{mk} \rangle = \sum_{K=L,R} [n_m(t)\Gamma_{MK,m} - W_{MK,m}]$$
(16)

and

$$\frac{i}{\hbar} \sum_{K=L,R} \sum_{k \in K} (V_{k1}^{(MK)} \langle b_{k2}^+ \rangle - V_{2k}^{(MK)} \langle b_{k1} \rangle) =$$
$$= -p_M(t) \sum_{K=L,R} [\frac{1}{2} (\Gamma_{MK,1} + \Gamma_{MK,2}) + i\Delta_{MK}] \quad (17)$$

where

=

$$\Gamma_{MK,m} = \frac{2\pi}{\hbar} \sum_{k \in K} |V_{km}^{(MK)}|^2 \delta(\varepsilon_k - \varepsilon_m), \qquad (18)$$

$$\Delta_{MK} = \frac{1}{\hbar} P \sum_{k \in K} \left[ \frac{|V_{k1}^{(MK)}|^2}{\varepsilon_k - \varepsilon_1} - \frac{|V_{2k}^{(MK)}|^2}{\varepsilon_k - \varepsilon_2} \right]$$
(19)

is the correction to the frequency of molecular transition  $(\varepsilon_2 - \varepsilon_1)/\hbar$  due to electron transfer between the molecule and lead K,

$$W_{MK,m} = \frac{2\pi}{\hbar} \sum_{k \in K} f_K(\varepsilon_k) |V_{km}^{(MK)}|^2 \delta(\varepsilon_k - \varepsilon_m) \quad (20)$$

$$= f_K(\varepsilon_m)\Gamma_{MK,m} \tag{21}$$

It should be noted that the second equality, Eq.(21), is valid only provided that molecular state  $\varepsilon_m$  is far from the Fermi level of lead K. The point is that the position of  $\varepsilon_m$  influences on the bath correlation frequency,  $\omega_c$ , for the relaxation parameter  $W_{MK,m}$  ( $\omega_c$  is the frequency interval at which the interaction of a system (molecule) with bath (metal leads) is essentially changed). Really, if molecular states  $\varepsilon_m$  are far from the Fermi levels of both leads,  $\omega_c$  for  $W_{MK,m}$  is the same as that for  $\Gamma_{MK,m}$ , which is determined by the frequency interval for the system-bath interaction matrix element  $V_{km}^{(MK)}$  and the density of states of metal leads. For this case  $\omega_c$  can be evaluated as 1-10 eV [20]. The situation is different if we assume that the molecular level position is pinned to the Fermi energy of a lead. In the latter case  $\omega_c$  for  $W_{MK,m}$ is determined also by the frequency interval at which  $f_K(\varepsilon)$  is essentially changed that is  $\sim k_B T/\hbar = 0.026$ eV for room temperature (see Eq.(20)). Since the value of  $\omega_c$  places a limit on the used approximation, according to which the relaxation parameters do not depend on exciting electromagnetic field (see Sec.VI), one can use Eq.(21) only in the case when the bath correlation frequency  $\omega_c$  is the same for both  $W_{MK,m}$  and  $\Gamma_{MK,m}$ .

One can easily see from Eqs.(10), (11), (16) and (17) that in the absence of energy transfer  $(\hat{V}_N)$ , equations for the populations of molecular states and molecular polarization form a closed set of the equations of motion.

### B. Calculation of terms related to energy transfer in the equations for $n_m$ and $p_M$

The calculation of terms related to energy transfer in Eqs.(10) and (11) is similar to that of Sec.III A. Invoking again the non-crossing approximation by omitting  $\hat{V}_P$  and  $\hat{V}_M$  terms in the equations of motion for the expectation values  $\langle b_M b_{k'k}^+ \rangle$  and  $\langle b_{k'k}(\hat{n}_2 - \hat{n}_1) \rangle$ , which appear on the right-hand sides of Eqs.(10) and (11), respectively, we get

$$\frac{d}{dt} \langle b_M b_{k'k}^+ \rangle = \frac{i}{\hbar} (\varepsilon_{k'} - \varepsilon_k - \varepsilon_2 + \varepsilon_1) \langle b_M b_{k'k}^+ \rangle + \frac{i}{\hbar} V_{kk'}^{(NK)} \{ f_K(\varepsilon_k) [1 - f_K(\varepsilon_{k'})] \langle b_M^+ b_M \rangle - f_K(\varepsilon_{k'}) [1 - f_K(\varepsilon_k)] \langle b_M b_M^+ \rangle \}$$
(22)

$$\frac{d}{dt} \langle b_{k'k}(\hat{n}_2 - \hat{n}_1) \rangle = \frac{i}{\hbar} (\varepsilon_k - \varepsilon_{k'}) \langle b_{k'k}(\hat{n}_2 - \hat{n}_1) \rangle + \\ + \frac{i}{\hbar} V_{k'k}^{(NK)} \{ f_K(\varepsilon_{k'}) [1 - f_K(\varepsilon_k)] + \\ + f_K(\varepsilon_k) [1 - f_K(\varepsilon_{k'})] \} p_M$$
(23)

Formally integrating the last equations, performing Markovian approximation and substituting the results into the corresponding terms containing  $\hat{V}_N$  on the right-hand side of Eqs.(10) and (11), we obtain

$$-\frac{2}{\hbar} \operatorname{Im} \sum_{K=L,R} \sum_{k \neq k' \in K} V_{k'k}^{(NK)} \langle b_M b_{k'k}^+ \rangle$$
$$= \sum_{K=L,R} [B_{NK}(\varepsilon_1 - \varepsilon_2, \mu_K) \langle b_M b_M^+ \rangle$$
$$- B_{NK}(\varepsilon_2 - \varepsilon_1, \mu_K) \langle b_M^+ b_M \rangle]$$
(24)

and

$$\frac{i}{\hbar} \sum_{K=L,R} \sum_{k \neq k' \in K} V_{kk'}^{(NK)} \langle b_{k'k}(\hat{n}_2 - \hat{n}_1) \rangle = -p_M(t) \sum_{K=L,R} \{ i\Delta_{NK} + \frac{1}{2} [B_{NK}(\varepsilon_1 - \varepsilon_2, \mu_K) + B_{NK}(\varepsilon_2 - \varepsilon_1, \mu_K)] \}$$
(25)

where

×

$$B_{NK}(\varepsilon_m - \varepsilon_n, \mu_K) = \frac{2\pi}{\hbar} \sum_{k \neq k' \in K} |V_{kk'}^{(NK)}|^2 \times \delta(\varepsilon_k - \varepsilon_{k'} + \varepsilon_m - \varepsilon_n) f_K(\varepsilon_k) [1 - f_K(\varepsilon_{k'})]$$
(26)

and

$$\Delta_{NK} = \frac{1}{\hbar} P \sum_{k \neq k' \in K} \{ f_K(\varepsilon_{k'}) [1 - f_K(\varepsilon_k)] + f_K(\varepsilon_k) [1 - f_K(\varepsilon_{k'})] \} \frac{|V_{kk'}^{(NK)}|^2}{\varepsilon_k - \varepsilon_{k'} + \varepsilon_2 - \varepsilon_1} \quad (27)$$

is the correction to the frequency of molecular transition  $(\varepsilon_2 - \varepsilon_1)/\hbar$  due to energy transfer between the molecule

and lead K. In deriving Eqs. (24) and (25) we have used the arguments, which are similar to those used above in the derivation of Eqs. (16) and (17).

