# The strong equivalence principle from a gravitational gauge structure

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#### Abstract

Gravitational self-interactions are assumed to be determined by the covariant derivative acting on the Riemann-Christoffel field strength. Once imposed on a metric theory, this Yang-Mills gauge constraint extends the equality of gravitational mass and inertial mass to compact bodies with non-negligible gravitational binding energy. Applied to generalized Brans-Dicke theories, it singles out Einstein's tensor theory and Nordström scalar theory for gravity but also suggests a way to implement a minimal violation of the strong equivalence principle.

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## Introduction

The weak equivalence principle (WEP) rests upon the empirical equality of the inertial and gravitational masses. It states that in an empty space, namely a space with no matter present and no physical fields except the (homogeneous) gravitational field, all test bodies fall with the same acceleration. In practice, somebody in a free-falling elevator would experience no apparent weight. From this " happiest thought ", Einstein inferred that all physical laws of special relativity (electromagnetism included) remain valid in a sufficiently small free-falling laboratory to eventually establish his succesful general theory of relativity. In this theory, the gravitational interactions of matter (and light) are characterized by a universal coupling to the metric tensor  $g_{\mu\nu}$ . For the relativistic action of a free particle, it simply amounts to substitute  $g_{\mu\nu}(x)$  for the Minkowski metric  $\eta_{\mu\nu}$ . Consequently any test particle propagates along stationary paths and the track is always a geodesic of the curved space-time, regardless of its mass. If the WEP is implemented without reference to the Einstein non-linear field equations, what then does privilege them ? The careful observation of free-fall for compact bodies (i.e bodies containing non-negligible gravitational self-interactions, contrary to test bodies) should in principle provide an answer to this question.

The strong equivalence principle (SEP) naturally extends the universality of physical results to local gravitational experiments [1]. It simply states that the free-fall of a compact body in empty space is also independent of its gravitational binding energy. Here, the ratio of internal gravitational binding energy to the total mass energy defines the "compactness". From its typical value for human-size bodies, one easily concludes that present Eötvös-like experiments only test the WEP, not the SEP. Yet, the Lunar Laser Ranging experiment beautifully corroborates the equivalence principle for celestial bodies : the Moon orbit around the Earth does not appear to be polarized toward the Sun. In the first part of this paper, we raise this heuristic hypothesis about non-linear effects of gravity at the level of a fundamental principle.

The WEP only rules the kinematics of point-like particles, not the dynamics of the gravitational fields. In fact, even after the Einstein field equations had been set up, it was thought that one had to demand in addition that the geodesic equation be the equation of motion of a test particle. Eventually, it was realized that this can be deduced directly from the general covariant conservation of the energy-momentum tensor which is always valid in Einstein's theory.

The SEP also governs the kinematics of extended bodies and appears to distinguish Einstein's theory from other relativistic theories of gravity. An effective violation of the SEP might be introduced through the explicit breakdown of energy-momentum conservation for compact bodies. In the second part of this paper, we suggest a covariant way to implement a minimal violation of the SEP without reference to the energy-momentum tensor for the matter fields.

Both matter and self-interaction couplings are necessarily involved in the free-fall of bodies which carry a non-negligible amount of internal gravitational binding energy. Let us then first turn toward Yang-Mills theories where these couplings are precisely related.

### Self-interaction Gauge Structure

#### A. The case for strong interactions

Non-abelian gauge fields carry themselves the quantum numbers with which they interact. Indeed, in the classical equations of motion

$$\partial_{\mu}F^{\mu\nu} = -gj^{\nu} - ig[A_{\mu}, F^{\mu\nu}] \tag{1}$$

the first term on the rhs is the (gauge-invariant but not conserved) matter current, while the second one represents the non-linear self-coupling effect of the Yang-Mills fields [2]. Consequently, these gauge fields act themselves as a source and their self-interaction is fixed by their coupling to matter. In particular, the effective colour charges of the octet gluons (the gauge fields A associated with strong interactions) and triplet quarks (the fermionic matter fields F) are equal up to well-defined SU(3) group theory factors. LEP precision measurements around the electroweak scale provide the corresponding quadratic Casimir operators [3].

