Notes on Certain Newton Gravity Mechanisms of Wave Function Localisation and Decoherence

Lajos Diósi

Research Institute for Particle and Nuclear Physics H-1525 Budapest 114, POB 49, Hungary

September 21, 2018

Abstract

Both the additional non-linear term in the Schrödinger equation and the additional non-Hamiltonian term in the von Neumann equation, proposed to ensure localisation and decoherence of macro-objects, resp., contain the same Newtonian interaction potential formally. We discuss certain aspects that are common for both equations. In particular, we calculate the enhancement of the proposed localisation and/or decoherence effects, which would take place if one could lower the conventional lengthcutoff and resolve the mass density on the interatomic scale.

1 Introduction

Experts in history of science may perhaps know what von Neumann's approach would be to the concept of a fully quantised Universe. His measurement theory yields perfect statistical interpretation of the quantum state as long as there exists a classical — non-quantised — sector of the Universe. The challenge of a fully quantised Universe has been attracting many theorists even in the lack of pressing experimental evidences. Where might those evidences — or at least indications — come from? That must be the combination of extreme high energies and extreme high gravitating mass densities. As a consequence, the mainstream concept of a quantised Universe targets a quantised cosmology through the quantisation of the Einstein theory of space-time. Despite theoretical efforts through the past decades, that big step has not been done so far. Experts do not agree what the bottle-neck is. It may be our concept of the quantum or our concept of the space-time. Both, certainly. I used to emphasise one: the bottle-neck is the quantum. The von Neumann theory of measurement becomes useless if the whole Universe is quantised. To make a shortcut to our subject, we cite a figure from ref. [1], with the Schrödinger equation of the Universe written in the middle, see fig. 1. Our failure to interpret the universal wave function Ψ may not be related to relativity. The formal argument of the figure is almost categoric: one of the three partially unified theories is



Figure 1: Scheme of Physics' Building. c =velocity of light, G =Newton's gravitational constant, \hbar =Planck constant. The corners of the triangle represent the three fundamental theories, the sides correspond to partially unified theories while the middle symbolises the fully unified theory.

missing. Then, why not, the bottle-neck may be the missing unified theory for quantum mechanics (\hbar) and Newtonian gravity (G) — once called Newtonian Quantum Gravity. We may assume that the path upto a relativistic theory of a quantised Universe goes through the non-relativistic theory of Newtonian Quantum Gravity explaining the quantised motion of common macroscopic objects. In particular, we can make a small step toward the theory of quantised Universe if we establish a theory of "spontaneous" measurement of quantised non-relativistic macro-objects.

For the past 20 years, many authors have considered the possible role of Newtonian gravity in resolving the apparent controversy between the common classical motion of macro-objects and their quantum mechanical description [1]-[14]. I will focus on specific old proposals where the standard Schrödingervon-Neumann equations of quantum mechanics are modified by concrete gravitational terms of simple and transparent mathematical structure.

2 Two mechanisms, two models, one Newtonian structure

The studies of our interest concentrated on two inter-related elements of classical behaviour of a rigid macro-object: precise center of mass *localisation* and *decoherence* (decay) of superposition between separate positions. To guarantee the first, the attractive Newtonian self-consistent gravitational field was introduced into the Schrödinger equation [2, 11]. To guarantee the second, a universal decay mechanism was postulated for superpositions between separate positions, scaled by the difference between the corresponding Newtonian field strengths [4, 5, 8]. In both *localisation* and *decoherence* mechanisms, resp., the relevant

quantity is the Newtonian interaction

$$U(X, X') = -G \int \frac{f(\mathbf{r}|X)f(\mathbf{r}'|X')}{|\mathbf{r}' - \mathbf{r}|} d\mathbf{r} d\mathbf{r}'$$
(1)

between two mass densities corresponding to two configurations X, X' of the macro-objects that form our quantum system. Typically for rigid objects, position X contains the center of mass coordinates $\mathbf{x}_1, \mathbf{x}_2, \ldots$ and the rotation angles $\theta_1, \theta_2, \ldots$. For simplicity, we shall consider spherically symmetric or point-like objects, to discuss their translational degrees of freedom. Hence X stands for $\mathbf{x}_1, \mathbf{x}_2, \ldots$ only.

With the help of the interaction potential (1), we construct the Schrödinger-Newton equation for the wave function $\psi(X)$ of the massive objects [2, 11]:

$$i\hbar \frac{d\psi(X)}{dt} = \text{standard q.m. terms} + \int U(X, X') |\psi(X')|^2 dX' \ \psi(X) \ . \tag{2}$$

The second term on the rhs leads to stationary solitary solutions. The Schrödinger-Newton eq. ensures the stationary localisation of the objects. Yet, the equation can not account for the expected decoherence of macroscopic superpositions like $|X\rangle + |Y\rangle$.

An alternative, irreversible, equation serves this latter purpose. We start from the von Neumann equation which is equivalent with the standard Schrödinger equation. It evolves the density matrix $\rho(X, Y)$ rather than the wave function $\psi(X)$. The construction of the von-Neumann-Newton equation reads [4, 5]:

$$\frac{d\rho(X,Y)}{dt} = \text{standard q.m. terms} + \frac{U(X,X) + U(Y,Y) - 2U(X,Y)}{2\hbar}\rho(X,Y) .$$
(3)

The second term on the rhs contributes to an exponential decay of the superposition $|X\rangle + |Y\rangle$, with the following decoherence time [4, 5, 8]:

$$\frac{2\hbar}{2U(X,Y) - U(X,X) - U(Y,Y)} .$$
 (4)

To avoid misunderstandings, we emphasise that the Schrödinger-Newton eq. (2) and the von-Neumann-Newton eq. (3) are two *alternative* equations to modify the standard quantum mechanics for macro-objects. In our notes, we shall treat these two separate equations parallel to each other because the gravitational terms depend on the same Newton interaction (1) in both equations. [The desired two effects, localisation plus decoherence, have been realised in ref. [5] through a single stochastic Schrödinger/von-Neumann-Newton equation based invariably on the structure U(X, X').]

3 Case study of a rigid ball

Following tradition, restrict ourselves for the study of a single rigid ball of mass M and radius R. Its mass density depends on the distance from the center of

mass \mathbf{x} :

$$f(\mathbf{r}|\mathbf{x}) = f(\mathbf{r} - \mathbf{x}) , \qquad (5)$$

where f is spherically invariant function. The Newtonian interaction (1) depends on the distance $\mathbf{x}' - \mathbf{x}$:

$$U(\mathbf{x}' - \mathbf{x}) = -G \int \int \frac{f(\mathbf{r} - \mathbf{x})f(\mathbf{r}' - \mathbf{x}')}{|\mathbf{r}' - \mathbf{r}|} d\mathbf{r} d\mathbf{r}' .$$
(6)

Through this section, we assume that the characteristic distances $|\mathbf{x}' - \mathbf{x}|$ are small compared to any other relevant length scales of the problem. Then we expand the interaction potential upto the first nontrivial order in $\mathbf{x}' - \mathbf{x}$ [2]:

$$U(\mathbf{x}' - \mathbf{x}) = U_0 + \frac{1}{2}M\omega_G^2 |\mathbf{x}' - \mathbf{x}|^2 , \qquad (7)$$

where ω_G is a certain gravitational frequency of self-interacting bulk matter [14]. We can write it into this simple form:

$$\omega_G^2 = \frac{4\pi}{3M} G \int f^2(\mathbf{r}) d\mathbf{r} \ . \tag{8}$$

At constant mass density $\bar{f} = 3M/4\pi R^3$ we obtain:

$$\omega_G^2 = \frac{4\pi}{3} G\bar{f} = \frac{GM}{R^3} .$$
 (9)

As we mentioned in sect. 1, the Newtonian interaction potential (1) plays the key role in the proposed mechanisms of localisation (2) or decoherence (3) of macro-objects. Using the approximation (7) for U(X, X'), we obtain the nonlinear Schrödinger-Newton eq. (2) for the wave function of the center-of-mass of our ball:

$$i\hbar \frac{d\psi(\mathbf{x})}{dt} = -\frac{\hbar^2}{2M} \Delta \psi(\mathbf{x}) + \frac{1}{2} M \omega_G^2 |\mathbf{x} - \langle \mathbf{x} \rangle|^2 \psi(\mathbf{x}) .$$
(10)

For simplicity, we assumed the absence of external potentials. This non-linear equation has exactly calculable solitary solutions. In the co-moving system, the quantum mechanical mean value $\langle \mathbf{x} \rangle$ is constant and the system becomes isomorfic with a harmonic oscillator of frequency ω_G . The width of its localised ground state is the following [2]:

$$\sqrt{\frac{\hbar}{M\omega_G}} = \left(\frac{\hbar^2}{GM^3}\right)^{1/4} R^{3/4} . \tag{11}$$

This could be the natural quantum mechanical localisation of the ball. As it is obvious from the eq. (10), nothing prevents the ball from getting into and then remaining in the superposition of two localised ground states that are far from each other. These "cat" states of macro-objects can be excluded from the theory via the decoherence mechanism modelled by the von-Neumann-Newton master eq. (3). Applying again the approximation (7) for U(X, X'), the master equation reduces to the following form [4, 5]:

$$\frac{d\rho(\mathbf{x},\mathbf{y})}{dt} = \frac{i\hbar}{2M} (\Delta_x - \Delta_y)\rho(\mathbf{x},\mathbf{y}) - \frac{1}{2\hbar} M\omega_G^2 |\mathbf{x} - \mathbf{y}|^2 \rho(\mathbf{x},\mathbf{y}) , \qquad (12)$$

where $\rho(\mathbf{x}, \mathbf{y})$ is the density matrix of the center of mass. This equation implies the decoherence time [cf. eq. (4)]

$$\frac{2\hbar}{M\omega_G^2}\frac{1}{|\mathbf{x} - \mathbf{y}|^2} = \frac{2\hbar R^3}{GM^2}\frac{1}{|\mathbf{x} - \mathbf{y}|^2}$$
(13)

for the decay of the superposition $|\mathbf{x}\rangle + |\mathbf{y}\rangle$ [4, 5, 8].

Most studies [1]-[14] agree that the heuristic mass density, e.g., postulating a bulk homogeneous ball, yields plausible localisation (11) and decoherence (13) scales. In general, the Newtonian localisation and decoherence can be ignored for atomic systems while the quantum dynamics of massive bodies becomes dominated by them. It turns out, however, that the predicted scales depend on the precise definition of the mass density $f(\mathbf{r}|X)$.

4 Point-like objects — divergence, early cutoff

The Newtonian self-energy U(X, X') diverges for point-like particles when, e.g.:

$$f(\mathbf{r}|\mathbf{x}) = M\delta(\mathbf{r} - \mathbf{x}) . \tag{14}$$

This divergence could paralyse both our localisation and decoherence models above. Interestingly, the Schrödinger-Newton eq. (2) remains regular for pointlike particles as well. But the von-Neumann-Newton eq. (3) becomes divergent. Let us follow the analysis by Gian-Carlo Ghirardi, Renata Grassi and Alberto Rimini [6]. The von-Neumann-Newton eq. does not conserve the energy. The rate of increase of the translational energy for a rigid ball can be exactly calculated:

$$\frac{dE}{dt} = \frac{G\hbar}{2M} \int f^2(\mathbf{r}) d\mathbf{r} = \frac{3}{8\pi} \hbar \omega_G^2 .$$
(15)

This rate diverges for a point-like object. Comparing the above "heating rate" with certain experimental evidences, Ghirardi et al. come to the conclusion that the cutoff on spatial mass density resolution must be as early as $a = 10^{-5}$ cm. (The present author used 10^{-12} cm, ignorantly, cf. [5] and also [15].) The cutoff can technically be realized by the corresponding regularisation of the Newtonian kernel 1/r or, alternatively, of the mass density $f(\mathbf{r}|X)$.

The Ghirardi et al. choice is the smoothened $f(\mathbf{r}|X)$:

$$f(\mathbf{r}|X) = (2\pi a^2)^{-3/2} \int \exp\left(-\frac{1}{2a^2}|\mathbf{r} - \mathbf{r}'|^2\right) f_0(\mathbf{r}'|X) d\mathbf{r}' , \qquad (16)$$

where $f_0(\mathbf{r}|X)$ is the microsopic mass distribution of the point-like or extended constituents. Eventually, Ghirardi and co-workers adapted their continuous spontaneous localisation (CSL) theory to the smoothened mass-density $f(\mathbf{r}|X)$. The "mass-proportional CSL", cf. e.g. [16], uses the simple contact potential:

$$U_{CSL}(X, X') = -\gamma \int f(\mathbf{r}|X) f(\mathbf{r}|X') d\mathbf{r}$$
(17)

rather than the original Newtonian version (1). In CSL, the strength-parameter γ is no longer related to Newton's G, although γ is considered a universal parameter. Its ultimate range is under careful investigation by Adler [17].

5 Interatomic resolution

How does the interaction potential (1) change if, not imposing the early cutoff 10^{-5} cm of sect. 4, we increase the resolution of the mass density toward the interatomic scales? For ball geometry, eq. (8) shows that the gravitational frequency ω_G grows with the spatial fluctuations of the mass density. To model the fine-structure beyond the constant average mass density \bar{f} , suppose the ball consists of identical atoms of mass m each. Assume, furthermore, that the atomic mass is blurred on a certain distance σ . One could take a spatial Gaussian distribution of linear spread σ . For our purposes, little homogeneous balls of radius σ will suitably represent the individual atoms. Suppose the scale σ is much smaller than the interatomic distance, yet much greater than the scale of the center of mass displacement $|\mathbf{x}' - \mathbf{x}| \ll R$. Then the total atomic contribution to the rhs of (8) yields:

$$\omega_G^2 = \frac{4\pi}{3} G \bar{f}_\sigma = \frac{GM}{\sigma^3} , \qquad (18)$$

where $\bar{f}_{\sigma} = 3m/4\pi\sigma^3$ is the average density of the blurred atoms. Comparing this result to (9), we can resume that the microscopic resolution of the mass density *enhances* the proposed Newtonian gravitational mechanisms. The enhancement can simply be characterised via replacing the Newton constant G by the following effective constant \tilde{G} :

$$\widetilde{G} = \frac{f_{\sigma}}{\overline{f}}G \ . \tag{19}$$

The higher the atomic density \bar{f}_{σ} the stronger will be the proposed gravitational localisation (11) and decoherence (13) effects.

Such enhancement depends on the geometry of the massive object. If the object is a rectangular slab rather than a ball, the the gravitational frequency ω_G (9) as well as the effective Newton constant \tilde{G} will be re-calculated easily.

6 Closing remarks

One witnesses a growing number and variety of proposals that point toward possible experiments in the near future that will test the predicted decoherences at least (see, e.g., [18]). Testing the proposed spontaneous mechanisms of macroscopic localization is, however, completely out of question (unless the test of decoherence is considered an indirect test of localisation as well). In some cases, the decoherence effects predicted by the Newtonian mechanism as in eqs.(3) and/or (4), would be too week to be observed [19]. This tendency may, however, change if the models resolve the mass density over interatomic scales, see e.g. sect. 5, provided the reasons of the earlier length-cutoff are somehow neutralised. We can thus see the issue of mass density resolution is definitive from the experimental viewpoint.

From the theoretical viewpoint, the divergence of the von-Neumann-Newton eq. and the corresponding decoherence time for point-like particles represents a serious issue. Any cutoff turns the parameter-independent model into a less attractive one-parameter model. Yet, we do not know whether the Newtonian mechanism, i.e. the structure U(X, X'), plays a role in spontaneous decoherence or, alternatively, the simple contact structure $U_{CSL}(X, X')$ of Ghirardi et al. is the real one, while CSL has been a two-parameter model from the beginning.

If, in the spirit of fig. 1, we stress that the Newtonian mechanisms are responsible for the emergent classical behaviour of the massive non-relativistic quantised matter then the intellectual perspective includes not only the modification of the standard quantum mechanics but the refinement of our concept of gravity and space-time. The Schrödinger-Newton and the von-Neumann-Newton eqs. represent the modification of quantum mechanics. So far we have not modified or refined our concept of gravity. This perspective may overcome the encountered difficulties of the the modified quantum equations. It should lead to an autonomous theory of some, still unknown, new quality of physical phenomena. I wrote this in 1992 and put a question mark below the edge connecting \hbar and G. It marks the radically new phenomenon that will follow from the autonomous theory — provided such theory exists and we discover it. As it happened already for the Dirac and Einstein theories. Yet, it is not clear which scenario wins: shall we verify the new physics through the noise it generates (cf. decoherence) or through the radically new phenomena that we become able to predict theoretically.

This work was supported by the Hungarian OTKA Grant 49384.

References

 L. Diósi, Gravity and Quantization for each other p299-304 in: Quantum Chaos - Quantum Measurement, eds.: P.Cvitanovic, I.Percival and A. Wirzba (Kluwer, Dordrecht, 1992).

- [2] L. Diósi, Phys. Lett. 105A, 199-202 (1984).
- [3] L. Diósi and B. Lukács, Annln. Phys. 44, 488-492 (1987).
- [4] L. Diósi, Phys. Lett. **120A**, 377-381 (1987).
- [5] L. Diósi, Phys. Rev. A40, 1165-1174 (1989).
- [6] G.C. Ghirardi, R. Grassi and A. Rimini, Phys. Rev. A42, 1057 (1990).
- [7] K.R.W. Jones, Mod. Phys. Lett. A10, 657 (1995); Austral. J. Phys. 48, 1055 (1995); D.A. Bernstein, E. Giladi, K.R.W. Jones, Mod. Phys. Lett. A13, 2327 (1998).
- [8] R. Penrose, Gen. Rel. Grav. 28, 581 (1996).
- [9] R. Penrose, Phil. Trans. R. Soc. Lond. A356, 1927 (1998).
- [10] J. Anandan, Found. Phys. **29**, 333 (1999).
- [11] I.M. Moroz, R. Penrose, and P. Tod, Class. Quant. Grav. 15, 2733 (1998).
- [12] D. Kumar and V. Soni, Phys. Lett. 271A, 157 (2000); V. Soni, Pramana J. Phys. 59, 375 (2002).
- [13] S. De Filippo, F. Maimone and A.L. Robustelli, Physica A 330, 459 (2003).
- [14] T. Geszti, Phys. Rev. A69, 032110 (2004).
- [15] L. Diósi, Braz. J. Phys. **35**, 260 (2005).
- [16] A. Bassi and G.C. Ghirardi, Phys. Rep. 379, 257 (2003).
- [17] S. L. Adler, in this volume; Los Alamos e-print quant-ph/0605072.
- [18] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A59, 3204 (1999); W. Marshall, C. Simon, R. Penrose and D. Bouwmeester, Phys. Rev. Lett. 91, 130401 (2003); D. Vitali, S. Mancini, L. Ribichini, et al., J. Opt. Soc. Am. B20, 1054 (2003); C. Henkel, M. Nest, P. Domokos, and R. Folman, Phys. Rev. A70, 023810 (2004); I. Wilson-Rae, P. Zoller and A. Imamoglu, Phys. Rev. Lett. 92, 075507 (2004); J. Christian, Phys. Rev. Lett. 95 160403 (2005); D. Kleckner et al., Phys. Rev. Lett. 96 173901 (2006).
- [19] J. Z. Bernád, L. Diósi and T. Geszti, Los Alamos e-print quant-ph/0604157.