PONDEROMOTIVE CONTROL OF QUANTUM MACROSCOPIC COHERENCE

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Abstract

It is shown that because of the radiation pressure a Schrödinger cat state can be generated in a resonator with oscillating wall. The optomechanical control of quantum macroscopic coherence and its detection is taken into account introducing new cat states. The effects due to the environmental couplings with this nonlinear system are considered developing an operator perturbation procedure to solve the master equation for the field mode density operator.

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1 Introduction

One of the fundamental aspects of quantum mechanics is the existence of interference among quantum states which signs the difference between a superposition of states and a mixture of states. The quantum theory may adequately well describe macroscopic objects by means of a linear superposition of states with macroscopically distinguishable properties. Recently, due to the improved technology, there was a growing interest on the possibility of observing such superposition states, commonly known as Schrödinger cats [1]. Good candidates for these macroscopic states are the coherent states of an e.m. field mode. The properties of superposition of two generic coherent states has been studied in Ref. [2] and the simplest superposition of even and odd coherent states was introduced in Ref. [3]. A review of these states is given in Ref. [4]. Within the field of optics several proposals for the generation of linear superpositions of coherent states in various nonlinear processes [5, 6] and in quantum non-demolition measurements [7] have been made. It is worthy to note that the field in a cat state has a lot of advantages in optical communication [8]. However, by coupling the system to its enviroment, as in the act of measurement, one always introduces dissipation and decoherence effects, which tend to destroy any quantum features [9].

In the common scheme of the Kerr-like medium modeled by an anharmonic oscillator, it was shown [10] that the photon number distribution and interferences in phase space are highly sensitive to even small dissipative coupling. This fact, plus the smallness of the $\chi^{(3)}$ nonlinearity, makes the prospect of experimentally producing and detecting such states highly questionable in these media.

On the other hand, it is well known [11] that an empty optical cavity with a moving mirror may mimic a Kerr-like medium when it is illuminated with coherent light. The effect of intensity dependent optical path is due, in this case, to the radiation pressure force.

In this paper we shall present such a model as an alternative one for the generation of Schrödinger cats. We will show that, with the appropriate measurement technique, it could also be useful for revealing quantum macroscopic coherence.

2 The Model

We consider a linear Fabry-Perot empty cavity with one fixed partially reflecting end mirror and one perfectly reflecting mirror, which can move (undergoing harmonic oscillations) under the influence of radiation pressure. If L is the equilibrium cavity length, the resonant frequency of the cavity will be

$$\omega_c = \pi \frac{c}{L} n \tag{1}$$

where n is an integer number determined by the frequency of the input light and c is the speed of light. We assume that the retardation effects, due to the oscillating mirror, in the intracavity field are negligible. We will also neglect the correction to the radiation pressure force due to the Doppler frequency shift of the photons [12]. Thus we are able to write the Hamiltonian of the whole system as

$$H = \hbar\omega_c a^\dagger a + \hbar\omega_m b^\dagger b + H_{int} \tag{2}$$

where a, $a^\dagger$are the boson operators of the resonant cavity mode and b, $b^\dagger$ are the boson operators of the oscillating mirror with the mass m and the angular frequency ω_m. This latter will be many order of magnitude smaller than ω_c to ensure that the number of photons generated by the nonstationary Casimir effect [14] as consequence of the Casimir forces [13] in the resonator with moving boundaries is completely negligible. H_{int} accounts for the fact that the intracavity photon changes its energy, by ω_c, as the oscillating mirror moves [15]

$$H_{int} = -\hbar G a^\dagger a (b + b^\dagger) \tag{3}$$

with the coupling constant given by

$$G = \frac{\omega_c}{L} \left(\frac{\hbar}{2m\omega_m} \right)^{1/2} . \tag{4}$$

From the Hamiltonians (2) and (3) we can derive, using the BCH formula for the Lie algebra [16], the time evolution operator in the following form

$$U(t) = e^{iE(t)(a^\dagger a)^2} e^{iF(t) a^\dagger a \hat{x}(t)} \left[e^{-i\omega_c a^\dagger a t/\omega_m} e^{-ib^\dagger b t} \right] \tag{5}$$

where

$$\hat{x}(t) = b e^{it/2} + b^\dagger e^{-it/2} \tag{6}$$

is the mirror quadrature operator, while

$$E(t) = \kappa^2 [t - \sin t] ; \quad F(t) = 2\kappa \sin(t/2) ; \quad \kappa = G/\omega_m , \tag{7}$$

with t the time scaled by ω_m, i.e. we have replaced $\omega_m t$ by t. Furthermore, from now on, we will consider the evolution operator omitting the free motion of the two modes a and b, i.e. the term inside the square brackets on Eq. (5).

3 Generation of Schrödinger Cat States

From Eq. (5) one can immediately recognizes that the time evolution introduces anharmonicity due to the presence of the nonlinear term $(a^\dagger a)^2$ whose strenght depends also on time [17]. It is also easy to see that at each time for which $F(t) = 0$ the two subsystems are disentagled. Furthermore due to its macroscopicity we should consider the oscillating mirror initially in a thermal state at temperature T

$$\rho_T = (1 - z) \sum_n z^n |n\rangle\langle n| ; \quad z = \exp\left(-\frac{\hbar\omega_m}{k_B T} \right) , \tag{8}$$

with $z/(1-z) = N_{th}$ that represents the mean number of excitations of the mechanical oscillator, i.e. the number of thermal phonons. Thus starting from an initial coherent state $|\alpha_0\rangle$ for the radiation mode we have

$$\rho(t^*) = e^{iE(t^*)(a^\dagger a)^2}|\alpha_0\rangle|\langle\alpha_0| \otimes \rho_T e^{-iE(t^*)(a^\dagger a)^2} \tag{9}$$

with

$$t^* = 2\pi m_1; \quad m_1 \in N \tag{10}$$

so that

$$F(t^*) = 0; \quad E(t^*) = \kappa^2 2\pi m_1. \tag{11}$$

Now in order to see the cat states, the following condition must be fulfilled [5]

$$E(t^*) = \frac{\pi}{2} + 2\pi m_2; \quad m_2 \in N \tag{12}$$

so that combining Eqs.(11) and (12) one gets

$$\kappa^2 = \frac{1}{m_1}(\frac{1}{4} + m_2) \tag{13}$$

which can be read as a restriction on the possible values of the various external parameters. Thus if the above conditions are satisfied, we have

$$\rho(t^*) = \frac{1}{2}\left[e^{-i\pi/4}|\alpha_0\rangle + e^{i\pi/4}|-\alpha_0\rangle\right]\left[\langle-\alpha_0|e^{-i\pi/4} + \langle\alpha_0|e^{i\pi/4}\right] \otimes \rho_T\,; \tag{14}$$

however, this is not the only way to create a quantum superposition in this system. In fact, let us consider the times t' for which

$$E(t') = \frac{\pi}{2} + 2\pi m; \quad m \in N\,. \tag{15}$$

In these cases, obviously, $F(t')$ is not necessarily zero then, the reconstruction of the superposed coherent states is impossible due to the entanglement between the two subsystems. One can now use a conditional measurement to create the desired states, performing a sort of quantum state engineering [18].

Let us suppose that the mirror's quadrature $\hat{x}(t)$ is measured [19], giving the result y_t. The state of the radiation field after the measurement is found by projecting the system's state onto the eigenstate $|y_t\rangle$

$$\rho_{after}(t) = \mathcal{C} e^{iE(t)(a^\dagger a)^2} e^{iF(t)a^\dagger a y_t} |\alpha_0\rangle\langle y_t|\rho_T|y_t\rangle\langle\alpha_0| e^{-iF(t)a^\dagger a y_t}|\ e^{-iE(t)(a^\dagger a)^2}\,, \qquad (16)$$

where $\mathcal{C}$ is a normalization constant

$$\mathcal{C} = (\langle y_t|\rho_T|y_t\rangle)^{-1}\,. \qquad (17)$$

At the times t' we have, from Eqs. (15) and (16)

$$\begin{aligned} \rho_{after}(t') = \;& \tfrac{1}{2} \left[e^{-i\pi/4}|\alpha_0 e^{iF(t')y_{t'}}\rangle + e^{i\pi/4}|-\alpha_0 e^{iF(t')y_{t'}}\rangle \right] \\ \otimes \;& \left[\langle -\alpha_0 e^{iF(t')y_{t'}}| e^{-i\pi/4} + \langle \alpha_0 e^{iF(t')y_{t'}}| e^{i\pi/4} \right]\,, \end{aligned} \qquad (18)$$

which is a superposition of coherent states whose phase depends on the measurement process; and further, if the result of the measurement is

$$y_{t'} = \frac{\pi}{2}\frac{1}{F(t')}\,, \qquad (19)$$

it is possible to recover in Eq. (18) the generalized even and odd coherent states like those discussed in [20, 21] which show quantum interference as well as other particular features.

