## Experimental Study of A Photon as A Subsystem of An Entangled Two-Photon State

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## Abstract

The state of the signal-idler photon pair of spontaneous parametric down conversion is a typical nonlocal entangled pure state with zero entropy. The precise correlation of the subsystems is completely described by the state. However, it is an experimental choice to study only one subsystem and to ignore the other. What can we learn about the measured subsystem and the remaining parts? Results of this kind of measurements look peculiar. The experiment confirms that the two subsystems are both in mixed states with entropy greater than zero. One can only obtain statistical knowledge of the subsystems in this kind of measurement.

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One of the most surprising consequences of quantum mechanics is the entanglement of two or more distant particles. The first example of a two-particle entangled state was suggested by Einstein, Podolsky, and Rosen in their famous *gedankenexperiment* in 1935 [1]. The EPR state is a pure state of two spatially separated particles which can be written as,

$$\left|\Psi\right\rangle = \sum_{a,b} \delta\left(a+b-c_{0}\right)\left|a\right\rangle\left|b\right\rangle \tag{1}$$

where a and b are the momentum or the position of particle 1 and 2 respectively and  $c_0$  is a constant. It is clear that state (1) is a two-particle state; however, it cannot be factored into a product of the state of particle 1 and the state of particle 2. This type of states was defined by Schrödinger as *entangled states* [2].

One, perhaps the most easily accessible, example of an entangled state is the state of a photon pair emitted in Spontaneous Parametric Down Conversion (SPDC). SPDC is a nonlinear optical process from which a pair of signal-idler photon is generated when a pump laser beam is incident onto an nonlinear optical crystal. The signal-idler two-photon state can be calculated by first order perturbation from the SPDC nonlinear interaction Harmiltonion [3],

$$|\Psi\rangle = \sum_{s,i} \delta\left(\omega_s + \omega_i - \omega_p\right) \delta\left(\mathbf{k}_s + \mathbf{k}_i - \mathbf{k}_p\right) a_s^{\dagger}(\omega(\mathbf{k}_s)) a_i^{\dagger}(\omega(\mathbf{k}_i)) \mid 0\rangle$$
(2)

where  $\omega_j$ ,  $\mathbf{k}_j$  (j = s, i, p) are the frequency and wavevectors of the signal (s), idler (i), and pump (p) respectively,  $\omega_p$  and  $\mathbf{k}_p$  can be considered as constants, usually a single mode laser is used for pump,  $a_s^{\dagger}$  and  $a_i^{\dagger}$  are the respective creation operators for signal and idler photon. The delta functions of the state ensure energy and momentum conservation. It is indeed the conservation laws that determine the values of an observable for the pair. Quantum mechanically, state (2) only provides precise momentum (energy) *correlation* of the *pair* but no precise momentum (energy) determination for the signal photon and the idler photon. In EPR's language: the momentum (energy) of neither the signal nor the idler is determined by the state; however, if one is known to be at a certain value the other one is determined with certainty. Notice also that state (2) is a pure state. It provides a complete description of the entangled two-photon system. Following the creation of the pair, the signal and idler may propagate to different directions and be separated by a considerably large distance. If it is a free propagation, the state will remain unchanged except for the gain of a phase, so that the precise momentum (energy) *correlation* of the *pair* still holds. The conservation laws guarantee the precise value of an observable with respect to the pair (not to the individual subsystems). It is in this sense, we say that the entangled two-photon state of SPDC is *nonlocal*. Quantum theory does allow a complete description of the precise *correlation* for the spatially separated subsystems, but no complete description for the physical reality of the subsystems defined by EPR. It is in this sense, we say that quantum mechanical description (theory) of the entangled system is *nonlocal*.

So far, our discussion involves no measurement.

