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Abstract

This paper describes two tests which are able to distinguish deterministic forms of seasonality from non-stationary seasonal fluctuations. The first one tests for time variations in the deviations of seasonal dummies from an overall mean. The second for time variations in the seasonal dummies at each seasonal frequency. The asymptotic distribution of the tests is derived under weak assumptions which allow for a wide variety of weakly dependent non-explosive processes. The tests are applied to three data sets with different seasonal characteristics.

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1 Introduction

There has been a renewed interest in macroeconomics for the study of seasonal fluctuations in economic activity. Traditionally, seasonal fluctuations have been considered a nuisance which obscure the more important components of the series (presumably the growth and cyclical components, see e.g. Burns and Mitchell (1946)) and seasonal adjustment procedures have been devised and implemented to eliminate them (see e.g Shiskin, Young and Musgrave (1965)).

With the work of Hansen and Sargent (1990), Ghysels (1988), Barsky and Miron (1989) seasonal fluctuations have come back to the mainstream of macroeconomic research and recent work by Braun and Evans (1990), Chattarjee and Ravikumar (1990) has started to document their properties in relation to business cycles and to the main body of neoclassical growth theory.

In analyzing the economic properties of seasonal fluctuations the existing literature has proceed in two ways. One branch has assumed that the most important component of these fluctuations is deterministic or periodic with unchanged periodicity (see e.g. Barsky and Miron (1989) or Miron (1990)) and has derived implications based on this unverified assumption. Another has proceeded under the assumption of seasonal unit roots and has tested this assumption against the stationary alternative along the lines of Dickey, Hasza and Fuller (1984), Hylleberg, Granger, Engle and Yoo (1990) or Osborn (1990).

We find both approaches unsatisfactory for different reasons. First, although plots of the data indicate the presence of stable seasonal fluctuations in many macroeconomic variables (Christmas has been the major retail season for many years), their intensity has changed over time. Therefore the assumption of constant deterministic seasonal patterns is problematic and may induce serious specification biases. It appears unwise to proceed under the assumption of deterministic seasonality without testing this assumption. On the other hand, seasonal unit roots are hard to justify because in the very long run they imply that summer becomes winter and viceversa, and apart from few cases (see the energy consumption series examined by Granger, Engle and Hallman (1989), the Japanese consumption and incomes series examined by Engle, Granger, Hylleberg and Lee (1991) or the industrial production series examined by Canova (1991) and some of the GDP series analyzed by Hylleberg, Jorgensen and Sorensen (1991)) changes in the features and in the location of seasonal peaks and troughs are rare events for aggregate macroeconomic variables. Unit root tests also have low power in small samples and the failure to reject a seasonal unit root does not imply that the unit root approach is correct.

We believe that there is a larger economic scope in testing deterministic vs. slowly changing seasonal patterns where changes occur primarily in the intensity of the fluctuations. However, the stability of seasonal fluctuations is an issue that has not been addressed so far in the literature. To the best of our knowledge only Hansen and Sargent (1990) examine the closely related question of whether deterministic periodic models represent real data better than stochastically driven seasonal processes.

The task of this paper is to propose tests which are able to distinguish constant deterministic forms of seasonality from non-stationary seasonal fluctuations. The test builds on the work of Nyblom (1989) and Hansen (1990a) on the structural stability of regression coefficients and is related to the work of Kwiatkowski and Schmidt (1990) which tests stationarity vs. nonstationarity of a series and to the work of Saikkonen and Luukkonen (1989) and Tanaka (1990) who examine the null hypothesis of a moving average unit root in a time series. It is also complementary to the work of Ghysels (1991) which shows the existence of dependencies between stages of the business cycle and seasonal fluctuations.

Two test statistics are derived. The first tests the null hypothesis of constant deterministic seasonals (as deviations from an overall mean) against the alternative of seasonal dummies which shift over time as a martingale. The second tests the null hypothesis of constant deterministic seasonals against the alternative of seasonal dummies which shift as a martingale at one particular seasonal frequency. These alternatives are fairly general, allowing the test to be powerful against several forms of non-stationary seasonality, including seasonal unit roots as well as simple structural breaks. Our statistic is robust to the presence of heteroskedasticity and serial correlation in the residuals of the regression. The asymptotic distribution of the test is derived under mild assumptions which allow for a wide variety of weakly dependent non-explosive processes.

We apply the tests to three different data sets. The first is a now standard one originally examined by Barsky and Miron (1989). We are interested in establishing if their maintained hypothesis that quarterly seasonal fluctuations in US macro variables are well approximated by deterministic patterns is appropriate or not. The second data set used is the set of quarterly industrial production indices for eight industrialized countries used in Canova (1991). The third is a data set on stock returns on value weighted indices for seven industrialized countries. This last data set deserves special attention because "January effects" and other abnormal periodic patterns in stock returns have been repeatedly documented and known for a long time (see Thaler (1987) for a survey of these anomalies). It is therefore of interest to examine whether the knowledge of these patterns has changed their properties, or, in other words, if information about the existence of periodic patterns has led to structural changes due to profit taking activities.

The results indicate that for 16 of the 25 series examined by Barsky and Miron the assumption of unchanged seasonality is problematic. We also show that is some cases the economic significance of these changes is substantial. Similarly the seasonal patterns of the European industrial production indices have important stochastic seasonal components. On the other hand, we find that the seasonal pattern of stock returns has substantially changed only in Japan and in the UK.

The rest of the paper is organized as follows: the next section describes the model and its relation with the existing literature. Section 3 presents the test statistics for the null hypothesis of constant deterministic seasonals as deviations from an overall mean. Section 4 derives the test statistics for deviations from constant deterministic seasonals at each seasonal frequency. Applications to economic data appear in section 5. Conclusions are summarized in section 6.

2 The Model

Our is a linear time series model with seasonality. Seasonality will be represented by intercepts which are season-dependent, while we assume that the regression slope parameters are constant across seasons. An easy way to represent such a model is to write it as a multivariate regression. Suppose there are $s > 1$ "seasons" in a year (in a model with quarterly data s=4). Define y_t to be the $s \times 1$

vector containing the dependent variable for each season in year t. Similarly define x_t to be the $k \times s$ matrix whose columns are the regressors for each season in year t. This gives the multivariate regression model

$$y_t = \alpha + x_t'\beta + e_t \qquad t = 1, 2, \ldots, n \tag{1}$$

The error e_t is an $s \times 1$ vector, representing the regression errors for each season. The assumption that seasonality is incorporated only through the intercept is represented in (1) by the $s \times 1$ vector α whose elements represent the intercept in each season, while the $k \times 1$ vector β is common across seasons. The number of years is n so that there are ns total observations. The regressors x_t in (1) may include lagged dependent variables, subject to the qualifications given in section 3.4. Model (1) can be easily extended to cover the case of multivariate time series regressions by taking y_t to be a $ps \times 1$ vector. This extension is straightforward and will not be pursued here as it unnecessarily complicates the notation .

Equation (1) is estimated by ordinary least squares (OLS). In many applications, there are no independent variables (only the intercepts α) so OLS on (1) is equivalent to taking the average of the dependent variable by season.

