New Journal of Physics
The open access journal at the forefront of physics
Deutsche Physikalische Gesellschaft DPG IOP Institute of Physics

Spin dynamics in rolled-up two-dimensional electron gases

To cite this article: Maxim Trushin and John Schliemann 2007 *New J. Phys.* **9** 346

New Journal of Physics
The open–access journal for physics

Spin dynamics in rolled-up two-dimensional electron gases

Maxim Trushin and John Schliemann
Institute for Theoretical Physics, University of Regensburg, D-93040 Regensburg, Germany
E-mail: maxim.trushin@physik.uni-regensburg.de

New Journal of Physics **9** (2007) 346
Received 16 May 2007
Published 28 September 2007
Online at http://www.njp.org/
doi:10.1088/1367-2630/9/9/346

Abstract. A curved two-dimensional electron gas with spin–orbit interactions due to the radial confinement asymmetry is considered. At a certain relation between the spin–orbit coupling strength and curvature radius the tangential component of the electron spin becomes a conserved quantity for *any* spin-independent scattering potential that leads to a number of interesting effects such as persistent spin helix and strong anisotropy of spin relaxation times. The effect proposed can be utilized in non-ballistic spin-field-effect transistors.

Contents

1. Introduction

Spin–orbit coupling is one of the key ingredients for electrical control and manipulation of spins in semiconductor nanostructures and therefore is a major topic of both experimental and theoretical studies in semiconductor spintronics. A paradigmatic example for a spintronics device is the spin-field-effect transistor (SFET) proposed by Datta and Das over 15 years ago [1]. The original proposal envisaged a two-dimensional electron gas (2DEG) in a semiconductor quantum well with Rashba spin–orbit coupling [2, 3]. This contribution to spin–orbit interaction stems from an asymmetry of the confining potential in the growth direction and can be particularly pronounced for materials such as InAs. Most noteworthy, the strength of the Rashba term can be tuned in experiments via a gate voltage across the quantum well [4]–[8]. This is in contrast to the Dresselhaus coupling, another effective contribution to spin–orbit interaction in 2DEGs resulting from the lack of inversion symmetry in zinc-blende III–V semiconductors [9]. In particular, for typical parameters of realistic materials it is in principle possible to tune the Rashba coupling to be equal in magnitude to the Dresselhaus coupling [10]. In this situation an additional conserved spin quantity arises which opens the perspective to possibly operate an SFET also in the diffusive regime [11], apart from other interesting effects such as persistent spin helix [12] and strong anisotropy of spin relaxation times [13]. In the present paper, we investigate a similar interplay between the Rashba coupling and the effects of a *finite curvature* of a cylindrical 2DEG. Such curved systems have been produced recently by several independent groups [14]–[17] and studied regarding their magnetotransport properties [17]–[20]. Our theoretical results obtained here predict the existence of a conserved spin component for appropriately tuned system parameters, very analogously to the balancing of Rashba and Dresselhaus coupling in a flat 2DEG. Moreover, within the framework of second quantization, this observation can be extended to a full su(2) algebra of conserved quantities, in full analogy to recent findings for a flat 2DEG [12]. Finally, we also discuss our results with respect to the *zitterbewegung* of electrons due to spin–orbit coupling in two-dimensional semiconductor structures [21].

2. Results and discussion

Let us consider the Hamiltonian describing electrons in a rolled-up layer of radius R depicted in figure 1(a). Following Rashba [2, 3], we rely on the effective mass model, and, hence, the Hamiltonian reads

$$H = H_{\rm kin} + H_{\rm SO} + V(z,\varphi) + U(R), \tag{1}$$

where $U(R)$ is the radial confining potential $U(r)$ at $r = R$, $V(z,\varphi)$ is the *arbitrary* scalar potential describing, for example, the influence of impurities or imperfections. The spin–orbit coupling term has the form [22]

$$H_{\rm SO} = \alpha\left(\sigma_\varphi k_z - \sigma_z q_\varphi/R\right), \tag{2}$$

