Summary.—With the exception of the Teffé strain which will be discussed in another publication, no trace of sexual isolation is observed between strains of *D. willistoni* from different parts of Brazil and from Guatemala. When Brazilian males are offered a choice of females of their own and of other Brazilian strains, inseminations occur at random. If a mixture of Brazilian and Guatemalan females are confined either with Brazilian or with Guatemalan males, a greater proportion of Brazilian than of Guatemalan females are inseminated. The mating of Brazilian and Guatemalan flies is, therefore, selective rather than random; however, the particular type of selectivity here observed does not constitute a barrier to gene exchange.

1 Dobzhansky, Th., *Amer. Natur.*, **71**, 404–420 (1937).

2 Dobzhansky, Th., *Genetics and the Origin of Species*, New York, 1937, 1941.

3 Mayr., E., *Systematics and the Origin of Species*, New York, 1942.

4 The expression "physiological isolating mechanisms" used in some papers of Dobzhansky was intended to distinguish between isolating mechanisms based on inherent properties (physiology) of the organisms concerned and geographical isolation. Since this expression has led to some misapprehension, the adjective "reproductive," instead of "physiological," is used to convey the same idea.

5 Phillips, J. C., *J. Exp. Zool.*, **18**, 69–146 (1915).

6 Gordon, M., *Trans. N. Y. Acad. Sci.*, **5** (2), 63–77 (1943).

7 Dobzhansky, Th., and Koller, P., *Biol. Zentr.*, **58**, 589–607 (1938).

8 Stalker, H. D., *Genetics*, **27**, 238–257 (1942).

9 Patterson, J. T., Stone, W. S., and Griffen, A. B., *Univ. Texas Public.*, **4228**, 162–200 (1942); Stone, W. S., *Ibid.*, 16–22; Griffen, A. B., *Ibid.*, 68–73; Mainland, G. B., *Ibid.*, 74–112.

10 Copulation in *D. willistoni* begins as early as 8 hours after the hatching from pupae; to collect virgins bottles were emptied at 4-hour intervals, but it was found expedient to use also females hatched overnight since the proportion fertilized is small.

THE COMPRESSIBILITY OF MEDIA UNDER EXTREME PRESSURES

BY F. D. MURNAGHAN

DEPARTMENT OF MATHEMATICS, THE JOHNS HOPKINS UNIVERSITY

Communicated July 11, 1944

It is well known that Hooke's Law, which postulates a linear relation between stress and strain, has a very limited range of applicability even when the applied stress is a uniform pressure. We have in previous papers[1,2] furnished a formula which is valid over a much greater range than Hooke's Law; this formula agreed well with experimental results up to pressures as high as 50,000 atmospheres (the highest for which measurements were then available). Since then Bridgman[3] has published measurements up to 100,000 atmospheres and this, combined with theoretical considerations of a fundamental character, has caused us to reconsider the whole question. Hooke's Law and our formula may both be derived from the principle of energy conservation; the difference in the results is due to the fact that for infinitesimal deformations (to which the validity of Hooke's Law is limited) it is correct to say that stress is the gradient, with respect to strain, of the energy per unit volume. For non-infinitesimal deformations this simple relation must be replaced by a more complicated one. The feature common to both theories is the *assumption* that the energy density is a function of the strain; it being agreed that the strain is measured from the position of zero stress. This assumption, which has apparently never been seriously questioned, has the quality of an "action at a distance" theory; we assume that the energy of deformation is furnished by a knowledge of the relationship of the actual position of the medium to a *remote* position in which the medium was unstressed. We propose here to discard this action at a distance theory and to replace it by a *differential* theory in which we are concerned merely with the variation of the energy as we pass from *any* position of the medium to an infinitesimally near position. Thus we must be prepared to confront the situation where the *initial* position of the medium is one in which the medium is under stress.

This being understood, a fundamental question arises: are the elastic *constants* of the medium really constant? The elastic constants are simply coefficients which occur in the statement of Hooke's Law and we mean by the word *constant* that they do not depend on the *increment* of the stress. But do they depend on the initial value of the stress? All experimental evidence points to the fact that they do, and many determinations have been made of the variation of such things as compressibility with pressure. It is fair to say that the reason that Hooke's Law is so limited in its range of applicability is that it neglects the dependence of the elastic constants on the initial stress. This dependence of the elastic constants on the initial stress cannot be overemphasized. For example, when we say that an elastic medium is isotropic, we mean that certain relations hold amongst the elastic constants (so that *two* are sufficient to furnish the statement of Hooke's Law); it is clear, then, that a medium which is elastically isotropic under zero stress may fail to be elastically isotropic when stress is applied. We should certainly expect this lack of isotropy to appear when the applied stress is (as in the case of the Young's modulus experiment) not merely a uniform pressure or traction. We feel certain that it is the reliance of the classical theory of elasticity upon the hope that a stretched wire is elastically isotropic, if the unstretched wire is, that is responsible for its complete failure to predict the yield point phenomenon.

