## Mechanical Mixing in Nonlinear Nanomechanical Resonators

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(October 27, 2018)

Nanomechanical resonators, machined out of Silicon-on-Insulator wafers, are operated in the nonlinear regime to investigate higher-order mechanical mixing at radio frequencies, relevant to signal processing and nonlinear dynamics on nanometer scales. Driven by two neighboring frequencies the resonators generate rich power spectra exhibiting a multitude of satellite peaks. This nonlinear response is studied and compared to  $n^{th}$ -order perturbation theory and nonperturbative numerical calculations.

68.60.Bs, 46.70.Hg, 95.10.Fh

Mechanical devices in combination with modern semiconductor electronics offer great advantages as for example their robustness against electrical shocks and ionization due to radiation. In outstanding work by Rugar and Grütter [1] the importance for applications in scanning probe microscopy of mechanical cantilevers was demonstrated. Greywall *et al.* [2] investigated noise evasion techniques for frequency sources and clocks with microscopical mechanical resonators. The main disadvantage of mechanical devices so far is the low speed of operation. This has been overcome with the realization of nanomechanical resonators, which allow operation at frequencies up to 500 MHz [3–7].

In the present work we realize such a nanomechanical resonator to study its nonlinear dynamics and its mechanical mixing properties. Mixing is of great importance for signal processing in common electronic circuits. Combining signal mixing with the advantages of mechanical systems, i.e. their insensitivity to the extremes of temperature and radiation, is very promising, especially when considering the high speed of operation currently becoming available. Here we present measurements on such a nonlinear nanomechanical resonator, forced into resonance by application of two different but neighboring driving frequencies. We also present a theoretical model, based on the Duffing equation, which accurately describes the behavior of the mechanical resonator. The model gives insight into the degree of nonlinearity of the resonator and hence into the generation of higherharmonic mechanical mixing.

The starting materials are commercially available Silicon-on-insulator (SOI) substrates with thicknesses of the Si-layer and the SiO<sub>2</sub> sacrificial layer of 205 nm and 400 nm, respectively (Smart-Cut wafers). The gate leads connecting the resonator to the chip carrier are defined using optical lithography. In a next step the nanomechanical resonator is defined by electron beam lithography. The sample is dry-etched in a reactive-ion etcher (RIE) in order to obtain a mesa structure with clear-cut walls. Finally, we perform a hydro-fluoric (HF) wet-etch step in order to remove the sacrificial layer below the resonators and the metallic etch mask. The last step of processing is critical point drying, in order to avoid surface tension by the solvents. The suspended resonator is shown in a scanning electron beam micrograph in Fig 1(a): The beam has a length of  $l = 3 \ \mu m$ , a width of  $w = 200 \ nm$ , and a height of  $h = 250 \ nm$  and is clamped on both sides. The inset shows a close-up of the suspended beam. The restoring force of this Au/Si-hybrid beam is dominated by the stiffer Si supporting membrane. The selection of the appropriate HF etch allows for attacking only the Si and thus the minute determination of the beam's flexibility and in turn the strength of the nonlinear response.

The chip is mounted in a sample holder and a small amount of  ${}^{4}\text{He}$  exchange-gas is added (10 mbar) to ensure thermal coupling. The sample is placed at 4.2 K in a magnetic field, directed in parallel to the sample surface but perpendicular to the beam. When an alternating current is applied to the beam a Lorentz force arises perpendicular to the sample surface and sets the beam into mechanical motion. For characterization we employ a spectrum analyzer (Hewlett Packard 8594A): The output frequency is scanning the frequency range of interest ( $\sim 37$  MHz), the reflected signal is tracked and then amplified (setup  $\alpha$  in Fig. 1(b), reflectance measured in mV). The reflected power changes when the resonance condition is met, which can be tuned by the gate voltages  $V_q$  in a range of several 10 kHz. The mixing properties of the suspended nanoresonators are probed with a different setup comprising two synthesizers (Marconi 2032 and Wavetek 3010) emitting excitations at constant, but different, frequency (setup  $\beta$  in Fig. 1(b)). Here, the reflectance is measured in dBm for better comparison of the driving amplitudes and the mixing products. The reflected power is finally amplified and detected by the spectrum analyzer.