where $k_z = -\mathrm{i}\frac{\partial}{\partial z}$ and $q_\varphi = -\mathrm{i}\frac{\partial}{\partial\varphi}$ are the longitudinal and angular momentum operators respectively, $\sigma_\varphi = -\sigma_x \sin\varphi + \sigma_y\cos\varphi$, σ_z are the corresponding Pauli matrices, and α is the spin–orbit coupling constant. The kinetic term reads

$$H_{\rm kin} = \frac{\hbar^2 k_z^2}{2m^*} + \varepsilon_0 q_\varphi^2, \tag{3}$$

where $\varepsilon_0 = \hbar^2/(2m^* R^2)$ is the size confinement energy, and m^* is the effective electron mass.

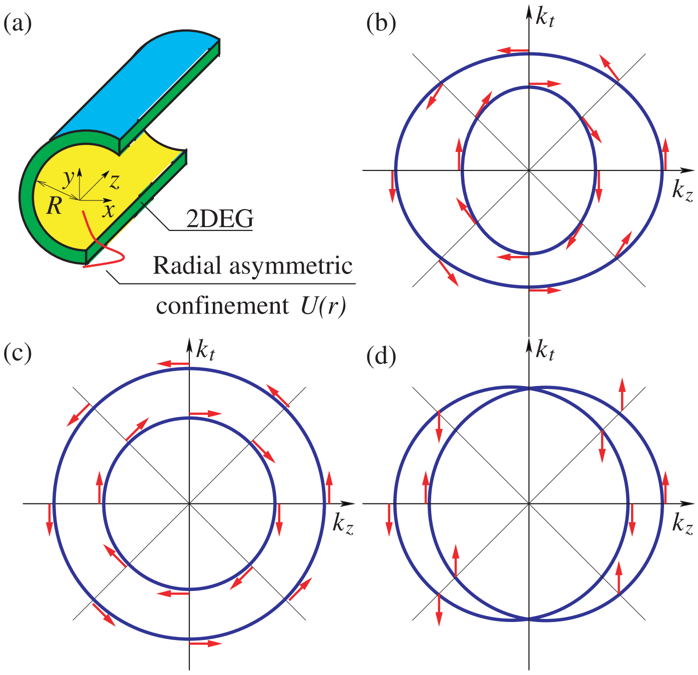

Figure 1. (a) The system under consideration: a rolled-up 2DEG with spin–orbit coupling induced by the asymmetric radial confinement $U(r)$. (b) In general case, Fermi contours of the rolled-up 2DEG are anisotropic. Here, k_z and k_t are the longitudinal and tangential components of the electron momenta, respectively. The arrows show the spin orientation in the eigenstates (7) and (8). (c) In the planar case $R \gg \hbar^2/(2m^*\alpha)$ the Fermi contours represent just two concentric circles, i.e. the dispersion law is isotropic. Here, the spin orientation depends on the momentum. (d) At $\alpha = -\hbar^2/(2m^*R)$ the Fermi contours are two circles as well. Here, the spin orientation *does not* depend on the momentum within a spin-split subband, i.e. the tangential component of the electron spin is conserved.

The spin dynamics can be described utilizing the commutation relations between the spin projection operators $s_z = \frac{1}{2}\sigma_z$, $s_r = \frac{1}{2}(\sigma_x \cos\varphi + \sigma_y \sin\varphi)$, $s_\varphi = \frac{1}{2}(-\sigma_x \sin\varphi + \sigma_y \cos\varphi)$ and the Hamiltonian (1). The corresponding equations read

$$\frac{\mathrm{d}s_z}{\mathrm{d}t} = -\frac{\alpha}{\hbar}k_z \begin{pmatrix} 0 & \mathrm{e}^{-\mathrm{i}\varphi} \\ \mathrm{e}^{\mathrm{i}\varphi} & 0 \end{pmatrix}, \tag{4}$$

$$\frac{\mathrm{d}s_r}{\mathrm{d}t} = \frac{\mathrm{i}}{\hbar} \begin{pmatrix} -\mathrm{i}\alpha k_z & \mathrm{e}^{-\mathrm{i}\varphi}\left(\varepsilon_0 + \frac{\alpha}{R}\right)\left(\frac{1}{2} - q_\varphi\right) \\ \mathrm{e}^{\mathrm{i}\varphi}\left(\varepsilon_0 + \frac{\alpha}{R}\right)\left(\frac{1}{2} + q_\varphi\right) & \mathrm{i}\alpha k_z \end{pmatrix}, \tag{5}$$

$$\frac{\mathrm{d}s_\varphi}{\mathrm{d}t} = \frac{\varepsilon_0 + \alpha/R}{\hbar} \begin{pmatrix} 0 & \mathrm{e}^{-\mathrm{i}\varphi}\left(\frac{1}{2} - q_\varphi\right) \\ -\mathrm{e}^{\mathrm{i}\varphi}\left(\frac{1}{2} + q_\varphi\right) & 0 \end{pmatrix}. \tag{6}$$

Note that the left-hand sides of equations (4)–(6) are nothing other than the corresponding spin precession frequency operators.

Let us have a look at the special case $\alpha = -\varepsilon_0 R$. Here equation (5) becomes diagonal, whereas the right-hand side of equation (6) vanishes. The latter means that the tangential spin s_φ does not precess at all, i.e. s_φ is the conserved quantity for *arbitrary* potential $V(z,\varphi)$. It is important to emphasize here, that in the planar case with Rashba coupling none of all the possible spin projections are conserved.

The effect has the following geometrical interpretation. On the one hand, the spin rotation angle in the 2DEG with Rashba spin–orbit coupling depends explicitly on the length L of the electron path, namely $\Delta\theta_{\mathrm{so}} = 2m^*\alpha L/\hbar^2$. On the other hand, the spin rotation angle of an electron moving adiabatically along the arc of radius R is $\Delta\theta_{\mathrm{g}} = L/R$. Here, the index g means 'geometrical'. Now, one can see easily that in the special case $1/R = -2m^*\alpha/\hbar^2$ the spin rotation angle of geometrical origin $\Delta\theta_{\mathrm{g}} = -2m^*\alpha L/\hbar^2$ completely compensates the spin rotation angle $\Delta\theta_{\mathrm{so}}$ which is due to the spin–orbit coupling alone.

The phenomena found here is similar to what is proposed by Schliemann *et al* [11] for the planar 2DEG in the presence of *both* Rashba and Dresselhaus interactions. The interplay between them can lead to the conservation of the spin quantity $\Sigma = \frac{1}{\sqrt{2}}(\sigma_x \pm \sigma_y)$, that might be utilized in non-ballistic SFETs. In contrast to [11], for us it is enough that the spin–orbit coupling stems from the asymmetry of the confinement $U(r)$ only, and the bulk spin–orbit effects are not necessary. Nevertheless, all the proposals regarding the non-ballistic SFET [11] are valid for the device studied here as well.

To show this, we consider the Hamiltonian (1) at $V(z,\varphi) + U(R) = 0$. Then, its eigenstates are

$$\psi^+ = \begin{pmatrix} \mathrm{i}\sin\gamma\,\mathrm{e}^{-\mathrm{i}\varphi/2} \\ \cos\gamma\,\mathrm{e}^{\mathrm{i}\varphi/2} \end{pmatrix} \mathrm{e}^{\mathrm{i}(k_z z + l_\varphi \varphi)}, \tag{7}$$

$$\psi^- = \begin{pmatrix} \cos\gamma\,\mathrm{e}^{-\mathrm{i}\varphi/2} \\ \mathrm{i}\sin\gamma\,\mathrm{e}^{\mathrm{i}\varphi/2} \end{pmatrix} \mathrm{e}^{\mathrm{i}(k_z z + l_\varphi \varphi)}, \tag{8}$$

where $\tan 2\gamma = -\alpha k_z/[(\varepsilon_0 + \alpha/R)l_\varphi]$, and the spectrum reads

$$E_\pm = \frac{\hbar^2 k_z^2}{2m^*} + \varepsilon_0 l_\varphi^2 + \frac{\varepsilon_0}{4} + \frac{\alpha}{2R} \pm \sqrt{\alpha^2 k_z^2 + \left(\varepsilon_0 + \frac{\alpha}{R}\right)^2 l_\varphi^2}. \tag{9}$$

