

Observers and Measurements in Noncommutative Spacetimes

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Abstract

We propose a "Copenhagen interpretation" for spacetime noncommutativity. The goal is to be able to predict results of simple experiments involving signal propagation directly from commutation relations. A model predicting an energy dependence of the speed of photons of the order E/E_{Planck} is discussed in detail. Such effects can be detectable by the GLAST telescope, to be launched in 2006.

PACS 04.60.Pp, 02.40.Gh, 11.30.Cp, 03.65.Ta, 98.70.Rz, 11.10.Nx

Keywords: quantized spacetime, in-vacuo dispersion, gamma-ray bursts, quantum measurement

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1 Introduction

One of experimentally observable signatures of quantum gravity may be a Planck-scale suppressed variation of the photon velocity

$$v \approx 1 + \xi E/E_{Planck} \quad (1)$$

where ξ is a factor of order 1 (see [1] for a review of potentially observable effects) This effect may be detected by observing short-duration γ -ray bursts occurring at cosmological distances [2]. The GLAST space telescope, to be launched in 2006, will be sensitive enough for such an experiment [3]. In fact, already existing data on TeV γ -ray flares in active galaxies set a bound $|\xi| \lesssim 250$ [4, 5].

In light of these exciting experimental developments, it is very interesting to understand possible theoretical origins of in-vacuo dispersion relations like (1). One popular proposal is to say that quantum gravity will result in some sort of spacetime noncommutativity,

$$[x_\mu, x_\nu] \neq 0, \quad (2)$$

and try to relate in-vacuo dispersion to particular forms of postulated commutation relations. Such studies usually proceed via constructing field theory on noncommutative spacetimes, discussing wave packets, etc. (see e.g. [6]).

In contrast, we would like to build a theoretical scheme which would allow to discuss signal propagation and derive dispersion relations directly from kinematics encoded in the deformed commutation relations, without recourse to dynamics.

First of all, this will require a careful analysis of the logical structure of measurements in noncommutative spacetime. So far such analysis in a form satisfactory for our purposes has not been carried out. The scheme that we discuss is very similar in spirit to the Copenhagen interpretation of quantum mechanics. Similarly to how the Copenhagen interpretation relies on the existence of classical observers to interpret quantum mechanical phenomena, we will require the existence of special-relativistic observers who measure events happening in quantum spacetime.

Validity of the Copenhagen interpretation may be explained by strong decoherence phenomena occurring when a microscopic quantum system interacts with a macroscopic measuring device. Such decoherence will always occur when there is a macroscopic / microscopic separation of scales. We thus believe that it should be possible to analyze elementary experimental consequences of quantum spacetime structure, such as Eq. (1), in terms of a suitable ‘‘Copenhagen interpretation’’.

After this work has been completed and reported at a local seminar, paper [15] appeared, where a similar approach to noncommutative spacetime is advocated.

2 Observers and events

As we have already mentioned, we assume existence of classical inertial observers. For any two such observers we may speak of their spacetime coordinates in each other’s coordinate system, as well as of their relative velocity. These variables will be assumed to have definite values. All inertial observers are assumed to be equivalent.

Our observers are classical, but spacetime will be quantum. This means that its elementary element is not a point, but is rather described by a state $|\psi\rangle$ in a Hilbert

space H . We call such an element an “event”, $|\psi\rangle$ the event wavefunction, and H the Hilbert space of events.

A possible spacetime events can be a particle decay, a particle collision, or (the example we mostly use below) a photon emission. All such events will have wavefunctions associated with them.

The most elementary thing an observer can do is to observe (or measure, we will use the two terms interchangeably) an event. To do that, each observer is equipped with a set of quantum mechanical Hermitean operators \hat{x}_μ , one for each spacetime coordinate. The results of the measurement of a coordinate x_μ is a real random variable X with distribution $p(X)$ such that its moments are given by

$$\int X^k p(X) dX = \langle \psi | (\hat{x}_\mu)^k | \psi \rangle, \quad k = 0, 1, 2, \dots \quad (3)$$

In particular, the average value and the standard deviation are given by

$$\overline{X} = \langle \psi | \hat{x}_\mu | \psi \rangle, \quad (4)$$

$$\overline{(\Delta X)^2} = \langle \psi | (\hat{x}_\mu)^2 | \psi \rangle - \langle \psi | \hat{x}_\mu | \psi \rangle^2. \quad (5)$$

Interesting things may happen when two different observers observe the same event. To relate their viewpoints, we must have a rule which tells us how the wavefunction transforms when the observer changes. In other words, we must have a unitary transformation

$$|\psi_A\rangle \xrightarrow{\mathcal{U}} |\psi_B\rangle \quad (6)$$

of wavefunctions of the same event between observers A and B.