In Fig. 2 the radio-frequency (rf) response of the beam near resonance is depicted for increasing magnetic field strength B=0, 1, 2, ..., 12 T. The excitation power of the spectrum analyzer was fixed at -50 dBm. The mechanical quality factor,  $Q = f/\delta f$ , of the particular resonator

under test in the linear regime is Q = 2330. As seen the profile of the resonance curve changes from a symmetric shape at moderate fields to an asymmetric, sawtooth shape at large field values, characteristic of an oscillator operated in the nonlinear regime.

This behavior can be described by the Duffing equation

$$\ddot{x}(t) + \mu \dot{x}(t) + \omega_0^2 x(t) + \alpha x^3(t) = F(t)$$
(1)

with a positive prefactor  $\alpha$  of the cubic term being the parameter of the strength of the nonlinearity [8]. In Eq. (1)  $\mu$  is the damping coefficient of the mechanical system,  $\omega_0 = 2\pi f_0$ , where  $f_0$  is the mechanical eigenfrequency of the beam, and x(t) its elongation. In our case the external driving F(t) is given by the Lorentz force:

$$F(t) = \frac{lB}{m_{\text{eff}}}I(t) = \frac{lB}{m_{\text{eff}}}I_0\cos(2\pi f t), \qquad (2)$$

where  $l = 1.9 \cdot 10^{-6}$  m is the effective length and  $m_{\text{eff}} = 4.3 \cdot 10^{-16}$  kg is the effective mass of the resonator. *B* is the magnetic field and  $I_0$  the input current corresponding to the amplitude of the driving power.

Solving Eq. (1) and computing the amplitude of the oscillation as a function of the driving frequency f for several excitation strengths reproduces the measured curves shown in Fig. 2. The solutions at large power exhibit a region where three different amplitude values coexist at a single frequency. This behavior leads to a hysteretic response in the measurements at high powers (e.g. -50 dBm) [7], as shown in the inset of Fig. 2, where we used an external source (Marconi) to sweep the frequencies in both directions. If the frequency is increased (inverted triangles ( $\bigtriangledown$ ) in the inset), the resonance first follows the lower branch, and then suddenly jumps to the upper branch. When sweeping downwards from higher to lower frequencies (triangles ( $\bigtriangleup$ )), the jump in resonance occurs at a different frequency.

Turning now to the unique properties of the nonlinear nanomechanical system: By applying two separate frequency sources as sketched in Fig. 1(b) (setup  $\beta$ ) it is possible to demonstrate mechanical mixing, as shown in Fig. 3(a). The two sources are tuned to  $f_1 = 37.28$  MHz and  $f_2 = 37.29$  MHz with constant offset and equal output power of -48 dBm, well in the nonlinear regime. Without applying a magnetic field the two input signals are simply reflected (upper left panel). Crossing a critical field of  $B \simeq 8$  T higher-order harmonics appear. Increasing the field strength further a multitude of satellite peaks evolves. As seen the limited bandwidth of this mechanical mixer allows effective signal filtering.

Variation of the offset frequencies leads to the data presented in Fig. 3(b): Excitation at -48 dBm and B = 12 T with the base frequency fixed at  $f_1 = 37.290$  MHz and varying the sampling frequency in 1 kHz steps from  $f_2 = 37.285$  MHz to 37.290 MHz yields satellites at the offset frequencies  $f_{1,2} \pm n\Delta f$ ,  $\Delta f = f_1 - f_2$ . The dotted line is taken at zero field for comparison, showing only the reflected power when the beam is not set into mechanical motion. At the smallest offset frequency of 1 kHz the beam reflects the input signal as a broad band of excitations.

We model the nanomechanical system as a Duffing oscillator (1) with a driving force

$$F(t) = F_1 \cos(2\pi f_1 t) + F_2 \cos(2\pi f_2 t), \qquad (3)$$

with two different, but neighboring, frequencies  $f_1$  and  $f_2$  and amplitudes  $F_i = l B I_i / m_{\text{eff}}$ .