One can see from Eqs.(10) and (24) that in the presence of energy transfer  $(\hat{V}_N)$ , equations for the populations of molecular states and molecular polarization do not form a closed set of the equations of motion. They must be supplemented, at least, with equations for the expectation values of tetradic variables  $\langle b_M b_M^+ \rangle$  and  $\langle b_M^+ b_M \rangle = N_M$ , where  $\langle b_M b_M^+ \rangle$  and  $N_M$  are related by the following equation:

$$\langle b_M b_M^+ \rangle = N_M - n_2 + n_1 \tag{28}$$

# C. Equation for $\langle b_M b_M^+ \rangle$

Using Eq.(9), straightforward operator algebra manipulations yield for  $\langle b_M b_M^+ \rangle$  in RWA

$$\frac{d\langle b_M b_M^+ \rangle}{dt} = -\frac{2}{\hbar} \operatorname{Im} \sum_{K=L,R} \sum_{k \in K} (V_{k1}^{(MK)} \langle b_{2k} b_M^+ \rangle - V_{2k}^{(MK)} \langle b_{k1} b_M^+ \rangle) + \frac{2}{\hbar} \operatorname{Im} \sum_{K=L,R} \sum_{k \neq k' \in K} V_{k'k}^{(NK)} \langle b_M b_{k'k}^+ \rangle - \operatorname{Im} \{\Omega^*(t) \exp[i\omega t - i\varphi(t)] p_M\}$$
(29)

where the second term on the right-hand side of Eq.(29) has been calculated above, Eq.(24). The first term on the right-hand sides of Eq.(29) is associated with the electron transfer process. To evaluate it we consider the equations of motion for the expectation values  $\langle b_{2k}b_M^+\rangle$  and  $\langle b_{k1}b_M^+\rangle$ , omitting  $\hat{V}_P$  and  $\hat{V}_N$  interactions and keeping only resonant terms:

$$\frac{d\langle b_{2k}b_M^+\rangle}{dt} = \frac{i}{\hbar}(\varepsilon_k - \varepsilon_1)\langle b_{2k}b_M^+\rangle + \frac{i}{\hbar}V_{1k}^{(MK)}[\langle b_M b_M^+\rangle - f_K(\varepsilon_k)(1 - n_2)]$$
(30)

$$\frac{d\langle b_{k1}b_M^+\rangle}{dt} = \frac{i}{\hbar}(\varepsilon_2 - \varepsilon_k)\langle b_{k1}b_M^+\rangle + \frac{i}{\hbar}V_{k2}^{(MK)}[(1 - f_K(\varepsilon_k))n_1 - \langle b_M b_M^+\rangle]$$
(31)

Integrating Eqs.(30) and (31), performing Markovian approximation and substituting the results into the first term on the right-hand side of Eq.(29), we get

$$-\frac{2}{\hbar} \operatorname{Im} \sum_{K=L,R} \sum_{k \in K} (V_{k1}^{(MK)} \langle b_{2k} b_M^+ \rangle - V_{2k}^{(MK)} \langle b_{k1} b_M^+ \rangle) =$$
$$= \sum_{K=L,R} [-\langle b_M b_M^+ \rangle (\Gamma_{MK,1} + \Gamma_{MK,2}) + (1 - n_2) W_{MK,1} +$$
$$+ (\Gamma_{MK,2} - W_{MK,2}) n_1]$$
(32)

where  $\Gamma_{MK,m}$  and  $W_{MK,m}$  were defined in Eqs.(18), (20) and (21).

### D. A closed set of the equations of motion

Using Eqs.(10), (11), (16), (17), (18), (19), (20), (21), (24), (25), (26), (27), (28), (29), (32) and switching to the system that rotates with instantaneous frequency  $\omega(t)$ ,  $\tilde{p}_M(t) = p_M(t) \exp\{i[\omega t - \varphi(t)]\}$ , we obtain a closed set of equations for the quantities that vary slowly with time during the period of a light wave

$$\frac{dn_m}{dt} = (-1)^m \operatorname{Im}[\Omega^*(t)\tilde{p}_M] + W_{Mm} - \Gamma_{Mm}n_m - (-1)^m [B_N(\varepsilon_2 - \varepsilon_1)N_M - (n_1 - n_2 + N_M)B_N(\varepsilon_1 - \varepsilon_2)]$$
(33)

$$\frac{d\tilde{p}_M}{dt} = i[\omega(t) - \omega_0]\tilde{p}_M + \frac{i}{2}\Omega(t)(n_1 - n_2) - \frac{1}{2}\Gamma_{MN}\tilde{p}_M \quad (34)$$

$$\frac{dN_M}{dt} = (\Gamma_{M1} - W_{M1})n_2 + W_{M2}(1 - n_1) + \operatorname{Im}[\Omega^*(t)\tilde{p}_M] - [B_N(\varepsilon_2 - \varepsilon_1) + \tilde{\Gamma}_{M1} + \tilde{\Gamma}_{M2}]N_M + (n_1 - n_2 + N_M)B_N(\varepsilon_1 - \varepsilon_2)$$
(35)

where

$$\Gamma_{Mm} = \sum_{K=L,R} \Gamma_{MK,m}, \ W_{Mm} = \sum_{K=L,R} W_{MK,m} \quad (36)$$

$$\sum_{K=L,R} B_{NK}(\varepsilon_m - \varepsilon_n, \mu_K) = B_N(\varepsilon_m - \varepsilon_n)$$
(37)

$$\Gamma_{MN} = \Gamma_{M1} + \Gamma_{M2} + B_N(\varepsilon_2 - \varepsilon_1) + B_N(\varepsilon_1 - \varepsilon_2)$$
(38)

and  $\omega_0 \equiv (\varepsilon_2 - \varepsilon_1)/\hbar + \sum_{K=L,R} (\Delta_{NK} + \Delta_{MK})$  is the frequency of the molecular transition with the corrections due to energy and electron transfer between the molecule and the leads.