When we consider infinitesimal transformations, we see that we must have the usual set of translation generators \hat{P}_μ , rotation generators \hat{J}_{ij} , and boosts \hat{J}_{0i} , all acting Hermiteanly on the Hilbert space of events. Finite transformation are of course obtained by exponentiation.

It is instructive to compare the above setup with the standard Quantum Mechanics example of quantum spin measurement, which contains analogues of all of the above steps. Spin measurements are performed by classical observers, each having his coordinate systems. Coordinate systems of different observers are related by the rotation group. Each observer measures spin components using a set of Hermitean operators $\{\hat{S}_x, \hat{S}_y, \hat{S}_z\}$ which act on normalized spin states $|\psi\rangle$ lying in a Hilbert space H . Finally, spin states seen by different observers are related by a unitary representation of the rotation group.

An important point to notice is that we may use classical relations between different observers’ coordinate systems when discussing spin measurements. This is of course due to the fact that the discussed effects are of order 1 (think about localizing the spin in the z -direction and then measuring the z -component in the system rotated by 90°) and are insensitive to possible minor uncertainties in rotation angles. The same point of view can be applied to spacetime noncommutativity — we are interested in cumulative effects which have to be insensitive to possible Planck-scale uncertainty in, say, the distance between the two observers.

To discuss physics, it remains to postulate commutation relations between the measurable operators \hat{x}_μ and the change of observer generators $\hat{P}_\mu, \hat{J}_{\mu\nu}$. The usual commutative

spacetime would correspond to the usual relations of the Heisenberg and Poincaré algebras. In noncommutative spacetime this structure will be deformed. As we will see below, not all deformations are allowed on physical grounds.

In the rest of the paper we will mainly be concerned with deformations of the Heisenberg algebra. The Lorentz algebra can be included, and in the examples we will check that it is possible to introduce Hermitean boost generators. However, at present we are unable to utilize such inclusion in order to restrict allowed forms of noncommutativity.

3 Example of a measurement

Consider two observers situated distance L from each other in the x_1 direction. This means that if $|\psi_A\rangle$ is the wavefunction of a spacetime event as seen by observer A, then the same event for observer B will be represented by the wave function

$$|\psi_B\rangle = e^{i\hat{P}_1 L} |\psi_A\rangle. \quad (7)$$

We now consider a Gedanken experiment which is supposed to model real experiments measuring variable speed of signal propagation. Analysis of this experiment will also be used in the next section to limit possible forms of deformations of the algebra.

Suppose that, at the origin of the coordinate system of observer A, a photon was emitted in direction of observer B. The instance of emission will be represented in our framework by a wavefunction of a spacetime “event” $|\psi_A\rangle$. By assumption,

$$\langle\psi_A|\hat{x}_\mu|\psi_A\rangle = 0, \quad \mu = 0, 1, 2, 3. \quad (8)$$

How is the energy E of the photon to be represented in the wavefunction of its emission event? We make a plausible assumption that it is to be related to the *localization* of the event. More energetic photons resolve spacetime better and must have better localized emission event wavefunctions. Moreover, we assume that a quantitative relation holds:

$$\overline{(\Delta x_\mu)^2} = \langle\psi_A|(\hat{x}_\mu)^2|\psi_A\rangle \sim 1/E^2, \quad \mu = 0, 1, 2, 3, \quad (9)$$

so that the localization of the emission event is of the order of the wavelength of the photon.

Now, the emitted photon will propagate and somehow interact with the noncommutative structure of spacetime (which will of course depend on the deformed part of the algebra, which we have not discussed so far). Details of this process cannot be determined without discussing dynamics of fields in noncommutative spacetimes. Eventually, the photon arrives at the position of observer B and is detected by him. It is this process of detection of the emitted photon that constitutes an act of observing its emission events by observer B.