Before presenting our results of a numerical solution of Eq. (1) for the driving forces (3) we perform an analysis based on  $n^{th}$ -order perturbation theory [9] to explain the generation of higher harmonics. Expanding

$$x = x_0 + \epsilon x_1 + \epsilon^2 x_2 + \dots, \qquad (4)$$

where we assume that the (small) parameter  $\epsilon$  is of order of the nonlinearity  $\alpha$ , and inserting this expansion into Eq. (1) yields equations for the different orders in  $\epsilon$ . In zeroth order we have

$$\ddot{x}_0 + \mu \dot{x}_0 + \omega_0^2 x_0 = F_1 \cos(2\pi f_1 t) + F_2 \cos(2\pi f_2 t) , \quad (5)$$

to first-order  $\ddot{x}_1 + \mu \dot{x}_1 + \omega_0^2 x_1 + \alpha x_0^3 = 0$ , and similar equations for higher orders. After inserting the solution of Eq. (5) into the first-order equation and assuming  $f_1 \approx f_2 \approx f_0 = \omega_0/2\pi$ , two types of resonances can be extracted: One resonance is located at  $3f_0$  which we, however, could not detect experimentally [10]. Resonances of the other type are found at frequencies  $f_i \pm \Delta f$ . Proceeding along the same lines in second-order perturbation theory we obtain resonances at  $5f_0$  and  $f_i \pm 2\Delta f$ . Accordingly, owing to the cubic nonlinear term,  $n^{th}$ -order resonances are generated at  $(2n + 1)f_0$  and  $f_i \pm n\Delta f$ . While the  $(2n + 1)f_0$ -resonances could not be observed, the whole satellite family  $f_i \pm n\Delta f$  is detected in the experimental power spectra Fig. 3(a,b).

The perturbative approach yields the correct peak positions and, for B < 4 T, also the peak amplitudes. However, in the hysteretic, strongly nonlinear regime a nonperturbative numerical calculation proves necessary to explain quantitatively the measured peak heights. To this end we determined the parameters entering into Eq. (1) in the following way: The damping is estimated from the quality factor Q = 2330 which gives  $\mu = 50265$  Hz. The eigenfrequency is  $f_0 = 37.26$  MHz as seen from Fig. 2 in the linear regime. The nonlinearity  $\alpha$ is estimated from the shift [9]

$$\delta f(B) = f_{\max}(B) - f_0 = \frac{3\alpha [\Lambda_0(B)]^2}{32\pi^2 f_0} \tag{6}$$

in frequency  $f_{\text{max}}$  at maximum amplitude in Fig. 2. In zero order the displacement of the beam is given by  $\Lambda_0 =$ 

 $lI_0B/(4\pi f_0\mu \ m_{\rm eff})$ . Relation (6) yields with  $I_0 = 1.9 \cdot 10^{-5}$ A a value of  $\alpha = 9.1 \cdot 10^{28} \ ({\rm ms})^{-2}$ .

We first computed x(t) by numerical integration of the Duffing equation with driving (3) and  $F_1 = F_2 = lBI_0/m_{\rm eff}$ ,  $I_0 = 2.9 \cdot 10^{-5}$ A. We then calculated the power spectrum from the Fourier transform  $\hat{x}(\omega)$  of x(t)for large times (beyond the transient regime). For a direct comparison with the measured power P in Fig. 3 we employ  $P \simeq RI_{\rm imp}^2$ . Here R is the resistance of the electromechanical circuit and  $I_{\rm imp} = [4\pi f_0 \mu \ m_{\rm eff}/(lB)]\hat{x}(\omega)$ in close analogy to the zero-order relation between displacement  $\Lambda_0$  and  $I_0$ .

The numerically obtained power spectra are displayed in Fig. 4: (a) shows the emitted power for the same parameters as in Fig. 3(a), but with B = 4, 8, 9, 10, 11, and 12 T. Corresponding curves are shown in Fig. 4(b) for fixed B and various  $\Delta f$  for the same set of experimental parameters as in Fig. 3(b). The positions of the measured satellite peaks,  $f_i \pm n\Delta f$ , and their amplitudes are in good agreement with the numerical simulations for the entire parameter range shown. Even small modulations in the peak heights to the left of the two central peaks in Fig. 3(b) seem to be reproduced by the calculations in Fig. 4(b). (Note that the height of the two central peaks in Fig. 3 cannot be reproduced by the simulations, since they are dominated by the reflected input signal.)