We want to test model (1) to discover if the seasonal intercepts α have changed over time. It may seem reasonable at a first glance to test constant seasonals against time-varying seasonals of a stationary form, such as an AR(1) process, in the spirit of Watson and Engle (1985). Notice, however, that stochastic fluctuations in α are indistinguishable from stochastic fluctuations in the error e_t. Thus testing for stochastic variations in α of an AR(1) form is equivalent to testing for serial correlation in the regression error at the seasonal frequency. Tests of this form are well understood and do not need further elaboration in this paper.

Instead, we can test in the direction of long run changes in the seasonal patterns. As mentioned above, stochastic fluctuations in the error are equivalent to stochastic fluctuations in the intercept. Therefore we can construct our test focusing equivalently either on the seasonal intercepts or on the regression error.

The statistical theory for testing constant coefficients against parameter instability is well developed, so we will discuss the test in that framework. To test the assumption of constant seasonality, we decompose the intercept α into an overall mean and deviations from this mean. We can write this as:

$$\alpha = P\mu + D\gamma \tag{2}$$

where P is an $s \times 1$ vector of ones, $\mu = \frac{P'\alpha}{s}$ is the overall mean, γ is the $(s-1) \times 1$ vector of deviations from μ for the first $s-1$ seasons, and D is a $s \times (s-1)$ matrix:

$$D = \begin{pmatrix} I_{s-1} \\ -P'_{s-1} \end{pmatrix}$$

Note that although the left and the right side of (2) are equivalent parametrizations of the model, the decomposition of the right hand side is useful for γ captures all of the seasonal fluctuations and it is distinct from μ which represents a level effect.

Since we are interested in testing for the presence of nonconstant seasonality, we will maintain the assumption that the overall mean μ is constant and test for nonstationarity in the seasonal parameters γ. We can represent the null and the alternative hypotheses by writing the coefficients process as:

$$\begin{aligned} \gamma_t &= \gamma + \tau\,\xi_t \\ \xi_t &= \xi_{t-1} + \epsilon_t \end{aligned} \tag{3}$$

$$H_0 : \tau = 0 \qquad\qquad H_1 : \tau > 0$$

where the error ϵ_t is a martingale difference sequence. Under the null hypothesis, deviations of the seasonal intercepts from an overall mean are constant at the value γ. For $\tau > 0$, deviations of the seasonal intercepts from an overall mean will change in the long-run, although the changes may be either slow and gradual, or swift (a structural break). Since for some variables of interest the changes may be modest in scope, we desire a test which is powerful especially for small values of τ. In section 3.6 we provide the locally most powerful test of H_0 against H_1. The finding that γ is time varying does not necessarily conflict with the observation that "Christmas" does not migrate from December. A small value of τ does not induce major changes in the rough features of the seasonal fluctuations, but will allow the magnitude of the seasonal cycles to change over time. On the other hand, a large value of τ may induce changes in the magnitude, location and features of the seasonal cycles.

The specification for seasonal change given in (3) implies that the original intercepts α in (1) are not independent. In fact, when $\tau > 0$, at least two elements of α are I(1) processes, and they are cointegrated. Without this restriction, non constancy in the mean would be included in the alternative hypothesis, and our tests would not be able to distinguish unit roots at zero frequency from unit roots at seasonal frequencies.

As mentioned above, the null of constant deterministic seasonality against the alternative of time varying seasonals may be thought of in several different ways, each producing exactly the same statistics.

The first interpretation is that the null is parameter constancy (the seasonal intercepts γ are the parameters) and the alternative is random walk parameters. This test in a more general form has been discussed at length in the statistics literature. For recent treatments, see Nyblom (1989) and Hansen (1990a).

The second interpretation is that the null is that e_t is stationarity, and the alternative is that e_t has a seasonal unit root. Kwiatkowski and Schmidt (1990) have proposed a test of this form of stationarity vs. a unit root. Our model differs from Kwiatkowski and Schmidt by testing stationarity vs. a seasonal unit root.

The third interpretation is that the null is a unit moving average root. Denote by ℓ the standard lag operator and let the operator $\Delta_s = (1 + \ell + \ell^2 + \ldots + \ell^{s-1}) = \frac{(1-\ell^s)}{1-\ell}$. Applying this operator to equation (1) we find

$$\Delta_s y_t = \Delta_s x_t' \beta + \Delta_s e_t \tag{4}$$

Thus the assumption of no unit roots at seasonal frequencies corresponds to $s-1$ unit moving average seasonal roots in the differenced equation. This test is examined in Saikkonen and Luukkonen (1989) and Tanaka (1990). Our model differs from these treatments in that we are testing the assumption of a moving average seasonal roots.

3 Test Statistics

3.1 General Case

Nyblom (1989) has shown how to derive the statistic for the test of H_0 against H_1. The criterion function for estimation of (1)-(3) by OLS is

$$\sum_{t=1}^{n} q_t(\mu, \gamma, \beta)$$

where

$$q_t(\mu, \gamma, \beta) = -(y_t - P\mu - D\gamma - x_t'\beta)'(y_t - P\mu - D\gamma - x_t'\beta).$$

Take the derivative of q_t with respect to the parameters to be tested, evaluated at the OLS estimates:

$$\begin{aligned} \left.\frac{\partial q_t(\mu, \gamma, \beta)}{\partial \gamma}\right|_{\hat{\mu},\hat{\gamma},\hat{\beta}} &= D'(y_t - P\hat{\mu} - D\hat{\gamma} - x_t'\hat{\beta}) \\ &= D'(y_t - \hat{\alpha} - x_t'\hat{\beta}) \\ &= D'\hat{e}_t \end{aligned}$$

In a maximum likelihood context, these are known as the "scores". In the present context, note that these derivatives are simply the residuals from regression (1) multiplied by the constant matrix D. Construct cumulative scores:

$$\hat{S}_i = \sum_{t=1}^{i} \hat{e}_t, \tag{5}$$

and let $\hat{S}_i^* = D'\hat{S}_i$. The test statistic is then given by

$$\begin{aligned} L &= n^{-2} \sum_{i=1}^{n} \hat{S}_i^{*\prime} \hat{\Omega}^{-1*} \hat{S}_i^* \\ &= n^{-2} \sum_{i=1}^{n} \hat{S}_i' D(D'\hat{\Omega}D)^{-1} D'\hat{S}_i \end{aligned} \tag{6}$$

where $\hat{\Omega}$ is a consistent estimate of

$$\Omega = \lim_{n \to \infty} \frac{1}{n} E[S_n S_n'] \qquad S_n = \sum_{i=1}^{n} e_i \tag{7}$$

This test statistic is quite simple to calculate. The null of constant seasonality is rejected in favor of non-constant seasonality for *large* values of L. In section 3.6 we develop a large sample distribution theory for this test statistic. The only context-dependent issue is how to form $\hat{\Omega}$. We examine several special cases.

3.2 No serial correlation

If the error e_t in (1) is not serially correlated, then

$$\Omega = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} E(e_t e_t')$$

giving a natural estimate

$$\hat{\Omega} = \frac{1}{n} \sum_{t=1}^{n} \hat{e}_t \hat{e}_t'$$

Of course, if e_t is serial uncorrelated, it stands to reason that $E(e_t e_t')$ should be a diagonal matrix, since the elements of e_t are simply regression errors in different seasons in any year. Thus $\Omega = \text{diag}(\sigma_1^2, \ldots, \sigma_s^2)$, where s is the number of seasons per year, and the natural estimator of Ω is

$$\hat{\Omega} = \text{diag}(\hat{\sigma}_1^2, \ldots, \hat{\sigma}_s^2), \qquad \hat{\sigma}_j^2 = \frac{1}{n} \sum_{t=1}^{n} \hat{e}_{jt}^2$$

This construction allows the error to be heteroskedastic across seasons (that is, for the variance of the regression error to be season-dependent).

It may be reasonable to make the more stringent assumption that the regression are homoskedastic across seasons, in which case

$$\Omega = I_s \sigma^2, \qquad \sigma^2 = \lim_{n\to\infty} \frac{1}{ns} \sum_{t=1}^{n} E(e_t' e_t),$$

and the natural estimator is

$$\Omega = I_s \hat{\sigma}^2, \qquad \hat{\sigma}^2 = \frac{1}{ns} \sum_{t=1}^{n} (\hat{e}_t' \hat{e}_t).$$

In this case the test statistic simplifies to

$$L = \frac{1}{n^2 \hat{\sigma}^2} \sum_{i=1}^{n} \hat{S}_i' D(D'D)^{-1} D' \hat{S}_i.$$

3.3 Serial Correlation

In many cases, there is no reason to believe that the error e_t is not serially correlated. In applications which simply measure the role of seasonality in univariate time series, there may be no regressors in the model other the seasonal intercepts (no x_t in (1)), so all of the stochastic variation is absorbed in the error term. In these cases, the covariance matrix estimators of the previous section will yield biased estimates of the matrix Ω.

A general purpose estimator of Ω as defined in (7) can be obtained non-parametrically using a kernel. Take a positive semi-definite kernel window $w(.)$ such as the Bartlett or Parzen window (the Bartlett is $w(x) = 1 - |x|$) and bandwidth parameter m and construct

$$\hat{\Omega} = \sum_{k=-m}^{m} w(\frac{k}{m})\frac{1}{n}\sum_{t=1}^{n} \hat{e}_{t+k}\hat{e}_t'$$

where the second summation is over all t such that $1 \leq t+k \leq n$. This estimator is of the form recommended by Newey and West (1987). A demonstration of the consistency of $\hat{\Omega}$ under quite weak conditions is given in Hansen (1990b). $\hat{\Omega}$ is asymptotically robust to general heteroskedasticity and serial correlation. Its non-parametric form, however, induces a slow rate of convergence relative to correctly specified parametric estimators.

3.4 Lagged Dependent Variables

The distribution theory under the null hypothesis is not affected if the regressors x_t include lagged dependent variables. But if the lagged variables are able to capture one or more seasonal unit roots, the test may have no power. Essentially, what must be excluded are lags of the dependent variable which may capture seasonal patterns. This may be easier to see if we rewrite (1) in the case of no regressors as:

$$y_i = \alpha' d_i + e_i \tag{8}$$

where y_i is now a scalar, d_i is a $s \times 1$ seasonal dummy vector and $i = 1, \ldots, ns$. We could consider adding lags of y_t, i.e.

$$B(\ell)y_i = \alpha' d_i + e_i \tag{9}$$

where $B(\ell) = 1 - \beta\ell - \ldots - \beta_m\ell^m$. So long as $m = 1$, the autoregressive polynomial $B(\ell)$ will not be able to extract seasonal unit roots. But if $m \geq 2$, $B(\ell)$ may absorb at least one of the seasonal unit roots under the alternative hypothesis. Therefore the residuals $\hat{e}_t$ may not display significant stochastic trend, and the test will not be consistent (i.e. it will not reject with high probability under the alternative).

This discussion should not be interpreted as suggesting that all lagged dependent variables should be excluded from (1) or (3). Indeed, exclusion of lagged dependent variables means that the non-parametric estimator $\hat{\Omega}$ will have to capture all of the covariance structure of the process. These estimators frequently perform better if prewhitening is done. In the present context, this can be achieved by inclusion of one lag of the dependent variable. This will soak up much of the covariance structure of the process, but should have no adverse effect upon the power of the test.

3.5 Individual Significance Tests

The statistic L tests the joint hypothesis that none of the seasonal seasonal deviations from the overall mean have changed over the sample period. If the joint test rejects the null hypothesis, it may be of interest to know which season displays the non-constant behavior.

Individual stability tests are quite simple to construct. Denote by $\hat{S}_{ji}$ the j-the element of the vector $\hat{S}_i$ given in (5), and by $\hat{\Omega}_{ii}$ the i-th diagonal element of $\hat{\Omega}$. Then, the locally most powerful test for non-stationarity in the j-th seasonal is:

$$L_j = n^{-2} \sum_{i=1}^{n} \frac{\hat{S}_{ji}^2}{\hat{\Omega}_{jj}} \tag{10}$$

3.6 Distribution Theory

Denote by $[a]$ the greatest integer less or equal than a.. We will require the following assumption.

Assumption: For some $p > 2,\ q \geq 2,\ \delta > 0$

i. $\beta \xrightarrow{P} \beta_0$,

ii. $\sup_t E|e_t|^p < \infty$,

iii. $\sup_t E|x_t' x_t|^{\frac{q+\delta}{2}} < \infty$

iv. $\sup_{0 \leq r \leq 1} \frac{1}{\sqrt{n}} \sum_{t=1}^{[nr]} E(x_t - \bar{x}) = O(1)$, where $\bar{x} = \frac{1}{n} \sum_{t=1}^{n} x_t$

v. (x_t, e_t) is α-mixing with mixing coefficients α_m such that $\sum_{m=1}^{\infty} \alpha_m^{1-c} < \infty$, where $c = max(q, \frac{p}{2})$

vi. $\hat{\Omega} \xrightarrow{P} \Omega$

The conditions are quite weak, allowing for a wide of weakly dependent, non-explosive processes. Note that condition *iv* is implied if Ex_t is constant.

Denote by $B(r)$ a vector Brownian motion with covariance matrix Ω, $W(r)$ a vector Brownian motion with covariance I_m, $W^*(r)$ a m dimensional Brownian bridge, i.e. $W^*(r) = W(r) - rW(1)$ and by "$\Rightarrow$" weak convergence. Define the distribution:

$$L_m = \int_0^1 W^*(r)'W^*(r)dr$$

which is parametrized solely by m, the dimensionality of the vector Brownian Bridge W^*.

Theorem 1: Under the null hypothesis,

i. $\frac{1}{\sqrt{n}}\hat{S}_{[nr]} \Rightarrow B(r) - rB(1)$;

ii. $L \xrightarrow{D} L_{s-1}$

(A proof of the theorem appears in appendix A.)