4 Quasiprobability and Marginal Distribution

The evolved density operator of the whole system can be easily constructed by using the time evolution operator of eq. (5)

$$\rho(t) = U(t)|\alpha_0\rangle\langle\alpha_0| \otimes \rho_T U^\dagger(t)\,, \qquad (20)$$

and then the evolution can be described for example, in terms of the Q-function

$$Q(\alpha,\beta,t) = \langle\alpha|\langle\beta|\rho(t)|\beta\rangle|\alpha\rangle = e^{-|\alpha|^2-|\alpha_0|^2-|\beta|^2}(1-z)\sum_{j=0}^{\infty} z^j|\beta|^j$$

$$\times \quad \left| \sum_{n=0}^{\infty} \frac{(\alpha^* \alpha_0)^n}{n!} \exp\left\{ \left[iE(t) - \frac{1}{2} F^2(t) \right] n^2 + iF(t) n e^{-it/2} \beta^* \right\} \sum_{r=0}^{j} \frac{\left(iF(t) n e^{it/2} \right)^r}{r! \sqrt{(j-r)!}} \right|^2 , \tag{21}$$

where the variables α, β are referred to the radiation and to the mirror respectively. However, since the distinguishing element of a linear superposition of coherent states is the presence of interference fringes in the marginal distribution, we are interested in that one, for the particular times discussed in the previous Section. Its definition, for a generic state $\rho^{field}(t)$ of the radiation field, is given by

$$P(X) = \langle X | \rho^{field}(t) | X \rangle \,, \tag{22}$$

where $|X\rangle$ are eigenstates of the quadrature operator $X = (a + a^\dagger)/2$, while ρ^{field} should be intended as $\mathrm{Tr}_m\{\rho\}$ with Tr_m the trace over the mirror degrees of freedom. In the case of Eq. (14) we can integrate over the degree of freedom of the mirror to obtain the marginal distribution of the field mode as [10]

$$\begin{aligned} P(X) &= \left| \langle X | \frac{1}{\sqrt{2}} \left[e^{-i\pi/4} |\alpha_0\rangle + e^{i\pi/4} | - \alpha_0\rangle \right] \right|^2 \\ &= \frac{1}{2} \left[P_+(X) + P_-(X) + 2\sqrt{P_+(X) P_-(X)} \sin\left(4X |\alpha_0| \sin\left(\arg \alpha_0\right) \right) \right] , \end{aligned} \tag{23}$$

$$P_\pm(X) = \left(\frac{2}{\pi} \right)^{1/2} \exp\left[-2X - |\alpha_0|^2 \mp 2X(\alpha_0 + \alpha_0^*) - \frac{1}{2}(\alpha_0^2 + \alpha_0^{*2}) \right] ;$$

while in the case of Eq. (18) the marginal distribution for the field mode is in effect a conditional probability

$$P(X|y_{t'}) = \left| \langle X | \frac{1}{\sqrt{2}} \left[e^{-i\pi/4} |\alpha_0 e^{iF(t')y_{t'}}\rangle + e^{i\pi/4} | - \alpha_0 e^{iF(t')y_{t'}}\rangle \right] \right|^2 , \tag{24}$$

whose explicit expression is the same as in Eq. (23), apart an extra phase factor in the coherent sate which gives the interference pattern along a direction depending on the result of the measurement as well.

5 Damped Mode Equation and Solutions

Let us now consider the proposed model as an open system interacting with the "rest of Universe" [22]. We will study only the case in which the radiation mode relaxes much faster than the mirror (the opposite case, i.e. the mirror that relaxes much faster than the cavity mode, does not show any quantum features due to the thermalization effects). Moreover, since, in order to see the Schrödinger cats, we are interested to short time behaviour (i.e. times much shorter than the typical radiation relaxation time), we can consider the mirror practically not affected by any damping. Hence, the master equation for the whole system will be taken in the form

$$\dot{\rho} = \frac{i}{\hbar}[\rho, H] + \chi(\rho)\,, \tag{25}$$

where [23]

$$\chi(\rho) = \frac{\gamma}{2}[2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a]\,, \tag{26}$$

and where we have considered the number of thermal photons to be negligible at optical frequencies. In our model, the damping constant γ takes into account the loss of photons through the fixed mirror, so it is related to its transmissivity Tr by the relation $\gamma = cTr/2L$ with c the speed of light. However, since we are using a scaled time we should replace $\gamma/\omega_m \to \gamma$. Now, the undamped system is an exact solvable system with the free evolution operator $U(t)$ given by Eq. (5) and obeying the equation $i\dot{U}(t) = HU(t)$. Then, introducing a new density operator R, in a form similar to the interaction representation, i.e. $\rho = URU^\dagger$, we may rewrite Eq. (25) as

$$\dot{R} = U^\dagger\chi(URU^\dagger)U = \tilde{\chi}(R)\,, \tag{27}$$

where the operator $\tilde{\chi}(R)$ is obtained by the following recepie: all the additional operators a_i in the initial operators $\chi(\rho)$ are replaced by $\tilde{a}_i = U^\dagger a_i U$, while the operator ρ is replaced by R. We could write down the solution of the Eq. (27) in the form $R = R_0 + Y$, where R_0 is a constant operator, i.e. $\dot{R}_0 = 0$, and the operator Y satisfies the equation corresponding to (27)