The numerical results in Fig. 4(a) show clearly the evolution of an increasing number of peaks with growing magnetic field, i.e. increasing driving amplitude. As in the experiment, the spectra exhibit an asymmetry in number and height of the satellite peaks which switches from lower to higher frequencies by increasing the magnetic field from 8 T to 12 T. This behavior can be understood from Eq. (6) predicting a shift  $\delta f$  in resonance frequency with increasing magnetic field. This shift is reflected in the crossover in Figs. 3(a) and 4(a). For B = 8 T the amplitudes of the satellite peaks are larger on the left than on the right side of the two central peaks. As the field is increased the frequency shift drives the right-hand-side satellites into resonance increasing their heights.

The power spectra in Fig. 3(a) and 4(a) are rather insensitive to changes in magnetic field for B < 8 T compared to the rapid evolution of the satellite pattern for 8 T < B < 12 T. Our analysis shows that this regime corresponds to scanning through the hysteretic part (inset Fig. 2) in the amplitude/frequency (or amplitude/*B*field) diagram, involving abrupt changes in the amplitudes. The resonator studied is strongly nonlinear but not governed by chaotic dynamics. Similar setups should allow for entering into the truly chaotic regime.

In summary we have shown how to employ the nonlinear response of a strongly driven nanomechanical resonator as a mechanical mixer in the radio-frequency regime. This opens up a wide range of applications, especially for signal processing. The experimental results are in very good agreement with numerical calculations based on a generalized Duffing equation, a prototype of a nonlinear oscillator. Hence these mechanical resonators allow for studying nonlinear, possibly chaotic dynamics on the nanometer scale.

We thank J.P. Kotthaus for helpful discussions. We acknowledge financial support by the Deutsche Forschungsgemeinschaft (DFG).

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FIG. 1. (a) Scanning electron beam micrograph of the electromechanical resonator with a length  $l = 3 \ \mu m$ , width  $w = 200 \ nm$ , and height  $h = 250 \ nm$ . The Si-supporting structure is covered by a thin Au-sheet (50 nm thick); the electrodes on the left and right allow tuning of the elastic properties. Inset shows a magnification of the beam. (b) Experimental setup for sampling the mechanical properties of the suspended beam: For characterization we employ a spectrum analyzer scanning the frequency range of interest ( $\alpha$ ). Mechanical mixing is analyzed by combining two synthesizers  $(f_1, f_2)$  and detecting the reflected power ( $\beta$ ).

FIG. 2. Characterization of the nonlinear response of the suspended beam by ramping the magnetic field from 0 T up to 12 T, obtained with the spectrum analyzer operated with output power level of -50 dBm (setup  $\alpha$ ). Inset shows the measured hysteresis:  $\nabla$  correspond to an increase in frequency and  $\triangle$  represent the lowering branch.

FIG. 3. (a) Two synthesizers (setup  $\beta$  in Fig. 1(b)) running at frequencies of  $f_1 = 37.28$  MHz and  $f_1 = 37.29$  MHz with constant offset (output power -48 dBm) induce higher-order harmonics as a result of mechanical mixing by the nanoresonator in the nonlinear regime (B > 8 T). (b) Excitation with two frequencies at -48 dBm and B = 12 T: Base frequency is  $f_1 = 37.290$  MHz, while the sampling frequency is varied in 1 kHz steps from  $f_2 = 37.285$  MHz to 37.290 MHz. As seen the spacing of the harmonics follows the offset frequency  $\Delta f = f_1 - f_2$ . The dotted line is taken at B = 0 T showing pure reflectance of the beam without excitation of mechanical motion.

FIG. 4. Calculation of the power spectra from the numerical solution of Eqs. (1), (3) for the same driving frequencies as used in Fig. 3. (a) Variation of magnetic field B = 4,8,9,10,11, and 12 T. (b) Variation of offset frequency at B = 12 T. Note that the two central peaks of Fig. 3 are not reproduced by the theory, since they stem from the reflected input signal.

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