Note that the expectation values of s_z, s_φ calculated for the eigenstates (7) and (8) are, in general, momentum dependent (see figures 1(b) and (c)). Therefore, the electron spin becomes randomized due to the momentum scattering, and any given spin-polarization of the electron beam vanishes at the lengths of the order of the electron mean free path. At $\alpha = -\varepsilon_0 R$ the tangential spin-polarization remains unchanged for any spin-independent scattering (see figure 1(d)). Thus, assuming two spin-polarized contacts at the ends of the rolled-up 2DEG, one can modulate the electric current via Rashba constant α as discussed in [11] in great detail.

As an important property of the system studied here, the spinors in equations (7) and (8) depend explicitly on the spatial coordinate φ, i.e. spin and orbital degrees of freedom are entangled. This observation corresponds to the fact that tangential momentum operator q_φ/R does not commute with the spin operators s_φ and s_r, differently from the situation in a planar 2DEG with spin–orbit coupling of, e.g. Rashba and Dresselhaus type. This property of rolled-up 2DEGs has essentially geometrical origin since, generally speaking, an electron spin moving along a path with finite curvature R changes its direction depending on the adiabaticity parameter $2\alpha m^* R/\hbar^2$, see [23]. However, the expectation values of s_φ, s_z and s_r within the eigenstates (7) and (8) are independent of the angle coordinate φ.

Another promising application of the effect proposed is the observation of the persistent spin helix studied recently in [12]. In fact, the exact su(2) symmetry necessary for the persistent spin helix can be found not only in flat 2DEGs with both Rashba and Dresselhaus interactions but in rolled-up 2DEGs with Rashba interaction alone. Indeed, the exact su(2) symmetry is generated by the following operators

$$S^{+} = \sum_{k_z,l_\varphi} c^{\dagger}_{k_z+k_R,l_\varphi,-} c_{k_z-k_R,l_\varphi,+}, \tag{10}$$

$$S^{-} = \sum_{k_z,l_\varphi} c^{\dagger}_{k_z-k_R,l_\varphi,+} c_{k_z+k_R,l_\varphi,-}, \tag{11}$$

$$S^{z} = \sum_{k_z,l_\varphi} \left(c^{\dagger}_{k_z,l_\varphi,-} c_{k_z,l_\varphi,-} - c^{\dagger}_{k_z,l_\varphi,+} c_{k_z,l_\varphi,+} \right), \tag{12}$$

where $c_{k_z,l_\varphi,s}$ are the annihilation operators of the particles with the spin-index $s \in \{\pm\}$, and $k_R = m^*\alpha/\hbar^2$. These operators and the Hamiltonian written as

$$H = \sum_{k_z,l_\varphi,s} E_s(k_z, l_\varphi) c^{\dagger}_{k_z,l_\varphi,s} c_{k_z,l_\varphi,s} \tag{13}$$

obey the following commutation relations

$$\left[H, S^{\pm}\right] = 0, \quad \left[H, S^{z}\right] = 0, \tag{14}$$

$$\left[S^{+}, S^{-}\right] = 2S^{z}, \quad \left[S^{z}, S^{\pm}\right] = \pm S^{\pm}. \tag{15}$$

Thus, the operators $S^{\pm}$ and S^{z} commute with the Hamiltonian and form a representation of su(2), and all findings of [12] are valid for our system as well.