Part i says that the cumulative sums of the residuals can be approximated in distribution by a vector Brownian bridge. Part ii gives the large sample distribution theory for the test statistic. The representation of the limit distribution is in terms of a vector Brownian bridge. The distribution is a multivariate generalization of the large sample distribution of the Von Mises goodness of fit statistic (see, for example Anderson and Darling (1952)). This distribution is non-standard, but depends only on the parameter s, the number of seasons. Critical values can be calculated by simulation or more direct means; see Nyblom (1989) or Hansen (1990a) for more discussion. We report critical values in Table 1. The first row of the table (seasons=1) gives the critical values for individual significance tests discussed in section 3.5. The other rows give critical values for the joint tests. For example, if quarterly data are used, the value of the statistic L should be compared against the critical values given in the third line.

Monte Carlo studies in Nyblom (1989) and Hansen (1990a) suggest that the asymptotic distribution is an excellent approximation in small samples. These papers also give Monte Carlo evidence on the power of the test against random walk and structural break alternatives. Tanaka (1990) discusses power using analytic methods.

4 Testing for Deviations at Seasonal Frequencies

4.1 General Approach

The null hypothesis of unchanged stationary seasonality implies two testable implications: that the seasonal dummies are constant over time; and that there are no unit roots at seasonal frequencies. The previous section developed tests of the null of constant seasonal dummies against the alternative of random walk dummies. While there are many attractive features of this approach (for example, it allows for examination of stability by season), it has the disadvantage of not distinguishing at which seasonal frequency the unit root appears. In some applications, it maybe also be interesting to know which seasonal frequency accounts for the nonconstant behavior of the dummy. In this section we present a test of the null of constant seasonality against the alternative of a unit root at one seasonal frequency. The spirit of the test is similar to Hylleberg, Engle, Granger and Yoo (1990). The major difference is that they take the null hypothesis of stationary stochastic seasonality.

We find that it is easier to present these tests by re-writing equation (1) in the scalar form

$$y_i = d_i'\alpha + x_i'\beta + e_i \quad i = 1, \ldots, T \tag{11}$$

where $T = ns$ is the total number of observations, d_i is an $s \times 1$ dummy variable indicating the season and where e is $\mathcal{N}(0, I\sigma_e^2)$.

We now present a general method for testing for the presence of non-stationarity. To derive the test statistic, assume that (11) holds and let $e = (e_1, \ldots, e_T)'$ be given by:

$$e = u + \tau\nu = u + \tau C\eta \tag{12}$$

where u is $\mathcal{N}(0, I\sigma_u^2)$, η is $\mathcal{N}(0, I)$, and C is a $T \times T$ constant matrix. Then e is $\mathcal{N}(0, I\sigma_u^2 + \tau^2 CC')$ The process $\{e_i\}$ is i.i.d. when $\tau = 0$ which we take to be the null hypothesis

$$H_o : \tau = 0 .$$

The GLS criterion function (in obvious notation) is

$$(y - G\alpha - X\beta)'(I\sigma_u^2 + \tau^2 CC')^{-1}(y - G\alpha - X\beta) . \tag{13}$$

where G is a matrix with dummy variables. The Lagrange multiplier statistic is found by taking the derivative of (13) with respect to τ^2, and evaluating the answer at $\tau^2 = 0$ and the OLS estimates. This is proportional to

$$\hat{e}'CC'\hat{e} = \sum_{i=1}^{T} S_i^2$$

where $C'\hat{e} = S = (S_1, ..., S_T)'$. The test statistics for deviations at frequency ω can be written as:

$$L_\omega = \frac{1}{T^2\sigma^2}\sum_{t=1}^{T} S_t^2 \tag{14}$$

In order to allow for general forms of weakly dependent serial correlation in the regression error e_i, we need an estimate of the "long-run" variance of the regression error $\{e_i\}$, *i.e.* an estimate of the form

$$\hat{\sigma}^2 = \sum_{k=-m}^{m} w(\frac{k}{m})\frac{1}{T}\sum_t \hat{e}_{i+k}\hat{e}_i \ .$$

4.2 Testing for Deviation from Constancy at Frequency π

The LM test statistic (14) is a generalization of the test statistic (6) and is completely determined by the transformation matrix C. Testing for deviation from constancy at a particular frequency means choosing C so that the process ν_i (and thus e_i when $\tau \neq 0$) has a unit root at that frequency. For the sake of simplicity we will present the tests for $s = 4$ (so that π and $\frac{\pi}{2}$ are the seasonal frequencies). In appendix B we discuss the selection of C for $s = 12$. We first explore a test for constant seasonality at frequency π. ν_i has a unit root at π if it has the representation

$$\nu_i = -\nu_{i-1} + \epsilon_i$$

with ϵ_i iid. In this case,

$$C = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ 1 & -1 & 1 & \dots & 0 \\ & & & \dots & \end{pmatrix} .$$

Set $q_i = (-1)^i \hat{e}_i$. The process S_t can be written as

$$S_t = (-1)^t \sum_{i=t}^{T} q_i = (-1)^t \sum_{i=1}^{t-1} q_i \equiv (-1)^t Q_{t-1}.$$

where $Q_t = \sum_1^t q_i$. The second equality holds since $\sum_{i=1}^{T} q_i = \sum_{i=1}^{T} (-1)^i \hat{e}_i = 0$ due to the presence of seasonal dummies in the regression (11). This allows us to form the LM test statistic for the presence of a unit root at frequency $\omega = \pi$:

$$L_\pi = \frac{1}{T^2 \hat{\sigma}^2} \sum_{t=1}^{T} Q_t^2$$

A derivation similar to that of Theorem 1 yields

Theorem 2: Under the null hypothesis, $L_\pi \xrightarrow{D} L_1$.

The test statistic for the presence of a unit root at frequency π has the same asymptotic distribution as the test for a unit root in a single dummy variable. Critical values can be found in the first row of table 1.

4.3 Testing for Deviation from Constancy at Frequency $\frac{\pi}{2}$

ν_i has a pair of (complex conjugate) unit roots at $\frac{\pi}{2}$ if it has the representation

$$\nu_i = -\nu_{i-2} + \epsilon_i$$

with ϵ_i iid. In this case,

$$C = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ -1 & 0 & 1 & \dots & 0 \\ 0 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ & & & \dots & \end{pmatrix} .$$

With a little algebra, we find that

$$S_t = \sum_{j=0}^{(T-t)/2} (-1)^j \hat{e}_{t+2j} = \sum_{j=0}^{t/2} (-1)^j \hat{e}_{t-2j}$$

$$= \sin(\frac{t\pi}{2}) \sum_{j=1}^{t} \sin(\frac{j\pi}{2}) e_j + \cos(\frac{t\pi}{2}) \sum_{j=1}^{t} \cos(\frac{j\pi}{2}) e_j .$$

And therefore

$$S_t^2 = \sin(\frac{t\pi}{2})^2 \left(\sum_{j=1}^{t} \sin(\frac{j\pi}{2}) \hat{e}_j \right)^2 + \cos(\frac{t\pi}{2})^2 \left(\sum_{j=1}^{t} \cos(\frac{j\pi}{2}) \hat{e}_j \right)^2 .$$