$$C_A = 2.89 \pm 0.01 \text{ (stat.)} \pm 0.21 \text{ (syst.)}$$
 (2a)

$$C_F = 1.30 \pm 0.01 \text{ (stat.)} \pm 0.09 \text{ (syst.)}$$
 (2b)

which are in very good agreement with the non-abelian gauge structure of QCD ( $C_A = 3$ ,  $C_F = 4/3$ ) and rule out any abelian vector gluon model ( $C_A = 0$ ,  $C_F = 1/2$ ). They amount to a universal running of the three-gluon coupling defined by Eq.(1). This running as a function of energy points then at another genuine property of the strong interactions, namely quark and gluon confinement into hadronic bound states.

For pure gauge fields, i.e. without matter, Eq.(1) simply becomes the Yang-Mills conditions

$$D_{\mu}F^{\mu\nu} = 0. \tag{3}$$

The covariant derivative introduced in Eq.(3) defines then the field strength

$$[D_{\mu}, D_{\nu}] \equiv iF_{\mu\nu} \tag{4}$$

in a way similar to the definition of the curvature tensor for gravity ...

#### B. The case for gravitational interactions

If the gravitational field interacts with the matter fields through the general covariance which turns the ordinary derivative  $\partial_{\mu}$  into the covariant derivative  $D_{\mu}$ , any test body moves following geodesics, regardless of its mass or internal structure. It results from this kinematics in curved space-time that the WEP is fulfilled. However, the gravitational field may also interact with itself, like non-abelian fields do. Indeed, the affine connection  $\Gamma^{\lambda}_{\mu\nu}$  corresponds to the gauge field  $A_{\mu}$  and the Riemann-Christoffel tensor  $R^{\sigma}_{\lambda\mu\nu}$  corresponds to the non-abelian field strength  $F^{\mu\nu}$  with

$$[D_{\mu}, D_{\nu}]^{\sigma}_{\ \lambda} \equiv -R^{\sigma}_{\ \lambda\mu\nu}.$$
(5)

Compact bodies which differ by their binding energy could therefore fall with different accelerations if the gravitational self-interaction was not universal. But the present null results on an anomalous polarization of the Moon orbit around the Earth (the Nordtvedt effect [4]) plead in favour of a single three-graviton vertex known with a precision of  $10^{-3}$ . Inspired by the strong interaction gauge theory, we impose then the Yang-Mills conditions (3) on the curvature tensor :

$$D_{\sigma}R^{\sigma}_{\ \lambda\mu\nu} = 0\,,\tag{6}$$

in regions where matter is absent. Note that Eq.(6) does not require the introduction of a metric but only defines a Riemannian manifold for which the parallel-displacement field is sourceless. Thus geodesics are not defined (yet) and the motion of a body cannot be determined from Eq.(6) alone. However the resulting constraints naturally embody the gravitational self-interaction in the cubic  $\Gamma\Gamma\Gamma$  terms. If the affine connection  $\Gamma$  corresponds to the gauge potential A, gravitons gravitate the way gluons glue. Consequently, a gravity theory in which Eq.(6) holds may incorporate a suitable version of the SEP in the following sense : it is an intrinsic property of gravitational dynamics, entirely determined by the geometry of space-time. If the twenty constraints of Eq.(6) turned out to over-determine the field equations of the theory, then this would imply a violation of the SEP. In other words, Eq.(6) is not the full expression of the SEP but only a necessary covariant condition to ensure its implementation. In particular, Eq.(6) does not imply the WEP while the SEP does by definition. This latter point becomes obvious once a metric is introduced.

Let us express the covariant contraint (6) for the particular case of an asymptotically flat isotropic metric expanded in the weak stationary field V(r) = -GM/r:

$$g_{00} = 1 + 2\alpha \left(\frac{V}{c^2}\right) + 2\beta \left(\frac{V}{c^2}\right)^2 + \mathcal{O}\left(\frac{1}{c^6}\right)$$

$$g_{ij} = -\delta_{ij} \left\{ 1 - 2\gamma \left(\frac{V}{c^2}\right) + \frac{3}{2}\delta \left(\frac{V}{c^2}\right)^2 + \mathcal{O}\left(\frac{1}{c^6}\right) \right\}.$$
(7)

The  $\alpha, \beta, \gamma$  and  $\delta$  dimensionless coefficients, normalized to one for the Schwarzschild solution of general relativity, have to be determined experimentally [5]. From such a parametrization, we easily obtain the following relations

$$D_{\sigma}R^{\sigma}_{\ 00n} = -\left(\frac{1}{2c^4}\right)(4\beta - \alpha\gamma - 3\alpha^2)\partial_n(\partial_l V\partial^l V) \tag{8a}$$

$$D_{\sigma}R^{\sigma}_{\ell m n} = +\left(\frac{1}{c^4}\right)\left(6\delta - 6\gamma^2 - \alpha\gamma + \alpha^2\right)\left(\partial_l\partial_m V \partial_n V - \partial_l\partial_n V \partial_m V\right) \tag{8b}$$

if the  $\mathcal{O}(1/c^6)$  terms are neglected. So, the third order differential equations (6) for the metric  $g_{\mu\nu}$  manifestly contain unphysical solutions. For example, the exact solution with  $g_{00} = 1$  (i.e.  $\alpha = \beta = 0, \ \delta = \gamma^2$ ) possesses no gravitational redshifts [6] and violates thereby the WEP. This nicely illustrates the fact that Eq.(6) is only a necessary condition to fulfill the SEP. Thus, at this level Eq.(6) should not be regarded as the fundamental gravitational field equations derived from some variational principle in a new theory of gravity [7], but rather as further tensorial constraints on any metric theory incorporating by definition the WEP.

### The SEP in Metric Theories

Any metric theory of gravitation postulates the geodesic motion of test bodies such that the WEP corresponds to the zeroth-order condition  $\alpha \equiv 1$ . The higher-order constraints imposed by Eq.(6) reduce then respectively to

$$\eta \equiv 4\beta - \gamma - 3 = 0 \tag{9a}$$

$$\eta' \equiv 6\delta - 6\gamma^2 - \gamma + 1 = 0. \tag{9b}$$

Consequently, the Yang-Mills conditions (3) applied to the curvature field strength go beyond Einstein's heuristic hypothesis and imply that the non-linear parameters ( $\beta$  and  $\delta$ ) are independently determined by the intrinsic space curvature ( $\gamma$ ). The recent conjunction experiment with Cassini spacecraft [8] provides the range  $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ . If the conditions (9) prove physical, we may already infer that  $|\beta - 1| < 10^{-5}$  and  $|\delta - 1| < 10^{-4}$  in the vicinity of the Sun. These theoretical constraints are much stronger than the existing observational bounds from the solar system [9]. At present, the secular advance of Mercury's perihelion proportional to the combination  $(-\beta + 2\gamma + 2)/3$  yields a weaker constraint on the Eddington parameter  $\beta$ ,  $|\beta - 1| < 3 \times 10^{-3}$ . On the other hand, the crucial parameter  $\delta$  which appears only at the second order in the light deflection angle (consistently expressed in terms of the physical impact parameter b) :

$$\Delta = \left(\frac{4GM}{c^2b}\right) \left\{ \frac{(1+\gamma)}{2} + \frac{(8+8\gamma-4\beta+3\delta)}{16} \frac{\pi GM}{c^2b} \right\}$$
(10)

is far from being constrained nowadays. Based on optical interferometry between two microspacecraft, the LATOR experiment [10] aims at measuring  $\delta$  with a precision of  $10^{-3}$ . But this experiment would simultaneously reach the impressive level of accuracy of 1 to  $10^8$  for the parameter  $\gamma$ .

We are of course pleased to recover, as expected, the well-known Nordtvedt condition (9a) derived from a phenomenological relation [11] between the gravitational mass  $(m_{gr})$  and the inertial mass  $(m_{in})$ :

$$\frac{m_{gr}}{m_{in}} \approx 1 + \eta \left(\frac{\Omega}{mc^2}\right) \tag{11}$$

for compact bodies with non-negligible gravitational binding energy  $\Omega$ . Although the fraction of gravitational self-energy is only  $4.5 \times 10^{-10}$  for our planet, the Lunar Laser Range experiment confirms indeed that Earth and Moon fall toward the Sun at equal rates with a precision of about  $2 \times 10^{-13}$ . So, the gravitational binding energy equally contributes to the inertial mass and to the gravitational mass with a precision given by [9]

$$|\eta^{obs.}| = (4.4 \pm 4.5) \times 10^{-4}.$$
 (12)

However the general covariance of Eq.(6) unavoidably requires a second condition (9b) to be also fulfilled !

