

Production of the $X(3872)$ in charmonia radiative decays

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Abstract

We discuss the possibilities of producing the $X(3872)$, which is assumed to be a $D\bar{D}^*$ bound state, in radiative decays of charmonia. We argue that the ideal energy regions to observe the $X(3872)$ associated with a photon in e^+e^- -annihilations are around the $Y(4260)$ mass and around 4.45 GeV, due to the presence of the S -wave $D\bar{D}_1(2420)$ and $D^*\bar{D}_1(2420)$ threshold, respectively. Especially, if the $Y(4260)$ is dominantly a $D\bar{D}_1$ molecule and the $X(3872)$ a $D\bar{D}^*$ molecule, the radiative transition strength will be quite large.

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1 Introduction

Since its discovery by the Belle Collaboration [1], the $X(3872)$, which is extremely close to the $D^0\bar{D}^{*0}$ threshold, has stimulated a lot of efforts, both experimental and theoretical. It is regarded as one of the most promising candidates for a hadronic molecule, which are formed of two or more hadrons — analogous to the deuteron, the shallow bound state made of a proton and a neutron. The quantum numbers of the $X(3872)$ have been determined to be $J^{PC} = 1^{++}$ [2], in accordance with the hadronic molecular interpretations which can be either an S -wave bound state [3, 4, 5, 6] or a virtual state in the $D\bar{D}^*$ system [7]. Another puzzling new charmonium state is the $Y(4260)$ with quantum numbers $J^{PC} = 1^{--}$, which was observed by the BaBar Collaboration [8]. It is difficult to be put in the vector family of the $c\bar{c}$ in potential models. Various interpretations were proposed. One intriguing possibility is that the main component of the $Y(4260)$ is a $D\bar{D}_1(2420)$ bound state [9, 10, 11].¹ For a comprehensive review of the $X(3872)$, $Y(4260)$ and other XYZ states observed in the last decade, we refer to Ref. [14].

So far the $X(3872)$ has been observed in several different processes. The discovery was made in B -meson decays in the processes $B^\pm \rightarrow K^\pm J/\psi\pi^+\pi^-$ by the Belle Collaboration [1] and later confirmed by the BaBar Collaboration [15]. It was also observed in the proton–antiproton annihilations $p\bar{p} \rightarrow J/\psi\pi^+\pi^-X$ by both the CDF [16] and D0 [17] Collaborations, and in proton–proton collisions by the LHCb Collaboration [2, 18]. It is quite natural to search for the $X(3872)$ also in the decays of higher charmonia, especially the 1^{--} states, which can be easily and copiously produced in electron-positron collisions at, e.g., the Beijing Electron-Positron Collider II (BEPC-II). However, so far no evidence of the $X(3872)$ in the radiative charmonium decays has been reported. In this paper, we will investigate the production of the $X(3872)$ in the radiative decays of charmonium states, which include the $\psi(4040)$, the $\psi(4160)$, the $Y(4260)$ and the $\psi(4415)$, which are all in the energy range of the BESIII experiment [19] at the BEPC-II. As will be shown later on, among the vector charmonium(-like) states, the $Y(4260)$ is the most promising one for producing the $X(3872)$, if the long-distance part of its wave function is dominated by the $D\bar{D}_1$ hadronic molecule component — note that the mass of the $Y(4260)$ is located close to the S -wave $D\bar{D}_1$ threshold.

Our paper is organized as follows: In Sec. 2, based on a nonrelativistic effective field theory (NREFT), we will identify the most important mechanism for the $X(3872)$ production, namely the triangle loops with the coupling of the initial charmonium(-like) state with charmed mesons being S -wave. Using the effective Lagrangians given in Sec. 3, we will calculate the partial decay widths of the radiative transitions of the charmonia, especially parameter-free predictions for the $Y(4260) \rightarrow X(3872)\gamma$ and will be made, and the results will be given in Sec. 4. A brief summary will be given in the last section.

2 Identifying the most important mechanism

In general, a hadronic molecule is not a pure two-meson state since it can couple to other components, such as a $q\bar{q}$ or a compact multiquark state, when these have the same quantum numbers. Thus, such a hadronic molecule can be produced through either the compact quark component or the hadronic constituents. It is a process-dependent question and in some cases one of those two mechanisms is

¹Notice that there are two D_1 states of similar masses, and the one in question should be the narrower one, i.e. the $D_1(2420)$, because it is not sensible to discuss a constituent with a width comparable or even larger than the range of forces [12, 13].

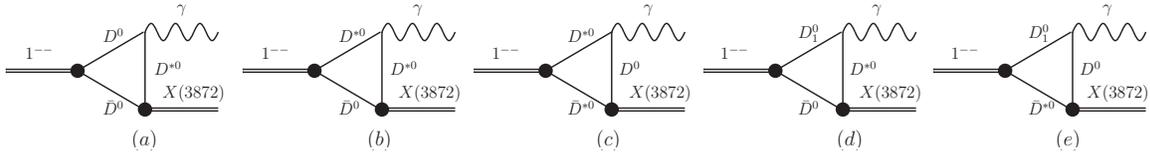


Figure 1: Relevant triangle loops for the production of the $X(3872)$ in the vector charmonium radiative decays. The charge-conjugated diagrams are not shown.

more important than the other. Let us take the $X(3872)$ as an example, which may be decomposed as

$$|X(3872)\rangle = \alpha_1 |c\bar{c}\rangle + \frac{\alpha_2}{\sqrt{2}} |D\bar{D}^* + c.c.\rangle . \quad (1)$$

Then the production amplitude is composed of two parts, $\mathcal{P}_{X(3872)} = \alpha_1 \mathcal{P}_{c\bar{c}} + \alpha_2 \mathcal{P}_{D\bar{D}^*}$, where $\mathcal{P}_{c\bar{c}}$ and $\mathcal{P}_{D\bar{D}^*}$ represents the production of the $c\bar{c}$ and $D\bar{D}^* + c.c.$, respectively (in the following, the charge conjugated channel will not be shown for simplicity but will be included in the numerical calculation). We assume that the $X(3872)$ is mainly a $D\bar{D}^*$ molecule, i.e. $|\alpha_2| \gg |\alpha_1|$. In this case, if $\mathcal{P}_{D\bar{D}^*}$ is not heavily suppressed, then the $X(3872)$ will dominantly be produced through the long distance $D\bar{D}^*$ component — see Refs. [20, 21] for a more detailed discussion.