$$\dot{Y} = \tilde{\chi}(Y + R_0)\,. \tag{28}$$

The operator R_0 represents to the free solution of the initial Eq. (25), i.e. without the term $\chi(\rho)$. Till now we only rewrote the master equation in another representation and it is still an exact equation. However, Eq. (27) is appropriate to apply the Born iteration procedure [24] provided that the the damping term $\chi(\rho)$ is small enough to be considered as a perturbative one (this could be the case since the parameter γ has to be small in order to achieve the Schrödinger cats). Then we could try to solve Eq. (28) simply by replacing in the r.h.s. the operator $\tilde{\chi}(Y + R_0)$ by $\tilde{\chi}(R_0)$, i.e. performing the first Born approximation. The solution is immediate, and the operator R assumes the form

$$R(t) = R_0 + \int_0^t \tilde{\chi}(R_0, \tau) d\tau\,. \tag{29}$$

It means that the initial density operator $\rho(t)$ becomes

$$\rho(t) = \rho_0(t) + \rho_\gamma(t)\,, \tag{30}$$

where the term $\rho_0(t)$ is the density operator of the free motion

$$\rho_0(t) = U(t)\rho_0(0)U^\dagger(t) \tag{31}$$

with initial density matrix $\rho_0(0) \equiv \rho(0)$. The correction term $\rho_\gamma(t)$ has the form

$$\rho_\gamma(t) = U(t)\left[\int_0^t \tilde{\chi}(R_0, \tau) d\tau\right] U^\dagger(t)\,, \tag{32}$$

or more explicitely

$$\begin{aligned} \rho_\gamma(t) &= \gamma \int_0^t d\tau \left\{ e^{-iF(t-\tau)\hat{x}(t-\tau) - 2iE(t-\tau)a^\dagger a} a \rho_0(t) a^\dagger e^{iF(t-\tau)\hat{x}(t-\tau) + 2iE(t-\tau)a^\dagger a} \right\} \\ &- \frac{\gamma}{2} t \left[a^\dagger a \rho_0(t) + \rho_0(t) a^\dagger a \right]\,. \end{aligned} \tag{33}$$

The range of validity of the above approximation is determined by the requierement $\rho_\gamma(t) << \rho_0(t)$. Below, it will become more clear that it works for $\gamma|\alpha_0|^2 t << 1$. It is also easy to check that $\mathrm{Tr}\{\rho_\gamma\} = 0$, then $\rho(t)$ is always normalized to unity. Let us now try to find the marginal distribution at the particular times t^* and t' discussed in Sec. 3. By means of Eqs. (33), (22) and (14), after lenghtly but straithforward algebra, one obtains

$$\begin{aligned}
&P(X) = \langle X|\mathrm{Tr}_m\{\rho_0(t^*) + \rho_\gamma(t^*)\}|X\rangle \\
&= \left(\frac{2}{\pi}\right)^{\frac{1}{2}} e^{-|\alpha_0|^2 - 2X^2} \sum_{p,q=0}^{\infty} \frac{2^{-(p+q)/2}}{p!q!} H_p(\sqrt{2}X) H_q(\sqrt{2}X) e^{i\arg\alpha_0(p-q)} \\
&\times|\alpha_0|^{p+q} \left\{ \frac{A_{q,p}}{2} + \frac{\gamma}{2}\left[A_{p,q} I_{p,q}(t^*)|\alpha_0|^2 - A_{q,p}\frac{p+q}{2}t^* \right]\right\}
\end{aligned} \tag{34}$$

where H_p are the Hermite polynomials,

$$A_{p,q} = \left[1 + i(-)^q - i(-)^p + (-)^{p+q}\right] , \tag{35}$$

and finally

$$I_{p,q}(t^*) = \int_0^{t^*} d\tau e^{-i[2E(t^*-\tau)](p-q)} . \tag{36}$$

In Eq. (34) the first term inside the curly brackets comes from ρ_0 and is related to the undamped motion, while the other is the perturbative term due to the environmental coupling. Due to the fact that at the times t^* the two subsystems (i.e. radiation cavity mode and mirror) are disentangled, the thermal effects do not destroy the cat state as can be seen in the above equations. The decoherence depends only on the leakage of photons through the fixed mirror.

In the case of cat states generated by conditional measurement the expression for the conditional probability in presence of damping has almost the same structure of Eq. (34), and can be obtained by using Eqs. (33), (22) and (18)

$$P(X|y_{t'}) = \langle X|\langle y_{t'}|\rho_0(t') + \rho_\gamma(t')|y_{t'}\rangle|X\rangle$$

$$= \mathcal{C}' \left(\frac{2}{\pi}\right)^{1/2} e^{-|\alpha_0|^2 - 2X^2} \sum_{p,q=0}^{\infty} \frac{2^{-(p+q)/2}}{p!q!} H_p(\sqrt{2}X) H_q(\sqrt{2}X) e^{i[\arg\alpha_0 + F(t')y_{t'}](p-q)}$$
$$\times |\alpha_0|^{p+q} \left\{ \frac{A_{q,p}}{2} \langle y_{t'}|\rho_T|y_{t'}\rangle + \frac{\gamma}{2}\left[A_{p,q}\tilde{I}_{p,q}(t')|\alpha_0|^2 - A_{q,p}\frac{p+q}{2}t'\langle y_{t'}|\rho_T|y_{t'}\rangle\right]\right\} \tag{37}$$