To illustrate this important point, let us first briefly revive the Einstein-Grossmann "Entwurf" [12]. This " outline " is based on a spatially flat metric,  $g_{ij} = -\delta_{ij}$ , and predicts an advance of Mercury's perihelion of about 18" per century [13], i.e. 5/12 of the observed value. In the weak field approximation, the corresponding Eddington parameters are  $\gamma = \delta = 0$  and  $\beta = 3/4$ , respectively. As a consequence, the theory obeys the Nordtvedt constraint (9a) but not (9b). This can easily be understood from the fact that the pure gravity kinetic term in the associated action functional involves ordinary derivatives of the metric field. Therefore, general covariance is lost and the action is only invariant with respect to arbitrary linear transformations.

On the other hand, both constraints (9a) and (9b) are satisfied by the Einstein final theory [14] with all Eddington parameters equal to unity ( $\beta \equiv 1, \gamma \equiv 1, \delta \equiv 1$ ), but also by the Nordström scalar theory [15]. The geometric reformulation [16] of this first consistent relativistic theory of gravitation leads indeed to a special conformally flat metric,  $g_{\mu\nu} = (1 + V/c^2)^2 \eta_{\mu\nu}$ , i.e. a finite set of Eddington parameters ( $\beta = 1/2, \gamma = -1, \delta = 2/3$ ). These particular values imply a retrogression of Mercury's perihelion of 1/6 of the observed magnitude as well as the vanishing of the deflection angle  $\Delta$  expressed in Eq.(10).

The Bianchi identities,  $D_{\sigma}R^{\sigma}_{\lambda\mu\nu} + D_{\nu}R^{\sigma}_{\lambda\sigma\mu} + D_{\mu}R^{\sigma}_{\lambda\nu\sigma} = 0$ , allow us to extend the analysis of the Yang-Mills conditions beyond the weak field approximation. Indeed, these identities imply that the basic Eq.(6) is equivalent to

$$D_{\nu}R_{\lambda\mu} - D_{\mu}R_{\lambda\nu} = 0. \tag{13}$$

A contraction of Eq.(13) yields the necessary condition  $D_{\nu}R^{\nu}{}_{\mu} = \partial_{\mu}R$ . A direct comparison with the contracted Bianchi identities,  $2 D_{\nu}R^{\nu}{}_{\mu} = \partial_{\mu}R$ , implies therefore a constant scalar curvature R. But our hypothesis of an asymptotically flat metric eventually requires R to be vanishing. Using then the standard decomposition of the Riemann tensor  $R^{\sigma}{}_{\lambda\mu\nu}$  into the Weyl tensor  $W^{\sigma}{}_{\lambda\mu\nu}$  and the Ricci tensor  $R_{\lambda\mu}$ , one easily infers that Eq.(13) is, in its turn, equivalent to

$$D_{\sigma}W^{\sigma}_{\ \lambda\mu\nu} = 0 \tag{14a}$$

$$R = 0. \tag{14b}$$

We immediately conclude from Eqs.(13) and (14) that both Einstein tensor theory  $(R_{\lambda\mu} = 0)$  and Nordström-Einstein-Fokker scalar theory  $(W^{\sigma}_{\lambda\mu\nu} = 0, R = 0 \text{ are in fact exact vacuum solutions of Eq.(6)}.$ 

At the  $1/c^4$  order in the weak field approximation, the tensorial Eqs.(14a) and (14b) respectively constrain two linear combinations of the  $\eta$  and  $\eta'$  parameters:

$$(2\eta - \eta') = 0 \tag{15a}$$

$$(\eta + \eta') = 0. \tag{15b}$$

Consequently,  $\eta'$  has definitely to be on an equal footing with the Nordtvedt parameter  $\eta$  defined in Eq.(11). This nicely confirms our intuition that Eq.(6) provides a covariant formalism for the SEP in the general framework of metric theories. Such a formalism seems to single out one pure tensor and one pure scalar theory of gravity. It is therefore worth checking if they both survive within a restricted class of metric theories which naturally emerge as low-energy approximations of superstring or Kaluza-Klein theories.