Both the short and long distance production of the $X(3872)$ in the radiative decays of the $\psi(4040)$ and $\psi(4160)$ are considered in Refs. [22, 23] in the framework of the so-called X-EFT [24]. Here, we will focus on the contribution from intermediate charmed meson loops, i.e. the quantity $\mathcal{P}_{D\bar{D}^*}$ defined above. The mechanism is shown in Fig. 1. Both the initial charmonium and the $X(3872)$ couple to a pair of charmed and anticharmed mesons. The $X(3872)$ couples to the $D\bar{D}^*$ pair in an S -wave. With the quantum numbers being 1^{--} , the initial charmonium can couple to either two S -wave charmed mesons in a P -wave, or one P -wave and one S -wave charmed mesons in an S - or D -wave. As we will show in the following, the mechanism with an S -wave coupling to the initial charmonium will greatly facilitate the production processes.

The charmed meson channels which could have significant effects are those close to the mass of the considered charmonium. In this work we will consider the $s_\ell^P = \frac{1}{2}^-$ (S -wave), and $s_\ell^P = \frac{3}{2}^+$ (P -wave) charmed mesons, where s_ℓ is the total angular momentum of the light quark system which includes the light quark spin and orbital angular momentum. The pertinent triangle loops are shown in Fig. 1. There is no D_2 analogue of diagrams (d,e) because, different from the D_1 case, its quantum numbers does not allow that all vertices are in S -wave. One should notice that although the $X(3872)$ can have a sizable D^+D^{*-} component [27, 28], because the magnetic coupling to the neutral charmed mesons is much larger than that to the charged ones, see, e.g. Ref. [25], we only consider the neutral charmed mesons in the loops.²

Because all the charmonia considered are close to the open charm thresholds in question, the intermediate charmed and anticharmed mesons are nonrelativistic. We are thus allowed to use a non-relativistic power counting, the framework of which was introduced for studying the intermediate

²In fact, there can be photonic coupling to the charged charmed mesons from gauging the $\psi D^{(*)}\bar{D}^{(*)}$ vertex and the kinetic energy of the charmed mesons, see e.g. [26]. However, they are of order $\mathcal{O}(v)$, thus less important, in the power counting scheme to be detailed in the following. Furthermore, the loops involving such vertices are divergent and hence need unknown counterterms. In contrast, Ref. [29] states that including the charged charmed mesons would largely increase the partial decay width of the $X(3872) \rightarrow \gamma J/\psi$ based on a vector meson dominance model in a flavor SU(4) formalism.

meson loop effects in certain hadronic transitions of charmonia in Refs. [30, 31, 32]. Being nonrelativistic, the velocity of the intermediate mesons v is much smaller than 1. Thus, the loop diagrams as shown in Fig. 1 can be organized through a velocity counting, where the three-momentum scales as v , the kinetic energy scales as v^2 , and each of the nonrelativistic propagators scales as v^{-2} . In leading order, the S -wave coupling is momentum independent and does not contribute any power to the velocity counting. The P -wave coupling scales as v [30] or as the external momentum [31, 32] depending on the process in question.

Let us focus on the last two diagrams of Fig. 1 first. The D meson has $s_\ell = 1/2$, and D_1 has $s_\ell = 3/2$. Thus, they can couple to $L = 2$ but not to $L = 0$, where L is the orbital angular momentum, in the heavy quark limit. As a result, only the D -wave charmonia can couple to the $D\bar{D}_1$ in an S -wave, and for the S -wave charmonia the coupling must be D -wave. Thus, if the initial state is a D -wave charmonium or has a significant $D\bar{D}_1$ molecular component (as might be the case for the $Y(4260)$), the loop integral scales as

$$\frac{v^5}{(v^2)^3} E_\gamma = \frac{E_\gamma}{v}, \quad (2)$$

where E_γ is the external photon energy. The decay amplitude is the product of the loop integral and the coupling constants for the three vertices. One sees that the amplitude is greatly enhanced for small velocity. It was shown in Ref. [33] that the value of the velocity should be understood as the average of two velocities which correspond to the two cuts in the triangle diagram. These two velocities may be estimated as $\sqrt{|m_1 + m_2 - M_i|/\bar{m}_{12}}$ and $\sqrt{|m_2 + m_3 - M_f|/\bar{m}_{23}}$, where m_2 is the mass of the charmed meson between the two charmonia, $m_{1(3)}$ is the mass of the meson between the initial (final) charmonium and the photon, $\bar{m}_{ij} = (m_i + m_j)/2$, and $M_{i(f)}$ is the mass of the initial (final) charmonium. Therefore, the amplitude is most enhanced when both the initial and final charmonia are close to the corresponding thresholds.