As indicated above, equations for the populations of molecular states and molecular polarization, Eqs.(33) and (34), form a closed set of the equations of motion if the energy transfer is absent  $(B_N(\varepsilon_m - \varepsilon_n) = 0)$ . When energy transfer is present they must be supplemented with Eq.(35) for the exciton population. On the other hand, in the absence of electron transfer  $(W_{Mm} =$  $\Gamma_{Mm} = 0)$  Eq.(35) coincides with Eq.(33) for  $n_2$ , which implies that  $N_M = n_2$ . Indeed,  $N_M = \langle \hat{c}_2^+ \hat{c}_1 \hat{c}_1^+ \hat{c}_2 \rangle =$  $\langle \hat{n}_2(1-\hat{n}_1) \rangle = \langle \hat{n}_2^2 \rangle = \langle \hat{n}_2 \rangle = n_2$  when the electron population on the molecule is conserved, i.e. when  $\hat{V}_M = 0$ . It is the combined effect of the electron and energy transfer, represented by the terms  $\hat{V}_M$  and  $\hat{V}_N$  in the Hamiltonian, that leads to the need to include Eq.(35) in the closed set of the equations of motion.

# E. Calculation of current and transferred charge

The electronic current I is given by the rate at which the number of electrons changes in any of leads, e.g. [19, 20]

$$I = e \frac{d}{dt} \sum_{k \in L} \langle \hat{n}_k \rangle = \frac{ie}{\hbar} \sum_{k \in L} \langle [\hat{H}, \hat{n}_k] \rangle \tag{39}$$

Evaluating the commutator in Eq.(39), we get

$$I = \frac{2e}{\hbar} \operatorname{Im} \sum_{m=1,2} \sum_{k \in L} V_{km}^{(MK)} \langle b_{mk} \rangle$$
$$= e \sum_{m=1,2} [n_m(t) \Gamma_{ML,m} - W_{ML,m}]$$
(40)

where we used Eq.(16). Correspondingly, the charge transferred during an electromagnetic pulse of finite duration is given by  $Q = \int_{-\infty}^{\infty} I(t) dt$ .

In the Appendix A we show that in the absence of the radiative and nonradiative energy transfer couplings,  $\hat{V}_P$  and  $\hat{V}_N$ , Eqs.(18), (20), (21), (33), (36) and (40) lead to the well known Landauer formula for the current [21].

# IV. CURRENT INDUCED BY A QUASISTATIONARY LIGHT PULSE

In this section we calculate the current induced in a molecular nanojunctions by a strong quasistationary light pulse. Here and in the next section we assume that the molecular energy gap  $\varepsilon_2 - \varepsilon_1$  is much larger than the voltage bias  $\mu_L - \mu_R$  and that the HOMO and LUMO energies,  $\varepsilon_1$  and  $\varepsilon_2$ , are positioned rather far ( $\gg k_B T$ ) from the Fermi levels of both leads, so that the dark (Landauer) current through the junction is small and may be disregarded. Using for this situation  $W_{MK,1} \approx \Gamma_{MK,1}, W_{MK,2} \approx B_{NK}(\varepsilon_1 - \varepsilon_2, \mu_K) \approx 0$ , we obtain from Eqs.(33), (34), (35) and (40)

$$\frac{dn_1}{dt} = -\operatorname{Im}[\Omega^*(t)\tilde{p}_M] + \Gamma_{M1}(1-n_1) + B_N(\varepsilon_2 - \varepsilon_1)N_M$$
(41)

$$\frac{dn_2}{dt} = \operatorname{Im}[\Omega^*(t)\tilde{p}_M] - \Gamma_{M2}n_2 - B_N(\varepsilon_2 - \varepsilon_1)N_M \quad (42)$$

$$\frac{d\tilde{p}_M}{dt} = i[\omega(t) - \omega_0]\tilde{p}_M + \frac{i}{2}\Omega(t)(n_1 - n_2) - \frac{1}{2}\Gamma_{MN}\tilde{p}_M \quad (43)$$

$$\frac{dN_M}{dt} = \operatorname{Im}[\Omega^*(t)\tilde{p}_M] - [B_N(\varepsilon_2 - \varepsilon_1) + \Gamma_{M1} + \Gamma_{M2}]N_M$$
(44)

1 3 7

$$I = e[(n_1 - 1)\Gamma_{ML,1} + n_2\Gamma_{ML,2}]$$
(45)

One can see from Eq.(45) that the current strongly increases when  $n_2, 1 - n_1 \sim 1$ , which can be realized for strong light fields. If we further assume that the pulse amplitude  $\mathcal{E}(t)$  and frequency  $\omega(t)$  change slowly on the time scale of all relaxation times as well as the reciprocal Rabi frequency, one can put all time derivatives on the left-hand sides of Eqs.(41), (42), (43) and (44) equal to zero, and the resulting stationary equations can be easily solved

$$n_{2} = \frac{\Omega^{2}(t)(\Gamma_{M2} + \Gamma_{M1})/(4\Gamma_{M2})}{\frac{(\Gamma_{M2} + \Gamma_{M1})^{2}}{4\Gamma_{M1}\Gamma_{M2}}\Omega^{2}(t) + (\Gamma_{MN}/2)^{2} + [\omega_{0} - \omega(t)]^{2}}$$
(46)

$$n_1 = 1 - n_2 \frac{\Gamma_{M2}}{\Gamma_{M1}} \tag{47}$$

$$N_M = \frac{\Gamma_{M2}}{\Gamma_{M1} + \Gamma_{M2}} n_2 \tag{48}$$

$$\tilde{p}_{M} = \frac{\Omega(t)}{2} \frac{i(\Gamma_{MN}/2) - [\omega(t) - \omega_{0}]}{\Omega^{2}(t) \frac{(\Gamma_{M2} + \Gamma_{M1})^{2}}{4\Gamma_{M1}\Gamma_{M2}} + (\Gamma_{MN}/2)^{2} + [\omega_{0} - \omega(t)]^{2}}$$
(49)