### The SEP in Tensor-Scalar Theories

Let us consider the following tensor-scalar (TS) action

$$S_{\rm TS}(\omega) = -\left(\frac{c^4}{16\pi}\right) \int d^4x \sqrt{-g} \left\{ \phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\} + S_{\rm Matter}(g_{\mu\nu},\psi) \tag{16}$$

involving in principle an arbitrary function  $\omega(\phi)$  of the scalar field  $\phi$ .

#### A. Constant- $\omega$

If  $\omega(\phi)$  is a constant parameter  $\omega_0$ , we recover the original Brans-Dicke (*BD*) theory [17]. In this theory, Eq.(11) simply amounts to a spatial variation of the Newton constant :

$$G_{\rm lab}(r) \approx G_{\infty} \left\{ 1 - \eta_{\scriptscriptstyle BD} \frac{V(r)}{c^2} \right\}$$
(17)

where  $G_{\infty}$  is the value of G measured far from the gravity source. Such a variation of the gravitational coupling obviously violates the universality of free-fall for compact bodies, i.e. the SEP. The inner structure of a compact body is indeed sensitive to changes of the gravitational "constant".

In the weak field approximation, we easily obtain

$$\eta_{BD} = 1 - \gamma$$

$$\eta'_{BD} = 2\gamma(\gamma - 1)$$
(18)

with  $\gamma = (\omega_0 + 1)/(\omega_0 + 2)$ . Consequently, Eqs.(9) are only fulfilled by general relativity ( $\omega_0 = \infty$ ). This result can easily be extended beyond the weak field approximation. Indeed, variations of the BD action with respect to the gravitational fields imply that the scalar  $\phi$  plays the role of a source for the Riemann-Christoffel tensor :

$$D_{\sigma}(\phi R^{\sigma}_{\ \lambda\mu\nu}) = -(\omega_0 + 1)(R_{\lambda\mu}\phi_{\nu} - R_{\lambda\nu}\phi_{\mu}). \tag{19}$$

The basic Yang-Mills condition (6) requires a constant scalar field and the tensor-scalar action (16) reduces to the Hilbert one for  $G\phi = 1$ . Note that the rhs of Eq.(19) vanishes for  $\omega_0 = -1$ , a value predicted by duality in the graviton-dilaton low-energy effective superstring action [18]. Such is not the case for the low-energy limit of a  $(4 + \varepsilon)$  dimensional Kaluza-Klein theory characterized by  $\omega_0 + 1 = 1/\varepsilon$ .

#### B. Variable- $\omega$

We have seen that the SEP requires  $\gamma_{BD} = +1$ . This is not a surprise since the Brans-Dicke theory obeys  $\beta_{BD} = 1$ , i.e. rules out the Nordström-Einstein-Fokker theory ( $\beta = 1/2$ ) from the start. But if  $\omega(\phi)$  is an arbitrary function of the scalar field,  $\beta$  is now proportional to its first derivative  $\omega'(\phi)$ . Remarkably, we still have an exact relation between the parameters  $\beta, \gamma$  and  $\delta$ :

$$\delta_{\rm TS} = \frac{4}{3}(\beta_{\rm TS} - 1) + \frac{1}{6}(8\gamma_{\rm TS}^2 - \gamma_{\rm TS} - 1) \tag{20}$$

such that one linear combination of  $\eta$  and  $\eta'$  can be directly expressed in terms of the space curvature parameter:

$$2\eta_{\rm TS} - \eta_{\rm TS}' = 2(1 - \gamma_{\rm TS}^2) \tag{21}$$

even if  $\omega(\phi)$  is not known! Consequently, a necessary condition to preserve the SEP in any tensorscalar theory is  $\gamma_{TS} = \pm 1$ . This generic result on the SEP is in agreement with [19]. Combined with the constraint (9a) and the relation (20), it singles out both Einstein and Nordström-Einstein-Fokker metrics at the <u>full</u>  $1/c^4$  order in the weak field approximation. The general static spherically symmetric solution of Eq.(6), given as an inverse-power-series in [20], proves that  $\alpha = \gamma = +1$  is sufficient to settle the Schwarzschild solution of general relativity. Yet, our covariant formalism (14) allows us to go again beyond such a result.