The appropriate statistic for the test of H_0 against the alternative of a unit root at $\omega = \frac{\pi}{2}$ is then

$$L_{\frac{\pi}{2}} = \frac{4}{T^2 \hat{\sigma}^2} \sum_{t=1}^{T} S_t^2 .$$

The reason for the factor of 4 in the numerator is for convenience, since then the asymptotic distribution is tabulated. To find this distribution, we first note that

$$\left(\frac{\sqrt{2}}{\sigma\sqrt{T}} \sum_{j=1}^{[Tr]} \sin(\frac{j\pi}{2}) e_j \ , \ \frac{\sqrt{2}}{\sigma\sqrt{T}} \sum_{j=1}^{[Tr]} \cos(\frac{j\pi}{2}) e_j \right) \Rightarrow (W_1(r), W_2(r))$$

where W_1 and W_2 are independent standard Brownian motions. It follows that:

Theorem 3. Under the null hypothesis, $L_{\frac{\pi}{2}} \xrightarrow{D} L_2$.

This is the same asymptotic distribution as that which results from testing for parameter instability for the two parameters case. We may therefore think of this as a "two degree of freedom" test, which coincides with our intuition that we are testing for the presence of a pair of conjugate unit roots. The appropriate critical values are given in the second row of Table 1.

Finally, by combining the results of these two subsections we see that ν_i has unit roots at both seasonal frequencies if it has a representation:

$$\nu_i = -\nu_{i-1} - \nu_{i-2} - \nu_{t-3} + \epsilon_i$$

with ϵ_i iid. In this case the L statistic for a joint test at frequency $\omega = \pi$ and $\frac{\pi}{2}$ converges in distribution to L_3. The joint test for deviations from constancy at all seasonal frequencies is the sum of the two tests statistics L_π and $L_{\frac{\pi}{2}}$ and has the same asymptotic distribution as our test for constancy of deviations from the overall mean described in section 3. Intuitively, this occur because, by taking deviations from an overall mean we essentially knock out the root at $\omega = 0$, leaving only $s-1$ possible roots in the process. Note that although the asymptotic distribution of the two tests is the same the value of the two statistics may be different.

5 Some applications

We apply the test statistics described in the previous sections to three different data sets. The first one has been originally examined by Barsky and Miron (1989) in their study of the relationship between seasonal and cyclical fluctuations. The data set includes 25 variables which covers practically all the major nonseasonally adjusted US macroeconomic variables (total fixed investment, fixed residential investments, fix nonresidential investments, fixed non residential structures, fixed non residential producer durables, total consumption, consumption of durables, consumption of nondurables, consumption of services, federal government expenditure, import and exports, final business sales, changes in business inventories, CPI, 1 month T-bill rates, M1, Unemployment, labor force, employment, monetary base, money multiplier, hours and wage rates). The original sources are described in the appendix of Barsky and Miron. The sample covers data from 1946,1 to 1985,4 except for M1 (starting date 1947,1), for unemployment and labor force (starting date 1948,1), employment (starting date 1951,1), the monetary base and the money multiplier (starting date 1959,1) and hours and wage (starting date 1964,1).

The second data set used is the vector of quarterly industrial production indices for eight European countries (UK, Germany, France, Italy, Spain, Austria, Belgium and Netherland) for the sample 1960,3 1989,2. Canova (1991) describes the original sources of the data.

The third data we examine is a set of monthly stock returns on value weighted indices for seven industrialized countries (US, Japan, Germany, France, UK, Italy, and Canada). This data set is obtained from the Citibase Tape and covers the period 1950,3-1989,9.

In constructing an estimate of the covariance matrix $\hat{\Omega}$, we use the Newey and West procedure using Bartlett windows with eight lags (i.e. two years of autocovariances). For the first two data sets we run the tests on the log differences because previous analyses have been undertaken using this transformation. In addition, one lag of the dependent variable is included among the regressors. The results of testing the null hypothesis of no structural change in the deterministic dummies are reported in tables 2, 3 and 4. The tables report significant dummies, the value of the L statistic for testing the stability of each dummy coefficient separately and of the L statistics for testing the stability of the vector of coefficients of deviations from an overall mean. In tables 2 and 3 we also report the values of the L_{π} and $L_{\frac{\pi}{2}}$ statistics. In table 4 we report the value of the $L_{\frac{\pi}{6}}$ statistics only since we are primarily interested in annual cycles. The L_{ω} statistics for monthly data are computed as described in appendix B. For four of the variables belonging to the first data set which display structural changes in their seasonal patterns (fixed investment, consumption, government expenditure and unemployment rate), we report in figure 1 plots recursive least square estimates of the dummy coefficients in the spirit of Franses (1990). Under the assumption of unchanged seasonal patterns the plot should depict four almost parallel lines. If lines intersect (e.g. spring becomes summer) unit root behavior at seasonal frequencies is likely to occur. If changes in seasonal patterns changed primarily in the intensity of the fluctuations, the lines should tend to converge or diverge.

The results indicate that for the first data set 24 out of the 25 variables display significant seasonal patterns (the one month T-bill rate is the only exception) and that for 16 of these the seasonal pattern has changed over time according to the joint L test. The nine variables which possess seasonal patterns which are well approximated by unchanged deterministic processes are fixed non residential investments, fixed nonresidential producer durables, consumption of durables, consumption of services, imports and exports, labor force and the wage rate. We find that changes occur in all of the four seasons but the most significant changes appear in the first quarter. We also find that for 12 variables the null of constant seasonality is rejected at both frequencies and that at the biannual frequency the test rejects the null in 19 cases. These results indicate that the comparison of deterministic seasonal and stochastic cyclical patterns as done in Barsky and Miron (1989) may not be appropriate since there are important time variations neglected in the analysis. They also agree with results recently obtained by Ghysels (1991) which shows that the seasonal pattern displayed by this set of macroeconomic variables tend to change with business cycle conditions with the major change occurring in the third quarter.

It is encouraging to observe that the individual dummy stability tests give similar conclusions as "eyeball" tests on the recursive estimates displayed in figure 1. The first quarter fixed investment dummy trends toward zero and the test rejects its constancy. The test also rejects the constancy of all government expenditure dummies, except for the third quarter dummy, a result which conforms with the plot of the recursive estimate. For the consumption series, the first and the fourth quarter dummies are the largest in absolute value, trend toward zero over time, and the test rejects their stability at the 1% level. A similar picture arises for the unemployment rate, except that it is the first and the second quarter dummies which are "large" in absolute value. In general, for all four variables considered in figure 1 there is a tendency for the overall mean to be constant, for seasonals to become milder and for the intensity of the fluctuations to be reduced with some dummy coefficient turning insignificant in the last two decades. In addition, for the consumption and employment series, the coefficients of the dummies of two quarters change sign throughout the sample even though their value is always close to zero. Despite these large changes, none of the variable examined display a significant change in the location of seasonal peaks and troughs over time. Since these pattern are very typical of those we found among all the variables in the sample, one conclusion that emerges is that the intensity of seasonal fluctuations has substantially subsided in the past two decades, but no seasonal inversion (summer becoming winter) has really occurred.

All variables in the second data set but the UK Industrial Production index clearly display seasonal patterns which are of a stochastic nature and for which a seasonal unit root may not be a bad approximation (i.e. the value of τ in (2) is large). The first and the fourth quarter dummies are those who most significantly change throughout the sample. The unit root seem to appear primarily at the biannual frequency. The estimated coefficients of the dummies over three different decades and the recursive least square plots (not presented for reasons of space) indicate changes in intensity, pattern and location of seasonal peaks and troughs over time.

Finally, all stock returns display some form of seasonality. The most significant seasonal dummies are for January returns (except for Germany and UK). July and August returns have significant coefficients in four European countries. When we test for the structural stability of individual coefficients we find that significant (at the 5% level) time variations have emerged only for returns on a value weighted index in Japan, UK and Italy. Jointly only the coefficients of the dummies in Japan and UK have significantly changed over the sample. In these two variables the rejection of the null hypothesis obtains at the annual frequency. Therefore, knowledge of the presence of predictable returns in four of the seven countries did not imply changes in these patterns, possibly indicating an inefficient propagation of information.

6 Conclusions

This paper proposes two tests to examine the structural stability of seasonal patterns over time. The tests are built on the null hypothesis of deterministic seasonality and exploits the properties of the cumulative scores in deriving the statistics of interest. We derive the asymptotic distribution of the statistics under general conditions which accommodate weakly dependent non-explosive processes.

We apply the test to three different data sets to examine whether deterministic dummies effectively capture the essence of existing seasonal variations. We find that in most cases of interest the quality of the approximation is poor and that significant time variations are present in the seasonal patterns of many time series. The presence of seasonal time variations partially invalidates some of the conclusions obtained by Barsky and Miron (1989), confirms recent findings of Ghysels (1991) and suggests the need for a more thorough and comprehensive examination of the statistical properties of macroeconomic variables.

Extension of the testing procedure presented in the paper to a vector of time series is straightforward. In that framework one can examine, e.g., whether at least one of the seasonal intercepts of the system has changed. The test can be carried out using the same asymptotic distribution developed in section 3. The only modification concerns the covariance matrix of the scores which is now of the form $\tilde{\Omega} = lim_{n \to \infty} \frac{1}{n} E[S_n \otimes S_n]$ where $\otimes$ is a Kroneker delta and the dimension of $B(r)$ is $m(s-1)$ where m is the number of time series included.

Appendix A

Proof of Theorem 1: We have

$$\hat{e}_t = e_t - (\hat{\alpha} - \alpha_0) - x_t'(\hat{\beta} - \beta_0) \tag{A.1}$$

By the first order conditions, $\frac{1}{n}\sum_{t=1}^{n} \hat{e}_t = 0$, so both the left and the right sides of (A.1) sum to zero. Denoting $\bar{e} = \frac{1}{n}\sum_{t=1}^{n} e_t$, this gives

$$0 = \bar{e} - (\hat{\alpha} - \alpha_0) - x_t'(\hat{\beta} - \beta_0). \tag{A.2}$$

Subtracting (A.2) from (A.1) we have

$$\hat{e}_t = (e_t - \bar{e}) - (x_t - \bar{x})'(\hat{\beta} - \beta_0)$$

This gives the following convenient expression for the cumulative sum of residuals:

$$\hat{S}_i = \sum_{t=1}^{i} \hat{e}_t = \sum_{t=1}^{i}(e_t - \bar{e}) - \sum_{t=1}^{i}(x_t - \bar{x})'(\hat{\beta} - \beta_0) \tag{A.3}$$

By the triangle inequality,

$$\sup_{0\le r\le 1} |\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}(x_t - \bar{x})| \tag{A.4}$$

$$\le \sup_{0\le r\le 1} |\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}(x_t - Ex_t)| + \sup_{0\le r\le 1} |\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}E(x_t - \bar{x})| + \sup_{0\le r\le 1} |\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}(E\bar{x} - \bar{x})|$$

$$\le 2 \sup_{0\le r\le 1} |\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}(x_t - Ex_t)| + \sup_{0\le r\le 1} |\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}E(x_t - \bar{x})|$$

$$\le 2 \sup_{0\le r\le 1} |\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}(x_t - Ex_t)| + O(1),$$

the final inequality being assumption (iv). This expression is bounded by the maximal inequality for α-mixing processes (Hansen, 1991, Corollary 3). Thus (A.4) is stochastically bounded and

$$\sup_r [\frac{1}{\sqrt{n}}\hat{S}_{[nr]} - \frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}(e_t - \bar{e})] = [\sup_{0\le r\le 1} \frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}(x_t - \bar{x})](\hat{\beta} - \beta_0) = o_p(1) \tag{A.5}$$

by assumption i. Finally, using the invariance principle for α-mixing processes (Herrndorf, 1984),

$$\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]}(e_t - \bar{e}) = \frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]} e_t - \frac{[nr]}{n}\frac{1}{\sqrt{n}}\sum_{t=1}^{[nr]} e_t$$

$$\Rightarrow B(r) - rB(1). \qquad (A.6)$$

(A.3) and (A.6) together give part i of the theorem. Part ii follows directly from the definition of L, assumption vi and the continuous mapping theorem.

Appendix B

In this appendix we report two types of results. First, we show how to construct a test for constancy against the presence of a single unit root at $\frac{\pi}{2}$ in the quarterly case. Second, we outline the procedure for deriving the form of C and S_t for testing the constancy of the dummies at each seasonal frequency in the monthly case.

ν_t has a single unit root at $\frac{\pi}{2}$ if it has either one of the two representations

$$\nu_t = -i\nu_{t-1} + \epsilon_t \qquad (B.1)$$

$$\nu_t = i\nu_{t-1} + \epsilon_t \qquad (B.2)$$

with ϵ_t iid. In this case, C has one of the two forms:

$$C = \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 \\ i & 1 & 0 & \ldots & 0 \\ -1 & i & 1 & \ldots & 0 \\ -i & 1 & i & \ldots & 0 \\ & & & \ldots & \end{pmatrix}.$$

$$C = \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 \\ -i & 1 & 0 & \ldots & 0 \\ -1 & -i & 1 & \ldots & 0 \\ i & 1 & -i & \ldots & 0 \\ & & & \ldots & \end{pmatrix}.$$

In the first case we set $q_i = (-i)^t\ \hat{e}_t$ in which case the process S_t can be written as

$$S_t = (-i)^t[\hat{e}_t + i\hat{e}_{t-1} - \hat{e}_{t-2} - i\hat{e}_{t-3} + \hat{e}_{t-4} + \ldots] = \sum_{s=0}^{t} (-i)^{t-s} e_{t-s}.$$

In the second case we set $q_i = (i)^t\ \hat{e}_t$ in which case the process S_t can be written as

$$S_t = (i)^t[\hat{e}_t - i\hat{e}_{t-1} - \hat{e}_{t-2} + i\hat{e}_{t-3} + \hat{e}_{t-4} + \ldots] = \sum_{s=0}^{t} (i)^{t-s} e_{t-s}.$$

Each of the two resulting statistics $L_{\omega=\frac{\pi}{2}}$ will then converge to L_1 in distribution.