where

$$\tilde{I}_{p,q}(t') = \int_0^{t'} d\tau \quad \exp\{-i[2E(t'-\tau) + F(t')F(t'-\tau)\sin(\tau/2)](p-q)\}$$
$$\times \quad \langle y_{t'} - F(t'-\tau)\sin(\tau/2)|\rho_T|y_{t'} - F(t'-\tau)\sin(\tau/2)\rangle\,, \tag{38}$$

and, due to Eq. (8), the following general expression holds [25]

$$\langle \mathcal{Y}|\rho_T|\mathcal{Y}\rangle = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} (1-z) \sum_{j=0}^{\infty} \frac{z^j}{2^j j!} e^{-2\mathcal{Y}^2} H_j^2(\sqrt{2}\mathcal{Y}) = \left(\frac{2}{\pi}\frac{1-z}{1+z}\right)^{\frac{1}{2}} \exp\left[-2\mathcal{Y}^2\frac{1-z}{1+z}\right]\,. \tag{39}$$

$\mathcal{C}'$ is a constant needed for the normalization after the projection in the measurement process, and it can be obtained by performing the integration over the X variable of Eq. (37) with the aid of the completness formula for the Hermite polynomials [25]

$$\mathcal{C}' = \left\{\langle y_{t'}|\rho_T|y_{t'}\rangle + \gamma|\alpha_0|^2\left[\tilde{I}_{p,q=p}(t') - \langle y_{t'}|\rho_T|y_{t'}\rangle t'\right]\right\}^{-1}\,. \tag{40}$$

It is easy to note that the correction term in both solutions (34) and (37) remains smaller than the undamped term provided $\gamma|\alpha_0|^2 t << 1$. Equation (37) shows a dependence of the decoherence effects also on the thermal state of the mirror (i.e. its temperature). In Figs. (1) and (2), we show respectively $P(X)$ and $P(X|y_{t'} = 0)$ (solid lines) of Eqs. (34) and (37) contrasted with the same in absence of damping (dashed lines). We may see that in the case of the cat state created at $t^* = 2\pi$, i.e. Fig. (1), the coherence has been almost totally washed out, due to the long time needed for the formation; while the conditional measurement could be used to generate the superposition at shorter time, in Fig. (2) $t' = 3\pi/2$, preserving the coherence effects. In this case, however, one should pay attention to the thermal effect of the mirror.

To this end, let us consider more closely the case of $y_{t'} = 0$, which is a high probable value for the mirror quadrature measurement. The normalization factor on the r.h.s. of Eq. (39) is a common factor that can be eliminated in Eq. (37) by using Eq. (40), while the exponentioal factor remains in the integral of Eq. (38) only. As z approaches the value 1, i.e. the temperature increases, it tends to becomes unity. This means that the thermal effects tend to destroy the coherence only up to a value of temperature, above which the interference fringes become insensitive (dotted line of Fig. (2)). Of course analogous discussions can be made for other values of the mirror quadrature $y_{t'}$.

We also note from both Fig. (1) and Fig. (2) that, as the dissipation becomes relevant, two gaussian peaks centered around the mean number of photons, and which are typical of the orthogonal quadrature, appear. This is essentially due to the rotation in the phase space introduced by the damping term ρ_γ. In fact, as can be seen in Eq. (33), it involves an integration over the time which leads to a distribution whose contributions come from various field phases, i.e. from different quadratures.

6 Detection of Quantum Coherence

In this section we will show that the above discussed model could also be used to reveal the quantum coherence.

According to Ref. [26], the photon number statistics of the radiation field could be opportunely used as signature of the presence of Schrödinger cat states. On the other hand, in the presented model, a measurement of the mirror's momentum $\hat{p}$ allows us to get the photon number statistics in an indirect way [27]. In particular the signal could be represented by the number $a^\dagger a$ of photons of the radiation mode, and the meter by the momentum of the movable mirror; the out of phase quadrature coupled to the photon number (see Eq. (3)).