The linear combination of  $\eta$  and  $\eta'$  in Eq.(21) is precisely associated with the tensor  $D_{\sigma}W^{\sigma}_{\lambda\mu\nu}$ in the weak field approximation (see Eq.(15a)). Following then Eq.(14), we still have the condition of vanishing scalar curvature at our disposal :

$$R = \left\{ \omega - 3 \frac{\omega' \phi}{(2\omega + 3)} \right\} \frac{\phi^{\alpha} \phi_{\alpha}}{\phi^2} = 0.$$
(22)

This second condition completely fixes the arbitrary functional  $\omega(\phi)$  appearing in the tensor-scalar action (16):

$$\omega(\phi) = -\left(\frac{3}{2}\right) \frac{G\phi}{(G\phi - 1)}.$$
(23)

A simple scalar field redefinition:

$$G\phi = 1 - \frac{\kappa\varphi^2}{6} > 0 \tag{24}$$

with  $\kappa = 8\pi G/c^4$  leads then immediately to the improved Einstein-massless scalar theory advocated by Deser [21]. The resulting action

$$S_{R=0} = -\left(\frac{1}{2\kappa}\right) \int d^4x \sqrt{-g}R + \int d^4x \sqrt{-g} \left\{ \left(\frac{\varphi^2}{12}\right) R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \right\}$$
(25)

corresponds indeed to gravity conformally coupled to a scalar field [22]. But we have shown that it may as well be considered as the generalized Brans-Dicke tensor-scalar theory with a vanishing scalar curvature R. From this latter point of view, the other condition (14a) is also satisfied if either the scalar (helicity-0) or the tensor (helicity-2) degree of freedom is frozen ( $\varphi \to 0$  or  $g_{\mu\nu} \to \eta_{\mu\nu}$ , respectively). In the first case, the action simply reduces to the Hilbert one and we are in the presence of the Einstein tensor theory. In the second case, a massless scalar freely propagates in the Minkowski space-time and we end up with the original Nordström scalar theory [15].

We conclude that only the Einstein and Nordström-Einstein-Fokker metric theories do comply with the SEP as defined by Eq.(6). From an observational viewpoint, it is obvious that perihelion advance and light deflection measurements ( $\Delta \neq 0$ ) exclude the latter in favour of the former. However, it would be more satisfactory if one could already discriminate them at the theoretical level.

#### Toward a Theory with Minimal Violation of the SEP

The sub-class of tensor-scalar theories defined by:

$$2\omega(\phi) + 3 = \frac{a}{(1 - G\phi)} \tag{26}$$

with  $a \neq 0$ , is rather interesting since  $\eta$  and  $\eta'$  are then separately expressed in terms of the crucial Eddington parameter  $\gamma$ :

$$\eta_a = (1 - \gamma^2) \frac{(1+a)}{2a}$$

$$\eta'_a = 2(1 - \gamma^2) \frac{(1-a)}{2a}.$$
(27)

For  $\gamma \neq \pm 1$ , the Yang-Mills conditions (3) imposed on the curvature tensor are not fulfilled and the value of the parameter *a* characterizes any violation of the SEP (see Fig.1).

The case a = -1 corresponds to the well-known "constant-G" theory of Barker [23]. Consequently, this theory violates the SEP since  $\eta = 0$  but  $\eta' \neq 0$ . Note that the same holds true for the complementary case a = +1 (i.e.,  $\eta' = 0$  but  $\eta \neq 0$ ).

