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Estimate of the soil water retention curve from the sorptivity and β parameter calculated from an upward infiltration experiment

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Abstract

The water retention curve ($\theta(h)$), which defines the relationship between the volumetric water content (θ) and the matric potential (h), is of paramount importance to characterize the hydraulic behaviour of soils. Because current methods to estimate $\theta(h)$ are, in general, tedious and time consuming, alternative procedures to determine $\theta(h)$ are needed. Using an upward infiltration curve, the main objective of this work is to present a method to determine the parameters of the van Genuchten (1980) water retention curve (α and n) from the sorptivity (S) and the β parameter defined in the 1D infiltration equation proposed by Haverkamp et al. (1994). The first specific objective is to present an equation, based on the Haverkamp et al. (1994) analysis, which allows describing an upward infiltration process. Secondary, assuming a known saturated hydraulic conductivity, K_s, calculated on a finite soil column by the Darcy's law, a numerical procedure to calculate S and β by the inverse analysis of an exfiltration curve is presented. Finally, the α and n values are numerically calculated from K_s, S and β. To accomplish the first specific objective, cumulative upward infiltration curves simulated with HYDRUS-1D for sand, loam, silt and clay soils were compared to those calculated with the proposed equation, after applying the corresponding β and S calculated from the theoretical K_s, α and n. The same curves were used to: (i) study the influence of the exfiltration time on S and β estimations, (ii) evaluate the limits of the inverse analysis, and (iii) validate the feasibility of the method to estimate α and n. Next, the $\theta(h)$ parameters estimated with the numerical method on experimental soils were compared to those obtained with pressure cells. The results showed that the upward infiltration curve could be correctly described by the modified Haverkamp et al. (1994) equation. While S was only affected by early-time exfiltration data, the β parameter had a significant influence on the long-time exfiltration curve, which accuracy increased with time. The 1D infiltration model was only suitable for $\beta < 1.7$ (sand, loam and silt). After omitting the clay soil, an excellent relationship ($R^2 = 0.99$, $p < 0.005$) was observed between the theoretical α and n values of the

synthetic soils and those estimated from the inverse analysis. Consistent results, with a significant relationship ($p < 0.001$) between the n values estimated with the pressure cell and the upward infiltration analysis, were also obtained on the experimental soils.

Keywords: Soil hydraulic properties; Hydraulic conductivity; Water retention curve.

INTRODUCTION

Philip (1957) called sorptivity, S, the first term of his power series expression of the cumulative infiltration of water into a soil. Sorptivity was therefore an index of the capacity of a porous medium to absorb or desorb water, dependent of its water content and diffusivity. The Philip (1957) two-term infiltration equation reduces in the case of horizontal infiltration, or absorption using his own term, to a linear relationship between cumulative infiltration, I, and the square root of time, t, such as $I=S\,t^{1/2}$. This parameter can be analytically calculated as a function of soil water content and diffusivity, which approximation has been found to give good results (Parlange et al., 1975). The sorptivity can be estimated at the early stages of infiltration, where suction or capillarity forces prevail over gravity. However, as the process progresses, gravity relevance gradually increases. In the latter case, sorptivity can be evaluated with the full infiltration equation using methods such as the disc infiltrometry. During the last three decades, the disc infiltrometer technique has been widely accepted for its versatility and simplicity. Up to date, several procedures to estimate S using a single disc and tension infiltration data have been developed: (i) methods based on the short-time transient state data (e.g. White et al. 1992, Vandervaere et al., 2000); (ii) methods combining both early-time transient and steady data, like the BEST method (Lassabatere et al., 2006; Lassabatere et al., 2009; Yilmaz et al., 2010); and (iii) methods based on the whole cumulative infiltration curve (Latorre et al., 2015).

The β parameter, which was initially defined by Haverkamp et al. (1994), is an integral shape constant that depends on soil diffusivity, hydraulic conductivity and initial and final volumetric water content. This is calculated through cumulative infiltration identification analysis (Haverkamp et al., 1994). Its value, only suitable for sand, loam and silt, ranges between 0.3 and 1.7 (Lassabatere et al. (2009). Although β can be analytically calculated from known hydraulic properties (Haverkamp et al., 1994), up to date there is not any experimental method that allows estimating its value.

The soil water retention curve, $\theta(h)$, is defined as the relationship between the soil volumetric water content (θ) and the matric potential (h) [L]. This soil function is, together with the hydraulic conductivity function, *K(h)*, one of the main properties that determine the water flow in the vadose zone. One of the most common functions used to describe the soil water retention curve is the unimodal van Genuchten (1980) equation, in which the θ is related to h through two empirical variables: the n and α, which represents a pore-size distribution parameter and a scale factor, respectively. The reference laboratory method to determine $\theta(h)$ is the pressure extractor (Klute, 1986), which estimates $\theta(h)$ from measured h and θ pairs. Although this technique has been improved by incorporating alternative methods to determine θ (Jones et al., 2005; Moret-Fernández et al., 2012) , the long time needed to conclude a measurement may limit its use. On the other hand, errors of the pressure plate apparatus may also limit its use in fine-textured soils (Solone et al., 2012). Other laboratory methods to estimate $\theta(h)$ are, for instance, the evaporation method that yields bot $\theta(h)$ and *K(h)* curves (Gardner and Miklich, 1962; Wind, 1968; Wendroth et al., 1993; Tamari et al., 1993), or methods based on the inverse numerical analysis of the transient water flow (Simunek and Van Genuchten, 1997; Simunek et al., 1998). These last techniques, that involve the inverse solution of the Richard's equation, are increasingly employed because of the short-time of the experiments and the ability to simultaneous estimate of *K(h)* and $\theta(h)$. These methods can be based on the analysis of downward infiltration (Simunek and van Genuchten, 1997) or upward infiltration processes. Among the different methods included in this last group, Hudson et al. (1996) suggested estimating the soil hydraulic properties from the inverse analysis of an upward flow experiment under laboratory conditions using a constant flux of water at the bottom of the soil sample. This laboratory technique was next improved by Young et al. (2002) who employed a Mariotte system and tensiometers installed along a 15-cm-long soil column. Using pressure head and cumulative flux data as auxiliary variables of the objective function, the soil hydraulic parameters were calculated with HYDRUS-1D by an optimization procedure. More recently, Moret-Fernández et al. (2016) developed a tension sorptivimeter that allowed estimating the soil hydraulic parameters from the inverse analysis of a multiple tension water absorption curve, without using tensiometers. The results demonstrated that the soil hydraulic parameters could be satisfactorily estimated if negative enough tensions were applied. Alternatively, medium negative soil tensions could be used (e.g., h = -30 cm) if K_s was previously estimated. At this aim, K_s was calculated according to the Darcy's law. In a similar way, Peña-Sancho et al. (2016) developed an alternative laboratory method in which, taking into account the hysteresis phenomena, the hydraulic properties were simultaneously estimated from a capillary wetting process at saturation followed by an evaporation process.

Although important efforts have been done to develop new methods to estimate $\theta(h)$ from the inverse analysis of an upward infiltration experiment, the information available in a soil water absorption process has not yet been completely deciphered. Thus, the main objective of this paper is to present a method to determine the van Genuchten (1980) parameters for water retention curve (α and n) from the S and β parameters defined in the 1D infiltration model by Haverkamp et al. (1994). To this end, firstly we present a modification of the Haverkamp et al. (1994) model to describe an upward infiltration, or exfiltration in the words of Eagleson (1978). To validate this model, upward infiltration curves simulated by HYDRUS-1D for four soils with different texture (sandy, loamy, silty and clayey) were compared to the corresponding curves calculated with the proposed equation. Assuming a known K_s, which can be obtained by the Darcy's law, the S and β estimated analytically were then compared to the corresponding values calculated by the inverse analysis of theoretical upward infiltration curves generated by HYDRUS-1D. Next, a procedure to calculate α and n from the previously calculated K_s, S and β was presented. Finally, the method was tested on sieved experimental soils of known hydraulic properties.

2. MATERIAL AND METHODS

2.1. Theory

The governing equation for one-dimensional Darcian upward flow in a variably saturated rigid porous medium is given by the following form of the Richard's equation (Philip, 1957)

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z} + K(\theta) \right] \quad (1)$$

where θ (L^3 L^{-3}) is the volumetric water content, t (T) is time, K the hydraulic conductivity (L T^{-1}), z is a vertical coordinate (L) positive upward, and $D(\theta)$ (L^2 T^{-1}) the diffusivity defined by Klute (1952) as

$$D(\theta) = K(\theta)\frac{dh}{d\theta} \quad (2)$$

where h (L) is the matric component of soil water potential. The respective initial and boundary conditions for upward infiltration are

$$\begin{aligned} &z = 0, t > 0, \theta = \theta_s \\ &z \geq 0, t = 0, \theta = \theta_i \\ &z \to \infty, t > 0, \theta = \theta_i \end{aligned} \quad (3)$$

where θ_s (L^3 L^{-3}) and θ_i (L^3 L^{-3}) are the saturated and initial volumetric water content, respectively.

For saturated and steady state condition, Eq (1) is reduced to the Darcy's law (Lichtner et al., 1996)

$$q = -K_s \frac{dH}{dz} \quad (4)$$

where q is the water flux density (L T^{-1}), K_s is the saturated hydraulic conductivity, and H=h+z (L) is the hydraulic head. Note that for saturated soils h ≥ 0.