In a type of measurements when "joint detections" are involved, for example a coincidence detection for the SPDC pair, it corresponds to the intensity correlation,  $\langle \Psi | \hat{I}_1 \otimes \hat{I}_2 | \Psi \rangle$ or the fourth order correlation of the fields,  $\langle \Psi | \hat{E}_1^{(-)} \hat{E}_2^{(-)} \hat{E}_2^{(+)} \hat{E}_1^{(+)} | \Psi \rangle$ . One may cooperate spin (polarization for photon) correlations into the coincidence joint detection too, as in the measurements for the EPR-Bohm state [4]. If the correlations of the pair have been built up in the entangled two-particle state from the beginning, it comes as no surprise that the intensity correlation reflects *perfect* correlation (EPR, EPR-Bohm or EPR-Bell type correlation) of the pair. The distance between the detectors would not matter. There is no *action-at-adistance* involved, if we are willing to give up the classical EPR reality. The entangled state indeed indicates and represents a very different physical reality: Does the signal photon or idler photon have a defined momentum (energy) in state (2)? No! Does the pair have a defined total momentum (energy) in state (2)? Yes! In state (2) the precise value of an observable is determined in the form of *total value* by conservation laws. In addition, one cannot "assume" or "imagine" two individual wavepackets each associated with the signal photon and the idler photon. It is a non-factorizable two-dimensional "wavepacket" associated with the entangled two-particle system [5]. For this very reason we have named the signal-idler pair the "biphoton". Many interesting phenomena involving biphoton have been demonstrated in two-photon interferometry and in two-photon correlation type experiments [6]. Several recent experiments have clearly shown that the two-photon interference is not the interference between two photons. It is not the signal and idler photon wavepackets but the two-dimensional biphoton wavepacket that plays the role [7].

Recently, there has been a lot of interest in a type of measurements in which only one subsystem of an entangled multi-particle state is measured and the remaining parts are left undisturbed. One of the popular misconceptions is to believe that the "state" of the undisturbed remaining parts is completely determined by this kind of "distance measurement": if the measurement of the subsystem either yields the result of a value for an observable or indicates the "state" of that subsystem, the undisturbed remaining parts is then "forced" into a "pure state". Do we have to accept *action-at-a-distance* in quantum theory?

It is a fact that the experimentalist can choose to look at one part of the entangled system and to ignore the other. The subsystems may well be separated spatially. For instance, one can use a photon counting detector to register a "click" event of the signal photon of the two-photon state of SPDC (not a "click-click" event!) to study only the properties of the signal and leave the idler undisturbed or to be predicted. What can we predict for the idler photon in this kind of measurement? Before answering this question, it may be better to ask first: "what can we learn about the signal photon in this kind of measurement?"

An interesting situation arises: while the two-photon state of SPDC is a pure state, the respective states of the signal and idler photon are not. The states of the individual signal and idler are both in thermal (mixed) states. This has been pointed out by several researchers, e.g. [8–11], from different perspectives. Significance of two parts in a mixed state constitute a quantum mechanical system in a pure state was emphasized by B. Yurke and M. Potasek [9] as an example of purely quantum thermalization, that is obtaining mixed states out of pure states in a Hamiltonian system. N.J. Cerf and C. Adami [11] introduced the *mutual* ( $S_{A:B} = S_{B:A}$ ) and *conditional* ( $S_{A|B}, S_{B|A}$ ) entropy (or information) for a two-particle system similar to the mutual and conditional entropies as defined in classical probability theory:

$$S = S_{A|B} + S_{B|A} + S_{A:B}, \quad S_A = S_{A|B} + S_{A:B}, \quad S_B = S_{B|A} + S_{A:B}.$$
 (3)

For an entangled two-particle system in a pure state (so that S = 0), the relations in (3) give,

$$S_A + S_{B|A} = 0, \qquad S_B + S_{A|B} = 0.$$
 (4)

The paradox of the whole system entropy S being zero while an entropy of its either part  $S_A$ and  $S_B$  are both positive (which is a formal expression of the statement that the information contained in the whole system is less than the information contained in its parts) is suggested resolvable by letting the conditional entropy to take on negative values.

In this paper we report an experimental work along the lines of this discussion. The reported experiment hinges on a typical Fourier spectroscopy measurement. The schematic setup is shown in Fig. 1. The measurement is based on a "click" type single photon detection; however, the photon source is an entangled two-photon source of SPDC: a 3mm BBO ( $\beta - BaB_2O_4$ ) crystal pumped by a 351.1nm CW Argon ion laser line. The orthogonally polarized signal-idler photon pairs are generated by satisfying the collinear degenerate (centered at wavelength 702.2nm) type-II phase matching condition [12]. The idler (extraordinary ray of BBO) is removed by a polarizing beamspliter PBS. The signal (ordinary ray of BBO) is to a Michelson interferometer. A photon counting detector is coupled to the output port of the interferometer through a 25mm focal lens. A 702.2nm spectral filter with Gaussian transmittance function (bandwidth 83nm FWHM) is placed in front of the detector. The counting rate of the detector is recorded as a function of the optical arm length difference,  $\Delta L$ , of the Michelson interferometer.