This solution corresponds to the molecular level and exciton populations as well as polarization adiabatically following the optical pulse. Substituting Eqs. (46) and (47)into Eq. (45), we get

$$I(t) = e \frac{\Gamma_{M2} + \Gamma_{M1}}{4\Gamma_{M1}\Gamma_{M2}} \frac{\Omega^2(t)(\Gamma_{ML,2}\Gamma_{MR,1} - \Gamma_{ML,1}\Gamma_{MR,2})}{\Omega^2(t)\frac{(\Gamma_{M2} + \Gamma_{M1})^2}{4\Gamma_{M1}\Gamma_{M2}} + (\frac{\Gamma_{MN}}{2})^2 + [\omega_0 - \omega(t)]^2}$$
(50)

At steady-state 
$$(\omega(t) = \omega, \Omega(t) = \Omega)$$
 and small fields,  
 $\Omega^2 \frac{(\Gamma_{M2} + \Gamma_{M1})^2}{4\Gamma_{M1}\Gamma_{M2}} \ll (\Gamma_{MN}/2)^2$ , this becomes

$$I = \frac{e\Omega^2}{4} \frac{\Gamma_{M2} + \Gamma_{M1}}{(\Gamma_{MN}/2)^2 + (\omega_0 - \omega)^2} \frac{\Gamma_{ML,2}\Gamma_{MR,1} - \Gamma_{ML,1}\Gamma_{MR,2}}{\Gamma_{M1}\Gamma_{M2}}$$
(51)

The last equation is similar to Eq.(50) of Ref. [16] with the only difference that the latter corresponds to the substitution of the sum  $\Gamma_{M2} + \Gamma_{M1}$  on the right-hand side of Eq.(51) by  $\Gamma_{MN} = \Gamma_{M2} + \Gamma_{M1} + B_N(\varepsilon_2 - \varepsilon_1) >$  $\Gamma_{M2} + \Gamma_{M1}$ . The difference may arise from the fact that Eq.(50) of Ref.[16] is obtained in the much used approximation of strong dephasing [22]  $N_M = \langle \hat{n}_2(1-\hat{n}_1) \rangle \approx$  $n_2(1-n_1)$ . For small fields the latter term is of order  $\Omega^4$ , since  $n_2, 1 - n_1 \sim \Omega^2$ . As a matter of fact, when the exciting field is weak, one can neglect the term  $B_N(\varepsilon_2-\varepsilon_1)n_2(1-n_1)\sim \Omega^4$  with respect to  $\Gamma_{M1}(1-n_1)\sim$  $\Omega^2$  and  $\Gamma_{M2}n_2 \sim \Omega^2$  on the right-hand sides of Eqs.(41) and (42), respectively. In other words, the approximation of strong dephasing  $N_M \approx n_2(1-n_1)$  disregards the depletion of state 2 due to energy transfer for small fields, and therefore results in some overestimation of the current. In contrast, our present approach takes the tetradic variable  $N_M$  into account exactly (in the framework of the Markovian approximation) and does describe the depletion of state 2 due to energy transfer.

For strong fields and near resonance excitation,  $\Omega^2(t) \frac{(\Gamma_{M2}+\Gamma_{M1})^2}{4\Gamma_{M1}\Gamma_{M2}} \gg (\Gamma_{MN}/2)^2, [\omega_0 - \omega(t)]^2, \text{ Eqs.}(46),$ (47), (48) and (49) describe the saturation effect

$$n_1 = n_2 = \frac{\Gamma_{M1}}{\Gamma_{M2} + \Gamma_{M1}},$$
 (52)

$$N_M = \frac{\Gamma_{M1}\Gamma_{M2}}{(\Gamma_{M2} + \Gamma_{M1})^2},\tag{53}$$

$$\tilde{p}_M = \frac{2\Gamma_{M1}\Gamma_{M2}}{(\Gamma_{M2} + \Gamma_{M1})^2} \frac{i(\Gamma_{MN}/2) - [\omega(t) - \omega_0]}{\Omega(t)}, \quad (54)$$

and Eq.(40) gives

$$I = e \frac{\Gamma_{ML,2}\Gamma_{MR,1} - \Gamma_{ML,1}\Gamma_{MR,2}}{\Gamma_{M2} + \Gamma_{M1}}$$
(55)

Eqs.(50) and Eq.(55) show that the optically induced current increases linearly with the pulse intensity  $\sim \Omega^2$  for weak fields, and saturates at the maximal value given by Eq.(55), for strong fields. As is easy to see from Eqs.(52) and (53), in the latter case  $N_M = n_2(1 - n_1)$ , since the strong dephasing limit where  $\tilde{p}_M$  can be disregarded is realized under saturation effect (see Eq.(54)).

# V. OPTICAL CONTROL OF CURRENT AND TRANSFERRED CHARGE WITH CHIRPED PULSES

In the previous section we have generalized the results of Refs.[12, 16] to the quasistationary strong electromagnetic field limit. As mentioned in the introduction, future generations of optical communication systems will employ coherent optical manipulations whose speed greatly exceeds that of currently available electronic devices. We next consider such coherent control processes.

Two well-known procedures based on a coherent excitation can, in principle, produce complete population inversion in an ensemble of two-level atoms. One of them is the  $\pi$ -pulse excitation [23], which makes use of the Rabi population oscillations. This approach has been successfully demonstrated in atoms as well as semiconductor quantum dots, often referred to as artificial atoms [24, 25, 26, 27]. The main disadvantage of the  $\pi$ -pulse excitation method is the requirement for resonant light source and the need of precise control of the pulse area [28].

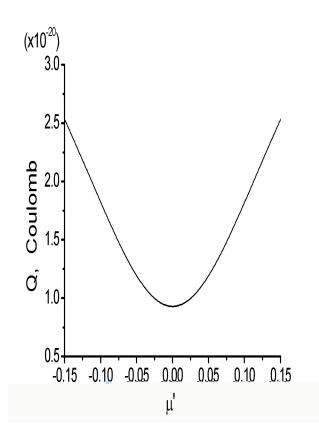
The second procedure, known as adiabatic rapid passage (ARP) [23, 28, 29, 30, 31, 32, 33, 34], enables us to transfer the entire population from ground  $|1\rangle$  to the excited  $|2\rangle$  electronic state. It is based on sweeping the pulse frequency through a resonance. The mechanism of ARP can be explained by avoided crossing of dressed (adiabatic) states

$$\Phi_{+}(t) = \sin \vartheta(t) |1\rangle + \cos \vartheta(t) |2\rangle$$

$$\Phi_{-}(t) = \cos \vartheta(t) |1\rangle - \sin \vartheta(t) |2\rangle$$
(56)

as a function of the instantaneous laser pulse frequency  $\omega(t)$  [28]. Here the mixing angle  $\vartheta(t)$  is defined (modulo  $\pi$ ) as  $\vartheta = (1/2) \arctan \frac{\Omega(t)}{\omega_0 - \omega(t)}$ , where  $\Omega(t)$  is the Rabi frequency. During the excitation, the mixing angle rotates clockwise from  $\vartheta(-\infty) = \pi/2$  to  $\vartheta(+\infty) = 0$  and the composition of adiabatic states changes accordingly. In particular, starting from state |1 >, the system follows the adiabatic (dressed) state  $\Phi_+(t)$  and eventually ends up in state |2 > [31]. A scheme based on ARP is robust since it is insensitive to pulse area and the precise location of the resonance. Therefore, we shall focus in what follows on ARP as a way to control optically induced charge transfer in molecular nanojunctions. The application of our formalism to the coherent optoelectronic properties of nanojunctions with quantum dots, using  $\pi$ -pulse excitation, [27], will be analyzed elsewhere.