Taking into account the Parlange et al. (1982) and Haverkamp et al. (1994) analysis, who using the Richards equation (Eq. 1) derived an analytical law predicting water infiltration into a soil, the 1-D upward cumulative infiltration curve, $I(t)$, measured on an infinite-length soil column with homogeneous initial water content can be described by the quasi-exact equation

$$\frac{2(1-\beta)\Delta K^2}{S_0^2}t = \frac{2\Delta K(I+K_i t)}{S_0^2} - \ln\left\{\frac{1}{\beta}\exp\left[\frac{2\beta\Delta K(I+K_i t)}{S_0^2}\right]+1-\frac{1}{\beta}\right\} \qquad (5)$$

where S_0 (L T$^{-0.5}$) is the sorptivity for θ_0; K_0 and K_i are the hydraulic conductivity values corresponding to θ_0 and θ_i, respectively, $\Delta K=K_i-K_0$, and β is an integral shape parameter. In our case, θ_0 and K_0 correspond to θ_s and the saturated hydraulic conductivity, K_s, respectively. The unique difference between Eq. (5) and the Haverkamp et al, (1994) model is a change of sign in the terms containing the hydraulic conductivities. These results agree with the exfiltration models proposed by (Boulet et al., 2000). As reported by Lassabatere et al. (2009) for downward infiltration, Eq. (5) should be only suitable for those soils (sand to silt) where the saturated-independent shape parameter, β, ranges between 0.3 and 1.7. The β shape coefficient is defined as (Haverkamp et al., 1994)

$$\beta = 2-2\frac{\int_{\theta_n}^{\theta_0}(K-K_n/K_0-K_n)(\theta_0-\theta_n/\theta-\theta_n)D(\theta)d\theta}{\int_{\theta_n}^{\theta_0}D(\theta)d\theta} \qquad (6)$$

Parlange (1975) demonstrated that, for homogeneous, uniform initial water content and infinite length soil column, the soil sorptivity, S (L T$^{-0.5}$), could be obtained from Eq. (1) as

$$S^2(\theta_0,\theta_n) = \int_{\theta_n}^{\theta_0} D(\theta)[\theta_0+\theta-2\theta_n]d\theta \qquad (7)$$

The unsaturated soil hydraulic properties can be described according to van Genuchten-Mualem model (Mualem, 1976; van Genuchten, 1980)

$$S_e(h) = \frac{\theta(h)-\theta_r}{\theta_s-\theta_r} = \frac{1}{\left(1+|\alpha h|^n\right)^m} \qquad (8)$$

$$K(\theta) = K_s S_e^{0.5}\left[1-\left(1-S_e^{1/m}\right)^m\right]^2 \tag{9}$$

where S_e is the effective water content, θ_r ($L^3\ L^{-3}$) denotes the residual volumetric water content and α (L^{-1}), *n,* and *m*=(*1-1/n*) are empirical parameters. Combining Eq. (2), (8) and (9), the diffusivity results (van Genuchten, 1980)

$$D(S_e) = \frac{(1-m)K_s}{\alpha m(\theta_s-\theta_r)} S_e^{1/2-1/m}\left[\left(1-S_e^{1/m}\right)^{-m}+\left(1-S_e^{1/m}\right)^m-2\right] \tag{10}$$

Combining Eq. (7) and Eq. (10), we obtain

$$S^2 = \frac{(1-m)K_s}{\alpha m(\theta_s-\theta_r)}\int_{\theta_n}^{\theta_0}\left[\theta_0+\theta-2\theta_n\right]S_e^{1/2-1/m}\left[\left(1-S_e^{1/m}\right)^{-m}+\left(1-S_e^{1/m}\right)^m-2\right]d\theta \tag{11}$$

2.2. Estimate of the *S*, β, α and *n* parameters from an upward infiltration curve

To estimate the S and β parameter from an exfiltration experiment (I), a 1-D homogeneous, uniform initial water content and finite soil column was considered. Saturated conditions in the bottom boundary followed by an overpressure step at the end of the water absorption process were considered. Under these conditions, the K_s can be easily estimated from the overpressure step at the end of the exfiltration process according to Eq. (4) (K_{Darcy}).

The exfiltration curve was simulated by rearranging Eq.(5) in the form of $f(I)=0$ for each considered time value. The problem to solve $I(t)$ was reduced to find the root of the $f(I)$ function (Latorre et al., 2015). To this end, the simple and robust bisection method was used (Burden et al., 1985). Large differences between nonlinear terms in Eq.(5) could lead to spurious solutions due to inaccurate results of fixed-precision arithmetic. To this end, the GMP (GNU Multiple Precision Arithmetic Library) library (Granlund, 2004) was used with 128 precision bits. Due to finite soil columns were considered and Eq. (5) required infinite soil columns, only infiltration times between t = 0 and the time just before the wetting front arrives at the top of the soil column were considered.

Assuming a known saturated hydraulic conductivity (with $K_s = K_{Darcy}$), the S and β parameters were optimized by minimizing the objective function, Q,

$$Q = \sum_{i=1}^{N} \left(\left(I_i - I(S, \beta, t_i) \right) \Delta t_i \right)^2 \tag{12}$$

where N is the number of measured (I, t) values, I_i and $I(S, \beta_i, t_i)$ are the experimental and the simulated upward cumulative infiltration data, respectively, and Δt_i is the time interval within each measurement (Latorre et al., 2015). To this end, the brute-force search (Pardalos and Romeijn, 2002) was employed. Given the two unknown variables β and S, the values of the objective functions were summarized as contours (response surfaces) for the S-β combination. The parameter combinations for the response surface were calculated on a rectangular grid, with β ranging from 0.3 to 1.7, S from 0.05 to 2.5 mm $s^{-0.5}$, and t from 0 to the end of the infiltration.

Under laboratory conditions, setting of the start time (t=0) can vary several seconds depending on user experience. This delay can have important consequences on the S and β optimization. To prevent this problem, and similarly to what reported by Latorre et al. (2015) for a downward infiltration, a procedure to remove the influence of the time-delay on the soil water absorption measurements was proposed. This consisted in optimizing Eq. (5) after removing several consecutive initial points, and moving the recalculated t_0 and I_0 to 0. The minimum Q value for the different initial times defined the actual time in which the absorption process starts. In this work, the delay-time intervals ranged between 0 and 2 s.

Once K_s, S and β values were determined, the α and n paremeters of the van Genuchten (1980) model were calculated by numerical solution of Eqs. (6) and (11), which represents a system of two equations with two unknown variables (α and n).

2.3. Numerical Experiments

The theoretical upward infiltration data used to validate this method were generated numerically by the HYDRUS-1D software (Simunek et al., 1996). A sand, loam, silt and clay soils as estimated by Carsel and Parrish (1988) were used during the simulations. The soil hydraulic parameters of the theoretical soils are summarised in Table 1 (Simunek et al., 2008). A 15 cm-high soil volume was discretized with a 1-D mesh of 1001 cells. Previously conducted numerical analysis demonstrated that, under this discretization, the solution was grid independent. The minimum time step used in the simulations was 0.0001 s. The initial pressure head of the homogeneous and isotropic column was -1.66 10^6 cm, and the tension at the base of the soil column was 0 cm. Atmospheric conditions with a maximal tension of 0 cm was imposed at the top boundary. According to HYDRUS 1D, this condition does not allow water to build up on the surface, preventing water overpressures on the soil surface. At the end of the soil water absorption process, once the wetting front arrived to the top boundary, an overpressure step of 5 cm was simulated for duration of 10 min. This additional step was subsequently used to calculate K_s (K_{Darcy}) with Eq. (4).

Two different numerical experiments were performed. The first one consisted in comparing the upward infiltration curve generated by HYDRUS with the corresponding curves simulated by Eq. (5). To this end, assuming a known $K_s = K_{Darcy}$, the exfiltration curves generated with Eq. (5) using the S and β parameters calculated from the theoretical K_s, α and n values (Table 1) (Eqs. 8, 9, 6 and 11) were compared to the corresponding curves simulated by HYDRUS-1D. The second numerical experiment consisted on testing an inverse approach. To this end, the theoretical S and β values (Eqs. 11 and Eqs. 6, 8 and 9, respectively) were compared to those calculated by optimization of the corresponding exfiltration curves generated by HYDRUS. In all cases, the exfiltration times of the HYDRUS curves ranged from zero to time just before the wetting front arrived to the soil surface.

Under real situations, experimental data is subject to several sources of uncertainty (i.e. water level measurement, initial and final water content, etc.), which are propagated to the hydraulic parameters estimates as well. In this work, we only considered the uncertainty due to water level measurement and its influence on the upward infiltration. This uncertainty arises due to the accuracy of the water level measurement sensor, and depends on the ratio between the water supply reservoir diameter and the soil cylinder diameter (Latorre et al., 2015). A preliminary experiment performed with a ±0.5 psi pressure transducer installed in a 2.0 cm-diameter water reservoir and connected to a 2.5 cm-diameter soil cylinder resulted to an upward infiltration measurement uncertainty of ±0.05 mm. The sensitivity analysis for the different S-β combination was performed around each inverse solution as part of a first order uncertainty analysis. The change of the corresponding objective function associated to the uncertainty source was first calculated and superimposed on the response surfaces in the form of a 0.05 mm contour line.

2.4. Experimental measurements

The upward infiltration curves were experimentally measured using a sorptivimeter device, similar to that described by Moret-Fernández et al. (2016). This consisted on a perforated rigid base (5 cm internal diameter -i.d.- and 0.7 cm high) contained in an aluminum receptacle of 10 cm diameter (Fig. 1). The top of the perforated base was covered with a 20 μm pore size nylon mesh, which was hermetically closed against an aluminium receptacle with an O-ring plus an aluminium ring (1 cm thick and 10 and 5 cm external and internal diameter, respectively). The bottom of the aluminium receptacle was connected to a Mariotte water-supply reservoir (30 cm high and 2.0 cm i.d.). A ±0.5 psi differential pressure transducer (PT) (Microswitch, Honeywell), connected to a datalogger (CR1000, Campbell Scientist Inc.), was installed at the bottom of the water-supply reservoir (Casey and Derby, 2002).