The SPDC spectrum closely resembles a rainbow ranging from red to blue. The spectrum collected by the 25mm focal lens is much wider then 83nm. It would be reasonable to expect a Gaussian spectrum with 83nm FWHM (determined by the spectrum filter used for the detector) from the above measurement. (This conjecture is different from the wrong belief or imagination that each individual of the signal and idler photons is associated with a Gaussian wavepacket.) On the contrary, we instead observed an "unexpected" result, the observed

spectrum is not Gaussian and its width is only 2.2nm (far from 83nm). The experimental data is reported in Fig. 2. The envelope of the sinusoidal modulations (in segments) is fitted very well by two "notch" functions (upper and lower part of the envelope). The width of the triangular base is about  $225\mu m$  which corresponds to roughly a spectral band width of 2.2nm.

To seek an explaination of this result, we must first examine the two-photon state of SPDC. We cannot assume a state for either the signal photon or idler photon. The single photon state is obtained by taking a partial trace of the two-photon state density operator, integrating over the spectrum of the idler and vice versa:

$$\hat{\rho}_s = tr_i \ \hat{\rho}, \quad \hat{\rho}_i = tr_s \ \hat{\rho} \tag{5}$$

with

$$\hat{\rho} \equiv |\Psi\rangle \langle\Psi|\,,\tag{6}$$

where  $\hat{\rho}$  the density matrix operator and  $|\Psi\rangle$  the two-photon state (2).

First, it is very interesting to find that even though the two-photon EPR state of SPDC is a pure state, i.e.,  $\hat{\rho}^2 = \hat{\rho}$ , the corresponding single photon state of the signal and idler are not, i.e.,  $\hat{\rho}_{s,i}^2 \neq \hat{\rho}_{s,i}$ . This accords with the earlier mentioned fact that the entropy of the system is zero (pure state) while each subsystem has an entropy greater than zero (mixed state). The zero entropy condition for a system in a pure state reflects the fact that the quantum state  $|\Psi\rangle$  provides a *complete* description of the system . On the other hand, the mixed state of each subsystem only reveals their statistical nature.

In the experiment, we realize a collinear degenerate type-II phase matching [12]. This means that the SPDC crystal orientation is such that the orthogonally polarized signal-idler pair with degenerate frequency  $\omega = \omega_p/2$ , are emitted collinearly. We select this direction by a set of pinholes during the experimental alignment process. Then the integral in Eq.(2) can be simplified to an integral over a frequency detuning parameter  $\nu$ , (the detailed calculation can be find in ref. [5]):

$$|\Psi\rangle = A_0 \int d\nu \ \Phi(DL\nu) \ a_s^{\dagger}(\omega+\nu) a_i^{\dagger}(\omega-\nu) \ |0\rangle.$$
(7)

where the *sinc*-like function  $\Phi(LD\nu)$  followed from Eq.(2) considering a finite length of the SPDC crystal [3]. It represents a spectral width of the two-photon state,

$$\Phi(DL\nu) = \frac{1 - e^{-iDL\nu}}{iDL\nu},\tag{8}$$

which is determined by the finite crystal length L and, specifically for the collinear degenerate type-II SPDC, by the difference of inverse group velocities for the signal (ordinary ray) and the idler (extraordinary ray):  $D \equiv 1/u_o - 1/u_e$ .

The constant  $A_0$  is found from the normalization condition  $tr\rho = \langle \Psi | \Psi \rangle = 1$  (dimensionless):

$$A_0 = \sqrt{\frac{DL}{4\pi}}.$$

Substituting  $|\Psi\rangle$  in the form of Eq.(7) into Eq.(5) the density matrix of signal is calculated to be,

$$\hat{\rho}_s = A_0^2 \int d\nu \ |\Phi(\nu)|^2 \ a_s^{\dagger}(\omega+\nu) |0\rangle \langle 0| \ a_s(\omega+\nu)$$
(9)

where

$$|\Phi(\nu)|^2 = \operatorname{sinc}^2 \frac{DL\nu}{2} \tag{10}$$

In (9) we consider a multimode (a continues frequency spectrum) entangled system with a single quantum, n = 1. The operator (9) describes the statistical distribution of this quantum. This is a good approximation since the coupling in SPDC is week and greater number states n > 1 that correspond to higher perturbation orders are extremely unlikely. On the other hand, n = 0 represents vacuum fields that do not result in detections [13].