As a particular example we shall consider a lightinduced charge transfer in molecular nanojunctions, using linear chirped pulses  $\omega(t) = \omega - \bar{\mu}(t - t_0)$  where  $\bar{\mu} = d^2 \varphi(t)/dt^2 = const.$ 



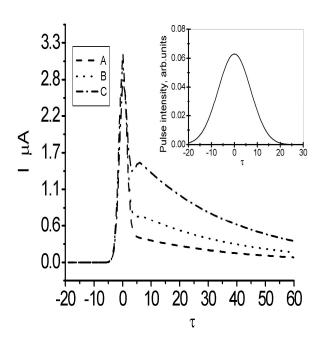


FIG. 3: The current I as a function of time  $\tau = \omega_0 t$  for the linear chirp rate  $\bar{\mu}/\omega_0^2 = 0$  (A), 0.07 (B) and 0.15 (C). Other parameters are identical to those of Fig.2. The figure illustrates how signatures of ARP increase the induced current. Inset, the square of electric field amplitude of the exciting pulse in arbitrary units.

FIG. 2: The charge Q transferred after the completion of the pulse action as a function of the linear chirp rate  $\mu' = \bar{\mu}/\omega_0^2$  ( $\omega_0$  is defined below Eq.(38)). Other parameters are as follows  $\omega = \omega_0 = 3$  eV,  $\Gamma_{M2}/\omega_0 = 0.03$ ,  $\Gamma_{M1}/\omega_0 = 0.04$ ,  $\Gamma_{MN} = \Gamma_{M1} + \Gamma_{M2}$ ,  $d\mathcal{E}_0/\hbar\omega_0 = 0.2$ ,  $\delta/\omega_0 = 0.1$ ,  $\Gamma_{ML,1} = 0.01$ eV,  $\Gamma_{ML,2} = 0.02$  eV.

### A. Numerical results

Figures 2 and 3 show the influence of  $\bar{\mu}$ , the chirp rate in the time domain, on the charge transferred during one electromagnetic pulse action. These results are obtained by numerical solution of Eqs.(41), (42), (43), (44) and (45) for a Gaussian pulse of the shape

$$E(t) \equiv \mathcal{E}(t) \exp(i\varphi(t)) = \mathcal{E}_0 \exp[-\frac{1}{2}(\delta^2 - i\bar{\mu})(t - t_0)^2]$$
(57)

and are displayed as a function of  $\bar{\mu}$ . We see that pulse chirping can increase the transferred charge (Fig.2) and the induced current (Fig.3) that can be explained by signatures of ARP (see below).

If chirped pulses are obtained by changing the separation of pulse compression gratings, the parameters  $\delta$  and  $\bar{\mu}$  are determined by the formulae [35, 36]:

$$\delta^{2} = 2\tau_{p0}^{2} [\tau_{p0}^{4} + 4\Phi^{\prime\prime 2} (\omega)]^{-1},$$
  
$$\bar{\mu} = -4\Phi^{\prime\prime} (\omega) \left[\tau_{p0}^{4} + 4\Phi^{\prime\prime 2} (\omega)\right]^{-1},$$
(58)

where  $\tau_{p0} = t_{p0}/\sqrt{2 \ln 2}$ ,  $t_{p0}$  is the pulse duration of the corresponding transform-limited pulse, and  $\Phi''(\omega)$ is the chirp rate in the frequency domain. The latter is defined by writing the electric field at frequency  $\tilde{\omega}$ as  $|E(\tilde{\omega})| \exp[i\Phi(\tilde{\omega})]$ , and expanding phase term  $\Phi(\tilde{\omega})$  in a Taylor series about the carrier frequency  $\omega \Phi(\tilde{\omega}) =$  $\Phi(\omega) + (1/2)\Phi''(\omega)(\tilde{\omega} - \omega)^2 + \dots$  Note that the local field in the junction reflects also plasmon excitation in the leads, and taking the incident pulse shape as affected only by the compression gratings used disregards the possible contribution of the near field responce of plasmonic excitations in the leads [17, 37]. Such effects will be considered elsewhere.

Figs.4 and 5 show the calculation results of the transferred charge Q as a function of the chirp rate in the frequency domain  $\Phi''(\nu) = 4\pi^2 \Phi''(\omega)$ . The calculated dependences  $Q(\Phi''(\nu))$  for curves A, B and C are confined to the values of an argument  $|\Phi''(\nu)| > 0$  corresponding to  $d\mathcal{E}_0/\hbar\omega_0 \leq 0.3$  (*d* is the molecular dipole moment, cf Eq.(6)), since our theory uses RWA. One can see that Qgrows rapidly for small  $|\Phi''(\nu)|$ . The growth of Q slows down for moderate  $|\Phi''(\nu)|$ , and then Q tends to a constant value for large  $|\Phi''(\nu)|$ . The larger is pulse energy, the larger is the value of  $|\Phi''(\nu)|$ , at which the growth of Q slows down. Fig.4 corresponds to the absence of the energy transfer  $(B_N(\varepsilon_2 - \varepsilon_1) = 0)$ , and Fig. 5 illustrates the influence of the energy transfer  $(B_N(\varepsilon_2 - \varepsilon_1) \neq 0)$ ,

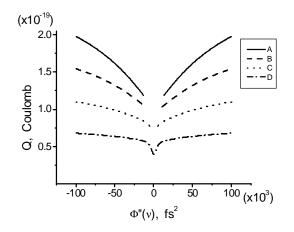


FIG. 4: The charge transferred after the completion of the pulse action as a function of the chirp rate in the frequency domain  $\Phi''(\nu)$ . The value of  $d\mathcal{E}_0/\hbar\omega_0 = 0.6$  (A), 0.5 (B), 0.4 (C) and 0.3 (D) for the transform-limited pulse. In the course of chirping the pulse energy is conserved so that  $\int_{-\infty}^{\infty} \mathcal{E}^2(t) dt = \mathcal{E}_0^2 \sqrt{\frac{\pi}{2} [\tau_{p0}^2 + \frac{4\Phi''^2(\omega)}{\tau_{p0}^2}]} = const$ , and  $\mathcal{E}_0$  decreases when  $|\Phi''(\nu)|$  increases;  $\tau_{p0} = 11$  fs. Other parameters are identical to those of Fig.2.

which diminishes the corresponding values of Q (see also Fig.6). The behavior and values of Q shown in Figs.2, 4, 5 and 6 can be rationalized by the theoretical consideration below.