To setup the sorptivimeter the perforated base plus nylon mesh should be previously saturated. To this end, the air-inlet of the Mariotte reservoir was levelled up to the top of the perforated base. Once the perforated base was saturated, all air trapped between the nylon mesh and the perforated base was removed with a syringe. To start the measurements, the soil to be analysed, which was contained in a stainless steel cylinder (5 cm- internal diameter -i.d.- and 5 cm-high) closed by the base with a 20 μm pore size mesh, was placed on the saturated porous base. Once both meshes come into contact with each other, the soils started to absorb water, which was reflected by the bubbling in the Mariotte reservoir. This part of the experiment finished when the wetting front arrived at the soil surface, time at which the bubbling in the Mariotte reservoir stopped. At this time, an overpressure step was introduced by raising the water reservoir to a desired height. The total pressure drop, dH, (Eq. 4) was calculated as the distance between the air-inlet tube of the reservoir and the top soil cylinder surface. The saturated hydraulic conductivity was calculated from the overpressure section of the cumulative absorption curve according to Eq.(4). The initial water content was measured gravimetrically, and the final water content was calculated as the sum of the initial water content plus the water absorbed by the soil at the time that a water sheet appeared on the top of the cylinder. Once the experiment finished, the cylinder was disassembled and the final water content was again measured gravimetrically.

Six different soils were employed: a coarse sand (80- 160 μm particle size) (*Ex-Sand_1*), a fine sand (250-500 μm μm particle size) (*Ex-Sand_2*), two different 2-mm sieved loam soils (*Ex-Loam_1 and Ex-Loam_2*), a 2-mm sieved clay loam (*Ex-Clay-Loam*) soil, and a 0.25-mm sieved clay (*Ex-Clay*) soil. Textural characteristics and organic carbon content of the selected soils are summarized in Table 2. Except for the sands, where a 10 cm-high cylinder was used, a 5 cm-high cylinder was employed for exfiltration experiments.

The S and β and α and n values calculated in the different soils from the upward infiltration curve were compared to the corresponding values calculated (Eqs. 8, 9, 6 and 11) from the soil hydraulic properties obtained with an independent method. Soil saturated hydraulic conductivity, K_{Darcy} (LT^{-1}), $K_s = K_{Darcy}$, was determined from an overpressure step at the end of the water absorption curve, and the α and n parameters were estimated by the TDR-pressure cell method (Moret-Fernández et. al, 2012). Pressure heads of 0.1, 0.2, 0.3, 0.4, 0.5 and 0.65 kPa were employed for the two sandy soils. For the remaining soils, pressure heads of 0.2, 0.5, 1, 2, 5, 25, 50 and 100 kPa were applied. The measured water retention curves were fitted, using the SWRC Fit Version 1.2. software (Seki, 2007) (http://seki.webmasters.gr.jp/swrc/), to the unimodal van Genuchten (1980) function (Eq. 8). Given the experimentant methods gives estimates of the drying branch of the water retention curve, the α parameters obtained from the TDR-cell measurements were converted to those corresponding to wetting branch using the empirical hysteresis index developed by Gebrenegus and Ghezzehei (2011). As reported by Likos et al. (2014), the assumption of constant n between the two water retention curve branches was adopted.

3.- RESULTS AND DISCUSSION

The soil saturated hydraulic conductivity values estimated by the Darcy law, K_{Darcy}, differed from the theoretical K_s values by 0.23%.Table 1 shows the theoretical S (Eq. 11) and β (Eqs. 6, 8 and 9) values calculated numerically for the synthetic sand, loam, silt and clay soils, using the corresponding theoretical K_s, α and n parameters. The robust fit (R^2 = 0.99; p < 0.0001) between the soil water absorption curves simulated by HYDRUS for the theoretical sand, loam and silt soils (Table 1) and those generated with the modified Haverkamp et al. (1994) model (Eq. 5) from the calculated S and β parameters (Table 1) and the corresponding K_{Darcy} values (Fig. 2), demonstrated that Eq. (5) can satisfactorily simulate upward infiltration curves. The divergence between the upward infiltrations curve simulated with HYDRUS for the theoretical clay and that obtained with Eq. (5) (Fig. 2) agrees with Lassabatere et al. (2009), who reported that the Haverkamp et al. (1994) model was only suitable for those soils where the shape parameter β ranges between 0.3 and 1.7 (sand, loam and silt). In our case, the β value calculated for the clay soil (Table 1) was higher than the threshold reported by Lassabatere et al. (2009), and therefore Eq. (5) overestimated the soil water absorption curve at long-times.

The $Q(S,\beta)$ response surface calculated for the theoretical loam soil showed that at short times (i.e. 100 s) the S tended to be β independent. This was reflected by the vertical valley found in the 100 and 500 s response surfaces (Fig. 3a and b). These results indicated that short absorption times may be enough to approach S, but insufficient to determine β. The S-β error maps progressively changed for increasing times (i.e. 8000 s) (Fig. 3c and d), where the above observed valley tended to transform to a well. In these cases, a unique minimum could be observed. The sensitivity analysis for S and β, defined by 0.05 mm error contour line (red line in Fig. 3) showed that the accuracy in S and β estimates also increased with longer times.

The significant relationship between the S (Eq. 11) calculated for theoretical the sand, loam, silt and clay soils (Table 1) and the corresponding values calculated from the inverse solution of Eq. (5) applied to the corresponding upward infiltration curve generated by HYDRUS (Fig. 4), demonstrated that Eq. (5) allowed accurate estimates of S in all soils, and a good description of the early-time stage of the upward infiltration curve (Fig. 2) for which S is independent from β (Fig. 3a).

If the clay soil is omitted from the regression analysis (black circle in Fig. 5), a robust relationship was found between the β values calculated from the soil hydraulic properties of Table 1 (Eqs. 6) and those estimated by minimizing the objective function (Eq. 12) for the corresponding synthetic exfiltration curves generated by HYDRUS (Fig. 5). These results agree to those reported by Lassabatere et al. (2009), who found that the Haverkamp et al. (1994) model was only suitable for β values ranging between 0.3 and 1.7 (sand, loam and silt). Taking into account these results, in the following only theoretical sand, loam and silt soils will be considered.

Given known K_s and S values, the analysis of an upward infiltration curve estimated with Eq. (5) for a theoretical loam soil column of infinite length (Table 1) and different values of β, showed that the β parameter had a significant influence on the medium- long-time upward infiltration (Fig. 6). Indeed, longer times involved greater differences between the upward infiltration curves. This behaviour contrasted with that observed in the S parameter, which was successfully derived from the early-time stage of the upward infiltration. These results would indicate that S and β affect two different sections of the upward infiltration curve. Given that K_s can be accurately calculated by Darcy law and S is well defined by the Haverkamp et al. (1994) model within the first infiltration steps, a response surfaces for the t-β combinations can be calculated using the known S and K_s values. These alternative error maps allow describing the time-evolution of the β parameter. All error maps showed an asymptotic form, in which the accuracy of β estimation within a confidence interval increases with time and depended on the soil type (Fig. 7). Coarser soil requires shorter exfiltration times. If a soil core with a 5 cm internal diameter (i.d.) is considered, the response surfaces for the exfiltrated water volume as function of β showed that, in general, estimations of β could be accurately achieved when the total volume of water absorbed by the soil was about 40 cm^3 (Fig. 7). This means that β could be accurately estimated from, for instance, the 5 cm i.d. by 5 cm-height cylinder commonly used for bulk density estimates. Analysis of the β–t response surfaces also showed that the β values decrease for coarser soils: $\beta_{sand} < \beta_{loam} < \beta_{silt}$. These results indicate that this parameter is inversely related to the shape factor of the van Genuchten (1980) soil water retention model: $n_{sand} > n_{loam} > n_{silt}$ (Table 1).

Once the K_s, β and S are estimated, the sensitivity analysis for numerical solution of the system of Eqs. (6) and (11) showed that the calculated α and n values presented a unique and well defined minimum (Fig. 8). An excellent relationship was also observed between the theoretical α and n values (Table 1) and those estimated from the optimization of the synthetic curves generated by HYDRUS (Fig. 9). These results indicate that the proposed method can be a feasible procedure to estimate the parameters α and n of the water retention curve. The comparison between theoretical and simulated $\theta(h)$ curves obtained for a loam from the S and β values enclosed within the 0.05 mm contour line of the S-β error maps soil are reported in Figure 3. For the loam soil accuracy of $\theta(h)$ estimation increases with longer infiltration times, and the most accurate estimates of $\theta(h)$ are obtained at the longest upward infiltration time ($t = 8000$ s).

From the practical point of view, three reasons led us to choose upward instead of downward infiltration process to estimate, under laboratory conditions, the soil hydraulic properties:

1.- Difficulties to hold the disc infiltrometer on the soil cylinder.

2.- Collapsing of the soil macropores due to infiltrometer weight (Moret and Arrúe, 2005), which can disturb the estimations of the soil hydraulic properties.

3.- Lost of hydraulic contact between infiltrometer disc and soil surface as the soil becomes saturated due to partial soil collapse.

These problems vanished in the upward infiltration method where the cylinder should not be hold and balanced, there was not weight on the soil surface, and the weight of the soil itself ensured the contact between the soil base and the top of the sorptivimeter.