Let us finally make some remarks regarding the *zitterbewegung* of electrons in rolled-up 2DEGs. Just as in the classic case of free relativistic electrons described by the Dirac equation, this phenomenon is nothing but a beating between different dispersion branches split in energy [21]. To investigate the *zitterbewegung* of electrons in rolled-up 2DEGs we find the components of the time dependent position operator in the Heisenberg picture which read

$$z_H(t) = z(0) + [z, R] + \tfrac{1}{2}\,[[z, R], R] + \tfrac{1}{6}\,[[[z, R], R], R] + \cdots, \tag{16}$$

$$\varphi_H(t) = \varphi(0) + [\varphi, R] + \tfrac{1}{2}\,[[\varphi, R], R] + \tfrac{1}{6}\,[[[\varphi, R], R], R] + \cdots, \tag{17}$$

where $R = -\mathrm{i}Ht/\hbar$. In the particular case $\varepsilon_0 = -\alpha/R$ neither of the position operator components contains oscillating terms, and the *zitterbewegung* is absent, similarly to the case of a flat 2DEG with balanced Rashba and Dresselhaus spin–orbit coupling [24].

The key problem regarding the present proposal is the experimental realization of the rolled-up 2DEGs fulfilling the required relation between R and α. In table 1, we present the values for Rashba constant which are necessary for the realization of the non-ballistic SFET proposed. In table 2, the curvature radius is calculated for a given α. Here, the Rashba constant is assumed to be the same as for the planar case. This is quite a rough assumption since the spin–orbit interactions can be changed because of the additional strain at the bending. Therefore, the Rashba constant should be remeasured for rolled-up 2DEGs even if its value for the planar case is already known.

Table 1. Critical Rashba constants $\alpha = -\hbar^2/(2m^* R)$ for some rolled-up structures reported in the literature.

Quantum well [references]	R	m^*/m	$\alpha = -\varepsilon_0 R$ (eV m)
AlGaAs/GaAs/AlGaAs [17]	8 μm	0.067	6×10^{-14}
AlGaAs/GaAs/AlGaAs [18]	4 μm	0.067	1.2×10^{-13}
SiGe/Si/SiGe [16, 25]	270 nm	0.19	6×10^{-13}

Table 2. Critical curvature radii $R = -\hbar^2/(2m^*\alpha)$ according to the Rashba parameters of some flat 2DEGs reported in the literature.

Quantum well [References]	α (eV m)	m^*/m	$\|R\|$
InAlAs/InGaAs/InAlAs [4]	7.2×10^{-12}	0.05	83 nm
InP/InGaAs/InP [6]	5.3×10^{-12}	0.041	150 nm
SiGe/Si/SiGe [25]	5.5×10^{-15}	0.19	33 μm

3. Conclusions

In conclusion, we have investigated the spin dynamics in rolled-up 2DEGs with interactions of Rashba type using the Hamiltonian which includes an arbitrary scattering potential as well. We have found that at a certain relation between the Rashba constant and radius of curvature, the tangential spin is conserved. This is the most striking feature of the rolled-up 2DEG as compared with its planar analogue. Apart from its fundamental importance, the effect proposed can be utilized in non-ballistic SFETs. In addition, su(2) spin rotation symmetry and *zitterbewegung* have been investigated.

Acknowledgment

We acknowledge financial support from Collaborative Research Center 689.

References

[1] Datta S and Das B 1990 Electronic analog of the electro-optic modulator *Appl. Phys. Lett.* **56** 665–7

[2] Rashba E I 1960 Properties of semiconductors with an extremum loop *Sov. Phys. Solid State* **2** 1109

[3] Bychkov Yu A and Rashba E I 1984 Properties of 2D electron gas with lifted spectral degeneracy *JETP Lett.* **39** 78

[4] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 Gate control of spin–orbit interaction in an inverted $In_{0.53}Ga_{0.47}As/In_{0.52}Al_{0.48}As$ heterostructure *Phys. Rev. Lett.* **78** 1335

[5] Hu C-M, Nitta J, Akazaki T, Takayanagi H, Pfeffer P and Zawadzki W 1999 Zero-field spin splitting in an inverted $In_{0.53}Ga_{0.47}As/In_{0.52}Al_{0.48}As$ heterostructure: band nonparabolicity influence and the subband dependence *Phys. Rev.* B **60** 7736