For a = +3, we recover the "zero-R" theory defined in Eq.(23), with  $\eta + \eta' = 0$  in the weak field approximation. We have seen from Eq.(25) that this covariant theory provides a smooth interpolation between the pure tensor and the pure scalar theories respecting the SEP (see the curve R = 0 in Fig.1). A minimal violation of the SEP is thus expected here.

In fact, an attractor mechanism toward  $G\phi = 1$  for  $a \gg 1$  in Eq.(26) has already been discussed in the Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology (i.e., with a timedependent evolution for the scalar auxiliary field) [24]. This mechanism is most easily described in the so-called Einstein frame where the scalar and not the ordinary particle follows the geodesic determined by the metric. The corresponding action

$$S_E = -\left(\frac{1}{2\kappa}\right) \int d^4x \sqrt{-g} \{R - 2g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\} + S_{\text{Matter}}\{A^2(\sigma)g_{\mu\nu},\psi\}$$
(28)

is obtained through a suitable conformal transformation on the metric originally defined in the Jordan frame (see Eq.16):

$$g_{\mu\nu} \to A^2(\sigma)g_{\mu\nu} \tag{29}$$



Figure 1: Tensor-scalar theories versus the Strong Equivalence Principle. Only the Einstein (E) and Nordström-Einstein-Fokker (NEF) theories comply with the gauge conditions (14). The dashed vertical line corresponds to the Brans-Dicke (BD) models while the dashed curves represent the Barker theory (a = -1) and its complementary (a = +1) defined by Eq.(26). Arrows indicate possible attractors within FLRW cosmology.

with  $G\phi = A^{-2}(\sigma)$  and  $2\omega(\phi) + 3 = (A'/A)^{-2}$ . In the Einstein frame (28), the  $\sigma$  field's cosmological evolution is just analogous [25] to the damped motion of a particle in the potential  $\ln A(\sigma)$ .

For the sub-class of tensor-scalar theories defined by Eq.(26), we obtain the corresponding conformal factor

$$A(\sigma) = \cosh\left(\frac{\sigma}{\sqrt{a}}\right),\tag{30}$$

such that general relativity ( $\sigma = 0$ ) is the only point of equilibrium. If a < 0, a singular relaxation toward A = 0 is generic. Yet, for a > 0,  $A(\sigma) \ge 1$  and only a smooth relaxation toward A = 1, i.e. general relativity, is possible. The "constant-G" theory (a = -1) exhibits indeed a singular attractor toward pure scalar gravity ( $A \to 0$ ). On the other, the "zero-R" theory (a = +3) with positive  $\omega(\phi)$  turns out to contain a natural attractor toward general relativity ( $A \to 1$ ). Consequently, cosmological dynamics provides a way to disentangle the two theories respecting the SEP in the static approximation (see arrows in Fig.1).

Let us therefore introduce matter  $(\psi)$ , radiation  $(\gamma)$  as well as a cosmological constant  $(\Lambda)$  in

the action  $S_{R=0}$  defined by Eq.(25). The resulting field equations read then:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa (T^{\varphi}_{\mu\nu} + T^{\psi}_{\mu\nu} + T^{\gamma}_{\mu\nu} + T^{\Lambda}_{\mu\nu})$$
$$D^{\mu}D_{\mu}\varphi = \frac{R}{6}\varphi . \tag{31}$$

The massless field  $\varphi$  behaves like a radiation field (i.e.,  $T^{\varphi} = T^{\gamma} = 0$ ). So, we immediately obtain the equation

$$R = -\kappa (T^{\psi} + T^{\Lambda}). \tag{32}$$

In the homogeneous FL cosmology, the RW metric is conformal to the Minkowski one (i.e.,  $W^{\sigma}_{\lambda\mu\nu} = 0$ ) such that Eq.(6) simply becomes

$$D_{\sigma}R^{\sigma}_{\ \lambda\mu\nu} = \frac{\kappa}{6} \left(g_{\lambda\nu}\partial_{\mu} - g_{\lambda\mu}\partial_{\nu}\right)T^{\psi} \tag{33}$$

when matter is present. Note that the scalar field  $(\varphi)$ , the radiation  $(\gamma)$  and the cosmological constant  $(\Lambda)$  do not modify the vacuum contraint (6) in this theory.