The saturated hydraulic conductivity and the van Genuchten (1980) water retention curve parameters measured with the Darcy's law and the TDR-pressure cell for the different experimental soils are summarized in Table 2. Experimental values were within the same order of magnitude that those reported by other authors for the same type of soils (Carsel and Parrish, 1988; Moret-Fernández et al., 2013). The robust agreement ($p < 0.0001$) between the experimental exfiltration curves and those generated with Eq. (5), using the measured K_{Darcy} and the optimized S and β values (Fig. 11), indicates that analytical model for upward infiltration (Eq.5) is flexible enough to fit also experimental soils. The asymthotic form of the t-β response surfaces calculated for the experimental soils (Fig. 12) indicates that the 5 cm high soil cylinder employed in the experiment was enough to accurate estimations of β. However, caution should be taken into account when using these small columns in soils with low α values, since presence of a zero flux boundary condition (top of sample) in close vicinity (5 cm) to the infiltrating surface (bottom of sample) could influence the upward flow of water. The S and β values calculated for the different experimental soils from the water retention parameters obtained from a wetting process (Table 2) and the corresponding values estimated from the inverse analysis are compared in Table 3. A significant relationship (y = 1.23x – 0.10; R^2 = 0.94; p < 0.001) was observed between the S values calculated with Eq. (11), using K_{Darcy} and the TDR-pressure cell data, and the corresponding values estimated from the optimization of the experimental upward infiltrations (Table 3). The slight differences between the S values should be attributed to the similar water retention curve parameters calculated by both methods. As reported for the theoretical soils, these results demonstrated that the inverse analysis of Eq.(5) also allowed reliable estimates of S, even for clay soils ($\beta > 1.7$). If the *Ex-Clay* soil is omitted, a good agreement (y = 1.10x - 0.05; R^2 = 0.99; p < 0.0002) was found between the β values calculated from the K_{Darcy} and the TDR-pressure cell data and those obtained from the inverse analysis of the experimental upward infiltrations (Table 3). As above mentioned, this different behaviour for the *Ex-Clay* soil could be attributed to inappropriate description of the exfiltration process by the analytical model (eq.5) when clay soils are considered for clay soil. After omitting the *Ex-Clay* soil, a non-significant relationship (y = 0.16x + 0.04 R^2 = 0.01; p < 0.96) was observed between the α values estimated from the upward infiltration and the corresponding pressure-cell values calculated for the wetting branch of $\theta(h)$ after applying the empirical hysteresis approach. This weak equivalence could be attributed to the hysteresis effect which probably was not well described by the empirical Gebrenegus and Ghezzehei (2011) model. An indirect confirmation for this hypothesis is given by the good correlation found for n that is less affected by hysteresis. These results suggest that cautions should be taken when using empirical models to characterize the hysteresis phenomenon. On the other hand, the lack of correlation in the α values could also be explained by the different wetting processes used in both methods; very slow in the PC and faster in the upward infiltration. The different wetting processes may affect the wet end section of *θ(h),* and consequently the α value, by modifying the contact angle of water with the soil particles, the amount of air entrapped in the pores, or the interconnection in the pore network (Bachmann and van der Ploeg, 2002; Or and Wraith, 1999; Maqsoud et al., 2004). A significant relationship was obtained between the *n* values estimated with both methods (Fig. 13). This strong relationship could be associated to the fact that *n* is more related to the soil textural characteristics (Jirku *et al.* 2013). Overall, the deviation between the *n* measured with the TDR-pressure cell and those obtained with the inverse numerical analysis was lower than 2.6%.

4.- CONCLUSIONS

A method to determine the parameters of the van Genuchten (1980) water retention curve (α and *n*) by inverse analysis of a single upward infiltration curve is proposed and tested. Firstly, the paper presents a modified version of Haverkamp et al (1994) model (Eq.5) that can be satisfactorily applied to upward infiltration processes. Assuming a known saturated hydraulic conductivity, which can be easily calculated on a finite soil column by the Darcy's law, the α and *n* parameters were calculated from soprtivity and β parameter estimated by the inverse analysis of Eq. (5). The method was satisfactorily validated on sand, loam and silt soils using synthetic exfiltration curves generated by HYDRUS-1D, and then tested on experimental soils of known hydraulic properties. Results showed that this technique was inexpensive, fast and simple to implement and allowed realible estimates of the soil hydraulic properties. On the other hand, because one of the major advantage of the inverse method is that it can be applied potentially to any transient flow experiment once the initial and boundary conditions are defined and the auxiliary variables to be measured are identified, the theoretical and practical advantages of the here presented specific transient experiment make the difference among alternative approaches. Although the results demonstrated that a 5 cm high soil column was enough to accurately estimate *S* and β, longer soil columns would probably allow more accurate estimations of the hydraulic properties. The intrinsic characteristics of the employed quasi-exact formulation, which is only useful for β values ranging between 0.3 and 1.7 (Lassabatere et al., 2009), limited its use to soils ranged between sand and silt textural groups. On the other hand, attention should be taken during the measurements of the soil wetting process, since inaccurate measurements of the cumulative soil water absorption curve can lead to incorrect estimations of S and β, and consequently inaccurate estimations of α and n parameters. Finally, further researches are needed to develop a more efficient optimization procedure and validate and test the method on undisturbed soil samples with different structural and textural characteristics.

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Figure captions

Figure 1. Sorptivimeter scheme

Figure 2. Comparison between the cumulative upward infiltration curves simulated by HYDRUS for the theoretical sand, loam, silt and clay soils (Table 1) and the corresponding curves generated with Eq. (5).

Figure 3. Response surface of S-β for a theoretical loam soil (Table 1) and an exfiltration times of (a) 100, (b) 500, (c) 2000 and (d) 8000 s. Thick red line indicates the experimental uncertainty contour line (0.05 mm) due to water level measurement.

Figure 4. Relationship between the theoretical sorptivity, S_{Th}, for different synthetic soils (Table 1) and the corresponding values estimated by minimizing the objective function (S_{Opt}) for the upward infiltration curves simulated by HYDRUS.

Figure 5. Relationship between the theoretical β (Eq. 6) (β_{Th}) for different synthetic soils (Table 1) and those estimated by minimizing the objective function (β_{Opt}) for the upward infiltration curves generated by HYDRUS.

Figure 6. Cumulative upward infiltration curves estimated with Eq. (5) using the K and S values of a theoretical loam soil (Table 1) and different values of β.

Figure 7. Response surfaces for the t-β and cumulative exfiltration vs. β combinations simulated fordifferent theoretical soils. Thick red line indicates the experimental uncertainty contour line (0.05 mm) due to water level measurement.

Figure 8. Sensitivity analysis for the estimation of the α and n parameters of a theoretical loam soil calculated from the numerical solution of the system of Eqs. (6) and (11). RMSE = root mean standard error for the comparison between the theoretical and the calculated α and n values.

Figure 9. Relationship between theoretical α and n values (α_{Th} and n_{Th}) and estimated (α_{Opt} and n_{Opt}) values calculated by minimizing the objective function for the corresponding synthetic curve generated by HYDRUS.

Figure 10. Comparison between the theoretical and simulated water retention curves calculated from the S and β values contained within the 0.05 mm contour line of the S-β error map (Fig. 3) modelled for a theoretical loam soil at (a) 100, (b) 500, (c) 2000 and (d) 8000 s of exfiltration times.

Figure 11. Experimental cumulative upward infiltration curves (I) measured for the four soils (Table 2) and the corresponding curves generated with Eq. (5).

Figure 12. Response surfaces for the β–t combination simulated for the experimental (a) coarse sand, and the 2-mm sieved (b) loam (*Ex-loam_1*) and (c) clay loam soils.

Figure 13. Relationship between the (a) α and (b) n values of the wetting branch of the water retention curve calculated on five experimental soils with the TDR-pressure cell (PC) and those estimated from the inverse analysis (Opt) of the corresponding cumulative upward infiltration curves.

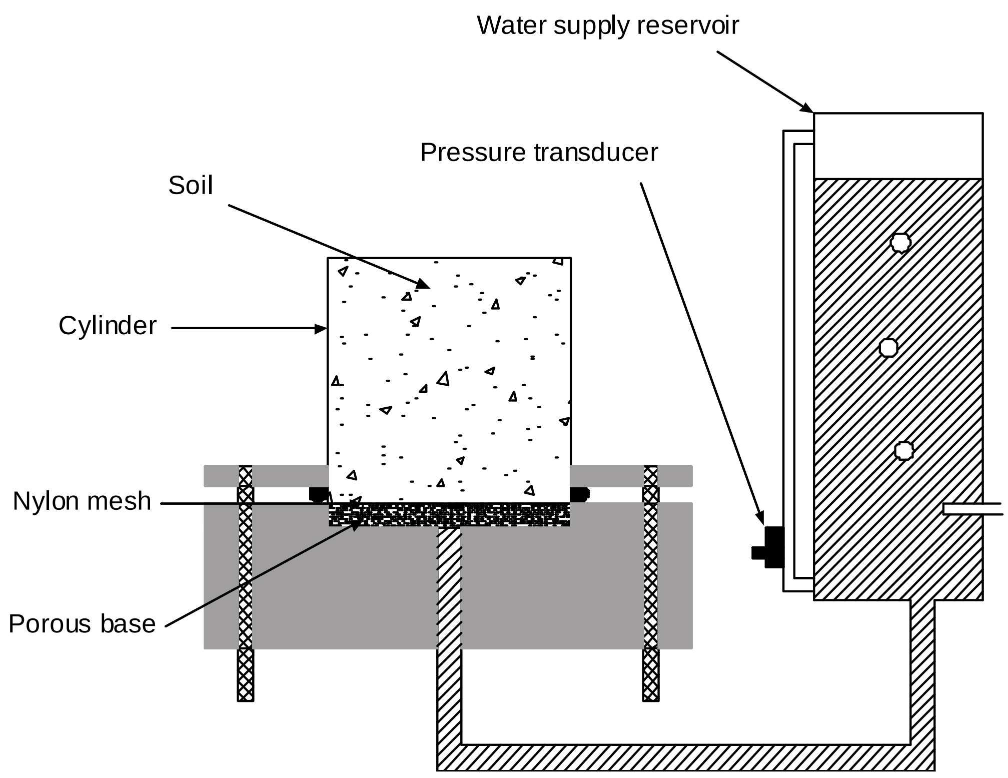
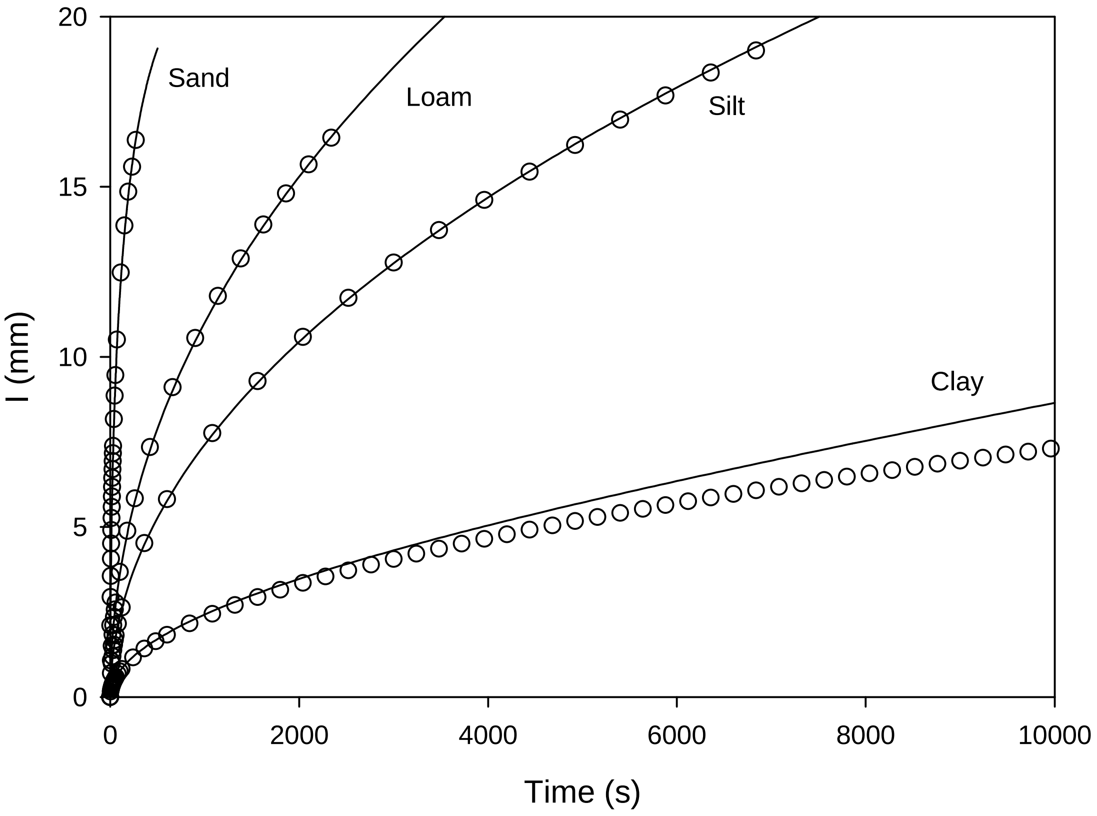