For diagrams (a), (b) and (c) of Fig. 1, the vertex involving the initial charmonium is in a P -wave. The momentum in that vertex has to be contracted with the external photon momentum q , and thus should be counted as q . The decay amplitude through this type of loops scales as

$$\frac{v^5}{(v^2)^3} \frac{q^2}{m_0} = \frac{E_\gamma^2}{m_0 v}, \quad (3)$$

where m_0 is a quantity of the dimension mass, and the factor of m_0^{-1} is introduced to make the above expression have the same dimension as that obtained in Eq. (2). This factor in fact accounts for the different dimensions of the coupling constants for the P -wave and S -wave vertices in diagrams (a, b, c) and (d, e), respectively, i.e. $m_0 = |g_4/g_3|$ where g_3 and g_4 are the coupling constants to be defined in Eq. (12) below. If all the coupling constants are of natural size, that is $m_0 \sim 1$ GeV, then this loop should be suppressed relative to the one in Eq. (2) for a soft photon. This is supported by the numerical results in Sec. 4. Notice that only neutral charmed mesons are involved so that the P -wave vertex, although it contains a derivative, will not get gauged and the triangle diagrams are gauge invariant.

If the initial charmonium is the $\psi(4040)$ or the $\psi(4415)$, which are the radial exceptions of J/ψ and thus S -wave charmonia, the coupling to the $D\bar{D}_1$ is in a D -wave in the heavy quark limit, as outlined above. In this case, the $\psi D\bar{D}_1$ vertex should be counted as v^2 . Using the same power counting, the loops in Fig. 1 (d, e) should scale as $v E_\gamma$, and thus are suppressed rather than enhanced for small values of v .

In the above discussions, we have neglected the width of the $D_1(2420)$, which presents a new scale. One concern is whether it would break the power counting established above. The width of the $D_1(2420)$ is 27.1 ± 2.7 MeV [34], thus $\Gamma_1 \lesssim |2b_{12}|$, where $b_{12} = m_1 + m_2 - M_i$. From Eq. (A.4),

which is the nonrelativistic scalar loop function where one of the intermediate mesons carries a finite constant width, one can conclude that the power counting scheme will not be modified by the presence of the finite width of the $D_1(2420)$ (as long as the width is sufficiently small).

3 Effective Lagrangians

Because the charmed mesons do not have definite charge parity, it is necessary to clarify the phase convention under charge conjugation to be used in our paper, which is

$$CD\mathcal{C}^{-1} = \bar{D}, \quad CD^*\mathcal{C}^{-1} = \bar{D}^*, \quad CD_1\mathcal{C}^{-1} = \bar{D}_1. \quad (4)$$

The $X(3872)$ has a positive C -parity, and the $Y(4260)$ as well as all the other vector charmonium states have negative C -parity. Thus, the flavor wave functions of the $X(3872)$ and $Y(4260)$ in terms of the charmed mesons are convention dependent. With the convention specified above, the $D\bar{D}^*$ and $D\bar{D}_1$ components of the $X(3872)$ and $Y(4260)$ can be written as ³

$$|X(3872)\rangle = \frac{1}{\sqrt{2}} |D\bar{D}^* + D^*\bar{D}\rangle, \quad |Y(4260)\rangle = \frac{1}{\sqrt{2}} |D_1\bar{D} - D\bar{D}_1\rangle. \quad (5)$$

Because we work with nonrelativistic kinematics for the charmed mesons and charmonia throughout this work, the two-component notation introduced in Ref. [25] is very convenient. In this simplified notation, the field for the ground state charmed mesons is $H_a = \vec{V}_a \cdot \vec{\sigma} + P_a$, where $\vec{\sigma}$ are the Pauli matrices, P_a and V_a annihilates the pseudoscalar and vector charmed mesons, respectively, and a is the flavor label for the light quarks. The quantum numbers of the light quark system in these two mesons are $s_\ell^P = \frac{1}{2}^-$. Under the convention specified in Eq. (4), the field annihilating the ground state mesons containing an anticharm quark is [36]

$$\bar{H}_a = \sigma_2 \left(\vec{V}_a \cdot \vec{\sigma}^T + \bar{P}_a \right) \sigma_2 = -\vec{V}_a \cdot \vec{\sigma} + \bar{P}_a. \quad (6)$$

The field for the $s_\ell^P = 3/2^+$ charmed mesons can be written as

$$T_a^i = P_{2a}^{ij} \sigma^j + \sqrt{\frac{2}{3}} P_{1a}^i + i \sqrt{\frac{1}{6}} \epsilon_{ijk} P_{1a}^j \sigma^k, \quad (7)$$

where P_{1a} and P_{2a} annihilate the charmed mesons $D_1(2420)$ and $D_2(2460)$, respectively. The charmed antimesons are collected in $\bar{T}_a^i = -\bar{P}_{2a}^{ij} \sigma^j + \sqrt{2/3} \bar{P}_{1a}^i - i \sqrt{1/6} \epsilon_{ijk} \bar{P}_{1a}^j \sigma^k$, where the convention $\mathcal{C}D_2\mathcal{C}^{-1} = \bar{D}_2$ is adopted. Under parity and charge conjugation and with the convention specified above, these fields transform as

$$H_a \xrightarrow{\mathcal{P}} -H_a, \quad H_a \xrightarrow{\mathcal{C}} \sigma_2 \bar{H}_a^T \sigma_2, \quad \bar{H}_a \xrightarrow{\mathcal{P}} -\bar{H}_a, \quad \bar{H}_a \xrightarrow{\mathcal{C}} \sigma_2 H_a^T \sigma_2, \quad (8)$$

$$T_a^i \xrightarrow{\mathcal{P}} T_a^i, \quad T_a^i \xrightarrow{\mathcal{C}} \sigma_2 \bar{T}_a^{iT} \sigma_2, \quad \bar{T}_a^i \xrightarrow{\mathcal{P}} \bar{T}_a^i, \quad \bar{T}_a^i \xrightarrow{\mathcal{C}} \sigma_2 T_a^{iT} \sigma_2. \quad (9)$$

Analogously, we can construct the field for the S -wave charmonia, which is $J = \vec{\psi} \cdot \vec{\sigma} + \eta_c$, where ψ and η_c annihilate the vector and pseudoscalar charmonia, respectively. The leading coupling of the S -wave charmonium with the charmed and anticharmed mesons reads as

$$\mathcal{L}_S = i \frac{g_2}{2} \left\langle \bar{H}_a^\dagger \vec{\sigma} \cdot \overleftrightarrow{\partial} H_a^\dagger J \right\rangle + \text{H.c.}, \quad (10)$$

