The Luttinger model following a sudden interaction switch-on

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The evolution of correlations in the exactly solvable Luttinger model (a model of interacting fermions in one dimension) after a sudden interaction switch-on is analytically studied. When the model is defined on a finite-size ring, zero-temperature correlations are periodic in time. However, in the thermodynamic limit, the system relaxes algebraically towards a stationary state which is well described, at least for some simple correlation functions, by the generalized Gibbs ensemble recently introduced by Rigol et al. [cond-mat/0604476]. The critical exponent that characterizes the decay of the one-particle correlation function is different from the known equilibrium exponents. Experiments for which these results can be relevant are also discussed.

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Experiments with cold atomic gases are motivating research into problems that, previously, would have looked highly academic. One such problem concerns the evolution of a quantum many-body system where interactions (or other parameters of the system) are time-dependent. An example is an interaction quench: an experiment where the strength of interactions is suddenly changed. This type of experiment is nowadays feasible thanks to the phenomenon known as Feshbach resonance [1, 2], which allows to tune the strength and sign of interactions in a cold atomic gas by means of a magnetic field. If the applied magnetic field is time dependent, the interactions become time-dependent. Alternatively, in optical lattices [3], it is possible to change the lattice parameters in a time-dependent fashion, which effectively amounts to varying the ratio of the interaction to the kinetic energy in time. On the theory side, the recent development of extensions of the density-matrix renormalization group (DMRG) algorithm [4, 5, 6, 7, 8] has spurred the interest in understanding the properties of quantum many-body systems out of equilibrium and, in particular, in the dynamics following a quench.

Because of these new possibilities, the evolution of observables and correlations following a sudden change of the system parameters is attracting much theoretical interest [9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. One interesting question that has been raised by a recent experiment in an array of 1D cold atomic gases [19] is whether after a quench a system possessing an infinite number of integrals of motion can exhibit relaxation towards a steady state or not. This question has been analyzed by the authors of Ref. [12], who have numerically shown that the steady state of an integrable gas of hard-core bosons is described by a generalized Gibbs distribution that maximizes the entropy with all possible constraints imposed by the existence of the (infinite number of) integrals of motion. Here the effect of suddenly turning on the interactions in the Luttinger model is analytically studied. It is shown that, when the model is defined on a finitesize ring, the asymptotic form of the two-point one-body and density correlations at zero temperature is periodic in time, and therefore the system exhibits no relaxation to a steady state with time-independent properties. In the termodynamic limit, however, the same correlation functions relax to a steady state, whose properties are different from those of the ground state. Indeed, the decay of the one-particle correlations with distance is governed by a critical exponent which is different from the known equilibrium exponents. Interestingly, one-particle and density correlations in the steady state can be obtained using the generalized Gibbs ensemble introduced by Rigol et al. in Ref. [12].

The Luttinger model (LM) describes a system of interacting Fermions in one dimension (1D). It was introduced by Luttinger [20] in 1963, but the correct exact solution was found in 1965 by Mattis and Lieb [21]. Asymptotic forms of one and two-particle correlations in equilibrium were obtained by Luther and Peschel [22]. Later, Haldane [23, 24, 25] proposed that this model describes the low-energy properties of a fairly broad class of systems in 1D known as Tomonaga-Luttinger liquids [23, 26, 27].

The Hamiltonian of the LM, $H_{LM}=H_0+H_2+H_4$, where $H_0=\sum_{p,\alpha}\hbar v_F p:\psi_\alpha^\dagger(p)\psi_\alpha(p)$: is the free-fermion Hamiltonian, and the interactions are described by

$$H_2 = \frac{2\hbar\pi}{L} \sum_q g_2(q) J_R(q) J_L(q),$$
 (1)

$$H_4 = \frac{\hbar \pi}{L} \sum_{q,\alpha} g_4(q) : J_{\alpha}(q) J_{\alpha}(-q) : ,$$
 (2)