Water supply reservoir
Pressure transducer
Soil
Cylinder
Nylon mesh
Porous base

Figure 1.


Sand
Loam
Silt
Clay
I (mm)
Time (s)
0
5
10
15
20
0
2000
4000
6000
8000
10000

610 **Figure 2.**

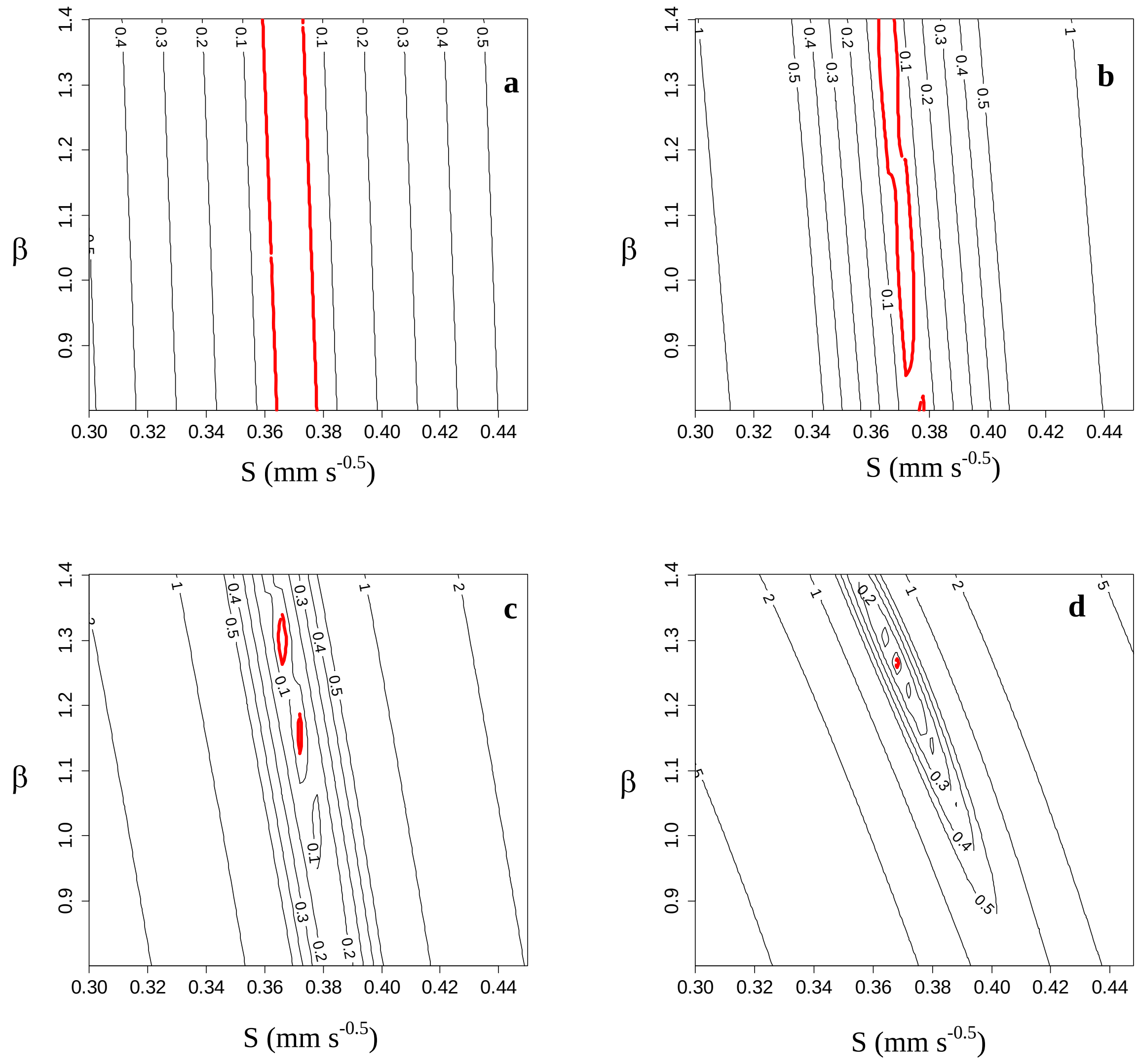
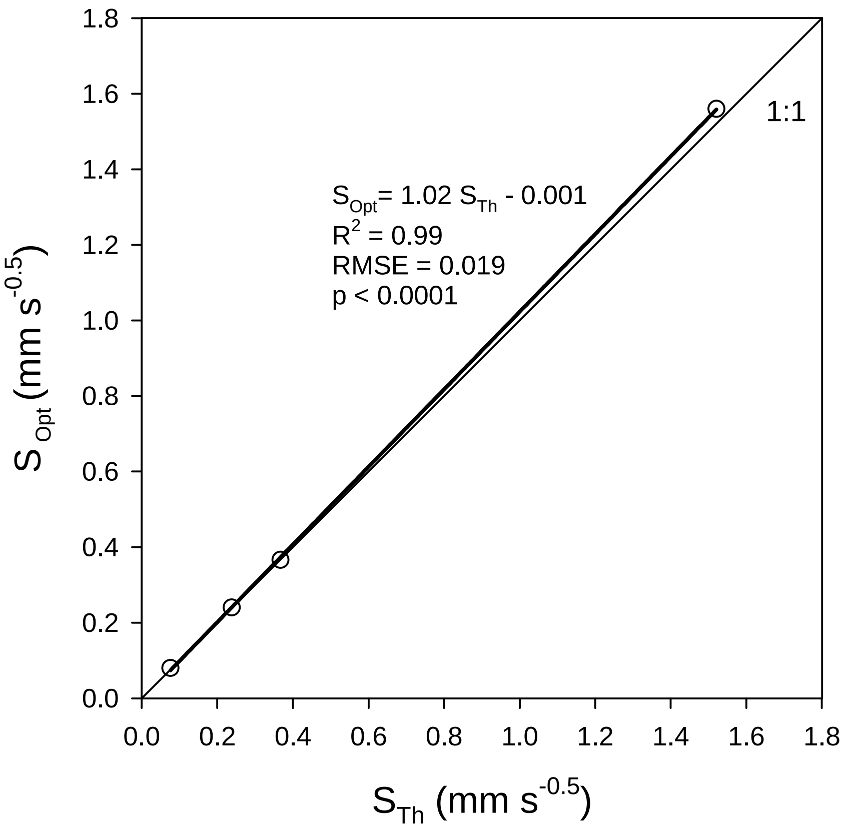
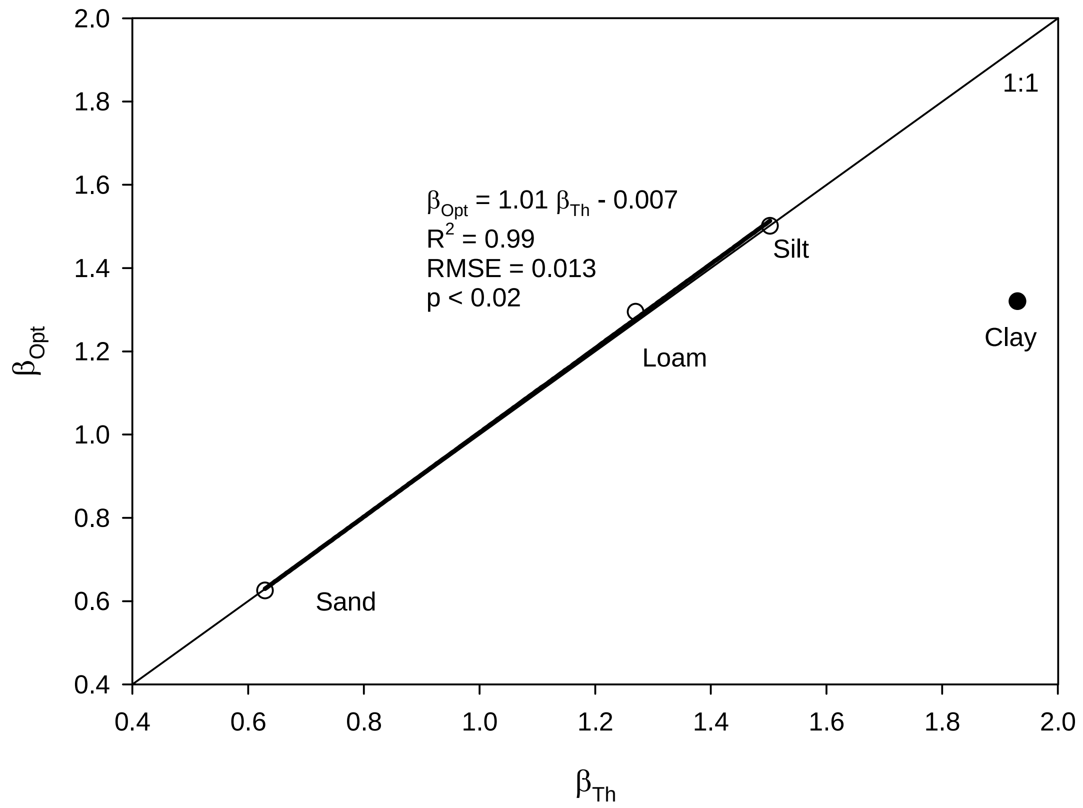