Fig. 7 illustrates the influence of detuning between the carrier pulse frequency  $\omega$  and the corrected frequency of the molecular transition  $\omega_0$  on the transferred charge Q.

To end this section we note that the current that corresponds to the expectation value of  $Q = 0.5 \cdot 10^{-19} C$  per pulse (corresponding to curve D in Fig.4) and to an estimated pulse repetition frequency of 82 MHz [27] results in a small but measurable value of about  $4 \times 10^{-12}$  ampere.

#### B. Analytical consideration

The problem under consideration above can be solved analytically in certain conditions.

1. Chirped pulse control of charge transfer in molecular nanojunctions as the Landau-Zener transition to a decaying level

Consider first an excitation of the molecular nanojunction with a linear chirped pulse  $\omega(t) = \omega_0 - \bar{\mu}t$  ( $t_0 = 0$ ,  $\omega = \omega_0$ ) of a constant amplitude ( $|\Omega| = const$ ) in the absence of energy transfer:  $B_N(\varepsilon_2 - \varepsilon_1) = \Delta_{NK} = 0$ ,

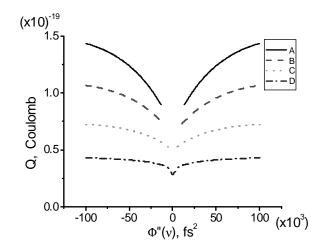


FIG. 5: The charge transferred after the completion of the pulse action as a function of the chirp rate in the frequency domain  $\Phi''(\nu)$  in the presence of energy transfer  $B_N(\varepsilon_2 - \varepsilon_1)/\omega_0 = 0.01$ . Other parameters are identical to those of Fig.4.

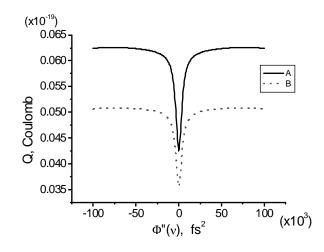


FIG. 6: The charge transferred after the completion of the pulse action as a function of the chirp rate in the frequency domain  $\Phi''(\nu)$  when  $d\mathcal{E}_0/\hbar\omega_0 = 0.1$  in the absence  $(B_N(\varepsilon_2 - \varepsilon_1)/\omega_0 = 0, \text{ curve A})$  and presence  $(B_N(\varepsilon_2 - \varepsilon_1)/\omega_0 = 0.01, \text{ curve B})$  of energy transfer. Other parameters are identical to those of Fig.4.

 $\Gamma_{MN} = \Gamma_{M2} + \Gamma_{M1} \ (\Gamma_{MN} \text{ was defined by Eq.(38)}).$  If in addition,  $\Gamma_{M1} = \Gamma_{M2} \equiv \Gamma_M$  and provided that level 1 is below and level 2 is above both Fermi energies, then it can be shown that  $n_1 = 1 - n_2$  (see Appendix B) and

$$Q = e(\Gamma_{ML,2} - \Gamma_{ML,1}) \int_{-\infty}^{\infty} n_2(t) dt$$
 (59)

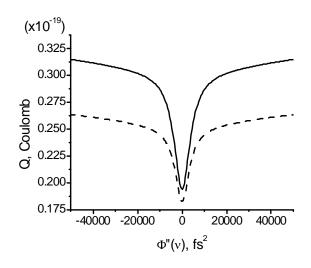


FIG. 7: Influence of the frequency detuning  $\omega_0 - \omega$  on the charge transferred after the completion of the pulse action for  $d\mathcal{E}_0/\hbar\omega_0 = 0.1$ .  $(\omega_0 - \omega)/\omega_0 = 0$  (solid line) and 0.05 (dashed line). Other parameters are identical to those of Fig.4.

Under these conditions our electron problem (Eqs.(41)-(44)) becomes mathematically equivalent to the Landau-Zener transition to a decaying level[45] solved analytically by Akulin and Schleich [38]. The magnitude  $\Gamma_{ML,2} \int_{-\infty}^{\infty} n_2(t) dt$  on the right-hand side of Eq.(59) represents the expectation value of the number of electrons passed from the molecule to the left lead after the completion of the pulse action, and  $\Gamma_{ML,1} \int_{-\infty}^{\infty} n_2(t) dt =$  $\Gamma_{ML,1} \int_{-\infty}^{\infty} (1 - n_1(t)) dt$  is the same for the electrons passed from the left lead to the molecule.

Using Eq.(59) and Eq.(25) of Ref.[38] for the magnitude  $I_{AS} \equiv \Gamma_M \int_{-\infty}^{\infty} n_2(t) dt$ , we obtain in terms of our representation

$$Q = e \frac{\Gamma_{ML,2} - \Gamma_{ML,1}}{\Gamma_M} I_{AS} = 2\pi e \frac{\Gamma_{ML,2} - \Gamma_{ML,1}}{\Gamma_M} \frac{\Omega^2}{4|\bar{\mu}|} \times \exp\left(\frac{-\pi\Omega^2}{4|\bar{\mu}|}\right) \left| W_{\frac{i\Omega^2}{4|\bar{\mu}|},-1/2}(-\frac{i\Gamma_M^2}{|\bar{\mu}|}) \right|^2$$
(60)

where  $W_{ia,-1/2}(z)$  is the Whittaker function [39]. The graph of  $I_{AS}$  as a function of Landau-Zener parameter and quenching parameter, which correspond to  $\Omega^2/|\bar{\mu}|$  and  $\Gamma_M^2/|\bar{\mu}|$ , respectively, in terms of our representation, can be found in Fig.1 of Ref.[38].