where the Fermi operators $\{\psi_{\alpha}(p), \psi_{\beta}^{\dagger}(p')\} = \delta_{p,p'}\delta_{\alpha,\beta}$ $(\alpha, \beta = L, R)$ and anti-commute otherwise. To avoid a degenerate ground state, anti-periodic boundary conditions are chosen: $\psi_{\alpha}(x+L) = -\psi_{\alpha}(x)$, $(\psi_{\alpha}(x) = \sum_{p} e^{is_{\alpha}px} \psi_{\alpha}(p)/\sqrt{L}$ is the Fermi field operator and $s_{R} = -s_{L} = +1$ and L the length of the system) so that $p = 2\pi(n - \frac{1}{2})/L$, and n is an integer. The "current" operators $J_{\alpha}(q) = \sum_{q} : \psi_{\alpha}^{\dagger}(p+q)\psi_{\alpha}(p):$, where $q = 2\pi m/L$, m being an integer; $: \dots :$ stands for the normal order prescription according to which all creation operators are to be found to the left of the annihilation operators and expectation values over the ground state

of H_0 are subtracted. Thus, the above model describes a system of fermions interacting via the four Fermion terms H_2 and H_4 . Fermions come in two chiralities, R standing for right moving and L for left moving particles, respectively. The dispersion is linear and therefore it is not bounded from below. To define a stable ground state for H_0 all single-particle levels with p < 0 are filled up for both chiralities, which yields a Dirac sea (i.e. an "infinite story" hotel) which will be denoted as $|0\rangle$. The coupling functions $g_2(q)$ and $g_4(q)$ are assumed to be finite for q = 0. Moreover, to ensure that the Hilbert space of H_{LM} and H_0 remain the same and, in particular, that their ground states have a finite overlap at finite L, $g_2(q)/(v_F + g_4(q)) \to 0$ faster than $|q|^{-1/2}$ as $|q| \to \infty$ and $|g_2(q)| < v_F + g_4(q)$ for all q [24].

The currents obey a Kac-Moody algebra [24, 26, 27]: $[J_{\alpha}(q), J_{\beta}(q')] = \frac{qL}{2\pi} \delta_{q+q',0} \ \delta_{\alpha,\beta}$. This fact allows to introduce, for $q \neq 0$, the following operators $b_0(q) = -i \left(2\pi/|q|L\right)^{1/2} \left[\theta(q)J_R(-q) - \theta(-q)J_L(q)\right]$ and $b_0^{\dagger}(q) = i \left(2\pi/|q|L\right)^{1/2} \left[\theta(q)J_R(q) - \theta(-q)J_L(-q)\right]$, which obey the standard algebra of boson ("phonon") operators. Moreover, there are two conserved operators $\delta N = N_R + N_L$, i.e. the number operator referred to the ground state $|0\rangle$, and the total current $J = N_R - N_L$, where $N_{\alpha} = J_{\alpha}(0)$. For fermions, the physical states obey the selection rule $(-1)^{\delta N} = (-1)^J$. In terms of the boson operators $b_0^{\dagger}(q), b_0(q)$ the Hamiltonian H_{LM} is quadratic but not diagonal. It can be diagonalized by means of a Bogoliubov ('squeezing') transformation [21]:

$$b(q) = \cosh \varphi(q) b_0(q) + \sinh \varphi(q) b_0^{\dagger}(-q), \quad (3)$$

$$b^{\dagger}(q) = \sinh \varphi(q) b_0(-q) + \cosh \varphi(q) b_0^{\dagger}(q).$$
 (4)

To render H_{LM} diagonal, we must choose $\tanh 2\varphi(q) = g_2(q)/[v_F + g_4(q)]$. Thus the Hamiltonian becomes [24] $H_{LM} = \sum_{q \neq 0} \hbar v(q) |q| b^{\dagger}(q) b(q) + \hbar \pi v_N \delta N^2/L + \hbar \pi v_J J^2/L$, where $v(q) = [(v_F + g_4(q))^2 - g_2^2(q)]^{1/2}$, $v_N = ve^{2\varphi}$, and [24] $v_J = ve^{-2\varphi}$, being v = v(0) and $\varphi = \varphi(0)$.

