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Near-field and far-field electric dipole radiation in the vicinity of a planar dielectric half space

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Abstract. We have used the full Sommerfeld integral formalism as well as an asymptotic formalism to study the near- and far-field radiation patterns of an electric dipole in the vicinity of a planar dielectric half space. We present systematic results for the polarization dependence of the radiation patterns in both half spaces and the ratio of the integrated power radiated into the two half spaces as a function of the relative refractive index as well as the dipole position. We find that the radiation patterns are highly structured and directed. Furthermore, the ratio of the integrated power increases significantly on increasing the relative refractive index, which can be exploited to enhance the sensitivity of spectroscopic studies of surface-bound molecules; however this ratio drops quickly for a dipole more than 0.2 wavelength from the interface.

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1. Introduction

Driven initially by a need to understand the radiation pattern from a radio antenna located near the earth's surface, the angular, radial and height dependence of the field strength from an oscillating electric dipole located some distance above a planar half space has attracted much interest over the years, starting with Sommerfeld in 1909 [1]. This problem is equally important in the context of collecting the maximum signal from fluorophores positioned in the vicinity of a planar interface. In addition to the far-field behaviour relevant to most radio and spectroscopic applications, the near-field effect of a substrate on, e.g., two-dimensional (2D) photonic crystal arrays formed near an interface is surely also important.

It is immediately obvious that the problem presents significant mathematical difficulties, since the phase velocities normal to the interface differ in the two media, yet the contours of constant phase must match at the interface. Qualitatively, what happens is that the oscillating dipole in a medium with a lower refractive index excites both homogeneous and evanescent modes in the higher refractive index medium and the presence of the additional mode allows matching of the Maxwell boundary conditions at the interface; the details are, however, all important. In addition to the mathematical complexity associated with the special functions entering the formal solution, numerical problems arise in its application.

In this paper, we studied the half space problem via simulation for a dipole located just above the interface that is oriented: (i) perpendicular to the interface; (ii) randomly in the plane of the interface; (iii) randomly in all directions. We systematically investigated the polarization dependence of the radiation patterns and the dependence of the ratio of the integrated power between the two half spaces on the relative refractive index as well as the distance of the dipole above the interface. From this information, one can obtain the behaviour for a dipole with any specified orientation. Clearly it is of primary interest to know how the power is directed. This problem has been extensively investigated, both theoretically [2]–[4] and experimentally [5, 6], but to our knowledge has not been treated sufficiently rigorously.

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Figure 1. The thick arrows show the contour used to evaluate the Sommerfeld integral.

2. Simulation methods

2.1. The Sommerfeld formalism

The near-field simulations carried out in this study were done using the Sommerfeld integral decomposition of a spherical wave [7],

$$\frac{e^{ik_i r}}{r} = i \int_0^\infty dk_\rho \frac{k_\rho}{k_z} J_0(k_\rho) e^{ik_z |z|}.$$
 (1)

Here, *r* is the distance between source and field points, $k_i = \sqrt{\varepsilon_i \mu_i} k_0$ is the magnitude of the wave vector of the medium, k_0 being that of the vacuum. J_0 is the first cylindrical Bessel's function and $k_z = \sqrt{k_i^2 - k_\rho^2}$. The subscript *i* is + or – depending on which side of the interface is being considered and the interface is taken to be the x-y plane. Equation (1) represents an integral decomposition of the spherical wave into a cylindrical wave, with wave number k_ρ , and a plane wave in the *z* direction, given by k_z . The contour of integration on the complex plane is shown in figure 1; it lies just below the real axis for values of k_ρ less than k_{max} which is chosen to be larger than the larger of the two values of k_i . In this way, the branch cuts and branch points (dictated by the expression for k_z defining out-going radiation [8]) are avoided. (Singularities can also occur on the real axis when layers are present.)

The electric Green's tensor has been derived and used by many authors [8]–[10] and is defined as

$$\ddot{G}(\vec{r} - \vec{r}') = \left(\vec{I} + \frac{\vec{\nabla}\vec{\nabla}}{k_i^2}\right) \frac{e^{ik_i|\vec{r} - \vec{r}'|}}{4\pi |\vec{r} - \vec{r}'|}.$$
(2)