When chirp is fast with respect to the rate of the electron transfer,  $\frac{\Gamma_M^2}{|\bar{\mu}|} \ll 1$ , one gets from Eq.(60)

$$Q = e \frac{\Gamma_{ML,2} - \Gamma_{ML,1}}{\Gamma_M} \left[ 1 - \exp\left(\frac{-\pi\Omega^2}{2\,|\bar{\mu}|}\right) \right], \qquad (61)$$

where we have used the integral representation [39] of the Whittaker function to calculate  $\left|\lim_{z\to 0} W_{ia,-1/2}(z)\right|^2 =$ 

 $(a\pi)^{-1}\sinh(\pi a)$ . The expression in the brackets on the right-hand side of Eq.(61) is simply the probability of Landau-Zener transition, which indeed is identical to the probability of the electron transfer from the excited molecule to the leads in the case of fast passage through the resonance. Indeed, in this case  $\Gamma_M \int_{-\infty}^{\infty} n_2(t) dt = \Gamma_M \int_0^{\infty} n_2(0) \exp(-\Gamma_M t) dt = n_2(0)$  where  $n_2(0)$  is the population of molecular state 2 immediately following the passage through the resonance. The highest charge transfer is therefore obtained if  $n_2(0) = 1$ . Eq.(61) shows that  $n_2(0)$  approaches 1 for strong interaction,  $\pi\Omega^2 \gg 2 |\bar{\mu}|$ , which corresponds to adiabatic rapid passage (ARP). In other words, when the interaction with light is short in comparison with the electron transfer, the transferred

charge is maximal when ARP is realized. Really,  $Q \to |e|$ if  $\left|\frac{\Gamma_{ML,2}-\Gamma_{ML,1}}{\Gamma_M}\right| \to 1$ . This issue is of importance for developing single-electron devices with optical gating based on molecular nanojunctions.

When  $\frac{\Omega^2 \Gamma_M^2}{4\bar{\mu}^2} \gg 1$ , the magnitude  $I_{AS}$  is given by [38]

$$I_{AS} = \frac{\pi \Gamma_M \Omega^2}{2 \left| \bar{\mu} \right| \sqrt{\Gamma_M^2 + \Omega^2}}.$$

and we get a simple formula for the charge transferred in the course of slow passage through the resonance (with respect to both the electron transfer rate and the reciprocal Rabi frequency)

$$Q = \frac{\pi \Omega^2 e(\Gamma_{ML,2} - \Gamma_{ML,1})}{2|\bar{\mu}|\sqrt{\Omega^2 + \Gamma_M^2}}$$
(62)

Eq.(62) gives  $Q = e(\Gamma_{ML,2} - \Gamma_{ML,1}) \frac{\pi\Omega}{2|\bar{\mu}|} \gg e$  at least for strong interaction when  $\Omega^2 \gg \Gamma_M^2$ .

# 2. Slow passage through the resonance and strongly chirped pulses

Eq.(62) can be obtained directly by integrating Eq.(50) with respect to time for  $\omega(t) = \omega_0 - \bar{\mu}t$  and  $\Omega = const$ . Indeed, integrating Eq.(50) yields

$$Q = \int_{-\infty}^{\infty} I(t)dt = \frac{\Gamma_{M2} + \Gamma_{M1}}{4\Gamma_{M1}\Gamma_{M2}} \times \frac{\pi\Omega^2 e(\Gamma_{ML,2}\Gamma_{MR,1} - \Gamma_{ML,1}\Gamma_{MR,2})}{|\bar{\mu}|\sqrt{\Omega^2 \frac{(\Gamma_{M2} + \Gamma_{M1})^2}{4\Gamma_{M1}\Gamma_{M2}} + (\Gamma_{MN}/2)^2}}$$
(63)

In the special case  $\Gamma_{M1} = \Gamma_{M2} \equiv \Gamma_M$  and  $B_N(\varepsilon_2 - \varepsilon_1) = 0$ , Eq.(63) leads to Eq.(62). As a matter of fact, Eq.(63) extends the case of slow passage through the resonance beyond the treatment of Ref.[38].

Eq.(63) can be used for the excitation of a bridging molecule by Gaussian pulses, Eqs.(57) and (58), as well when the pulses are strongly chirped [40]

$$2|\Phi''(\omega)| \gg \tau_{p0}^2 \tag{64}$$

For a strongly chirped pulse, one can ascribe to different instants of time the corresponding frequencies [40], i.e. different frequency components of the field are determined via values of the instantaneous pulse frequency  $\omega(t)$  for different instants of time. Then one can integrate  $\int_{-\infty}^{\infty} I(t)dt$  similar to Eq.(63), bearing in mind that  $\Omega(t) = (\frac{d\mathcal{E}_0}{\hbar}) \exp(-\frac{1}{2}\delta^2 t^2) \approx (\frac{d\mathcal{E}_0}{\hbar}) \exp(-\frac{\tau_{p0}^2}{4\Phi''^2(\omega)}t^2)$ is a much slower function of time than  $\omega(t) = \omega_0 - \bar{\mu}t \approx$  $\omega_0 + \frac{1}{\Phi''(\omega)}t$ . Using Eqs.(57), (58), (63) and (64), we then get

$$Q \approx \frac{\Gamma_{M2} + \Gamma_{M1}}{4\Gamma_{M1}\Gamma_{M2}} \times \\ \times \frac{\sqrt{\pi} (\frac{d}{\hbar})^2 \mu_0 c E_p \tau_{p0} e (\Gamma_{ML,2}\Gamma_{MR,1} - \Gamma_{ML,1}\Gamma_{MR,2})}{\sqrt{(\frac{d}{\hbar})^2 \frac{\mu_0 c E_p \tau_{p0}}{\sqrt{\pi} |\Phi''(\omega)|} \frac{(\Gamma_{M2} + \Gamma_{M1})^2}{4\Gamma_{M1}\Gamma_{M2}} + (\Gamma_{MN}/2)^2}}$$
(65)

where  $\mathcal{E}_0^2 \approx \frac{\mu_0 c E_p \tau_{P^0}}{\sqrt{\pi} |\Phi''(\omega)|}$ , since the magnitude  $\int_{-\infty}^{\infty} \mathcal{E}^2(t) dt = 2\mu_0 c E_p = const$  is conserved in the course of chirping. Here  $E_p$  is the pulse energy per unit area,  $\mu_0$  is the permeability constant, c - light velocity in vacuum. According to Eq.(65), in the case of slow passage through the resonance,  $Q \sim \sqrt{|\Phi''(\omega)|}$  for strong interaction when  $(\frac{d}{\hbar})^2 \frac{\mu_0 c E_p \tau_{P^0}}{\sqrt{\pi} |\Phi''(\omega)|} \frac{(\Gamma_{M2} + \Gamma_{M1})^2}{4\Gamma_{M1} \Gamma_{M2}} >> (\Gamma_{MN}/2)^2$ , and Q tends to a constant value for large  $|\Phi''(\omega)|$ . This elucidates the behavior observed in our simulations shown in Figs. 4 and 5 for moderate and large values of  $|\Phi''(\nu)|$ . In addition, Eq.(65) explains why the growth of Q slows down for larger value of  $|\Phi''(\nu)|$  if pulse energy increases.