Our purpose should be to detect the Schrödinger cat immediately after its generation inside the cavity, at time t^* (or t' if one uses conditional measurement generation); nevertheless in both cases the two subsystems, i.e. the mirror and the radiation mode, are disentangled (as can be seen in Eqs. (14) and (18)), so no information can be extracted in indirect way. Then, we must address the measurement to get something which is slightly different from the Schrödinger cat state, but still having quantum coherence features. To this end, let us consider at first the entanglement between the signal and the meter, which could be described by the correlation function defined as follow [28]

$$C_{s,m} = \frac{|\langle a^\dagger a \hat{p} \rangle - \langle a^\dagger a \rangle \langle \hat{p} \rangle|^2}{V_{a^\dagger a} V_{\hat{p}}} \,, \tag{41}$$

where V means the variance. This quantity shows how good is the scheme as a measurement device, and should be equal to one for a perfect scheme. By performing the expetaction values using Eqs. (33) and (5) we obtain

$$C_{s,m} = \frac{2|\alpha_0|^2\kappa^2 \left[\sin^2 t + 4\gamma \sin^2\left(\frac{t}{2}\right)\sin t\right]}{\left[\frac{1}{2} + N_{th} + 2|\alpha_0|^2\kappa^2 \sin^2 t\right](1-\gamma t) + |\alpha_0|^2\kappa^2 \frac{\gamma}{2}\left[2t - 8\sin t + 3\sin(2t)\right]} \,. \tag{42}$$

Thus $C_{s,m}$ is a function of t depending also on κ, which is a constant that contains all the external parameters. Fig. (3) illustrates the typical behaviour of $C_{s,m}$ versus t, showing the effects of dissipation as well as the thermal ones. From this figure it is obviously that higher values of $C_{s,m}$ for times closer to $0, \pi, 2\pi$ could be achieved by increasing the value of κ or $|\alpha_0|$, but we must take into account that the number of photons plays a delicate role in the dissipation effect.

Let us now consider a time at which the radiation is entangled with the mirror, then its state, in absence of loss, by Eq. (20), will be

$$\begin{aligned} \rho_0^{field}(t) &= \mathrm{Tr}_m\{\rho_0(t)\} \\ &= \int dy_t \langle y_t|\rho_T|y_t\rangle e^{iE(t)(a^\dagger a)^2}|\alpha_0 e^{iF(t)y_t}\rangle\langle \alpha_0 e^{iF(t)y_t}|e^{-iE(t)(a^\dagger a)^2} \,, \end{aligned} \tag{43}$$

and furthermore if $E(t)$ satisfies the condition (12) for that time, it becomes

$$\begin{aligned}\rho_0^{field}(t) &= \frac{1}{2}\int dy_t \langle y_t|\rho_T|y_t\rangle \\ \times \quad & \left(e^{-i\frac{\pi}{4}}|\alpha_0 e^{iF(t)y_t}\rangle + e^{i\frac{\pi}{4}}|-\alpha_0 e^{iF(t)y_t}\rangle\right)\left(e^{-i\frac{\pi}{4}}\langle-\alpha_0 e^{iF(t)y_t}| + e^{i\frac{\pi}{4}}\langle\alpha_0 e^{iF(t)y_t}|\right)\end{aligned} \tag{44}$$

which represents not a "pure" cat state, but that one whose phase is still convoluted with the mirror motion and at which we may refer as "pseudo-cat" state. This latter, however, has the advantage of being detected, since it does not imply any disentanglement. It is worth to remark that the dephasing effect due to the factor $\exp(iFy_t)$, which degrades the pure cat into a pseudo-cat state, is considerable only for those values of y_t contained under the gaussian state of the mirror. Then the temperature can emphasize this negative effect, since it introduces highest mirror number states, i.e. gaussians with larger width. On the other hand, in order to reduce this effect, it is also preferable to have the smallest possible values of $F(t)$. These are accessible only at times near to 2π (see eq. (7)). Thus, in order to realize the measurement, the choice of the mesurement time t and the value of κ should be made to fulfill simultaneously the following requirements: Eq. (12), the highest value of $C_{s,m}$, and the smallest value of F. Of course the detection should be performed at time much shorter than the typical cavity lifetime $\gamma^{-1} = 2L/cTr$, but also longer than the photon cavity fly time $2L/c$, to ensure the presence of photons inside the cavity.

Let us now suppose to have found the desired t and κ, then we revise the measurement strategy of Ref. [26] for the detection of quantum macroscopic coherence.

A coherent field $|\alpha_r\rangle$, the "reference", is added to the pseudo-cat state, immediately before the measurement, so that the resulting field in the cavity at the time of measurement is

$$\tilde{\rho}^{field}(t) = \frac{1}{\mathcal{N}} D(\alpha_r)\rho^{field} D^{-1}(\alpha_r)\,, \tag{45}$$

where D is the displacement operator and $\mathcal{N}$ is a normalization constant.

After the injection of the reference field, the photon number distribution in the cavity becomes

$$\mathcal{P}(n) = \langle n|\tilde{\rho}^{field}(t)|n\rangle = \frac{1}{\mathcal{N}}\frac{1}{2}\int dy_t \langle y_t|\rho_T|y_t\rangle \\ \times\left\{ \left| e^{-i\frac{\pi}{4}}\langle n|\alpha_0 e^{iF(t)y_t} + \alpha_r\rangle + e^{i\frac{\pi}{4}}\langle n| -\alpha_0 e^{iF(t)y_t} + \alpha_r\rangle\right|^2 \right\} + O(\gamma)\,, \tag{46}$$

where $O(\gamma)$ indicates the perturbative terms proportional to the first power of γ that we have omitted for space reasons.