For monthly data there are 12 roots on the unit circle. They are located at $\frac{j}{6}\pi$ where $j = 0, \ldots, 11$. and are given by $\pm 1, \frac{1}{2} \pm i\sqrt{3}2, -\frac{1}{2} \pm i\sqrt{3}2, \frac{i}{2} \pm i\sqrt{3}2, -\frac{i}{2} \pm i\sqrt{3}2$. Because of the "aliasing " problem roots at $(\frac{1}{6}\pi, \frac{11}{6}\pi)$, $(\frac{1}{3}\pi, \frac{5}{3}\pi)$, $(\frac{1}{2}\pi, \frac{3}{2}\pi)$, $(\frac{2}{3}\pi, \frac{4}{3}\pi)$ $(\frac{5}{6}\pi, \frac{7}{6}\pi)$, are associated with cycles corresponding to

12, 6, 4, 3, 2.4 months respectively. Also, because one root is at $\omega = 0$ there are only 6 distinguishable seasonal frequencies.

ν_t has a pair of (complex conjugate) unit roots at $\frac{\pi}{6}$ (annual frequency) if it has the representation

$$\nu_t = i\nu_{t-1} + \nu_{t-2} + \epsilon_t \tag{B.3}$$

with ϵ_t iid. ξ_t has a pair of (complex conjugate) unit roots at $\frac{\pi}{3}$ (semiannual frequency)if it has the representation

$$\nu_t = \nu_{t-1} - \nu_{t-2} + \epsilon_t. \tag{B.4}$$

ν_t has a pair of (complex conjugate) unit roots at $\frac{\pi}{2}$ (triannual frequency) if it has the representation

$$\nu_t = -\nu_{t-2} + \epsilon_t. \tag{B.5}$$

ν_t has a pair of (complex conjugate) unit roots at $\frac{2\pi}{3}$ (quarterly frequency) if it has the representation

$$\nu_t = -\nu_{t-1} - \nu_{t-2} + \epsilon_t. \tag{B.6}$$

ν_t has a pair of (complex conjugate) unit root at $\frac{5\pi}{6}$ if it has the representation

$$\nu_t = -i\nu_{t-1} + \nu_{t-2} + \epsilon_t \tag{B.7}$$

ν_t has a unit root at π (bimonthly frequency) if it has the representation

$$\nu_t = -\nu_{t-1} + \epsilon_t \tag{B.8}$$

By repeated substitutions it is immediate to derive the form of C and S_t in each of the six cases.

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Table 1: Asymptotic Critical Values for L

	Significance Level					
Seasons	1%	2.5%	5%	7.5%	10%	20%
1	0.748	0.593	0.470	0.398	0.353	0.243
2	1.070	0.898	0.749	0.670	0.610	0.469
3	1.350	1.160	1.010	0.913	0.846	0.679
4	1.600	1.390	1.240	1.140	1.070	0.883
6	2.120	1.890	1.680	1.580	1.490	1.280
11	3.270	2.990	2.750	2.600	2.490	2.220
12	3.510	3.180	2.960	2.810	2.690	2.410

Source: Hansen (1990a), Table 1.

Table 2: **Test for Structural Stability in the Seasonal Pattern of US macroeconomic variables. Sample 46,1-85,4**

Series		Quarter 1		Quarter 2		Quarter 3		Quarter 4	Joint	π	$\frac{\pi}{2}$
Fixed Investments	x	0.65(**)	x	0.25		0.70(**)		0.40	1.72(*)	0.20	3.96(*)
Fixed Residential Investments	x	0.53(**)	x	0.10		0.80(*)	x	0.06	1.29(**)	0.12	2.42(*)
Fixed Non Residential Investments	x	0.27	x	0.22	x	0.28	x	0.27	0.86	0.33	1.31(*)
Fixed Non Residential Structures	x	0.35	x	0.29	x	0.40	x	0.53(**)	1.32(**)	1.61(*)	0.35
Fixed Non Res. Producer Durables		0.29	x	0.33	x	0.23	x	0.23	0.91	0.57(**)	1.18(*)
Consumption	x	1.31(*)	x	0.66(**)	x	0.47(**)	x	1.11(*)	3.33(*)	1.65(*)	2.20 (*)
Consumption Durables	x	0.31	x	0.11		0.28	x	0.33	0.93	0.63(**)	0.72
Consumption Nondurables	x	1.01(*)	x	0.98(*)	x	0.80(*)	x	1.05(*)	3.78(*)	1.52(*)	3.50(*)
Consumption Services	x	0.96(*)	x	0.46	x	0.74(*)	x	0.87(*)	0.46	0.93(*)	0.82(**)
GNP	x	0.88(*)	x	0.74(*)	x	0.37	x	0.94(*)	2.26(*)	0.64(**)	3.94(*)
Government Expenditure	x	0.88(*)	x	0.74(*)	x	0.06	x	0.77(*)	2.35(*)	1.86(*)	0.80(**)
Imports		0.15	x	0.39	x	0.12		0.28	0.67	0.40	0.84(**)
Exports		0.17	x	0.21	x	0.46	x	0.19	0.68	0.27	0.26
Final Sales	x	1.24(*)	x	0.19	x	0.15	x	1.02(*)	2.31(*)	1.90(*)	2.31(*)
CPI		0.57(**)	x	0.59(**)	x	0.29		0.40	1.69(*)	0.22	0.72
T-bill rate		0.38		0.22		0.11		0.07	0.40	0.44	0.72
Business Inventories	x	0.27		0.93(*)		0.32		0.49(**)	1.95(*)	0.62(**)	2.43(*)
M1	x	0.06		1.39(*)	x	0.15	x	0.17	1.71(*)	1.51(*)	2.27(*)
Unemployment Rate	x	1.02(*)	x	0.95(*)		0.39	x	0.14	2.16(*)	1.44(*)	4.23(*)
Labor Force	x	1.00(*)	x	0.42	x	0.28	x	0.40	0.58	1.18(*)	1.29(*)
Employment	x	0.40	x	0.23	x	0.09		0.54(**)	1.11(**)	0.45	1.43(*)
Monetary Base	x	0.44	x	0.29	x	0.28	x	0.20	0.98	0.16	0.97(**)
Monetary Multiplier		0.42		0.83(*)		0.26	x	0.39	1.49(*)	0.33	3.32(*)
Hours	x	0.43	x	0.39	x	0.20	x	0.31	1.04(**)	0.05	1.19(*)
Wage	x	0.07	x	0.55(**)	x	0.24	x	0.59(**)	0.78	1.29(*)	0.40

Note: A "x" indicates a dummy which is signifiant at the 5% level. The numbers reported in the first four columns are the values of the L statistics for each quarter. The fift column reports the value of the L statistics for the joint test that three deviations from the overall mean are constant. The last two columns report the value of the L_ω statistics. A "*" indicates significance at 1%, a "**" significance at 5%.