The source point is at the position \vec{r}' and the field point at \vec{r} . In terms of tensor components the physical meaning of this Green's function is that $G_{\alpha\beta}$ is the α component of the electric field vector at the position \vec{r} due to a vector dipole of unit strength pointing in the β direction at the position \vec{r}' . The operator in equation (2) is now applied to the representation of equation (1) to

yield the desired decomposition. The interface electromagnetic boundary conditions

$$\hat{z} \times \left(\ddot{G}^{+} - \ddot{G}^{-}\right) = 0, \qquad \hat{z} \cdot \left(\varepsilon^{+}\ddot{G}^{+} - \varepsilon^{-}\ddot{G}^{-}\right) = 0,$$

$$\hat{z} \times \vec{\nabla} \times \left(\frac{\ddot{G}^{+}}{\mu^{+}} - \frac{\ddot{G}^{-}}{\mu^{-}}\right) = 0, \qquad \hat{z} \cdot \vec{\nabla} \times \left(\ddot{G}^{+} - \ddot{G}^{-}\right) = 0,$$
(3)

can then be applied. The waves in the z-direction separate naturally into TE and TM modes that propagate through the media completely independent of each other and are coupled only at the source. A derivation of the electric Green's tensor in the more general context of multiple layers in a stratified media is given in Paulus *et al* [11]. There it is shown how the boundary conditions are used and how the mode coupling at the source is taken into account. Some corrections and significant improvements will be published elsewhere [12] along with a derivation of the magnetic Green's function.

The magnetic field due to the electric dipole can be obtained from Maxwell's equations as

$$\vec{H}\left(\vec{r}\right) = -\frac{1}{k}\vec{\nabla}\times\vec{E}.$$
(4)

This equation can be applied locally to obtain the magnetic field from the appropriate components of the electric Green's tensor. It is correctly applied only to the z plane wave and ρ conical wave components of the representation before k_{ρ} integration and not to the integrated form that contains the results of 'folding' the two media together under the k_{ρ} integral. The TE (transverse electric) and TM (transverse magnetic) modes are thus treated separately and the results added up before k_{ρ} integration [8, 13]. In this way, the correct boundary conditions are satisfied. Using these considerations a magnetic Green's function \ddot{G}_H can be derived for the electric dipole with the physical meaning analogous to the electric Green's function as given above. These details will also be discussed elsewhere [12].

The radial part of the time averaged Poynting vector given by

$$\vec{S} = \frac{1}{2} \left(\vec{E} \times \vec{H}^* \right),\tag{5}$$

can then be calculated using the appropriate tensor components.

Here, we present numerical results using the Gauss–Kronrod method for numerically integrating on the contour in the k_{ρ} plane shown in figure 1. This is adequate for radial distance out to more than 25 wavelengths. For comparison results are given for far field using an asymptotic approach, which we now discuss.

2.2. Asymptotic approach

The far-field simulations carried out in this study were done using an asymptotic approach based on the Lorentz reciprocity theorem [4]. This method allows us to calculate the power $P^{p,s}(\theta, \varphi)$ radiated into the direction (θ, φ) in the differential solid angle $d\Omega = \sin \theta \, d\theta \, d\varphi$, associated with a dipole oriented at the angle (α, φ_0) . $P^{p,s}(\theta, \varphi)$ is normalized by the total integrated power of a dipole radiating in the vacuum. In order to obtain $P^{p,s}(\theta, \varphi)$, we require three basic quantities, $P^{p}_{\parallel}(\theta)$, $P^{s}_{\perp}(\theta)$. The subscripts stand for the dipole orientation, either parallel or



Figure 2. Schematic of the half space structure; k is the direction of observation.

perpendicular to the interface, and the superscripts stand for the polarization of the emissions. $P_{\perp}^{p}(\theta)$ corresponds to the signal emitted by a dipole oriented perpendicular to the interface, that is $(\alpha = 0, \varphi_0)$. $P_{\parallel}^{p}(\theta)$ and $P_{\parallel}^{s}(\theta)$ correspond to the p-polarized and s-polarized signal emitted in the $\varphi = 0$ and $\varphi = \pi/2$ planes, respectively, by a dipole oriented parallel to the interface at $\varphi_0 = 0$, that is $(\alpha = 0, \varphi_0 = 0)$. $P^{p,s}(\theta, \varphi)$ is given by

$$P^{p}(\theta,\varphi) = \cos^{2} \alpha P^{p}_{\perp}(\theta) + \sin^{2} \alpha \cos^{2} (\varphi - \varphi_{0}) P^{p}_{\parallel}$$

$$P^{s}(\theta,\varphi) = \sin^{2} \alpha \sin^{2} (\varphi - \varphi_{0}) P^{s}_{\parallel}$$
(6)

and

$$P^{u}(\theta,\varphi) = P^{p}(\theta,\varphi) + P^{s}(\theta,\varphi), \tag{7}$$

where $P^{u}(\theta, \varphi)$ corresponds to unpolarized radiation. Considering an incoming wave propagating from infinity in the opposite direction, as is done when using the Lorentz reciprocity approach, the three basic quantities, $P^{P}_{\parallel}(\theta)$, $P^{s}_{\perp}(\theta)$, $P^{P}_{\perp}(\theta)$, are proportional to the absolute amplitude of the ratios between the resultant electric fields at the dipole position, $E(z_{o})$, which can be calculated from Fresnel reflection and transmission coefficients, and the incoming electric field E_{in}