The case of uniform pressure is particularly simple since the stress tensor is scalar. The principle of conservation of mass, combined with Hooke's law for *infinitesimal* variations of stress, suffices provided we are willing to face the situation that the compressibility depends on the pressure. The principle of mass conservation yields $\frac{1}{v}\,\delta v = 3\,\frac{\partial}{\partial x}\,\delta x$ and Hooke's Law yields $p = -(3\lambda + 2\mu)\,\frac{\partial}{\partial x}\,\delta x$ and so

$$\frac{1}{v}\frac{dv}{dp} = -\frac{1}{\lambda + {}^2/_3\mu}.$$

Let us assume, as a first approximation, that $\lambda + {}^2/_3\mu$ is a linear function of p:

$$-v\frac{dp}{dv} = c(1 + kp)$$

so that

$$c = -\left(v\frac{dp}{dv}\right)_{p=0}; \qquad ck = -\frac{d}{dp}\left(v\frac{dp}{dv}\right)_{p=0}.$$

We find $\frac{v_0}{v} = (1 + kp)^{1/ck}$ or, on writing $\Delta v = (v_0 - v)$,

$$\frac{\Delta v}{v_0} = 1 - (1 + kp)^{-1/ck}.$$

This simple formula accounts reasonably well for the experimental results even for such a range of pressure as from 0 to 100,000 atmospheres. The data for Li given by Bridgman (loc. cit.) are approximated by setting $k = 0.153 \times 10^{-4}$, $ck = 2$ so that the relation connecting p and v has the simple form

$$0.153 \times 10^{-4}p = \left(\frac{v_0}{v}\right)^2 - 1; \qquad \frac{\Delta v}{v_0} = 1 - (1 + 0.153\cdot 10^{-4}p)^{-1/2}.$$

Bridgman tabulates $\frac{\Delta v}{v_0}$ at intervals of 10,000 atmospheres. The following table furnishes the comparison between the values obtained empirically and those calculated from the formula just given

$p =$	10^4;	2.10^4;	3.10^4;	4.10^4;	5.10^4;	6.10^4;	7.10^4;	8.10^4;	9.10^4;	10^5
$\frac{\Delta v}{v_0}$ (obs.)	0.074;	0.125;	0.164;	0.201;	0.237;	0.272;	0.305;	0.336;	0.366;	0.394
$\frac{\Delta v}{v_0}$ (calc.)	0.069;	0.125;	0.172;	0.212;	0.247;	0.278;	0.305;	0.330;	0.353;	0.371

The formula predicts that when $p = 2.10^5$ $\frac{\Delta v}{v_0}$ will be approximately 0.514; when $p = 10^6$ $\frac{\Delta v}{v_0}$ will be approximately 0.752. To compress Li to a density ten times its original density, a pressure of around 6.5×10^6 atmospheres is necessary.

In conclusion we add the remark that when the pressure is small enough so that its influence upon the elastic constants is negligible the appropriate formula is

$$\frac{v_0}{v} = e^{p/c}; \qquad \frac{\Delta v}{v_0} = 1 - e^{-p/c}.$$

ADDED IN PROOF. It is worthy of attention that the formula given here is, when the dimensionless constant ck is assigned the value 2, precisely that which leads to Laplace's well known law of density distribution throughout the interior of the earth. This law fits the observed facts remarkably well but it has been regarded by many authorities as merely empirical largely because its connection with the law of compressibility has not been clearly presented.

[1] "Finite Deformations of an Elastic Solid," *Amer. Jour. Math.*, **59**, 235–260 (1937).

[2] "The Compressibility of Solids," *von Kármán Anniversary Volume*, 1941, pp. 121–136.

[3] "Compressions and Polymorphic Transitions of Seventeen Elements to 10^5 Kg./Cm.^{2}," *Phys. Rev.*, **60**, 351–354 (1941).

NOTE ON THE DISCOVERY OF RED STARS

BY GUILLERMO HARO

HARVARD COLLEGE OBSERVATORY

Communicated July 26, 1944

During an investigation of colors, magnitudes, and spectral types in the Hercules-Vulpecula region, it was noted that a star at the approximate position $19^h10^m\!.6$, $+21° 37'$ (1855), $\lambda = 23°$, $\beta = +3°$ had a very large color index. A preliminary examination of blue and red plates indicated that the blue-red color index was between 6.5 and 7.0 magnitudes.

The blue-red and blue-yellow (international) color indices of this star were determined from polar comparison plates, the blue magnitude from Cramer Hi-Speed plates, the red magnitude from Eastman 103a-E plates with a ciné-red filter (effective wave-length near 6300 A), the yellow magnitude from 103a-G plates with a yellow filter. The 8-inch Ross-Lundin (IR) refractor was used for the red and the yellow, the 16-inch Metcalf (MC) refractor for the blue magnitudes. All plates were taken between May 26 and June 5, 1944. The results were as follows: