Generation of Closed Timelike Curves with Rotating Superconductors

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Abstract

The spacetime metric around a rotating SuperConductive Ring (SCR) is deduced from the gravitomagnetic London moment in rotating superconductors. It is shown that theoretically it is possible to generate Closed Timelike Curves (CTC) with rotating SCRs. The possibility to use these CTC's to travel in time as initially idealized by Gödel is investigated. It is shown however, that from a technology and experimental point of view these ideas are impossible to implement in the present context.

1 Introduction

In Newtonian physics, causality is enforced by a relentless forward march of an absolute notion of time. In special relativity things are even more restrictive; not only must you move forward in time, but the speed of light provides a limit on how swiftly you may move through space (you must stay within your forward light cone). In general relativity it remains true that you must stay within your forward light cone; however this becomes strictly a local notion, as globally the curvature of spacetime might "tilt" light cones from one place to another. It becomes possible in principle for light cones to be sufficiently distorted that an observer can move on a forward directed path that is everywhere timelike and yet intersects itself at a point in its "past"-this is a Closed Timelike Curve (CTC) [1] [2].

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2 Closed Timelike Curves in Stationary, Axisymmetric Metrics

The general metric for a stationary, axisymmetric solution of Einstein Field Equations (EFE) containing CTCs, with rotation is given by [3] [4]:

$$ds^{2} = -A(r)c^{2}dt^{2} + 2B(r)rcd\phi dt + C(r)r^{2}d\phi^{2} + D(r)(dr^{2} + dz^{2})$$
(1)

The range of the coordinates is: $t \in]-\infty, +\infty[, r \in [0, +\infty[, \phi \in [0, 2\pi[, and z \in]-\infty, +\infty[, respectively. The metric components are functions of r alone. The determinant of the metric tensor is assumed to be Lorentzian,$

$$g = det(g_{\mu\nu}) = -(AC + B^2)D^2 < 0, \qquad (2)$$

therefore

$$(AC + B^2) > 0. (3)$$

Landau demonstrated that condition, Equ.(3), is always fulfilled in physically real spacetime [5] [6]. If the metric becomes non-Lorentzian, space-time becomes unstable [7] and decays in very short intervals of time to the Minkowsky metric.

Since the angular coordinate, ϕ , is periodic, an azimuthal curve $\gamma = \{t = Cte, r = Cte, z = Cte\}$ is a closed curve of invariant length $s_{\gamma}^2 = C(r)(2\pi)^2$. If C(r) is negative then the integral curve with (t, r, z) fixed is a CTC.

3 Gravitomagnetic London Moment and Spacetime Metric around a Rotating SCR

When a SCR is set into rotation it generates a gravitomagnetic field, call gravitomagnetic London moment [8][9][10].

$$B_g = \frac{\rho^*}{\rho} 2\omega = \frac{\rho^*}{\rho} \nabla \times v_\phi \tag{4}$$

Where ρ^* is the Cooper pairs mass density, ρ is the SC bulk mass density, ω is SC's angular velocity, and v_{ϕ} is the tangential velocity of the SCR along the azimuthal direction. For commodity we define the Cooper pair fraction, $a = \rho^*/\rho$. This London-type gravitomagnetic field is constant within the interior surface of the ring. Since the gravitomagnetic field is originated from a vector potential A_g

$$B_g = \nabla \times A_g \tag{5}$$

We deduce that in a superconductor the gravitomagnetic vector potential is proportional to the azimuthal velocity of the ring.

$$A_g = av_\phi = ar\omega \tag{6}$$

Where r is the radial distance from the SCR's rotation axis. From the weak field approximation of EFE, which leads to the laws of gravitoelectromagnetism [11], we know that the gravitomagnetic vector potential determines the g_{0i} components of the metric tensor.

$$g_{0i} = \frac{4}{c} A_{gi} \tag{7}$$

Doing Equ.(7) into Equ.(6) we obtain

$$g_{0\phi} = \frac{4}{c} a r \omega. \tag{8}$$

Assuming a SCR with height much larger than its radius, R, which is equivalent to the assumption of an hollow infinitely long superconductive cylinder, we have no gravitomagnetic field outside the cylinder. Therefore knowing $g_{0\phi}$, and imposing a flat metric outside the SCR, i.e. for $r \geq R$, we deduce the other metric components in Equ.(1).

$$A(r) = 1 \tag{9}$$

$$B(r) = \frac{4}{c}ar\omega \tag{10}$$

$$C(r) = 1 - 8a \tag{11}$$

$$D(r) = 1 \tag{12}$$

The relativistic interval is:

$$ds^{2} = -c^{2}dt^{2} + \left(8a\frac{\omega r}{c}\right)rcd\phi dt + \left(1 - 8a\right)r^{2}d\phi^{2} + dr^{2} + dz^{2}$$
(13)

which as expected from the boundary conditions defined above, simplifies to the flat metric in the limit where r = R and $\omega = d\phi/dt$:

$$ds^{2} = -c^{2}dt^{2} + R^{2}d\phi^{2} + dr^{2} + dz^{2}$$
(14)

As we mentioned above, when the azimuthal metric component becomes negative CTCs become possible in certain regions.

$$1 - 8a < 0 \tag{15}$$

$$a > \frac{1}{8} \tag{16}$$

Therefore when the Cooper pair fraction in the superconductive material is higher than 1/8, azimuthal closed curves $\gamma = \{t = Cte, r = Cte, z = Cte\}$, designated by CTCs, are generated when the superconductor is set rotating. In general the Cooper pair fraction in common SCs is $a \sim 10^{-7}$. Therefore a Cooper pair fraction higher that 1/8 is extremely challenging and is not achievalble presently with any known superconductor. Making abstraction of current technological limitations, let us ask in what region of space, with respect to the SCR's rotation axis, will the CTCs be located? To answer this question we need to evaluate the constraints imposed by having a Lorentzian metric determinant .

Doing Equ.(9)-Equ.(11) into Equ.(3), we find five different cases: For $a \leq 1/8$, the SCR cannot generate CTCs and the metric Equ.(13) will be allowed for all r > 0. In the case, in which the SCR is capable to generate CTCs, i.e., a > 1/8 we have four possibilities depending on the value of a and of the angular velocity, ω :

- 1. If 1/8 < a < 1 and $\omega < 1/a$ leads to $r > r_{max}$.
- 2. If 1/8 < a < 1 and $\omega > 1/a$ leads to $r < r_{max}$.
- 3. If a > 1 and $\omega > 1/a$, then $r > r_{max}$.
- 4. If a > 1 and $\omega < 1/a$, then $r < r_{max}$.

Where

$$r_{max} = \frac{c}{4a\omega} (8a - 1)^{1/2} \tag{17}$$

From Equ.(17) we see that for conditions close to the boundary conditions of case 1., 2. 3., and 4., the radius, r_{max} , is approximately equal to the distance between the Earth and the Moon. Therefore in the case 1., and 3, the metric Equ.(13) is not Lorentzian inside any SCR having a realistic radius ($R << r_{max}$). In these cases the metric Equ.(13) decays to Minkowsky metric and $B_g = 0$ for $r < r_{max}$. For the case 2., and 4., however, SCR's with realistic size would be capable to host CTC's in their hollow region (if the challenging Cooper pair fraction could be achieved). The light cone structure and the requirements to use these CTCs for traveling in time will be investigated in the next two sections.

4 Lightcone Structure Along the Azimuthal Direction

In the examination of the lightcone structure, we will see in what follows that the azimuthal closed ϕ -curves (note that since we are here interested in

lightcones $dt \neq 0$) are indeed spacelike for certain values of a and timelike for others. Doing dr = dz = 0 in Equ.(13), for the case of lightcones, ds = 0

$$-c^{2}dt^{2} + \left(\frac{8a\omega r}{c}\right)rcd\phi dt + \left(1 - 8a\right)r^{2}d\phi^{2} = 0$$
(18)

Solving Equ.(18) with respect to the variable $cdt/rd\phi$ we obtain.

$$\frac{cdt}{rd\phi} = -\frac{1}{2} \left(-\frac{8a\omega r}{c} \pm \sqrt{\left(\frac{8a\omega r}{c}\right)^2 + 4\left(1 - 8a\right)} \right)$$
(19)

For $0 \le a < \frac{1}{8}$, which also includes the case of a non-superconductive material, a = 0, we have

$$\frac{cdt}{rd\phi} \sim \pm 1 \tag{20}$$

the lightcone is just the usual Minkowskian one.

For a = 1/8, we have:

$$\frac{cdt}{rd\phi} = \begin{cases} 0\\ \omega r/c \end{cases}$$
(21)

the lightcone becomes very narrow, since in general $\omega r \ll c$, it also dips and touches the ϕ axis.

For a > 1/8; the ϕ -curve is enclosed within the lightcone:

$$\frac{cdt}{rd\phi} = \frac{8a\omega r}{2c} \pm \epsilon \tag{22}$$

Where $\epsilon > \frac{8a\omega r}{2c}$. The lightcone is still very narrow. To enlarge it we would need to have $r\omega \sim c$, which also requires a >> 1/8 and $v_{\phi} \simeq c$. The curve is always timelike, and hence the propertime flows monotonically and never becomes imaginary, i.e. the curve does not reverse and proceed into the past lightcone. This timelike curve returns to its original location in spacetime, it is a closed timelike curve, as we expected for the considered Cooper pair fraction.

