than average". In fact, in this complete set of 134 cases of the strongest earthquakes, Uranus was 39 times within ± 1 hr. of the meridian (expected 22·3 times). That is significant at a level below 10^{-3}. This disposes of the starting point of Mr. Burr's criticism.

To state the plain facts: One is led to the conclusion that the position of Uranus within $\pm 15°$ of the meridian at the moment of great earthquakes can be regarded as significant and that there exist times of longer period (several years) when it is very highly significant. It is quite obvious that the strains and stresses in the Earth's crust are the primary cause of earthquakes, but it has been shown, given the presence of these factors of sufficient magnitude, that the timing of the event in a significant number of cases can be described by the position of Uranus. If it is necessary to explain this within the limits of present-day science, attention should be directed to the fact that Uranus is the only planet of which the direction of its axis of rotation coincides with the plane of its orbital revolution. A possible magnetic field would influence the solar plasma in a way quite different from all the other planets.

The earthquake which destroyed Agadir on February 29 this year occurred with Uranus only about 4° from the meridian. Anybody in Agadir, knowing of my communication in *Nature*, and being warned by the preceding minor shocks and the reported behaviour of animals[1], would have kept away from buildings at the time of Uranus being near the meridian, which was from about 10 hr. to 12 hr., a.m. or p.m. The destruction of the town occurred at 11 hr. p.m. local time. An unbiased approach to these problems, of which the correlations of Uranus are only a part and a first step, may help humanity.

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[1] In seismic regions, as for example in Japan, Russia and Italy, sensitive tiltmeters give valuable indications on the actual condition of the Earth's crust (tilt-storms); see Nos. 600–617 of the bibliography of the Commission Permanente des Marées Terrestres (Observatory, Brussels, 1959).

Probability and Statistics

If $k_1(x)$ is any non-negative function in L_1 $(-\infty, +\infty)$, let us write:

$$k_n(x) = \int_{-\infty}^{+\infty} k_{n-1}(x - z)k_1(z)\mathrm{d}z$$

for $n = 2, 3, \ldots$ Then the function:

$$\Delta[k_1(x)] = \sum_{n=1}^{\infty} k_n(x)$$

is defined almost everywhere (although it is possibly infinite for some, or all, x).

Suppose $f_1(x)$ is the frequency function of a random variable X, the mean value of which $\mu_1 = EX$ is strictly positive (it may have the value $+\infty$). Then the renewal density theorem provides conditions on $f_1(x)$ under which, as $x \to \infty$:

$$\Delta[f_1(x)] \to \frac{1}{\mu_1}$$

The function $\Delta[f_1(x)]$ is called the renewal density function, and its limiting behaviour has been an object of study from the earliest days of renewal theory. Prior to the important paper of Feller[1] there appears to have been some controversy concerning this behaviour. Feller provided sufficient conditions under which the renewal density theorem would hold. These conditions have been modified and simplified by Täcklind[2] and Smith[3,4]. The simplest sufficient conditions to date are those given in ref. 4; they are: (i) $f_1(x) \to 0$ as $x \to \infty$; (ii) $f_1(x) \in L_p(-\infty, +\infty)$ for some $p > 1$.

In the course of some work on theory of dams I have recently discovered[5] that if:

$$f_1(x) = \int_0^{\infty} \frac{e^{-(t+x)}x^{t-1}}{\Gamma(t)}\mathrm{d}t, \quad x \geqslant 0$$
$$= \quad 0 \quad , \quad x < 0$$

then $\mu_1 = 1$ and $\Delta[f_1(x)] \to 1$, that is, the renewal density theorem holds. However, $f_1(x)$ is in no class L_p for $p > 1$. Thus the conditions of ref. 4 are not necessary.

We have now proved that the following constitute necessary and sufficient conditions for the validity of the renewal density theorem. They are:

$$f_1(x) \to 0 \text{ as } x \to \infty \qquad (1)$$

For every (small) $\delta > 0$, if

$$a_\delta(x) = f_1(x) \text{ for } 0 < x < \delta,$$
$$= 0, \text{ otherwise,}$$
$$\text{then } \Delta[a_\delta(x)] \to 0 \text{ as } x \to \infty \qquad (2)$$

For every (small) $\delta > 0$, if:

$$\varphi_\delta(x) = \int_\delta^\infty e^{ixy} f_1(y)\mathrm{d}y$$

then $\varphi_\delta(x)$ belongs to some class L_p $(p > 0)$, where p may depend upon δ. (3)

This is the first time that necessary and sufficient conditions for this theorem have been established. It will be seen that they are substantially less restrictive than the sufficient conditions of ref. 4; for example, no restraint is placed on the behaviour of $f_1(x)$ for negative x.

Condition (2) does not seem easy to verify; but we have shown that it is satisfied if $f_1(x) = 0(x^{-N})$ in $(0,\delta)$ for some sufficiently large N; in particular, it is satisfied if $f_1(x)$ is monotone in $(0,\delta)$. Condition (3) is satisfied if $f_1(x)$ belongs to $L_p(\delta,\infty)$ for every $\delta > 0$, where p may depend upon δ, but must exceed unity.

The motivation for our conditions is that only the behaviour of $f_1(x)$ in the open interval $(0,\infty)$ should affect the behaviour of $\Delta[f_1(x)]$ for large x. Singularities of $f_1(x)$ at the origin should remain at the origin for $f_n(x)$ and not be displaced to larger values of x. Thus, for large values of x, $\Delta[f_1(x)]$ should not be affected by singularities of $f_1(x)$ at the origin.

Needless to say, the 'counter-example' quoted above satisfies the new necessary and sufficient conditions.

I intend to publish full details of the proof in the near future.

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[1] Feller, *Ann. Math. Statist.*, **12**, 243 (1941).
[2] Täcklind, *Skand. Akt.*, **28**, 68 (1948).
[3] Smith, *Proc. Roy. Soc. Edin.*, **64**, 9 (1954).
[4] Smith, *Proc. Camb. Phil. Soc.*, **51**, 629 (1955).
[5] Smith, *Hotelling Festschrift* (in the press).