### VI. CONCLUSION

In this work a theory for light-induced current by strong optical pulses in molecular-tunneling junctions have been developed. We have considered a molecular bridge represented by its highest occupied and lowest unoccupied levels, HOMO and LUMO, respectively, and have derived a closed set of equations for electron populations of molecular states, molecular polarization and molecular excitation (exciton population) when two types of couplings between the molecule and the metal leads are presented: electron transfer that gives rise to net current in the biased junction and energy transfer between the molecule and electron-hole excitations in the leads.

We have used this formalism to analyze a novel control mechanism by which the charge flow is enhanced by chirped pulses. For linear chirp and when the energy transfer between the molecule and electron-hole excitations in the leads is absent, this control model can be reduced to the Landau-Zener transition to a decaying level, which has an exact analytical solution.

The relaxation parameters in the derived closed set of the equations of motion do not depend on the exciting electromagnetic field. This is true if the Rabi frequency  $\Omega$  is much smaller than the bath correlation frequency,  $\omega_c$ . If molecular states  $\varepsilon_m$  are far from the Fermi levels of both leads,  $\omega_c$  is determined by the frequency interval for the system-bath interaction matrix elements  $V_{km}^{(MK)}$  and  $V_{kk'}^{(NK)}$  and the density of states of metal leads. The last can be evaluated as 1-10 eV [20]. As a matter of fact, the approximation of constant relaxation parameters, which do not depend on exciting electromagnetic radiation, is consistent to the RWA used in our theory.

The situation is different if we assume that the molecular level position is pinned to the Fermi energy of a lead, that may lead to highly nonlinear current voltage dependence [16]. In this case  $\omega_c$  is determined also by the frequency interval at which  $f_K(\varepsilon)$  is essentially changed that is  $\sim k_B T/\hbar$  (see Eqs.(20) and (26)). In the last case  $\Omega$  can be of the same order of magnitude as  $\omega_c$  in the RWA, and the dependence of the relaxation parameters on exciting electromagnetic field [20] must be included into the theory.

To end this discussion we note that in this work we have investigated a model process driven by light absorption in a molecular bridge connecting metal leads. As already discussed, the geometry considered is potentially advantageous because of the possible local field enhancement due to plasmon excitation in the leads. It should be emphasized however that other processes, not considered in this work, may play important roles in nanojunction response to incident light. First, direct electron-hole excitations of the metal leads [41, 42] may affect response in an adsorbed molecule that goes beyond the local field enhancement associated with plasmon excitation. Secondly, experimental realization of strong local excitations in nanojunctions requires careful consideration of heating and heat dissipation and conduction [43]. Heating may be kept under control by driving the junction using a sequence of well separated optical pulses, as envisioned in the proposed experiment, but it should be kept in mind that a more detailed consideration of this issue may be needed.

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### APPENDIX A

Calculate the steady-state current in the absence of the radiative and nonradiative energy transfer couplings,  $\hat{V}_P$  and  $\hat{V}_N$ . The corresponding solution of Eq.(33) is as follows:  $n_m = W_{Mm}/\Gamma_{Mm}$ . Substituting it into Eq.(40) and using Eq.(21), we get

$$I = e \sum_{m=1,2} \frac{\Gamma_{ML,m} \Gamma_{MR,m}}{\Gamma_{Mm}} [f_R(\varepsilon_m) - f_L(\varepsilon_m)]$$
(A1)

The last formula can be written

$$I = e \sum_{m=1,2} \frac{\Gamma_{ML,m} \Gamma_{MR,m}}{\Gamma_{Mm}} \int d\varepsilon [f_R(\varepsilon) - f_L(\varepsilon)] \delta(\varepsilon - \varepsilon_m) =$$
  
$$= \frac{e}{2\pi\hbar} \sum_{m=1,2} \lim_{\Gamma_{Mm}/2 \to 0} \Gamma_{ML,m} \Gamma_{MR,m} \int d\varepsilon \frac{f_R(\varepsilon) - f_L(\varepsilon)}{(\varepsilon - \varepsilon_m)^2\hbar^{-2} + [\Gamma_{Mm}/2]^2}$$
(A2)

using the well known representation for  $\delta(x)$ 

$$\delta(x) = \frac{1}{\pi} \lim_{\sigma \to 0} \frac{\sigma}{x^2 + \sigma^2}$$
(A3)

The limit  $\lim_{\Gamma_{Mm}/2\to 0}$  on the right-hand side of Eq.(A2) is consistent with the Markovian approximation in the sense that relaxation parameters  $\Gamma_{Mm}/2$  are small in comparison to the bath correlation frequency,  $\omega_c$ .

The term  $\lim_{\Gamma_{Mm}/2\to 0} \frac{1}{(\varepsilon - \varepsilon_m)^2 \hbar^{-2} + [\Gamma_{Mm}/2]^2}$  on the right-hand side of Eq.(A2) can be written

$$\lim_{\Gamma_{Mm}/2\to 0} \frac{1}{(\varepsilon - \varepsilon_m)^2 \hbar^{-2} + [\Gamma_{Mm}/2]^2} = G^r_{mm}(\varepsilon) G^a_{mm}(\varepsilon)$$
(A4)

where  $G_{mm}^{r}(\varepsilon)$  and  $G_{mm}^{a}(\varepsilon)$  are the retarded and advanced Green's functions, respectively, [16] in the Markovian approximation. Substitution of Eq.(A4) into Eq.(A2) leads to the well known Landauer formula for the current [21].

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### APPENDIX B

Let us show that  $n_1 = 1 - n_2$  when  $\Gamma_{M1} = \Gamma_{M2} \equiv \Gamma_M$ . Summing Eqs.(41) and (42) at given condition, we have

$$\frac{dy}{dt} = -\Gamma_M y \tag{B1}$$

where we denoted  $y = n_1 + n_2 - 1$ . The solution of the last equation is as follows

$$y(t) = y(0) \exp(-\Gamma_M t) \tag{B2}$$

where y(0) = 0. This gives  $n_1(t) + n_2(t) = 1$  even in the presence of energy transfer when  $B_N(\varepsilon_2 - \varepsilon_1) \neq 0$ .

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