In summary when the Cooper pair fraction is important enough for the rotating superconductor to generate CTCs, a > 1/8, then in the regions indicated in condition 1., 2., 3., and 4., the light cone will *open* in the azimuthal direction and will contain time-like directions for decreasing t pointing into the past, making travel into the past possible. These time-like circles γ are not geodesics. Since the total acceleration of the curve does not vanish, as we will see in the next section.

5 Acceleration Requirements for Gödel's time travel

If we consider the circle γ given by

$$x^{0} = ct = Cte, x^{1} = r = Cte, x^{3} = z = Cte$$
 (23)

In this case the interval Equ.(13) will reduce to:

$$ds^{2} = (1 - 8a)r^{2}d\phi^{2} = g_{\phi\phi}d\phi^{2}$$
(24)

with r belonging to the CTC's allowed domains defined in conditions 1., 2. 3., and 4..The first question to ask is wether there exist any closed time-like geodesics in the spacetime described by the metric inside the SCR's hole, Equ.(13). If so, it would be possible to execute time travel in a state of free fall. It turns out that the answer is no (at least for the stationary case, in which no angular acceleration is communicated to the SCR). We see that the tangent vector to the circle γ ,

$$\hat{e}_{\phi} = \frac{\partial}{\partial x^2} = \frac{\partial}{\partial \phi},\tag{25}$$

has the length squared

$$\left(\frac{\partial}{\partial\phi}\right)^2 = \hat{e}_{\phi}.\hat{e}_{\phi} = g_{\phi\phi} = g_{22} = (1-8a)r^2 \tag{26}$$

The quadri-acceleration vector A^{j} is given by

$$A^j = c^2 u^j_{:k} u^k \tag{27}$$

For the time-like unit vector

$$u^{j}u_{j} = 1, \qquad u^{j} = -\delta_{2}^{j}|g_{22}|^{-1/2}$$
 (28)

The semicolon in Equ.(27) indicates covariant differentiation. We then obtain that the acceleration A defined by

$$A^2 = g_{jk} A^j A^k \tag{29}$$

becomes [12]

$$A = \frac{1}{4}c^2 \frac{d\ln|g_{\phi\phi}|}{dr}.$$
(30)

Or doing Equ.(26) into Equ.(30)

$$A = \frac{2c^2}{r} \tag{31}$$

We see that A does not vanish (it becomes null as $r \to \infty$).

The total integrated acceleration over γ is

$$TA(\gamma) = \oint_{\gamma} A d\tau \tag{32}$$

Where A is the acceleration at any point of the curve, Equ.(31), and τ is the elapsed proper time along γ , $d\tau = ds/c = \left(\left|g_{\phi\phi}\right|\right)^{1/2} d\phi/c$. Notice that $TA(\gamma) = 0$ if and only if γ is a geodesic (closed gravitational field lines like the ones obtained through gravitomagnetic induction, $\nabla \times \vec{g} = -\dot{\vec{B}_g}$, are geodesics). Doing Equ.(24) and Equ.(31) into Equ.(32) we obtain:

$$TA(\gamma) = 4\pi c \sqrt{|1 - 8a|} \tag{33}$$

the total acceleration of the γ curve, is a measure of the total variation of velocity Δv needed to achieve a complete Gödel's round trip.

The total elapsed proper time $PT(\gamma)$ along γ will be:

$$PT(\gamma) = \oint_{\gamma} d\tau = \frac{2\pi r \sqrt{|1 - 8a|}}{c}$$
(34)

Therefore a CTC taken as the world line of an observer would enable her/him to travel into her/him own past if the acceleration were tolerable and the proper time for the round trip was less than its lifetime[13][14][15].

6 Conclusion

It seems that the rotation of a superconductive ring is capable to generate CTCs if the Cooper pair fraction, $a = \rho^*/\rho$, could be raised above the critical value of a = 1/8. This is a huge value with respect to the current Cooper pair fraction of presently known superconductors, $a \sim 10^{-7}$. Even if it becomes possible to design superconductors having this threshold of Cooper pairs fraction, we see from conditions 1., and 2., 3., and 4., that the possible values of Cooper pairs fraction a > 1/8, and the SCR's angular velocities ω are severely constrained by the fact that the metric must remain Lorentzian. CTCs will only be present inside the SCR's hole, with radius $R << r_{max}$, for values of (a, ω) defined by conditions 2., and 4. In other words conditions 1.,

and 3, form non-Lorentzian barriers which constrain the operation of the SCR in terms of allowed Cooper pair fraction and angular velocities. Outside this allowed regime of operation, spacetime would become unstable, and would decay to the Minkowskian metric in a very short time [7]. This would also mean that the gravitomagnetic London moment, Equ.(4), would be null in the non-Lorentzian regions.

The utilization of these CTC's to travel in time as Gödel first idealize seems to be unpracticable, since the total accelerations needed to perform the time travel, Equ.(31), is not technically achievable, and the total elapsed time for a round trip is negligibly small, Equ.(34).

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