Let us now consider separately two cases. When α_0 and α_r have the same phase the photon distribution, denoted by $\mathcal{P}_{in}(n)$, as consequence of the first term in Eq. (46), which is the dominant one, should appear as the sum of two quasi Poissonian distributions peaked around $n = |\alpha_0 + \alpha_r|^2$ and $n = |-\alpha_0 + \alpha_r|^2$, with the tails due to the smearing effect of the gaussian integral. In fact, in Eq. (46), the interference part will be negligible provide to have $|\alpha_r| >> 1$. An interesting situation arises when α_0 and α_r have the same amplitude, then

$$\mathcal{P}_{in}(n) = \frac{1}{\mathcal{N}}\frac{1}{2}\frac{|\alpha_0|^{2n}}{n!}\int dy_t \langle y_t|\rho_T|y_t\rangle \\ \times\left\{ \left[c_+^n e^{-|\alpha_0|^2 c_+} + c_-^n e^{-|\alpha_0|^2 c_-} + 2c_+^{n/2} c_-^{n/2} e^{-2|\alpha_0|^2}\Re\{-i(i)^n\}\right] \right\} + O(\gamma)\,, \tag{47}$$

where

$$c_\pm = 2 \pm 2\cos\left(F(t)y_t\right)\,. \tag{48}$$

In that case, neglecting the perturbation terms, $\mathcal{P}_{in}(n)$ consists of a very sharp distribution centered at $n = 0$, which is a δ-like peak for a pure cat state, and a distribution peaked around $n = 4|\alpha_0|^2$. The existence of two separate peaks in the in-phase sum field is the proof of the existence of two classical fields within the cavity. However, it does not prove that these two fields are in a coherent quantum mechanical superposition. So we need to consider also the case when α_0 and α_r are $\pi/2$ out of phase, for which we have

$$\mathcal{P}_{out}(n) = \frac{1}{\mathcal{N}}\frac{1}{2}\frac{|\alpha_0|^{2n}}{n!}\int dy_t \langle y_t|\rho_T|y_t\rangle$$
$$\times\left\{ \left[s_+^n e^{-|\alpha_0|^2 s_+} + s_-^n e^{-|\alpha_0|^2 s_-} + 2 s_+^{n/2} s_-^{n/2} e^{-2|\alpha_0|^2}\Re\{-i(-i)^n\}\right] \right\} + O(\gamma) \quad (49)$$

where now

$$s_\pm = 2 \pm 2\sin\left(F(t) y_t\right) \; . \quad (50)$$

In this case the interference in the term in Eq. (46) becomes important, in fact $\mathcal{P}_{out}(n)$, again neglecting the perturbation terms, exhibits a Poisson envelope with strong oscillations, signaling the coherence effect. The above discussed dephasing effect in the pseudo-cat tends to wash out the oscillations and to transform the Poisson envelope in a gaussian one. Of course in both cases (*in* and *out*) also the damping terms cause a degradation of the signal.

In Fig. (4) we show $\mathcal{P}_{in}(n)$ for a pseudo-cat state a) which resembles that one for a pure cat state, contrasted with the same in presence of damping at zero temperature b) and at finite temperature c). Fig. (5) illustrates the same situations for $\mathcal{P}_{out}(n)$. Both figures are obtained using $t = 0.84 \times 2\pi$ and $\kappa = 0.5$ for which one has $F = 0.48$ and $C_{s,m} = 0.85$ (at zero temperature, while it is reduced to 0.55 when $N_{th} = 2$).

The $\mathcal{P}_{in}(n)$ and $\mathcal{P}_{out}(n)$ distributions can actually be measured detecting the momentum of the mirror, of course the measurement process is destructive, hence the state has to be reprepared for each measurement, and a large number of measurements should be performed to reach the desired statistics. Then, from these output distributions, one can recognize a signature of quantum coherence as in Fig. (4) and Fig. (5), provided to have small dissipation and very low temperature, which is needed also to guarantee a sufficient signal meter correlation (Fig. (3)).