Let us now consider an interaction quench in the LM. Here I consider only the case where the coupling functions $g_2(q)$ and $g_4(q)$ are suddenly switched on at t=0. Thus, the initial state of the system will be described by a thermal distribution determined by the non-interacting Hamiltonian H_0 , $\rho(t=0) = \rho_0 = e^{-H_0/T}/Z_0$, where $Z_0 = \operatorname{Tr} e^{-H_0/T}$. However, for t > 0, the evolution is dictated by the full Hamiltonian H_{LM} . A more general type of quench corresponds to a sudden switch between two different forms of $g_2(q)$ and $g_4(q)$. Whereas the results described below can be generalized to such a case, I believe a quench from the non-interacting limit is most interesting because the spectrum of H_0 contains free fermions whereas the spectrum of H_{LM} does not [22, 24, 26, 27]. Thus, a sudden switch-on of the interactions describes a time-dependent destruction of the characteristic discontinuity of the momentum distribution at the Fermi points p = 0.

Equal time correlations of a given operator O(x),

$$C_O(x,t) = \langle e^{iH_{LM}t/\hbar}O^{\dagger}(x)O(0)e^{-iH_{LM}t/\hbar}\rangle_0$$
(5)
= Tr $\rho_0 e^{iH_{LM}t/\hbar}O^{\dagger}(x)O(0)e^{-iH_{LM}t/\hbar}$. (6)

Note that since $[H_0, H_{LM}] \neq 0$, $C_O(x, t)$ is explicitly time-dependent. Indeed, in the LM model time dependence stems from H_2 , since $[H_0, H_4] = 0$. H_2 describes scattering between fermions moving in opposite directions, and, as shown below, it produces entanglement between the excitation modes with q > 0 and q < 0.

The exact evolution of $b_0(q)$ has a fairly simple form:

$$b_0(q,t) = f(q,t)b_0(q) + g^*(q,t)b_0^{\dagger}(-q), \tag{7}$$

where $b_0(q,t) = e^{iH_{LM}t/\hbar}b_0(q)e^{-iH_{LM}t/\hbar}$, $f(q,t) = \cos v(q)|q|t - i\sin v(q)|q|t\cosh 2\varphi(q)$, and $g(q,t) = i\sin v(q)|q|t\sinh 2\varphi(q)$. Note that this form obeys the correct boundary condition $b_0(q,0) = b_0(q)$. Entanglement between modes of opposite q vanishes for $\varphi(q) = 0$ (i.e. $g_2(q) = 0$) in agreement with the above discussion.

The evolution of one-particle correlations (i.e. $O(x) = \psi_{\alpha}(x)$) can be obtained from Eq. (7) and the bosonization formula [22, 24, 26, 27]:

$$\psi_{\alpha}(x) = \frac{\eta_{\alpha}}{(2\pi a)^{1/2}} e^{is_{\alpha}\phi_{\alpha}(x)}, \tag{8}$$

being $\eta_R \neq \eta_L$ two different Pauli matrices that ensure the anti-commutation of the left and right-moving Fermi fields; $\phi_{\alpha}(x) = s_{\alpha}\varphi_{0\alpha} + 2\pi x N_{\alpha}/L + \Phi_{\alpha}^{\dagger}(x) + \Phi_{\alpha}(x)$, where $[N_{\alpha}, \varphi_{0\beta}] = i\delta_{\alpha,\beta}$, $\Phi_{\alpha}(x) = \sum_{q>0} (2\pi/qL)^{1/2} e^{-qa/2} e^{iqx} b_0^{\dagger}(q)$, and $a \to 0^+$. Setting $\sinh 2\varphi(q) = e^{-|qR_0|/2} \sinh \varphi$, where $R_0 \ll L$ is of the order of the range of the interactions, and replacing v(q) by its q=0 value [22], simplifies the calculations without altering the asymptotic form of the correlations. At T=0, for a system of size L, one-body correlations are given by the following expression [31]:

$$C_{\psi_R}(x,t>0|L) = G_R^{(0)}(x|L) \left[\frac{R_0}{d(x|L)} \right]^{\gamma^2} \times \left[\frac{d(x-2vt|L)d(x+2vt|L)}{[d(2vt|L)]^2} \right]^{\gamma^2/2}, \tag{9}$$

where $d(x|L) = L|\sin(\pi x/L)|/\pi$ is the cord function, $G_R^{(0)}(x|L) = i/[2L\sin\pi(x+ia)/L]$ the non-interacing correlation function, and $\gamma = \sinh 2\varphi$. The above expression is accurate asymptotically, i.e. for $d(x \pm 2vt|L), d(x|L), d(2vt|L) \gg R_0$. It can be seen that the one-particle correlations are periodic in time: $C_{\psi_R}(x,t+T_0|L) = C_{\psi_R}(x,t|L)$ with $T_0 = L/2v$. This implies that the finite-size LM does not relax, which is a consequence of the (approximately) linear dispersion of the

eigenmodes near q=0 along with the absence of any damping mechanisms in the LM (see discussion at the end). However, in the thermodynamic limit, $L \to \infty$, and $d(x|L) \to |x|$. Therefore, Eq. (9) becomes:

$$C_{\psi_R}(x,t>0) = \frac{i}{2\pi(x+ia)} \left| \frac{R_0}{x} \right|^{\gamma^2} \left| \frac{x^2 - (2vt)^2}{(2vt)^2} \right|^{\gamma^2/2}.$$
(10)

It is interesting to analyze the above expression in the limit where $2vt \ll |x|$, where it becomes

$$C_{\psi_R}(R_0 \ll 2vt \ll |x|) \approx \frac{iZ(t)}{2\pi(x+ia)},$$
 (11)

being $Z(t) = (R_0/2vt)^{\gamma^2}$ a time-dependent renormalization constant of the Fermi quasi-particles. Thus for short-times the system behaves as a Femi liquid, with a singularity at the Fermi points given by Z(t), which decreases with time. On the other hand, for $2vt \gg |x|$ the correlation takes a non-Fermi liquid form:

$$C_{\psi_R}(R_0 \ll |x| \ll 2vt) \approx \frac{i}{2\pi(x+ia)} \left| \frac{R_0}{x} \right|^{\gamma^2}$$
 (12)

In particular, in the limit $t \to +\infty$ one-particle correlations relax to the power-law in the right-hand side of Eq. (12). Notice that, although $C_{\psi_R}(x,t\to\infty)$ exhibits a power-law behavior, the latter is governed by an exponent that is different from the one that governs asymptotic ground-state correlations [22, 24], $\gamma_0^2 = 2 \sinh^2 \varphi < \gamma^2 = \sinh^2 2\varphi$ for $\varphi \neq 0$. The origin of this new exponent will be discussed below.

The different behavior of $C_{\psi_R}(x,t)$ for short and long times can be understood in terms of a 'light-cone' effect [11]: The initial state $|0\rangle$ has higher energy than the ground state of H_{LM} (see discussion further below). Therefore, it contains long wave-length phonons that propagate from time = 0 to time = t along lightcones where the role of speed of light is played by v. These excitations determine which points retain the same type of correlations found in $|0\rangle$ and which points acquire new correlations. The latter phenomenon and the overall structure of (10) bears some resemblance to results reported in Ref. [11]. Nevertheless, I have so far failed to extend the methods of [11] to the quench in the LM. There are two main differences: the initial state in the present case is non-critical, and therefore it does not have any characteristic (gap) energy scale as the initial states considered in [11]. Secondly, and more importantly, the critical exponent found above is different from the bulk or boundary exponents of the field operator $\psi_R(x)$. Indeed, this may be an indication that the quench in the LM belongs to a different universality class.