Table 3: Test for Structural Stability in the Seasonal Pattern of Quarterly Industrial Production Indices, Sample 60,1-89,2

Series	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Joint	π	$\frac{\pi}{2}$
France	0.28	x 0.62(**)	x 0.14	x 0.55(**)	1.56(*)	0.79(*)	0.56
Germany	x 0.31	x 0.18	0.37	x 0.97(*)	1.21(*)	0.68(**)	1.92(*)
UK	x 0.69(**)	0.16	x 0.19	x 0.97(*)	0.67	1.32(*)	1.58(*)
Italy	x 1.09(*)	x 0.89(*)	x 0.37	x 0.41	1.96(*)	1.52(*)	5.66(*)
Austria	x 0.69(**)	x 0.27	x 0.20	x 0.42	1.45(*)	0.11	1.26(*)
Belgium	0.62(**)	x 0.25	x 1.12(*)	x 0.61(**)	2.06(*)	0.55(**)	4.88(*)
Netherland	1.08(*)	x 0.84(*)	0.49(**)	x 0.89(*)	2.77(*)	0.38	12.65(*)
Spain	0.47(**)	x 0.27	x 0.36	x 0.84(*)	1.93(*)	0.77(*)	0.84(*)

Note: A "x" indicates a dummy which is signifiant at the 5% level. The numbers reported in the first four columns are the values of the L statistics for each quarter. The fifth column reports the value of the L statistics for the joint test that three deviations from the overall mean are constant. The last two columns report the value of the L_ω statistics. A "*" indicates significance at 1%, a "**" significance at 5%.

Table 4: Test for Structural Stability in the Seasonal Pattern of Monthly Stock Returns. Sample 50,1-89,9

Series	Significant Dummies	January	February	March	April	May	June	July	August	September	October	November	December	Joint	$\frac{\pi}{6}$
US	1	0.07	0.06	0.14	0.32	0.33	0.11	0.06	0.05	0.30	0.11	0.09	0.17	1.74	0.56
Japan	1,2,11	0.57(**)	0.39	0.14	0.11	0.08	0.30	0.26	0.12	0.18	0.52(**)	0.33	0.19	3.02(**)	1.78(*)
Germany	7	0.34	0.12	0.18	0.16	0.25	0.15	0.22	0.12	0.28	0.12	0.22	0.14	2.17	0.65
France	1,4,7	0.05	0.40	0.05	0.28	0.07	0.11	0.17	0.09	0.06	0.15	0.07	0.12	1.43	0.70
UK	7	0.44	0.33	0.28	0.21	0.24	0.10	0.08	0.41	0.11	0.12	0.42	0.41	2.88(**)	1.44(*)
Italy	1, 8	0.61(**)	0.10	0.13	0.14	0.11	0.12	0.05	0.26	0.07	0.16	0.15	0.22	1.84	0.70
Canada	1	0.08	0.15	0.19	0.14	0.16	0.11	0.05	0.26	0.06	0.11	0.10	0.08	1.25	0.31

Note: The numbers reported are the values of the L statistic. A "*" indicates significance at 1% level, a "**" significance at 5 % level.

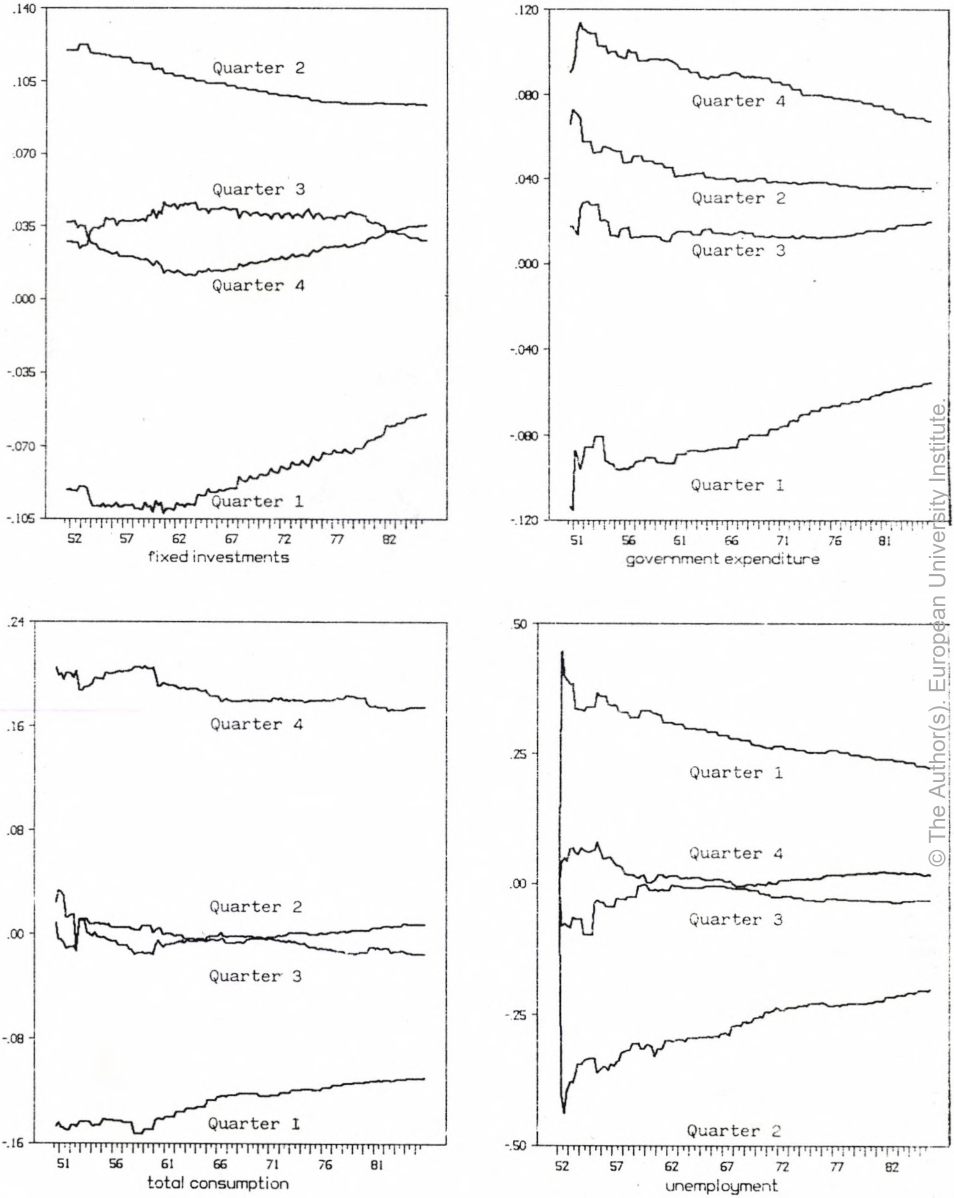

FIGURE 1: PLOT OF RECURSIVE ESTIMATES

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