By now, we can understand very well the experimental results. (1) For a spectrum of *sinc*-square function we do expect a double "notch" envelope in the measurement and the base of the triangle, which is determined by DL, is calculated to be  $225\mu m$  (we have considered the optical path difference is twice of the arm difference in Michelson interferometer),

corresponding to a 2.2nm bandwidth. The experimental result, from fitting, is about 225nm, which agrees well with the prediction [14]. (2) We see that the spectrum of the *signal* photon is dependent on the group velocity of the *idler* photon which is not measured at all in our experiment. However, this comes as no surprise, because the state of the signal photon is calculated from the two-photon state by integrating over the idler modes. (3) We also see immediately that  $\hat{\rho}_s^2 \neq \hat{\rho}_s$ , so the signal and idler single-photon states are both mixed states. It is then straightforward to evaluate numerically the Von Neuman entropy S [16] of the signal (or idler) subsystem,

$$S_s = -tr[\hat{\rho}_s \log \hat{\rho}_s],\tag{11}$$

based on the "double notch" fitting function. Note that operator (9) is diagonal. Taking its trace is simply to perform an integration over the frequency spectrum with the spectral density of Eq.(10). To compute the integral of Eq.(11) for the density matrix  $\hat{\rho}_s$  of Eq.(5), we replace variable  $\nu$  by a dimensionless variable  $DL\nu/2$  and evaluate the integral numerically. The calculation yields,

$$S_s \approx 6.4 > 0.$$

This again indicates the statistical mixture nature of the state of a photon (subsystem) in an entangled two-photon system.

Based on the experimental data, we conclude that the entropy of signal and idler are both greater then zero (mixed state); while the entropy of the signal-idler two-photon system is zero (pure state). This may mean that negative entropy is present somewhere in the system, perhaps in the form of the conditional entropy [11]. By definition of the conditional entropy, one is tempted to say that given the result of a measurement over one particle, the result of measurement over the other must yield negative information. This paradoxical statement is similar to and in fact closely related to the EPR "paradox". We suggest that the paradox comes from the same philosophy.

Conclusion: In these kind of measurements, in which the experiment only measures a subsystem of an entangled multi-particle system and leave the remaining parts undisturbed, one can only obtain statistical knowledge of the subsystems. Neither the measured subsystem nor the remaining parts is in pure state. The individual subsystems are described statistically by the quantum theory before the measurement and after the measurement. The measurements can never "force" the undisturbed subsystems into a pure state. Again, we emphasize that no *action-at-a-distance* in any format.

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- [13] Our approach therefore is quite different from the early "squeezed state" studies, e.g. [8,9] where the signal and idler radiation of SPDC is shown to be in the thermal (and hence mixed) state based on a single-mode model which allow all number-states but omits entanglement.
- [14] It is interesting to notice that apparently the same result, a triangular correlation function was also obtained in a two-photon interference experiment [15] based on *joint* coincidences detections. The two experiments are fundamentally different: the earlier one [15] demonstrates the overlap of two biphoton amplitudes while the present reveals the shape of a single-photon wave packet. Even though we understand, mathematicaly, why the results are identical, the physical meaning of this fact remains puzzling.
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## FIGURES

FIG. 1. Schematic of the experimental set up. A Michelson interferometer is used to study the spectrum of the signal of SPDC. The SPDC spectrum closely resembles a rainbow ranging from red to blue. A band pass spectral filter centered at 702.2nm with 83nm FWHM of a Gaussian transmittance function, is placed in front of the photon counting detector.

FIG. 2. Experimental data indicated a "double notch" envelope of the interference pattern. The X-axis,  $\Delta L$  in  $\mu m$ , is the optical arm difference of the Michelson interferometer. Each of the doted single vertical segment contains many cycles of sinusoidal modulations. The spike at  $\Delta L = 0$ , usually called "white light condition" for observing "white light" interference, is a broad band interference pattern which is determined by the spectral filter. Two vertical lines show +122.5 $\mu m$  and -122.5 $\mu m$ .



Figure 1. Dmitry V. Strekalov, Yoon-Ho Kim, and Yanhua Shih



Figure 2. Dmitry V. Strekalov, Yoon-Ho Kim, and Yanhua Shih