Finally, to effectively visualize the presence of interference fringes in the phase space, we would consider the marginal distribution for the pseudo-cat. This probability, obtained through the expectation value $\langle X|\mathrm{Tr}_m\{\rho_0(t)+\rho_\gamma(t)\}|X\rangle$ and using Eqs.(44) and (33), will be

$$P^{pc}(X)=\left(\frac{2}{\pi}\right)^{1/2}e^{-|\alpha_0|^2-2X^2}\sum_{p,q=0}^{\infty}\frac{2^{-(p+q)/2}}{p!q!}H_p(\sqrt{2}X)H_q(\sqrt{2}X)e^{i\arg\alpha_0(p-q)}$$
$$\times|\alpha_0|^{p+q}\left\{\frac{A_{q,p}}{2}+\frac{\gamma}{2}\left[A_{p,q}I_{p,q}(t)|\alpha_0|^2-A_{q,p}\frac{p+q}{2}t\right]\right\}\exp\left[-\frac{F^2(t)(p-q)^2(1+z)}{8(1-z)}\right], \tag{51}$$

where, with the superscript pc we refer to the pseudo-cat state. It is clear from the last exponential factor how the thermal phonons of the mirror tend to rapidly destroy the coherence effect.

In Fig. (6) we show the marginal distribution $P^{pc}(X)$ of Eq. (51) for various situations, using the above discussed values of parameters i.e. $t=0.84\times2\pi$, $\kappa=0.5$. From this picture we may note that the interference pattern of the pseudo-cat state is almost the same of the pure one and is still preserved at the time of measurement, even in the presence of loss provided to have a very small number of thermal excitations in the mechanical oscillator.

7 Conclusion

We have proposed the use of an optomechanical model for the generation of optical Schrödinger cat states. We have also presented a new scheme to reveal the quantum macroscopic coherence, based on the new states named pseudo-cats that could be intended as a sort of cat states which could be recognized before their "natural birth". Thus the model is substantially able to produce and also to detect interference effects without introducing different couplings, but one should pay attention to the different sources of dissipation.

We would also point out that the studied system could be implemented for example idealizing the movable mirror as a piezoelectric crystal [29]. The above used values of t, κ and γ (in the various figures), could be reached for example with the following set of parameters $\omega_c \approx 10^{16}\,\mathrm{s}^{-1}$, $\omega_m \approx 10^4\,\mathrm{s}^{-1}$, $m \approx 10^{-14}\,\mathrm{Kg}$, $L \approx 1.5\,\mathrm{m}$, $Tr \approx 10^{-6}$ and $T \approx 10^{-7} K$. Of course, other choices satisfying the above mentioned criteria can be made giving the same qualitative results. We are aware that a delicate point could be the realization of the mechanical oscillator with a very small mass, but we would remark that the mass parameter could also be interpreted as an effective value coming from the density of the vibrational modes of the mechanical oscillator [30]. Furthermore, the discussed model could be improved by inserting an active Kerr medium inside a cavity which enhances the nonlinear effects, slowing down the decoherence.

Finally, even if we have not coupled the system under study with an external readout apparatus able to measure the momentum of the moving mirror, we think that the presented model represents an interesting alternative way to approach, also in the experimental sense, the quantum macroscopic coherence phenomena.

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FIGURE CAPTIONS

Fig. 1 The marginal distribution $P(X)$ is plotted as function of the quadrature variable X for $\kappa = 0.5$, $|\alpha_0| = \sqrt{7}$, at $t^* = 2\pi$ and in two different cases: $\gamma = 0$ (dashed line) and $\gamma = 2 \times 10^{-2}$ (solid line).

Fig. 2 The marginal distribution $P(X|y_{t'})$ is plotted as function of the quadrature variable X for $y_{t'} = 0$, $\kappa = 0.52$, $|\alpha_0| = \sqrt{7}$, at $t' = 3\pi/2$ and in three different cases: $\gamma = 0, N_{th} = 0$ (dashed line); $\gamma = 2\times 10^{-2}, N_{th} = 0$ (solid line); $\gamma = 2\times 10^{-2}, N_{th} \geq 20$ (dotted line).

Fig. 3 The correlation coefficient $C_{s,m}$ is plotted against the time for $\kappa = 0.5$ in the case of $\gamma = 0, N_{th} = 0$ (dashed line); $\gamma = 10^{-2}, N_{th} = 0$ (solid line); $\gamma = 10^{-2}, N_{th} = 2$ (dotted line).

Fig. 4 The distribution $\mathcal{P}_{in}(n)$ vs. the photon number is plotted for a pseudo-cat with $\kappa = 0.5$, $|\alpha_0| = \sqrt{7}$ and $t = 0.84 \times 2\pi$ in the case of $\gamma = 0, N_{th} = 0$ a); $\gamma = 10^{-2}, N_{th} = 0$ b); $\gamma = 10^{-2}, N_{th} = 2$ c).

Fig. 5 The same of Fig. 4, but for $\mathcal{P}_{out}(n)$.

Fig. 6 The marginal distribution $P^{pc}(X)$ is plotted for $\kappa = 0.5$, $|\alpha_0| = \sqrt{7}$, $t = 0.84 \times 2\pi$ and the values of γ and N_{th} indicated in the figure. It is also compared with the distribution for a pure cat.