One may think that the relaxation behavior exhibited by $C_{\psi_R}(x,t)$ in the thermodynamic limit is because the field operator, $\psi_R(x)$, is a non-linear func-

tion of $b_0(q)$ and $b_0^{\dagger}(q)$. However, the (density) operator $J_R(x) = \partial_x \phi_R(x)/2\pi$ also exhibits relaxation. Setting $O(x) = J_R(x)$ in (6), the following is obtained using Eq. (7),

$$C_{J_R}(x,t|L) = -\frac{1}{4\pi^2} \left\{ \frac{1+\gamma^2}{[d(x|L)]^2} - \frac{\gamma^2}{2[d(x-2vt)]^2} - \frac{\gamma^2}{2[d(x-2vt)]^2} \right\} (13)$$

For finite L the density correlation function is again periodic in time. However, for $L \to \infty$, it shows relaxation: $C_{J_R}(x,t\to\infty|L)\to -(1+\gamma^2)/(4\pi^2x^2)$. This form again deviates from the ground state behavior, where the prefactor of $-1/(4\pi^2x^2)$ is $\cosh 2\varphi - \sinh 2\varphi = e^{-2\varphi}$ [24, 26, 27].

It is interesting to find that the above results in the $t\to\infty$ limit can be analytically obtained from the generalized Gibbs distribution introduced in Ref. [12], which is described by the following density matrix:

$$\rho_{gG} = \frac{1}{Z_{gG}} e^{\sum \lambda(q)I(q)}, \tag{14}$$

where $Z_{gG} = \text{Tr } e^{\sum \lambda(q)I(q)}$ and [H,I(q)] = [I(q),I(q')] = 0, that is, a set of independent integrals of motion. Since $[H_{LM},n(q)]=0$, where $n(q)=b^{\dagger}(q)b(q)$, the phonon occupancy operators seem as the most natural choice for I(q). The Lagrange multipliers $\lambda(q)$ are obtained from the condition [12]:

$$\langle n(q)\rangle_{t=0} = \langle 0|n(q)|0\rangle = \langle n(q)\rangle_{qG} = \text{Tr } [\rho_{qG}n(q)], (15)$$

where T=0 was assumed. Using (3,4), $\langle 0|n(q)|0\rangle=\sinh^2\varphi(q)$, which is a non-thermal distribution. However, $\lambda(q)$ do not need to obtained explicitly, as it suffices to realize that ρ_{gG} has the same form as the distribution in the canonical ensemble with $H/T=-\sum_q\lambda(q)n(q)$. One can also regard ρ_{gG} as a canonical distribution with a q-dependent temperature, $T(q)=-\hbar v(q)|q|/\lambda(q)$. Using this fact along with equations (3,4) and (8), I find that

$$C_{\psi_R}^{gG}(x) = \text{Tr } \rho_{gG} \ \psi_R^{\dagger}(x) \psi_R(0) = \lim_{t \to +\infty} C_{\psi_R}(x,t), \ (16)$$

$$C_{J_R}^{gG}(x) = \text{Tr } \rho_{gG} J_R(x) J_R(0) = \lim_{t \to +\infty} C_{J_R}(x, t).$$
 (17)

Thus, at least for these simple correlation functions, it seems that the generalized Gibbs distribution describes the stationary state of the LM after an interaction quench. The reason why the critical exponent γ^2 turns out to be different from the known equilibrium exponents can be thus explained in two different ways: mathematically, it is seen that in order to obtain the evolution of the operator $b_0(q)$, Eq (7), one has to do and undo the Bogoliubov transformation (3,4). However, these transformations do not cancel each other exactly (except at t=0) because of the phase factors $e^{\pm iv(q)|q|t}$ introduced by the time evolution operator. In contrast, in the equilibrium problem, since

the expectation value is taken over the ground state of H_{LM} , the Bogoliubov transformation is performed only once. Physically, in view of the results (16,17), the difference in exponent can be regarded a consequence of the non-equilibrium distribution of phonons $\langle 0|n(q)|0\rangle = \sinh^2\varphi(q)$, which is a constant of motion. Note as well that $\langle 0|n(q)n(q')|0\rangle - \langle 0|n(q)|0\rangle\langle 0|n(q')|0\rangle = \left[\sinh^4\varphi(q) + 2\cosh^2\varphi(q)\sinh^2\varphi(q)\right]\delta_{q,q'}$ is non-zero for q=q', since $|0\rangle$ is not an eigenstate of H_{LM} for $g_2(q)\neq 0$.