$$P_{\perp}^{p}(\theta) = \frac{3}{8\pi}n(\theta) \left| \frac{E_{\perp}^{p}(z_{0},\theta)}{E_{in}^{p}} \right|^{2}, \qquad P_{\parallel}^{p}(\theta) = \frac{3}{8\pi}n(\theta) \left| \frac{E_{\parallel}^{p}(z_{0},\theta)}{E_{in}^{p}} \right|^{2},$$

$$P_{\parallel}^{s}(\theta) = \frac{3}{8\pi}n(\theta) \left| \frac{E_{\parallel}^{s}(z_{0},\theta)}{E_{in}^{s}} \right|^{2},$$
(8)

where $0 < \theta < \pi/2$ corresponds to the upper half space and $\pi/2 < \theta < \pi$ corresponds to the lower half space (figure 2); here $n(\theta) = n_1$ for $0 < \theta < \pi/2$, $n(\theta) = n_2$ for $\pi/2 < \theta < \pi$.

3. Simulation results

We assume the upper half space has refractive index n_1 and the lower half space has refractive index $n_2(n_2 > n_1)$. The relative refractive index is defined as $n = n_2/n_1$ (figure 2).

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Figure 3. Far-field radiation patterns $P(\theta)$ versus θ . Per denotes a dipole oriented perpendicular to the interface; 2Pi denotes a dipole randomly oriented in the plane of the interface; and 4Pi denotes a randomly oriented dipole. The radiation patterns are independent of φ for the above cases. The polarization of the emitted signal is indicated by the different types of the lines. The same convention is adopted for the remaining plots.

3.1. Polarization dependence of the radiation patterns for a dipole located slightly above the interface (at $z_0 = 0.001$ nm) observed in the far-field and in the near-field for various observation distances

Figure 3 shows the s, p and unpolarized far-field radiation pattern on a logarithm scale for dipoles located at $z_0 = 0.001$ nm in the upper half space. The radiation pattern is highly structured and



Figure 4. Near-field radiation patterns $P(\theta)$ versus θ at five wavelengths from the source.

directed and $P(\theta)$ has a global maximum/minimum (with the in-plane averaged p-polarized emission having a minimum) at the critical angle between the media for all the above three cases.

Figures 4 and 5 show the near-field radiation patterns for the above cases calculated at five wavelengths and 25 wavelengths from the position of the dipole, respectively (we have carried out calculations at other observation distances which we do not give here for brevity). The global maximum/minimum of $P(\theta)$ approaches the critical angle with increasing observation distances. The rapidly oscillating behaviour of the $P(\theta)$ beyond the global maximum/minimum arises from



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Figure 5. Near-field radiation patterns $P(\theta)$ versus θ at 25 wavelengths from the source.

an interference of the homogenous modes with the travelling surface modes required to match the boundary conditions. The oscillation frequency increases and the amplitude decreases with increasing observation distances.

3.2. Ratio of the integrated power into the upper half space and the lower half space with respect to the relative refractive index for a dipole located at $z_0 = 0.001$ nm

We also calculated the integrated power emitted over the upper half space and the lower half space respectively; the ratio of these two powers is shown in figure 6. As the relative refractive

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Figure 6. Ratio of the integrated power versus relative refractive index. The curves are far-field results and the dots are near-field results.

index increases from 1.01 to 3.50, this ratio increases (i) from 1.51 to 8.81 for a dipole oriented perpendicular to the interface; (ii) from 1.24 to 47.4 for the unpolarized signal emitted by a dipole randomly oriented in the plane of the interface; and (iii) from 1.33 to 16.37 for the unpolarized signal emitted by a randomly oriented dipole. Clearly the relative refractive index plays an important role in determining the distribution of the radiation.

3.3. Position dependence of the integrated power ratio

Figure 7 shows the z_0 dependence of the ratio of the integrated power radiated into the upper half space and the lower half space calculated from the far-field radiation patterns. The ratio drops quickly and approaches a constant value when the dipole is more than about 0.2 wavelength away from the interface. The interface influences the radiation pattern dramatically only when the dipoles are close to the interface. Note the ratio of the integrated power calculated from the near-field radiation pattern will deviate from the far-field results at large z_0 (larger than 1.5 wavelengths in the present case), because the power radiated to the upper half space and lower half space observed at near field depends on the dipole position; for this reason such plots are not given.



Figure 7. Ratio of the integrated power versus dipole position in the far field.

4. Conclusions

We find that the electric dipole radiation pattern shows a number of interesting features: (i) it is highly structured and directed; (ii) as expected the radiation pattern observed at near fields evolves and approaches the radiation pattern observed at far field, but shows interesting structure along the way; (iii) the integrated power ratio increases significantly with increasing relative refractive index; and finally (iv) the power ratio drops quickly for dipoles more than 0.2 wavelength away from the interface.

We emphasize that when performing spectroscopic measurements, where the light is typically collected from the air side of substrate-anchored molecules, the signal collection can be significantly enhanced from using a transparent, high-dielectric-constant substrate and collecting the light emitted into the substrate. This fact is apparently not appreciated in various spectroscopy communities.

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