³In the literature, some authors write the wave function of the $X(3872)$ with a different relative sign of the two terms, $|X(3872)\rangle = \frac{1}{\sqrt{2}} |D\bar{D}^* - D^*\bar{D}\rangle$. This corresponds to a different convention for the C -parity transformation for the D^* , $\mathcal{C}D^*\mathcal{C}^{-1} = -\bar{D}^*$. Notice that only the flavor neutral mesons are eigenstates of the C -parity, the physical observables should be independent of the convention. For a detailed discussion in the case of the $X(3872)$, see Ref. [35].

where $A \overleftrightarrow{\partial} B \equiv A(\vec{\partial}B) - (\vec{\partial}A)B$ and $\langle \dots \rangle$ denotes the trace in flavor space. Notice that all the charmed meson and charmonium fields in the above Lagrangian and the ones in the following are nonrelativistic and have dimension $\text{mass}^{3/2}$.

Some of the 1^{--} charmonia in question are D -wave states. For instance, the $\psi(4160)$ is widely considered as the 2^3D_1 state [37, 38]. The field for the D -wave charmonia in two-component notation can be written as [23]

$$J^{ij} = \frac{1}{2} \sqrt{\frac{3}{5}} (\psi^i \sigma^j + \psi^j \sigma^i) - \frac{1}{\sqrt{15}} \delta^{ij} \vec{\psi} \cdot \vec{\sigma}, \quad (11)$$

where only the 1^{--} state relevant for our discussions is included. Considering parity, C -parity, spin symmetry and Galilean invariance, the leading order Lagrangian for the coupling of the D -wave charmonia to the charmed and anticharmed mesons can be written as

$$\mathcal{L}_D = i \frac{g_3}{2} \langle \bar{H}_a^\dagger \sigma^i \overleftrightarrow{\partial}^j H_a^\dagger J^{ij} \rangle + \frac{g_4}{2} \langle (\bar{T}^{j\dagger} \sigma^i H^\dagger - \bar{H}^\dagger \sigma^i T^{j\dagger}) J^{ij} \rangle + \text{H.c.}, \quad (12)$$

where the first term has already been introduced in Ref. [23] (g_3 is denoted by g in that paper).

In order to calculate the triangle diagrams depicted in Fig. 1, we need to know the photonic coupling to the charmed mesons. The magnetic coupling of the photon to the S -wave heavy mesons is described by the Lagrangian [39, 25]

$$\mathcal{L}_{HH\gamma} = \frac{e\beta}{2} \text{Tr} [H_a^\dagger H_b \vec{\sigma} \cdot \vec{B} Q_{ab}] + \frac{eQ'}{2m_Q} \text{Tr} [H_a^\dagger \vec{\sigma} \cdot \vec{B} H_a], \quad (13)$$

where $B^k = \epsilon^{ijk} \partial^i A^j$ is the magnetic field, Q is the light quark charge matrix, and Q' is the heavy quark electric charge (in units of the proton charge e). These two terms describe the magnetic coupling due to the light and heavy quarks, respectively. The E1 transition of the $\frac{3}{2}^+$ charmed mesons to the $\frac{1}{2}^-$ states may be parameterized in terms of a simple Lagrangian

$$\mathcal{L}_{TH\gamma} = \sum_a \frac{c_a}{2} \text{Tr} [T_a^i H_a^\dagger] E^i + \text{H.c.} \quad (14)$$

Note that here the coefficients are light-flavor-dependent.

At last, assuming that the $X(3872)$ and $Y(4260)$ are hadronic molecules, we parameterize their coupling to the charmed mesons in terms of the following Lagrangian

$$\mathcal{L}_{XY} = \frac{y}{\sqrt{2}} Y^{i\dagger} (D_{1a}^i \bar{D}_a - D_a \bar{D}_{1a}^i) + \frac{x}{\sqrt{2}} X^{i\dagger} (D^{*0i} \bar{D}^0 + D^0 \bar{D}^{*0i}) + \text{H.c.}, \quad (15)$$

where we assume that the $Y(4260)$ couples to the $D\bar{D}_1$ in an isospin symmetric manner so that the light flavor index a runs through u and d , and neglect all the other components except for the $D^0 \bar{D}^{*0}$ for the $X(3872)$.

4 Results and discussion

Considering a state slightly below an S -wave two-hadron threshold, the effective coupling of this state to the two-body channel is related to the probability of finding the two-hadron component in the physical wave function of the bound state, λ^2 , and the binding energy, $\epsilon = m_1 + m_2 - M$ [40, 41]

$$g_{\text{NR}}^2 = \lambda^2 \frac{16\pi}{\mu} \sqrt{\frac{2\epsilon}{\mu}} \left[1 + \mathcal{O}(\sqrt{2\mu\epsilon} r) \right], \quad (16)$$

where $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass and r is the range of forces, and the nonrelativistic normalization is used. After proper renormalization (see Ref. [42]), the coupling constants in Eq. (15) are given by the one in the above equation. Notice that the coupling constant gets maximized for a pure bound state, which has $\lambda^2 = 1$ by definition.