Let us finally consider how the above predictions could be experimentally observed. To date, there are no exact realizations of the LM in nature. However, one can exploit the fact that the LM describes the low-energy properties of Tomonaga-Luttinger liquids [24, 26, 27], of which several physical realizations in cold atomic gases are available [19, 28, 29]. Let us therefore consider a single-species cold Fermi gas confined to 1D in a strongly anisotropic trap [29]. In a single-species cold Fermi gas, the p-wave interaction is naturally negligible. One possibility to realize a sudden change of the interaction is to use a p-wave Feshbach resonance [29], which enhances the strength of this interaction. Alternatively, one can use a 1D dipolar Fermi gas, where interactions at long distances are described by the potential:

$$V_{\rm dip}(x) = \frac{1}{4\pi\epsilon_0} \frac{D^2(1 - 3\cos\theta)}{[x^2 + R_0^2]^{3/2}},\tag{18}$$

D being the dipolar momentum of the atoms and θ is the angle subtended by the direction of the atomic motion and an electric field (or magnetic, for magnetic dipoles) that polarizes the gas. In the above expression R_0 is of the order of the transverse size of the cloud. The Fourier transform of (18), $g_2(q) = g_4(q) \propto V_{\rm dip}(q) = \lambda(\theta)|qR_0|K_1(|qR_0|)$, where $\lambda(\theta) = D^2(1-3\cos\theta)/2\pi\epsilon_0R_0^2$ and $K_1(x)$ is the first order modified Bessel function. An sudden switch-on of $V_{\rm dip}(q)$ can be realized by deviating the electric field that polarizes the gas from the "magic" angle $\theta_m = \cos^{-1}(\frac{1}{3})$, for which the interaction vanishes (i.e. $\lambda(\theta_m) = 0$).

However, the full Hamiltonian for a TLL contains an infinite series of terms that spoil the integrability of the LM [23, 24]. Roughly speaking, these stem from the non-linearity of the fermion dispersion [32] and the fact that interactions couple right and left-moving modes in a way that is highly non-linear in terms of the boson fields $\phi_{\alpha}(x)$ (umklapp scattering) [23]. In a TLL all these deviations are irrelevant in the renormalizationgroup sense, which means that their effect on low-energy states is small. Nevertheless, after a sudden change of the interaction in the systems described above, highenergy excitations will be created that are not described by the LM. Exciting many fermions to levels very far from the Fermi level where the LM description is not accurate can be avoided by turning on the interaction to a value much smaller than the Fermi energy. On the other hand, low energy excitations will survive for longer

times and, since they dominate the long-time dynamics, the behavior of the correlations will be described by the above results. Thus, if the quench was conducted at zero temperature, since the atomic systems are finite, an approximately periodic behavior of correlations can be expected. However, Fermi gases are usually hard to cool down, and a situation where temperature is larger than level spacing (i.e. $T \gg \pi/L$) is perfectly reallistic. In this situation, one should consider correlations at finite T, neglecting finite size effects. The latter can be obtained from Eq. (9) upon replacing $L\sin(\pi x/L)/\pi$ by $(\hbar v_F/\pi T) \sinh(\pi T x/\hbar v_F)$, etc. Thus relaxation takes place because temperature induces a finite correlation length in the initial state and therefore correlations decay exponentially. One-body correlations that can be accessed through the momentum distribution, which can be measured in a time of light experiment. Thus the steady state momentum distribution following a sudden switch-on of interactions should differ from the equilibrium distribution at the same tempereature. A more detailed analysis will be given elsewhere [30].

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