Our main postulate is that whatever the process of propagation is, it must be consistent with the generator action as expressed by Eq. (7). In particular, this means that all observations performed by observer B will tell him that the photon was emitted at (average) time

$$\bar{t} = \langle\psi_B|\hat{x}_0|\psi_B\rangle. \quad (10)$$

Suppose now that *two* photons of *different* energies $E_{1,2}$ were emitted, with wavefunctions $|1_A\rangle$, $|2_A\rangle$ of their emission events satisfying (8). Thus, for observer A the emission

happened at the same spacetime point. However, emission times as perceived by observer B are given by (10) and will in general be different. Experimentally, this means that there will be *time delay* in arrival of the photons

$$\overline{\Delta t} = \overline{t_1} - \overline{t_2}. \quad (11)$$

The actual size of this delay will, as we will see later, be determined by the commutator $[\hat{x}_0, \hat{P}_1]$. Thus, we proceed to discuss deformations of the algebra.

4 Allowed deformations of the algebra

As we have already mentioned, we do not have any logical arguments which would restrict allowed deformations of the Lorentz part of the algebra (beyond obvious requirements of hermiticity and the Jacobi identities).

However, some restrictions do exist in the Heisenberg sector. Working in this sector means that we restrict attention to the class of observers who are at rest with respect to each other. In particular, the Gedanken experiment of the previous section is still possible.

First of all, the translation generators have to commute:

$$[\hat{P}_\mu, \hat{P}_\nu] = 0. \quad (12)$$

This is a consequence of our assumption that the observers are classical, and have definite coordinates in each other's coordinate systems. For instance, if observer A sees an event described by the wavefunction $|\psi\rangle$, then both wavefunctions

$$e^{i\hat{P}_1 L_1} e^{i\hat{P}_2 L_2} |\psi\rangle, \quad e^{i\hat{P}_2 L_2} e^{i\hat{P}_1 L_1} |\psi\rangle \quad (13)$$

correspond to the same event as seen by the observer having coordinates $(L_1, L_2, 0)$ in observer A's coordinate system. Thus the two wavefunctions have to coincide, which implies that \hat{P}_1 and \hat{P}_2 commute.

Further, notice a special role played by \hat{P}_0 . Namely, this operator does not change the world line of the observer, but merely shifts a reference point on it. The resulting observer is still the same. If we go back to the Gedanken experiment, the time delay between photons registered by observer B cannot depend on the choice of the reference point on his world line somewhere in the past. The same refers to all other measurements and observations he might perform. Mathematically, this means that \hat{P}_0 should have standard commutation relations with \hat{x}_μ . Thus we require as an axiom

$$[\hat{x}_\mu, \hat{P}_0] = i\delta_{\mu 0}. \quad (14)$$

5 Two-dimensional example

All essential features of the above setup can be already seen in 1+1 spacetime dimensions. The algebra in this case consists of 5 operators x , t , P_x , P_t , and the boost generator B

(in the rest of the paper we do not put carets over operators). It follows from the above discussion that we must have

$$[P_x, P_t] = 0, \quad [x, P_t] = 0, \quad [t, P_t] = i. \quad (15)$$

We have no restrictions on commutators with B . For having no preferred choice of deformation, let us just leave the Poincaré symmetry undeformed

$$[B, P_x] = iP_t, \quad [B, P_t] = iP_x. \quad (16)$$

The remaining 5 commutation relations $[x, t]$, $[x, P_x]$, $[t, P_x]$, $[x, B]$, $[t, B]$ must be consistent with the hermiticity of all operators and with the Jacobi identities. We also require that they respect the spatial reflection symmetry, which means that the commutators must preserve their form under the simultaneous changes $x \rightarrow -x$, $P_x \rightarrow -P_x$, $B \rightarrow -B$, $t \rightarrow t$, $P_t \rightarrow P_t$.

One sufficiently general way to construct such algebras is to start with the standard commutators

$$[x, t] = 0, \quad [x, P_x] = i, \quad [t, P_x] = 0, \quad [x, B] = it, \quad [t, B] = ix, \quad (17)$$

and then define Hermitean operators

$$t^{new} = t + F(P_x, P_t)x + xF(P_x, P_t), \quad (18)$$

$$x^{new} = G(P_x, P_t)x + xG(P_x, P_t). \quad (19)$$

Such an Ansatz produces commutation relations consistent with axiom (14). Consistency with reflection symmetry requires that

$$F(-P_x, P_t) = -F(P_x, P_t), \quad (20)$$

$$G(-P_x, P_t) = G(P_x, P_t). \quad (21)$$

For the rest of this section let us concentrate on perhaps the simplest interesting example which results from taking