The threshold of the D^0 and D^{*0} using the PDG fit values for the masses [34] is 3871.84 ± 0.20 MeV. The mass of the $X(3872)$ is 3871.68 ± 0.17 MeV [34]. With $M_Y = 4263_{-9}^{+8}$ MeV, and the isospin averaged masses of the D and D_1 mesons, we obtain the mass differences between the $X(3872)$ and $Y(4260)$ and their corresponding thresholds, respectively,

$$M_{D^0} + M_{D^{*0}} - M_X = 0.16 \pm 0.26 \text{ MeV}, \quad M_D + M_{D_1(2420)} - M_Y = 27_{-8}^{+9} \text{ MeV}. \quad (17)$$

Assuming that the $X(3872)$ and $Y(4260)$ are pure hadronic molecules, which corresponds to the probability of finding the physical states in the two-hadron states $\lambda^2 = 1$, we obtain

$$|x| = 0.97_{-0.97}^{+0.40} \pm 0.14 \text{ GeV}^{-1/2}, \quad |y| = 3.28_{-0.28}^{+0.25} \pm 1.39 \text{ GeV}^{-1/2}, \quad (18)$$

where the first errors are from the uncertainties of the binding energies, and the second ones are due to the approximate nature of Eq. (16). The range of forces is estimated by $r^{-1} \sim \sqrt{2\mu\Delta_{\text{th}}}$ where μ is the reduced mass and Δ_{th} is the difference between the threshold of the components and the next close one, which is $M_{D^{*+}} + M_{D^+} - M_{D^{*0}} - M_{D^0}$ for the $X(3872)$ and $M_{D_1} + M_{D^*} - M_{D_1} - M_D$ for the $Y(4260)$, respectively.

The value of β in the magnetic coupling of the S -wave charmed mesons is not precisely known. We will use the value $\beta^{-1} = 276$ MeV determined with $m_c = 1.5$ GeV in Ref. [25]. There is no experimental measurement on the radiative decays of the P -wave charmed mesons. However, there have been a few calculations using various quark models. Taking the predictions of $\Gamma(D_1^0 \rightarrow D^{(*)0}\gamma)$ in Refs. [43, 44, 45] as a guidance, the value for the coupling constant for the neutral charmed mesons c_0 is in the range [0.3, 0.5].

4.1 $\psi(4040) \rightarrow \gamma X(3872)$ and $\psi(4415) \rightarrow \gamma X(3872)$

The $\psi(4040)$ and $\psi(4415)$ were widely accepted as the $3S$ and $4S$ vector charmonium states, respectively [37]. In the heavy quark limit, spin symmetry requires that the S -wave charmonium couples to the $D^{(*)}\bar{D}_1$ in a D -wave. As shown in Sec. 2, such a D -wave vertex will cause the charmed meson loops to be suppressed. Thus, we will neglect these loops, and consider only the loops involving the S -wave charmed mesons D and D^* , which correspond to the diagrams shown in Fig. 1 (a), (b) and (c). Assuming that the two-body S -wave charmed mesons saturate the decay width of the $\psi(4040)$ and 90% of width of the $\psi(4415)$ — the only relatively well measured branching fraction is the sequential decay into the $D^0 D^- \pi^+ + c.c.$ through the $D\bar{D}_2(2460)$ which is $(10 \pm 4)\%$, we may obtain an upper limit for the coupling constant g_2 for both the $3S$ and $4S$ charmonium states,

$$|g_{2[3S]}| < 0.85 \text{ GeV}^{-3/2}, \quad |g_{2[4S]}| < 0.23 \text{ GeV}^{-3/2}. \quad (19)$$

As a result, the upper limits for the production of the $X(3872)$ are

$$\Gamma(\psi(4040) \rightarrow \gamma X(3872))_{(a,b,c)} < 0.25 \text{ keV}, \quad \Gamma(\psi(4415) \rightarrow \gamma X(3872))_{(a,b,c)} < 0.63 \text{ keV}, \quad (20)$$

which correspond to tiny branching fractions of order 10^{-5} .

However, even a small D -wave $c\bar{c}$ mixture would greatly enhance the decay width of the $\psi(4415)$. This is because the $\psi(4415)$ is only 10 MeV below the $D^*\bar{D}_1$ threshold, and the velocity, the relevant

parameter for the power counting, is as small as 0.04. Considering such an admixture, we obtain from the last two diagrams in Fig. 1

$$\Gamma(\psi(4415) \rightarrow \gamma X(3872))_{(d,e)} = 287 \sin^2 \theta (g_4 x \text{ GeV})^2 c_0^2 \text{ keV} \lesssim 89 \sin^2 \theta (g_4^2 \text{ GeV}) \text{ keV}, \quad (21)$$

where $c_0 \simeq 0.4$ is used, and $\sin \theta$ is the mixture of the D -wave component in the $\psi(4415)$ wave function. In Ref. [46], $\theta \approx 34^\circ$ is suggested from an analysis of the e^+e^- decay widths of the vector charmonia. We have assumed spin symmetry for the coupling of the initial charmonium to the charmed mesons.

4.2 $\psi(4160) \rightarrow \gamma X(3872)$

As discussed before, being the $2D$ charmonium state, the $\psi(4160)$ couples to a pair of S -wave charmed mesons in a P -wave, and to one S -wave and one P -wave charmed mesons in an S -wave. Thus all the diagrams shown in Fig. 1 contribute to its radiative decay into the $X(3872)$. For the diagrams (a), (b) and (c), we can derive an upper limit for their contributions. The upper limit for the coupling g_3 for the $\psi(4160)$ may be obtained by saturating its total decay width by two-body decays into a pair of S -wave charmed mesons. We obtain $|g_{3[2D]}| < 0.72 \text{ GeV}^{-3/2}$. Using this value, the contribution of the S -wave charmed mesons to the width of the $\psi(4160) \rightarrow \gamma X(3872)$ is less than 0.20 keV. We should mention that our numerical result for the width of the $\psi(4160) \rightarrow \gamma X(3872)$ is smaller than the estimate in Ref. [23] using a different method and using the BaBar measurement of the $X(3872) \rightarrow \gamma \psi'$ [47], which was not confirmed by the Belle Collaboration [48], as input.