Figure 3.



1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8
1:1
S_{Opt}= 1.02 S_{Th} - 0.001
R^2 = 0.99
RMSE = 0.019
p < 0.0001
S_{Opt} (mm $s^{-0.5}$)
S_{Th} (mm $s^{-0.5}$)

613 **Figure 4.**



2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
β_{Opt}
β_{Th}
1:1
β_{Opt} = 1.01 β_{Th} - 0.007
R^2 = 0.99
RMSE = 0.013
p < 0.02
Silt
Clay
Loam
Sand

614 **Figure 5.**

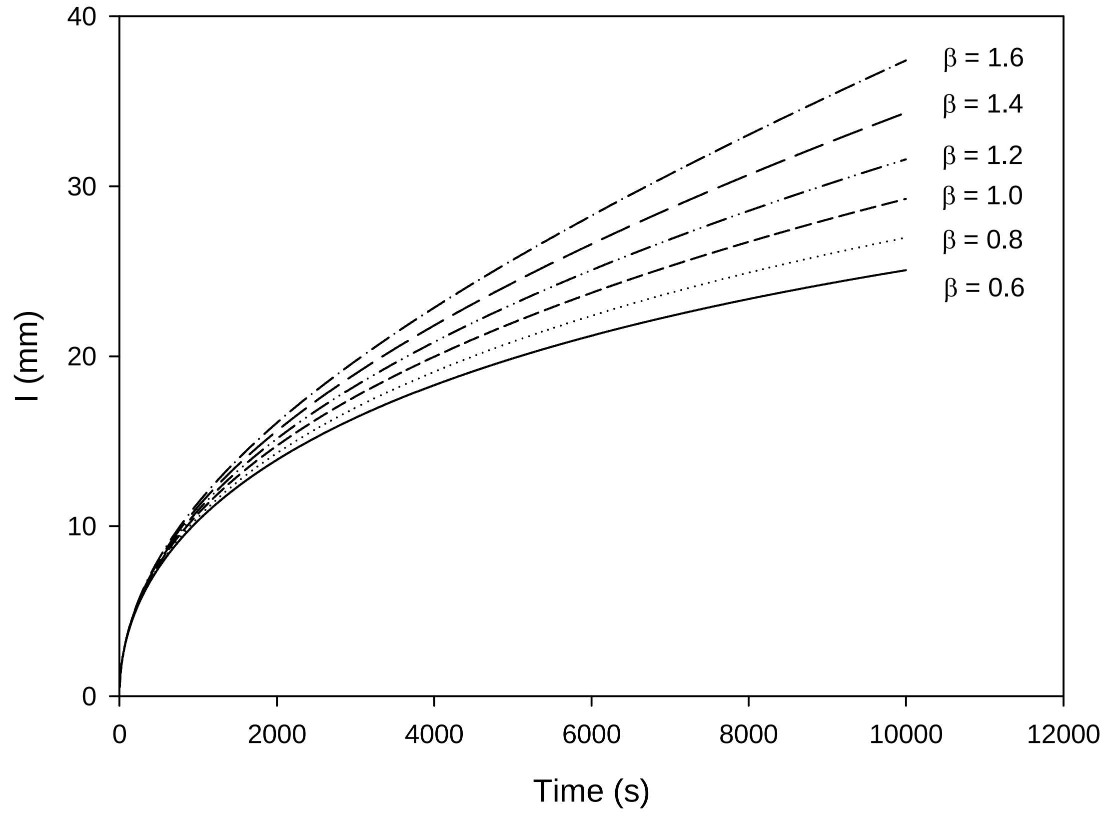

β = 1.6
β = 1.4
β = 1.2
β = 1.0
β = 0.8
β = 0.6
I (mm)
Time (s)
0
10
20
30
40
0
2000
4000
6000
8000
10000
12000

Figure 6.

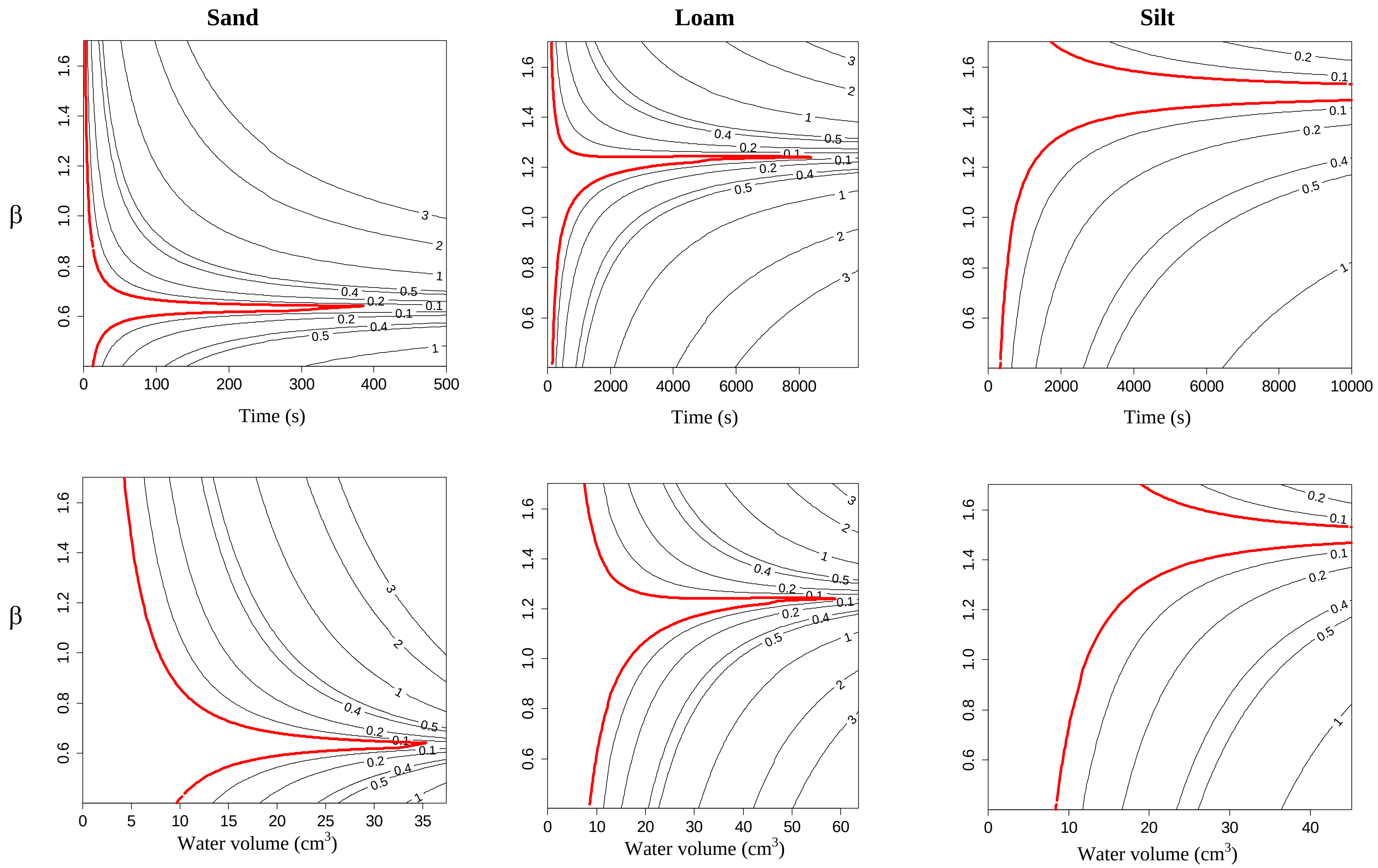
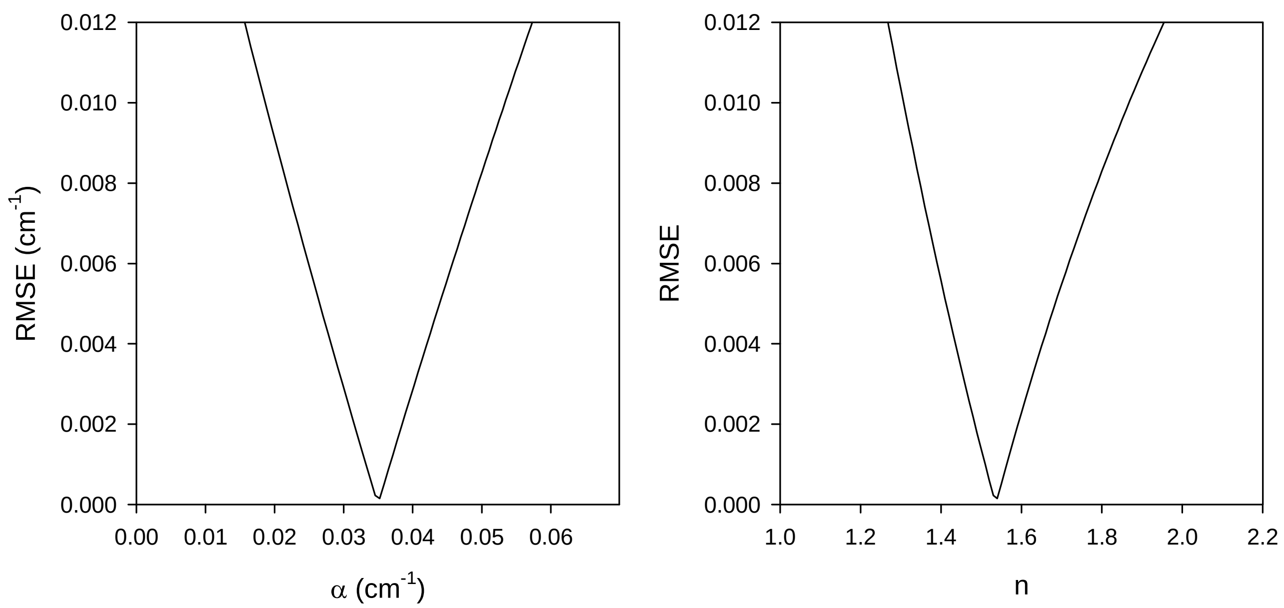


Figure 7.


RMSE (cm^{-1})
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0.00
0.01
0.02
0.03
0.04
0.05
0.06
α (cm^{-1})
RMSE
0.012
0.010
0.008
0.006
0.004
0.002
0.000
1.0
1.2
1.4
1.6
1.8
2.0
2.2
n

618 **Figure 8.**

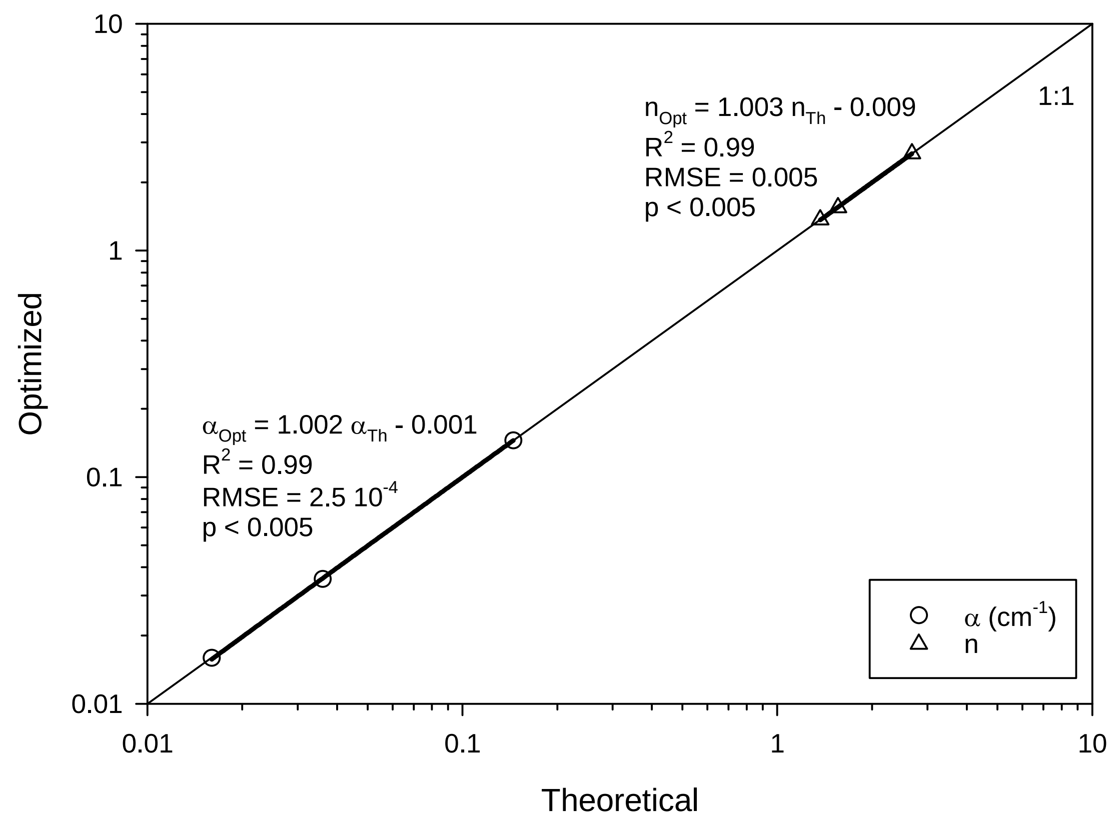

10
1
0.1
0.01
0.01
0.1
1
10
Optimized
Theoretical
1:1
n_{Opt} = 1.003 n_{Th} - 0.009
R^2 = 0.99
RMSE = 0.005
$p < 0.005$
α_{Opt} = 1.002 α_{Th} - 0.001
R^2 = 0.99
RMSE = 2.5 10^{-4}
$p < 0.005$
α (cm^{-1})
n

Figure 9.

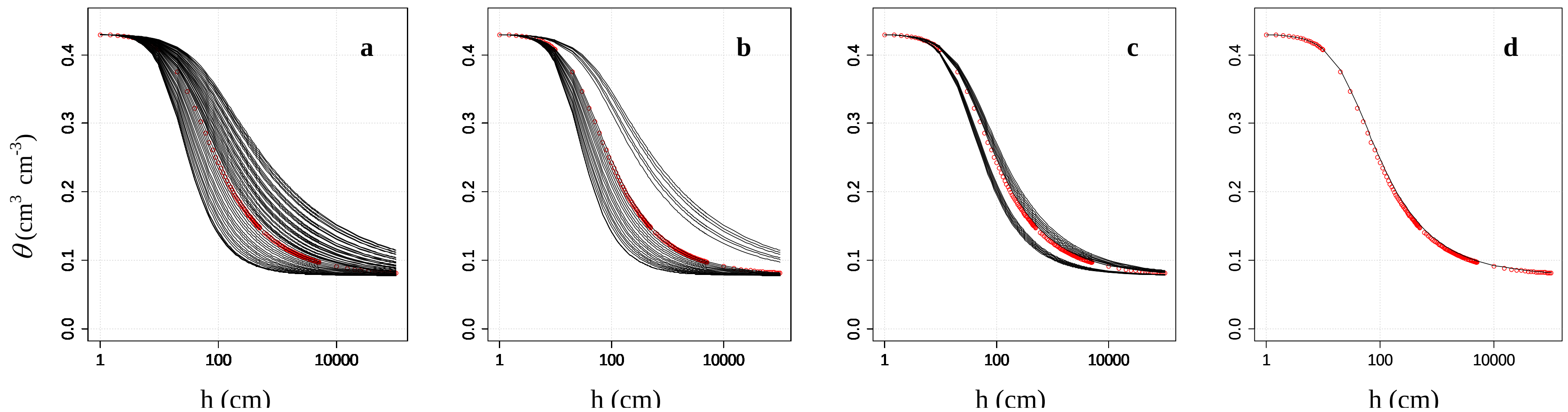
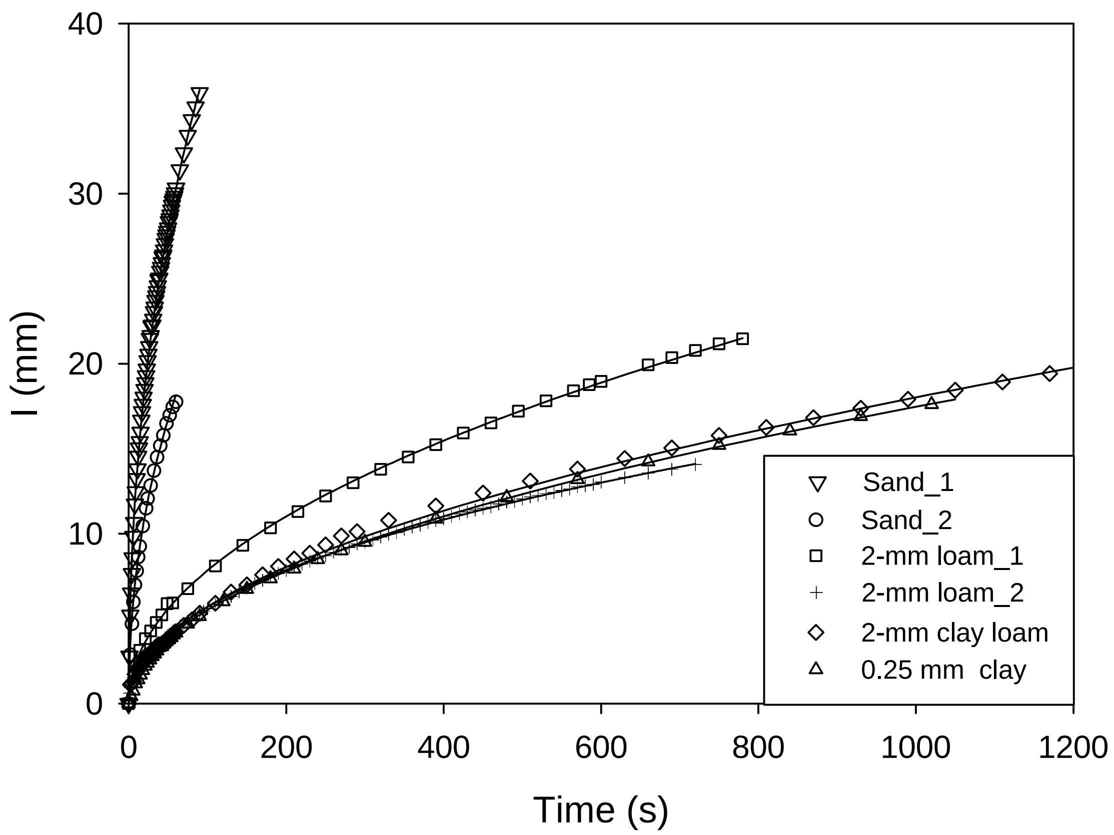


Figure 10.

620



621 **Figure 11.**

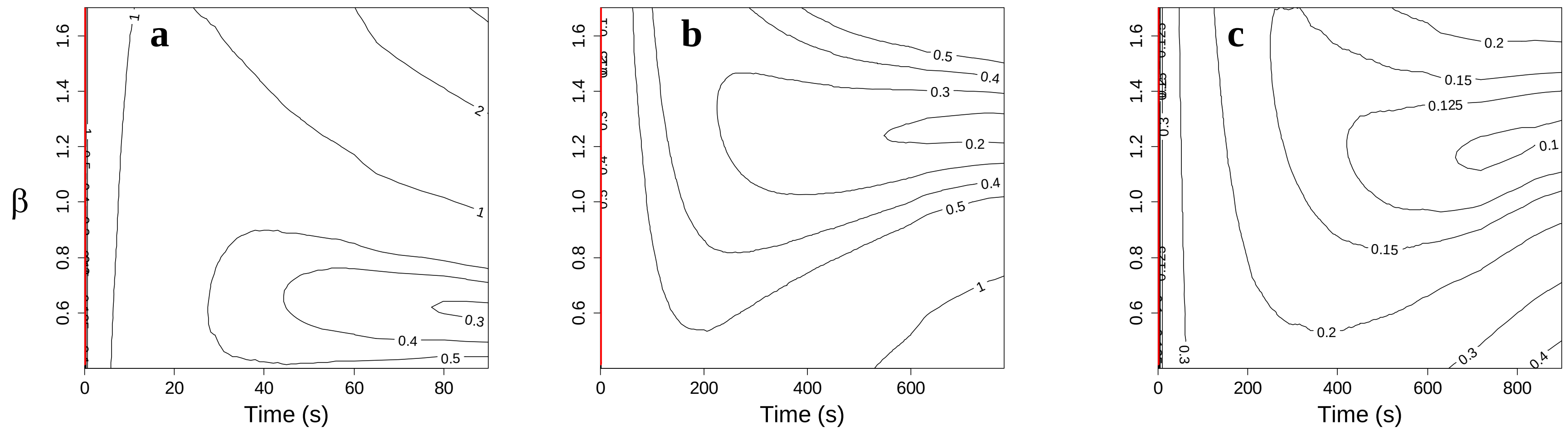


Figure 12.

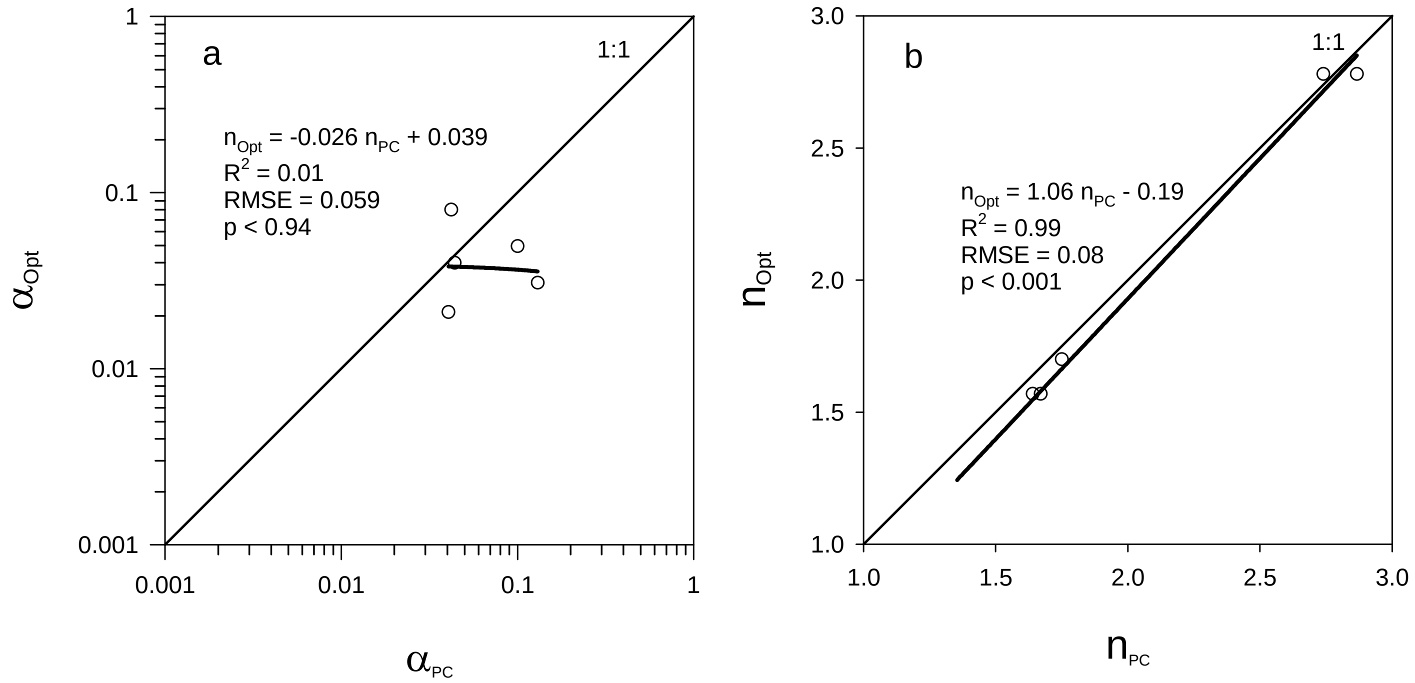

a
1:1
nOpt = -0.026 nPC + 0.039
R2 = 0.01
RMSE = 0.059
p < 0.94
αOpt
αPC
b
1:1
nOpt = 1.06 nPC - 0.19
R2 = 0.99
RMSE = 0.08
p < 0.001
nOpt
nPC

Figure 13.

Table 1. Values of initial (θ_i), saturated (θ_s) and residual (θ_r) water content, α and n parameters of the vanGenuchten (1980) water retention curve, hydraulic conductivity (K_s), sorptivity (S) (Eq. 11) and shape factor (β) (Eq. 6) calculated from the soil hydraulic properties, and total time of the HYDRUS-simulated soil water absorption curve applied on the theoretical sand and loam and silt soils.

	θ_i	θ_s	θ_r	α	n	K_s	S	β	*Absorption time*
	cm^3 cm^{-3}			cm^{-1}		mm s^{-1}	mm s$^{-0.5}$		s
Sand	0.045	0.43	0.045	0.145	2.68	8.25 10^{-2}	1.521	0.63	500
Loam	0.078	0.43	0.078	0.036	1.56	2.88 10^{-3}	0.367	1.27	2700
Silt	0.034	0.46	0.034	0.016	1.37	6.93 10^{-4}	0.238	1.50	8000
Clay	0.068	0.38	0.068	0.008	1.09	5.55 10^{-4}	0.076	1.93	10000

Table 2. Values of initial (θ_i), saturated (θ_s) and residual (θ_r) water content, α and n parameters of the vanGenuchten (1980) water retention curve measured with the TDR-pressure cell, saturated hydraulic conductivity (K_{Darcy}) measured with the Darcy method, soil textural properties and organic carbon content of the different experimental soils.

	θ_i	θ_s	θ_r	α	n	K_{Darcy}	Sand	Silt	Clay	Organic carbon
	cm^3 cm^{-3}			cm^{-1}		mm s^{-1}	g kg^{-1}			
Ex-Sand_1	0.02	0.38	0.02	0.05	2.87	0.1440	1000	-	-	-
Ex-Sand_2	0.02	0.35	0.02	0.05	2.74	0.0827	1000	-	-	-
Ex- loam_1	0.03	0.52	0.03	0.04	1.64	0.0123	280	470	250	11.7
Ex- loam_2	0.04	0.40	0.03	0.03	1.75	0.0178	422	409	169	4.3
Ex-Clay Loam	0.04	0.50	0.03	0.02	1.67	0.0032	205	497	298	19.9
Ex-Clay	0.03	0.43	0.03	0.05	1.25	0.0014	151	344	465	12.4

Table 3. Sorptivity (S) and β parameter calculated with Eq.(11) and Eq.(6), respectively, for the different experimental soils calculated from the soil hydraulic parameters converted to the wetting branch of the $\theta(h)$ measured with the TDR-pressure cell (PC) (Table 2), and the corresponding values estimated from the inverse analysis of the cumulative water absorption curve (Eq. 5).

	S_{PC}	$S_{Inv.\ Analysis}$	β_{PC}	$\beta_{Inv.\ Analysis}$
	mm $s^{-0.5}$			
Ex-Sand1	3.40	4.41	0.58	0.60
Ex-Sand2	2.42	2.71	0.61	0.60
Ex-Loam1	0.57	0.80	1.19	1.23
Ex-Loam2	1.06	0.60	1.09	1.14
Ex-Clay Loam	0.64	0.51	1.16	1.26
Ex-Clay	0.14	0.55	1.70	2.39