$$F = \frac{P_x}{2\kappa}, \quad G = \frac{1}{2}, \quad (22)$$

where $\kappa \sim E_{Planck}$ is the deformation scale. This gives commutation relations

$$[x, t] = ix/\kappa, \quad (23)$$

$$[x, P_x] = i, \quad (24)$$

$$[t, P_x] = iP_x/\kappa, \quad (25)$$

$$[x, B] = i\left(t - \frac{1}{2\kappa}(P_x x + x P_x)\right), \quad (26)$$

$$[t, B] = i\left(x + \frac{1}{2\kappa}(P_x t + t P_x) - \frac{1}{\kappa}P_t x - \frac{1}{2\kappa^2}(P_x^2 x + x P_x^2)\right). \quad (27)$$

By construction, this algebra can be realized in the Hilbert space of functions $\psi(p_x, p_t)$ with operators acting by

$$P_x = p_x, \quad (28)$$

$$P_t = p_t, \quad (29)$$

$$B = i(p_x \frac{\partial}{\partial p_t} + p_t \frac{\partial}{\partial p_x}), \quad (30)$$

$$x = i \frac{\partial}{\partial p_x}, \quad (31)$$

$$t = i \frac{\partial}{\partial p_t} + \frac{i}{2\kappa} (p_x \frac{\partial}{\partial p_x} + \frac{\partial}{\partial p_x} p_x). \quad (32)$$

Analogous algebras can of course be given in any spacetime dimensions. E.g. in 4d we may put

$$t^{new} = t + \frac{1}{2\kappa} \sum_{i=1}^3 (P_i x_i + x_i P_i), \quad (33)$$

keeping the usual x_i . The resulting commutation relations

$$[x_i, t] = i x_i / \kappa, \quad (34)$$

$$[t, P_i] = i P_i / \kappa, \quad \text{etc.} \quad (35)$$

are then symmetric under 3d rotations.

6 In-vacuo dispersion

Let us analyze the Gedanken experiment from Section 3 in terms of algebra (23)-(27). In the 4d case (34)-(35) will lead to the same observable effects.

Coordinates of the same emission event as seen by observers A and B are related by (7) as

$$\langle \psi_B | x | \psi_B \rangle = \langle \psi_A | e^{-i P_x L} x e^{i P_x L} | \psi_A \rangle. \quad (36)$$

Differentiating in L , we get

$$\frac{d}{dL} \langle \psi_B | x | \psi_B \rangle = \langle \psi_A | i [x, P_x] | \psi_A \rangle = -1. \quad (37)$$

Thus

$$\langle \psi_B | x | \psi_B \rangle = \langle \psi_A | x | \psi_A \rangle - L = -L \quad (38)$$

for a photon emitted at the origin of the A's coordinate system.

The same computation for observed time produces however a nontrivial relation

$$\langle \psi_B | t | \psi_B \rangle = -\frac{L}{\kappa} \langle \psi_A | P_x | \psi_A \rangle, \quad (39)$$

due to the non-vanishing commutator $[t, P_x]$.

Our algebra is realized in momentum representation by (28)-(32). In Section 3 we found it reasonable to assume that

$$\langle \psi_A | x^2 | \psi_A \rangle \sim 1/E^2, \quad (40)$$

where E is the photon energy. It follows that we must have

$$\langle \psi_A | P_x | \psi_A \rangle \sim E, \quad (41)$$

unless some cancellation occurs due to the symmetry of the wavefunction in P_x . However, such symmetry seems unlikely, because the emission wavefunction should encode the direction in which the photon is emitted. Thus we assume that (41) is true.

Eqs. (39) and (41) now show that if two photons of energies $E_{1,2}$ are emitted, observer B will detect (average) time-delay on arrival of the order

$$t_1 - t_2 \sim \frac{L}{\kappa} (E_1 - E_2), \quad (42)$$

which means that the speed of signal propagation varies with energy according to (1).

This is not, however, the end of the story. It turns out that apart from the time-delay, our model also incorporates another possible striking signature contemplated by quantum-gravity phenomenologists [1], namely spreading of the burst with energy. That is, spacetime noncommutativity induces intrinsic uncertainty in the arrival time of the photons. To find this uncertainty, we need to compute $\overline{(\Delta t)^2}$ for observer B.