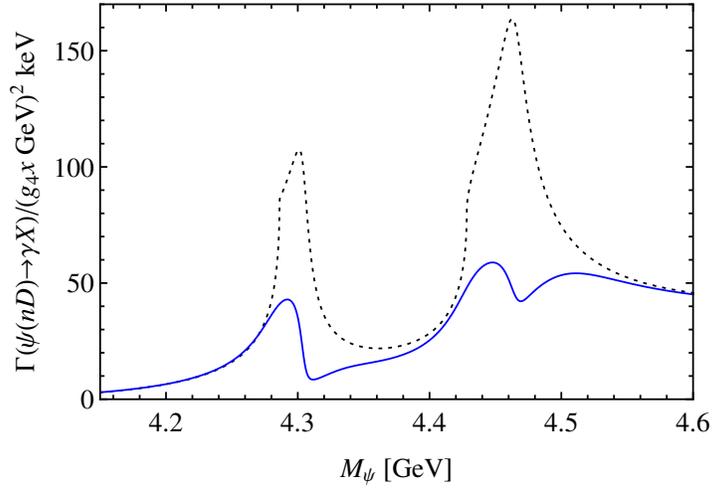


Figure 2: Dependence of the partial decay width of a D -wave charmonium into $\gamma X(3872)$ on the mass of the charmonium. The solid and dotted curves are obtained with and without taking into account the width of the $D_1(2420)$, respectively. Here, only the contributions from Fig. 1 (d) and (e) are included, and $c_0 = 0.4$ is used.

The value of g_4 , which is needed for evaluating the diagrams (d) and (e), is unknown. Thus, we express the result from these two diagrams in terms of g_4

$$\Gamma(\psi(4160) \rightarrow \gamma X(3872))_{(d,e)} = 19.4 (g_4 x \text{ GeV})^2 c_0^2 \text{ keV} \lesssim 6.0 (g_4^2 \text{ GeV}) \text{ keV}, \quad (22)$$

where we have taken $c_0 \simeq 0.4$. Expressing g_4 by $g_4 = g_3 m_0$, if $m_0 \sim 1$ GeV, then the approximate upper limit obtained from diagrams (d) and (e), 3 keV, is one order of magnitude larger than that from diagrams (a), (b) and (c). This can be understood from the power counting. The momentum of the photon in this decay is 280 MeV. Thus, the factor q/m_0 presents a suppression of the first three diagrams relative to the last two at the amplitude level. With the total width of the $\psi(4160)$ being 103 ± 8 MeV [34], a width of a few keV only amounts to a branching fraction of the order of 10^{-5} . Although larger than the 0.2 keV arising from the first three diagrams, it is still small so that an experimental observation will be difficult.

However, notice that the $\psi(4160)$ is far off the optimized region for the observation of the $X(3872)$. This can be seen from Fig. 2, which shows the dependence of the radiative decay width of a D -wave charmonium into the $\gamma X(3872)$ on the charmonium mass, where the solid and dotted curves represent the results with and without taking into account the finite width of the $D_1(2420)$, respectively. One sees pronounced peaks slightly above the $D\bar{D}_1$ and $D^*\bar{D}_1$ thresholds in the dashed curve. This is due to the closeness of the $X(3872)$ to the $D\bar{D}^*$ threshold, which makes the kinematics so special that $(c' - c)/(2\sqrt{-ac}) - a, c$ and c' are defined in Eq. (A.3) — is close to 1, and thus produces the maxima (recall that the imaginary part of $\arctan(i)$ is infinite, c.f. Eq. (A.2)). The pronounced peaks get smeared by the finite width of the $D_1(2420)$, as can be seen from the solid curve. Still, the width divided by g_4^2 peaks around 4.29 GeV and 4.45 GeV. Thus, as stated in Sec. 4.1, one might be able to make an observation through a D -wave admixture in the $\psi(4415)$.

4.3 $Y(4260) \rightarrow \gamma X(3872)$

We assume that the $Y(4260)$ is a $D\bar{D}_1$ molecule according to the suggestions of Refs. [9, 10, 11]. The production of the recently observed charged charmonium $Z_c(3900)$ [49, 50, 51] can be understood in this interpretation [11, 52] if it is a $D\bar{D}^*$ hadronic molecule [11, 53, 54, 55, 56]. Radiative decays of the $Y(4260)$ into a pair of charmed mesons was studied based on this assumption very recently [57]. In this picture, the radiative decay of the $Y(4260)$ into the $X(3872)$ will be a long-distance process, and the dominant decay mechanism is shown in Fig. 1 (d). With the the loop function given in the Appendix, we obtain the width

$$\Gamma(\psi(4260) \rightarrow \gamma X(3872))_{(d)} = 141_{-91}^{+136} (x^2 \text{ GeV}) c_0^2 \text{ keV}, \quad (23)$$

where the uncertainty is dominated by the use of Eq. (16), which is mainly due to neglecting the coupled channel $D^*\bar{D}_1$ in this case. The velocity counting is well controlled since $v \simeq 0.06$. Using Eq. (A.4), we have checked that including a finite constant width for the D_1 only causes a minor change of about 3%. The value of c_0 is in the range of [0.3, 0.5] using the width predictions in three different quark models [43, 44, 45]. Taking $c_0 = 0.4$, we plot the dependence of the width of the $Y(4260) \rightarrow \gamma X(3872)$ on the binding energy of the $X(3872)$ in Fig. 3, where the value of x is related to the binding energy via Eq. (16). Therefore, depending on the precise location of the $X(3872)$, the branching fraction can reach the order of 10^{-3} .