[6] Engels G, Lange J, Schäpers Th and Lüth H 1997 Experimental and theoretical approach to spin splitting in modulation-doped $In_xGa_{1-x}As/InP$ quantum wells for B → 0 *Phys. Rev.* B **55** 1958

[7] Matsuyama T, Kürsten R, Meissner C and Merkt U 2000 Rashba spin splitting in inversion layers on p-type bulk InAs *Phys. Rev.* B **61** 15588
[8] Grundler D 2000 Large Rashba splitting in InAs quantum wells due to electron wave function penetration into the barrier layers *Phys. Rev. Lett.* **84** 6074
[9] Dresselhaus G 1955 Spin-orbit coupling effects in zinc blende structures *Phys. Rev.* **100** 580
[10] Giglberger S *et al* 2007 Rashba and Dresselhaus spin splittings in semiconductor quantum wells measured by spin photocurrents *Phys. Rev.* B **75** 35327
[11] Schliemann J, Carlos Egues J and Loss D 2003 Nonballistic spin-field-effect transistor *Phys. Rev. Lett.* **90** 146801
[12] Bernevig B A, Orenstein J and Zhang S-C 2006 Exact su(2) symmetry and persistent spin helix in a spin–orbit coupled systems *Phys. Rev. Lett.* **97** 236601
[13] Averkiev N S and Golub L E 1999 Giant spin relaxation anisotropy in zinc-blende heterostructures *Phys. Rev.* B **60** 15582
[14] Prinz V Ya, Seleznev V A, Gutakovsky A K, Chehovskiy A V, Preobrazhenskii V V, Putyato M A and Gavrilova T A Free standing and overgrown InGaAs/GaAs nanotubes *Physica* E **6** 828
[15] Schmidt O G and Eberl K 2001 Thin solid films roll up into nanotubes *Nature* **410** 168
[16] Schmidt O G and Jin-Phillipp N Y 2001 Free-standing SiGe-based nanopipelines on Si (001) substrates *Appl. Phys. Lett.* **78** 3310
[17] Mendach S, Schumacher O, Heyn Ch, Schnüll S, Welsch H and Hansen W 2004 Preparation of curved two-dimesional electron systems in InGaAs/GaAs microtubes *Physica* E **23** 274
[18] Vorob'ev A B, Prinz V Ya, Yukecheva Yu S and Toropov A I 2004 Magnetotransport properties of two-dimensional electron gas on cylindrical surface *Physica* E **23** 171
[19] Shaji N, Qin H, Blick R H, Klein L J, Deneke C and Schmidt O G 2007 Magnetotransport through two dimensional electron gas in a tubular geometry *Appl. Phys. Lett.* **90** 42101
[20] Friedland K-J, Hey R, Kostial H, Riedel A and Ploog K H 2007 Measurements of ballistic transport at nonuniform magnetic fields in cross junctions of a curved two-dimensional electron gas *Phys. Rev.* B **75** 45347
[21] Schliemann J, Loss D and Westervelt R M 2005 Zitterbewegung of electronic wave packets in iii-v zinc-blende semiconductor quantum wells *Phys. Rev. Lett.* **94** 206801
[22] Mararill L I, Romanov D A and Chaplik A V 1998 Ballistic transport and spin–orbit interaction of two-dimensional electrons on a cylindrical surface *JETP* **86** 771
[23] Trushin M and Chudnovskiy A 2006 Curved one-dimensional wire as a spin switch *JETP Lett.* **83** 318
[24] Schliemann J, Loss D and Westervelt R M 2006 Zitterbewegung of electrons and holes in iii-v semiconductor quantum wells *Phys. Rev.* B **73** 085323
[25] Wilamowskii Z, Jantsch W, Malissa H and Rössler U 2002 Evidence and evaluation of the Bychkov-Rashba effect in SiGe/Si/SiGe quantum wells *Phys. Rev.* B **66** 195315