We have

$$\overline{(\Delta t)^2} = \langle \psi_B | t^2 | \psi_B \rangle - \langle \psi_B | t | \psi_B \rangle^2. \quad (43)$$

To find $\langle \psi_B | t^2 | \psi_B \rangle$, we need to differentiate

$$\langle \psi_A | e^{-iP_x L} t^2 e^{iP_x L} | \psi_A \rangle \quad (44)$$

in L *twice*, each time using the commutator $[t, P_x] = iP_x/\kappa$. The result is

$$\langle \psi_B | t^2 | \psi_B \rangle = \langle \psi_A | t^2 - \frac{L}{\kappa} (tP_x + P_x t) + \frac{L^2}{\kappa^2} P_x^2 | \psi_A \rangle. \quad (45)$$

This results in an estimate of

$$\overline{(\Delta t)^2} \sim 1/E^2 + L/\kappa + L^2 E^2/\kappa^2 \quad (46)$$

for the uncertainty of the arrival time.

Note that the analogous computation for the coordinate measurement gives

$$\langle \psi_B | x^2 | \psi_B \rangle = \langle \psi_A | x^2 | \psi_A \rangle + L^2. \quad (47)$$

It follows that

$$\overline{(\Delta x)^2} = \langle \psi_B | x^2 | \psi_B \rangle - \langle \psi_B | x | \psi_B \rangle^2 \sim 1/E^2 \quad (48)$$

for observer B just as for observer A. Thus the assumed form of noncommutativity does not increase the uncertainty of the space coordinate measurement.

The γ -ray burst observations we have in mind will involve typical values

$$L \sim 1Gpc, \quad E \sim 10MeV, \quad (49)$$

which leads to the uncertainty

$$\Delta t \sim LE/\kappa \sim 10^{-4}s, \quad (50)$$

the other two terms in (46) being negligible. Thus we see that the uncertainty is comparable to the average time-delay given by (42).

7 Discussion and outlook

In this paper we set up a scheme which allows to discuss observations in noncommutative spacetimes. The scheme involves classical observers perceiving events by accessing their wavefunctions. The (deformed) Poincaré group acts on the Hilbert space of events and is used to relate wavefunctions of an event as seen by different observers.

The standard commutation relations between observable operators, such as the event coordinate operators x_μ , and the Poincaré generators may be deformed, as an effective description of unknown physics at the Planck scale. However, an important consequence of logical consistency is that the commutators with P_0 be undeformed. Observable effects in signal propagation, such as in-vacuo dispersion, result from deformed commutators between x_μ and P_i .

The coordinate part of the algebra that we considered as an example in Section 5

$$[x_i, t] = ix_i/\kappa, \quad [x_i, x_j] = 0 \quad (51)$$

is well known as the κ -Minkowski spacetime. This spacetime was much studied recently, especially in connection with the Doubly Special Relativity theories [7, 8]. The energy-momentum sector of these DSR theories is symmetric under the action of a Hopf algebra, the so-called κ -Poincaré quantum algebra. The coproduct structure allows then for a unique reconstruction of the spacetime sector, which turns out to be the κ -Minkowski spacetime [9, 10, 11, 12].

However, taken as a whole, our treatment of spacetime noncommutativity is rather different from previous ones. In particular, noncommutative spacetimes discussed in the literature (starting with the classic paper [13]) typically do not satisfy our constraint that $[x_\mu, P_0] = i\delta_{\mu 0}$. Thus the observational interpretation of those types of noncommutativity is unclear to us. It is also unclear if imposing the coproduct structure on the algebra, like in the DSR theories, can be justified by any arguments apart from appealing for greater symmetry (although see [14]).

An advantage of our scheme is that it can be used to intuitively understand expected observable effects directly from commutation relations. However, such kinematical arguments can of course only give order-of-magnitude estimates. Detailed understanding of the structure of wavefunctions of events corresponding to real physical processes, such as photon emission, can only be produced by dynamical considerations.

One should thus try as a next step to develop (classical) field theory consistent with deformed commutation relations of the type discussed in this paper. Perhaps the existing literature on field theory in noncommutative spacetimes, such as [6], can be of help in this undertaking.

Acknowledgements

I am grateful to Giovanni Arcioni, Jeroen van Dongen, Erik Verlinde, Jung-Tai Yee, and especially Kostas Skenderis for useful discussions and suggestions. This work was supported by the Stichting voor Fundamenteel Onderzoek der Materie (FOM).

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