4.4 Using angular distributions to distinguish different loop contributions

We have argued that the triangle loops with all the intermediate states being the S -wave charmed mesons are suppressed relative to the ones with one S -wave and one P -wave charmed mesons when the initial charmonium is a D -wave state. This is based on the assumption that the coupling constants are of natural size so that $m_0 = |g_4/g_3| \sim 1$ GeV. If g_4 is unnaturally small, then these two kinds of

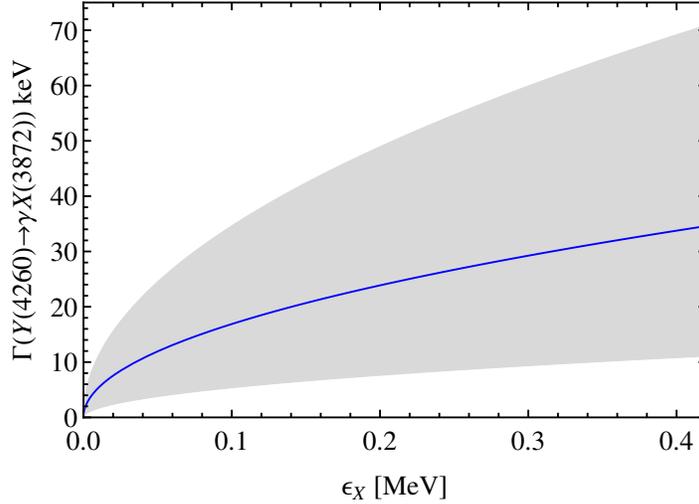


Figure 3: Dependence of the width of the $Y(4260) \rightarrow X(3872)\gamma$ in terms of the binding energy of the $X(3872)$, $\epsilon_X = M_{D^0} + M_{D^{*0}} - M_X$. Here the $D_1^0 D^{*0} \gamma$ coupling constant is taken as $c_0 = 0.4$.

mechanisms might be comparable. One can check which one is dominant by measuring certain angular distribution. This is because the two different types of loops have a different angular dependence — the one with two S -wave vertices does not depend on any angle with respect to the photon three momentum while the other does, as can be seen from the expressions

$$\begin{aligned} \mathcal{A}_{(d,e)} &= A (\vec{\epsilon}_\psi \times \vec{\epsilon}_\gamma) \cdot \vec{\epsilon}_X, \\ \mathcal{A}_{(a,b,c)} &= B \hat{q} \cdot \vec{\epsilon}_\psi (\hat{q} \times \vec{\epsilon}_X) \cdot \vec{\epsilon}_\gamma + C \hat{q} \cdot \vec{\epsilon}_X (\hat{q} \times \vec{\epsilon}_\psi) \cdot \vec{\epsilon}_\gamma, \end{aligned} \quad (24)$$

where \hat{q} is the unit vector along the three momentum of the photon, and $\vec{\epsilon}_\psi$, $\vec{\epsilon}_\gamma$ and $\vec{\epsilon}_X$ are the corresponding polarization vectors. The expressions for A , B and C in terms of loop functions are given in Appendix B. Because the vector charmonium produced in e^+e^- collisions is transversely polarized, the angle between the photon momentum and the ψ polarization vector can be related to that with respect to the beam axis. The relation follows from

$$\overline{\sum_{\lambda=1,2} \left| \hat{q} \cdot \vec{\epsilon}_\psi^{(\lambda)} \right|^2} = \frac{1}{2} \sin^2 \theta_q, \quad \overline{\sum_{\lambda=1,2} \left| \hat{q} \times \vec{\epsilon}_\psi^{(\lambda)} \right|^2} = \frac{1}{2} (1 + \cos^2 \theta_q), \quad (25)$$

where θ_q is the angle between the photon momentum and the beam axis. Thus, we have the angular distribution from diagrams (a,b,c)

$$\frac{d\Gamma_{(a,b,c)}}{d \cos \theta_q} \propto \overline{\sum_{\lambda=1,2} \left(2|B|^2 \left| \hat{q} \cdot \vec{\epsilon}_\psi^{(\lambda)} \right|^2 + |C|^2 \left| \hat{q} \times \vec{\epsilon}_\psi^{(\lambda)} \right|^2 \right)} \propto 1 + \rho \cos^2 \theta_q, \quad (26)$$

where $\rho = (|C|^2 - 2|B|^2) / (2|B|^2 + |C|^2)$. For the $\psi(4160) \rightarrow \gamma X(3872)$, the value is $\rho = -0.98$ so that the angular distribution is almost $\sim \sin^2 \theta_q$.⁴ Thus, when the long-distance part dominates the production of the $X(3872)$, one may use the angular distribution to distinguish the P -wave $D^{(*)} \bar{D}^{(*)}$ threshold and S -wave $D_1 \bar{D}^{(*)}$ threshold effects. A similar idea of using angular distributions to probe the structure of the $X(3872)$ was already proposed in Refs. [22, 23].

⁴The value of ρ shows that $|B| \gg |C|$, which is due to a strong cancellation between different loops in C .

5 Summary

In this paper, we have investigated the production of the $X(3872)$ in the radiative decays of excited charmonia. These states include the $\psi(4040)$, $\psi(4160)$, $\psi(4415)$ and the $Y(4260)$, which are the $3S$, $2D$, $4S$ charmonium and a conjectured $D\bar{D}_1$ molecule, respectively. Assuming the $X(3872)$ is a $D\bar{D}^*$ bound state, we considered its production through the mechanism with intermediate charmed mesons. Using a NREFT, we argue that the meson loops with all the vertices being in an S -wave should provide the most prominent contributions. We present a power counting that is confirmed by our numerical studies. It predicts that the closer to the threshold of the open charm intermediate states the initial charmonium is located, the more important the loops are. In this context, the production rate in the decays of the S -wave charmonia $\psi(4040, 4415)$, contrary to that for the D -wave charmonium $\psi(4160)$, should be small since they couple to the $D^{(*)}\bar{D}_1$ in a D -wave and $D^{(*)}\bar{D}^{(*)}$ in a P -wave. The production in the $Y(4260)$ decays will be strongly enhanced compared to all the other transitions studied in this work, if the $Y(4260)$ is a $D\bar{D}_1$ molecule, as suggested in Refs. [9, 10, 11], since the S -wave coupling constant is maximized in such a case. Especially, if the mechanism for the production of $Z_c(3900)$ in $Y(4260) \rightarrow \pi Z_c$ proposed in Ref. [11] is correct, the $X(3872)$ must be copiously produced in $Y(4260) \rightarrow X(3872)\gamma$.