For an initial value of the field  $\varphi$  smaller than the Planck mass (i.e.,  $\varphi_{\text{initial}} \lesssim \kappa^{-\frac{1}{2}}$ ), in agreement with Eq.(24), the time-dependent solution of Eqs.(31) implies an upper bound on the present space curvature,  $(\gamma - 1)_{\text{today}} \lesssim 10^{-4}$ . Current observations in the vicinity of the Sun already put a more severe constraint on  $\gamma$ . The LATOR experiment could in principle probe the theory defined by Eq.(25) over four orders of magnitude.

### Conclusion

The weak equivalence principle (WEP) settles the kinematics of test particles (space-time tells mass how to move), but not the dynamics of gravity (how mass tells space-time to curve). So it is quite remarkable that a covariant formulation of the strong equivalence principle (SEP) applied to generalized Brans-Dicke gravity models singles out one non-linear tensor theory ( $R_{\sigma\nu} = 0$ ) and one linear scalar theory ( $W^{\sigma\lambda\mu\nu} = 0, R = 0$ ). They may be considered as the analogs of the non-abelian QCD and abelian QED gauge theories, respectively: gluons carry colours ( $D_{\mu}F^{\mu\nu} = 0$ ) but photons do not carry electric charge ( $\partial_{\mu}F^{\mu\nu} = 0$ ). From a theoretical point of view, the abelian alternative to general relativity should not come as a surprise. The WEP alone implies that all non-gravitational fields couple in the same way to gravity. The strong version of the equivalence principle simply extends this universality of coupling to the gravitational field itself. However the photon, like any massless particle, does not couple to a conformally flat metric field. Consequently, the survival of a scalar theory complying with the SEP is obvious at the theoretical level, though ruled out at the phenomenological one.

If the SEP proves to be fundamental, then Eq.(6) itself should arise from a new theory of gravity, in a way similar to Eq.(3). The latter is derived from the pure Yang-Mills action functional which is an integral over the square of the curvature,  $F^{\mu\nu}F_{\mu\nu}$ . Here, no attempt has been made

to obtain the former from a variational principle. We simply note that if one considers, again by analogy, the conformally invariant Weyl theory in regions devoid of any matter :

$$S_{\text{Weyl}} = \xi \int d^4x \sqrt{-g} \{ W^{\sigma\lambda\mu\nu} W_{\sigma\lambda\mu\nu} \}, \qquad (34)$$

the resulting fourth-order differential equations for the metric [26]:

$$2D_{\nu}D_{\sigma}W^{\sigma\lambda\mu\nu} + W^{\sigma\lambda\mu\nu}R_{\sigma\nu} = 0 \tag{35}$$

contain non-trivial solutions [27] but reduce to the same alternative as for the tensor-scalar models once the constraint (14) are imposed by hand.

In the presence of matter fields, any strict analogy with the Yang-Mills equation of motion (3) has to break down at some point since the affine connection is itself constructed from the first derivatives of the metric tensor, while the gauge fields are not expressed in terms of more fundamental fields. However, an elegant formulation of gravity resembling the Yang-Mills theory has been advocated in [28, 29]. In this purely geometric formulation, the action is independent of the metric : the Lagrangian is exclusively quadratic in the field strength of an appropriate gauge group G having the Lorentz group as a subgroup. Here, the metric is no more a fundamental field than a hadron field is a fundamental field in QCD [29]. Indeed, four of the gravitational gauge potentials which belong to the adjoint representation of G are subsequently identified with the vierbein. The original action reduces then to the Einstein-Hilbert one with the addition of a cosmological term, in agreement with our Eq.(13). Thus, the SEP might indeed be a useful guiding gauge principle to single out such a higher-order action functional for gravitational interactions.

On the other hand, if the SEP turns out to be only approximate, the covariant condition (6) suggests a simple and natural way to implement a minimal violation of it within a sub-class of tensor-scalar theories characterized by an attractor mechanism toward general relativity. In particular, Eq.(33) nicely illustrates how matter is allowed to change Eq.(6).

### Acknowledgements

This work was supported by the Belgian Federal Office for Scientific, Technical and Cultural Affairs through the Interuniversity Attraction Pole P5/27.

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