We also show that the measurement of the angular distribution of the radiated photon in $e^+e^- \rightarrow Y(4160) \rightarrow \gamma X(3872)$ should be sensitive to the underlying transition mechanisms.

In this study, the $\psi(4415)$ was assumed to be an S -wave charmonium. However, if it has a sizable mixing with a D -wave $c\bar{c}$ component or an S -wave $D^*\bar{D}_1$ component (notice that it is only 10 MeV below the $D^*\bar{D}_1$ threshold), then it can also decay into the $X(3872)\gamma$ through the enhanced loops with S -wave couplings. Based on our calculation, we strongly suggest to search for the $X(3872)$ associated with a photon in the energy region around the $Y(4260)$ and 4.45 GeV in the e^+e^- collisions.

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A Loop functions

When we neglect the widths of all the intermediate mesons, the decay amplitudes can be expressed in the scalar three-point loop function

$$I(m_1, m_2, m_3, \vec{q}) = i \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - m_1^2 + i\epsilon) [(P-l)^2 - m_2^2 + i\epsilon] [(l-q)^2 - m_3^2 + i\epsilon]}, \quad (\text{A.1})$$

where m_i ($i = 1, 2, 3$) are the masses of the particles in the loop. This loop integral is convergent. Since all the intermediate mesons in the present case are highly nonrelativistic, the explicit expression

is derived as

$$\begin{aligned}
& I(m_1, m_2, m_3, \vec{q}) \\
&= \frac{-i}{8m_1 m_2 m_3} \int \frac{d^d l}{(2\pi)^d} \frac{1}{\left(l^0 - \frac{\vec{l}^2}{2m_1} + i\epsilon\right) \left(l^0 + b_{12} + \frac{\vec{l}^2}{2m_2} - i\epsilon\right) \left[l^0 + b_{12} - b_{23} - \frac{(\vec{l}-\vec{q})^2}{2m_3} + i\epsilon\right]} \\
&= \frac{\mu_{12}\mu_{23}}{16\pi m_1 m_2 m_3 \sqrt{a}} \left[\arctan\left(\frac{c' - c}{2\sqrt{a}(c - i\epsilon)}\right) + \arctan\left(\frac{2a + c - c'}{2\sqrt{a}(c' - a - i\epsilon)}\right) \right], \quad (\text{A.2})
\end{aligned}$$

where $\mu_{ij} = m_i m_j / (m_i + m_j)$ are the reduced masses, $b_{12} = m_1 + m_2 - M$, $b_{23} = m_2 + m_3 + q^0 - M$ with M the mass of the initial particle, and

$$a = \left(\frac{\mu_{23}}{m_3}\right)^2 \vec{q}^2, \quad c = 2\mu_{12}b_{12}, \quad c' = 2\mu_{23}b_{23} + \frac{\mu_{23}}{m_3} \vec{q}^2. \quad (\text{A.3})$$

For more information about the loop function, we refer to Refs. [32, 42]. The two arctangent functions correspond to the two cuts in the triangle diagram [33].

In the following, we give the expression for the loop with one of the mesons having a finite width. By assigning a constant width Γ_1 to the meson with a mass m_1 , the first propagator in Eq. (A.2) is modified to

$$\frac{1}{l^0 - \vec{l}^2 / (2m_1) + i\Gamma_1 / 2}.$$

Thus, the first cut of the triangle diagram involving m_1 will be influenced, and the scalar loop integral becomes

$$\begin{aligned}
& I(m_1, m_2, m_3, \vec{q}) \\
&= \frac{\mu_{12}\mu_{23}}{16\pi m_1 m_2 m_3 \sqrt{a}} \left[\arctan\left(\frac{c' - c}{2\sqrt{a}(c - i\mu_{12}\Gamma_1)}\right) + \arctan\left(\frac{2a + c - c'}{2\sqrt{a}(c' - a - i\epsilon)}\right) \right]. \quad (\text{A.4})
\end{aligned}$$

B Coefficients in the decay amplitudes

$$\begin{aligned}
A &= \sqrt{\frac{5}{6}} N g_4 x c_0 E_\gamma \left[I(m_{D_1^0}, m_{D^0}, m_{D^{*0}}, \vec{q}) + I(m_{D_1^0}, m_{D^{*0}}, m_{D^0}, \vec{q}) \right] \\
B &= \frac{4}{3} \sqrt{\frac{2}{15}} i N e g_3 x \vec{q}^2 \left(\beta + \frac{1}{m_c} \right) \left[5I^{(1)}(m_{D^0}, m_{D^0}, m_{D^{*0}}, \vec{q}) + 2I^{(1)}(m_{D^{*0}}, m_{D^{*0}}, m_{D^0}, \vec{q}) \right] \\
C &= \frac{2}{3} \sqrt{\frac{2}{15}} i N e g_3 x \vec{q}^2 \left[5 \left(\beta - \frac{1}{m_c} \right) I^{(1)}(m_{D^{*0}}, m_{D^0}, m_{D^{*0}}, \vec{q}) \right. \\
&\quad \left. - \left(\beta + \frac{1}{m_c} \right) I^{(1)}(m_{D^{*0}}, m_{D^{*0}}, m_{D^0}, \vec{q}) \right] \quad (\text{B.5})
\end{aligned}$$

where $N = \sqrt{M_X M_\psi}$ accounts for the nonrelativistic normalization, and the expression for the vector loop integral $I^{(1)}(m_1, m_2, m_3, \vec{q})$ can be found in Ref. [32].

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